

# Executive Stock Options as a Screening Mechanism

Cvitanić (Caltech)  
Cadenillas (U of Alberta)  
Zapatero (USC)

# Background

- ▶ Stock options are a very important component of compensation packages:

# Background

- ▶ Stock options are a very important component of compensation packages:
  - ▶ Not only executives, but managers in general

# Background

- ▶ Stock options are a very important component of compensation packages:
  - ▶ Not only executives, but managers in general
- ▶ There used to be strong accounting incentives that favor the use of options for compensation

# Background

- ▶ Stock options are a very important component of compensation packages:
  - ▶ Not only executives, but managers in general
- ▶ There used to be strong accounting incentives that favor the use of options for compensation
  - ▶ Not anymore

# Background

- ▶ Stock options are a very important component of compensation packages:
  - ▶ Not only executives, but managers in general
- ▶ There used to be strong accounting incentives that favor the use of options for compensation
  - ▶ Not anymore
  - ▶ However they still are a standard component of compensation packages

# Background

- ▶ Stock options are a very important component of compensation packages:
  - ▶ Not only executives, but managers in general
- ▶ There used to be strong accounting incentives that favor the use of options for compensation
  - ▶ Not anymore
  - ▶ However they still are a standard component of compensation packages
- ▶ Why?

# Background

- ▶ Stock options are a very important component of compensation packages:
  - ▶ Not only executives, but managers in general
- ▶ There used to be strong accounting incentives that favor the use of options for compensation
  - ▶ Not anymore
  - ▶ However they still are a standard component of compensation packages
- ▶ Why?
- ▶ (Another interesting question we do not explore here is their price...).



# Some explanations

- ▶ Around convexity of options and its two effects:

# Some explanations

- ▶ Around convexity of options and its two effects:
  - ▶ Options are riskier

# Some explanations

- ▶ Around convexity of options and its two effects:
  - ▶ Options are riskier
  - ▶ They compound positive performance

# Some explanations

- ▶ Around convexity of options and its two effects:
  - ▶ Options are riskier
  - ▶ They compound positive performance
- ▶ Lazear (2001) argues in favor of “sorting:

# Some explanations

- ▶ Around convexity of options and its two effects:
  - ▶ Options are riskier
  - ▶ They compound positive performance
- ▶ Lazear (2001) argues in favor of “sorting”:
  - ▶ Options are a cheaper way to compensate optimistic employees.

# Some explanations

- ▶ Around convexity of options and its two effects:
  - ▶ Options are riskier
  - ▶ They compound positive performance
- ▶ Lazear (2001) argues in favor of “sorting”:
  - ▶ Options are a cheaper way to compensate optimistic employees.
  - ▶ Oyer and Schaefer (2004) find empirical support in favor of sorting

# Some explanations

- ▶ Around convexity of options and its two effects:
  - ▶ Options are riskier
  - ▶ They compound positive performance
- ▶ Lazear (2001) argues in favor of “sorting”:
  - ▶ Options are a cheaper way to compensate optimistic employees.
  - ▶ Oyer and Schaefer (2004) find empirical support in favor of sorting
- ▶ Darrough and Stoughton (1988) show that non-linear compensation schemes can provide a better self-selection mechanism than linear schemes

# More explanations

- ▶ Ittner, Lambert and Larcker (2002) argue in favor of equity-based compensation is to attract new employees



# More explanations

- ▶ Ittner, Lambert and Larcker (2002) argue in favor of equity-based compensation is to attract new employees
  - ▶ “Option-based contracts are (...) more attractive to employees with higher skill levels who have greater ability to take actions that cause their options to finish in the money”

# More explanations

- ▶ Ittner, Lambert and Larcker (2002) argue in favor of equity-based compensation is to attract new employees
  - ▶ “Option-based contracts are (...) more attractive to employees with higher skill levels who have greater ability to take actions that cause their options to finish in the money”
- ▶ Finally, Arya and Mittendorf (2005) show that options provide firms with a tool to screen the true ability of the executive

# More explanations

- ▶ Ittner, Lambert and Larcker (2002) argue in favor of equity-based compensation is to attract new employees
  - ▶ “Option-based contracts are (...) more attractive to employees with higher skill levels who have greater ability to take actions that cause their options to finish in the money”
- ▶ Finally, Arya and Mittendorf (2005) show that options provide firms with a tool to screen the true ability of the executive
  - ▶ Options will only be accepted by executives who truthfully claim a high ability

# More explanations

- ▶ Ittner, Lambert and Larcker (2002) argue in favor of equity-based compensation is to attract new employees
  - ▶ “Option-based contracts are (...) more attractive to employees with higher skill levels who have greater ability to take actions that cause their options to finish in the money”
- ▶ Finally, Arya and Mittendorf (2005) show that options provide firms with a tool to screen the true ability of the executive
  - ▶ Options will only be accepted by executives who truthfully claim a high ability
- ▶ Our paper follows this line of reasoning

# More explanations

- ▶ Ittner, Lambert and Larcker (2002) argue in favor of equity-based compensation is to attract new employees
  - ▶ “Option-based contracts are (...) more attractive to employees with higher skill levels who have greater ability to take actions that cause their options to finish in the money”
- ▶ Finally, Arya and Mittendorf (2005) show that options provide firms with a tool to screen the true ability of the executive
  - ▶ Options will only be accepted by executives who truthfully claim a high ability
- ▶ Our paper follows this line of reasoning
  - ▶ We provide more structure and can analyze this argument in more detail

# Model: Stock

- ▶ We modify the standard Geometric BMP to accommodate the effect of the decisions of the manager

# Model: Stock

- We modify the standard Geometric BMP to accommodate the effect of the decisions of the manager

$$dS_t = \delta a_t dt + \alpha \sigma_t S_t dt + \sigma_t S_t dW_t$$

# Model: Stock

- ▶ We modify the standard Geometric BMP to accommodate the effect of the decisions of the manager

$$dS_t = \delta a_t dt + \alpha \sigma_t S_t dt + \sigma_t S_t dW_t$$

where  $a$  and  $\sigma$  are choices of the manager and  $\alpha$  and  $\delta$  exogenous parameters



# Model: Stock

- ▶ We modify the standard Geometric BMP to accommodate the effect of the decisions of the manager

$$dS_t = \delta a_t dt + \alpha \sigma_t S_t dt + \sigma_t S_t dW_t$$

where  $a$  and  $\sigma$  are choices of the manager and  $\alpha$  and  $\delta$  exogenous parameters

- ▶  $a$  is the level of (costly) effort the executive exercises

# Model: Stock

- ▶ We modify the standard Geometric BMP to accommodate the effect of the decisions of the manager

$$dS_t = \delta a_t dt + \alpha \sigma_t S_t dt + \sigma_t S_t dW_t$$

where  $a$  and  $\sigma$  are choices of the manager and  $\alpha$  and  $\delta$  exogenous parameters

- ▶  $a$  is the level of (costly) effort the executive exercises
- ▶  $\sigma$  is the costless choice of volatility (projects with different risk)

# Model: Stock

- ▶ We modify the standard Geometric BMP to accommodate the effect of the decisions of the manager

$$dS_t = \delta a_t dt + \alpha \sigma_t S_t dt + \sigma_t S_t dW_t$$

where  $a$  and  $\sigma$  are choices of the manager and  $\alpha$  and  $\delta$  exogenous parameters

- ▶  $a$  is the level of (costly) effort the executive exercises
- ▶  $\sigma$  is the costless choice of volatility (projects with different risk)
- ▶  $\alpha$  is a measure of the benefits of taking more risk, and it is a characteristic of the firm

# Model: Stock

- ▶ We modify the standard Geometric BMP to accommodate the effect of the decisions of the manager

$$dS_t = \delta a_t dt + \alpha \sigma_t S_t dt + \sigma_t S_t dW_t$$

where  $a$  and  $\sigma$  are choices of the manager and  $\alpha$  and  $\delta$  exogenous parameters

- ▶  $a$  is the level of (costly) effort the executive exercises
- ▶  $\sigma$  is the costless choice of volatility (projects with different risk)
- ▶  $\alpha$  is a measure of the benefits of taking more risk, and it is a characteristic of the firm
  - ▶ Proxy for growth opportunities available to the firm

# Model: Stock

- ▶ We modify the standard Geometric BMP to accommodate the effect of the decisions of the manager

$$dS_t = \delta a_t dt + \alpha \sigma_t S_t dt + \sigma_t S_t dW_t$$

where  $a$  and  $\sigma$  are choices of the manager and  $\alpha$  and  $\delta$  exogenous parameters

- ▶  $a$  is the level of (costly) effort the executive exercises
- ▶  $\sigma$  is the costless choice of volatility (projects with different risk)
- ▶  $\alpha$  is a measure of the benefits of taking more risk, and it is a characteristic of the firm
  - ▶ Proxy for growth opportunities available to the firm
- ▶  $\delta$  can be interpreted as an indicator of the type (quality) of the executive

# Model: Executive

- ▶ The utility receives compensation that consists of  $n$  options with strike price  $K$

# Model: Executive

- ▶ The utility receives compensation that consists of  $n$  options with strike price  $K$ 
  - ▶  $K = 0$  is stock

# Model: Executive

- ▶ The utility receives compensation that consists of  $n$  options with strike price  $K$ 
  - ▶  $K = 0$  is stock
- ▶ The executive is risk-averse and chooses  $a$  and  $\sigma$  to maximize utility



# Model: Executive

- ▶ The utility receives compensation that consists of  $n$  options with strike price  $K$ 
  - ▶  $K = 0$  is stock
- ▶ The executive is risk-averse and chooses  $a$  and  $\sigma$  to maximize utility

$$e(K, n) := \max_{a, \sigma} E \left[ \log \{ n(S_T - K)^+ \} - \frac{1}{2} \int_0^T a_t^2 dt \right]$$

# Model: Executive

- ▶ The utility receives compensation that consists of  $n$  options with strike price  $K$ 
  - ▶  $K = 0$  is stock
- ▶ The executive is risk-averse and chooses  $a$  and  $\sigma$  to maximize utility

$$e(K, n) := \max_{a, \sigma} E \left[ \log \{ n(S_T - K)^+ \} - \frac{1}{2} \int_0^T a_t^2 dt \right]$$

- ▶ Logarithmic utility is necessary for tractability reasons

# Model: Executive

- ▶ The utility receives compensation that consists of  $n$  options with strike price  $K$ 
  - ▶  $K = 0$  is stock
- ▶ The executive is risk-averse and chooses  $a$  and  $\sigma$  to maximize utility

$$e(K, n) := \max_{a, \sigma} E \left[ \log \{ n(S_T - K)^+ \} - \frac{1}{2} \int_0^T a_t^2 dt \right]$$

- ▶ Logarithmic utility is necessary for tractability reasons
  - ▶ As a result,  $n$  is irrelevant for incentive purposes

# Model: Executive

- ▶ The utility receives compensation that consists of  $n$  options with strike price  $K$ 
  - ▶  $K = 0$  is stock
- ▶ The executive is risk-averse and chooses  $a$  and  $\sigma$  to maximize utility

$$e(K, n) := \max_{a, \sigma} E \left[ \log \{ n(S_T - K)^+ \} - \frac{1}{2} \int_0^T a_t^2 dt \right]$$

- ▶ Logarithmic utility is necessary for tractability reasons
  - ▶ As a result,  $n$  is irrelevant for incentive purposes
  - ▶ But not to satisfy the participation constraint

# Model: Executive

- ▶ The utility receives compensation that consists of  $n$  options with strike price  $K$ 
  - ▶  $K = 0$  is stock
- ▶ The executive is risk-averse and chooses  $a$  and  $\sigma$  to maximize utility

$$e(K, n) := \max_{a, \sigma} E \left[ \log \{ n(S_T - K)^+ \} - \frac{1}{2} \int_0^T a_t^2 dt \right]$$

- ▶ Logarithmic utility is necessary for tractability reasons
  - ▶ As a result,  $n$  is irrelevant for incentive purposes
  - ▶ But not to satisfy the participation constraint
- ▶ The intuition of our results seems robust to more general settings

# Model: Firm with complete information

- ▶ The firm is risk-neutral and interested in the expected value of the stock minus compensation

# Model: Firm with complete information

- ▶ The firm is risk-neutral and interested in the expected value of the stock minus compensation

$$h(K, n) := \lambda E[S_T] - nE[(S_T - K)^+]$$

# Model: Firm with complete information

- ▶ The firm is risk-neutral and interested in the expected value of the stock minus compensation

$$h(K, n) := \lambda E[S_T] - nE[(S_T - K)^+]$$

where  $\lambda$  represents the relative importance of the expected value of the stock with respect to the compensation package



# Model: Firm with complete information

- ▶ The firm is risk-neutral and interested in the expected value of the stock minus compensation

$$h(K, n) := \lambda E[S_T] - nE[(S_T - K)^+]$$

where  $\lambda$  represents the relative importance of the expected value of the stock with respect to the compensation package

- ▶ Compensation has to satisfy the participation constraint

# Model: Firm with complete information

- ▶ The firm is risk-neutral and interested in the expected value of the stock minus compensation

$$h(K, n) := \lambda E[S_T] - nE[(S_T - K)^+]$$

where  $\lambda$  represents the relative importance of the expected value of the stock with respect to the compensation package

- ▶ Compensation has to satisfy the participation constraint

$$A(R) := \left\{ (K, n) \in [0, \infty)^2 : \max_{a, \sigma} E \left[ \log \{ n(S_T - K)^+ \} - \frac{1}{2} \int_0^T a_t^2 dt \right] \geq R \right\}$$

# Model: Firm with complete information

- ▶ The firm is risk-neutral and interested in the expected value of the stock minus compensation

$$h(K, n) := \lambda E[S_T] - nE[(S_T - K)^+]$$

where  $\lambda$  represents the relative importance of the expected value of the stock with respect to the compensation package

- ▶ Compensation has to satisfy the participation constraint

$$A(R) := \left\{ (K, n) \in [0, \infty)^2 : \max_{a, \sigma} E \left[ \log \{ n(S_T - K)^+ \} - \frac{1}{2} \int_0^T a_t^2 dt \right] \geq R \right\}$$

- ▶ Objective of the firm

# Model: Firm with complete information

- ▶ The firm is risk-neutral and interested in the expected value of the stock minus compensation

$$h(K, n) := \lambda E[S_T] - nE[(S_T - K)^+]$$

where  $\lambda$  represents the relative importance of the expected value of the stock with respect to the compensation package

- ▶ Compensation has to satisfy the participation constraint

$$A(R) := \left\{ (K, n) \in [0, \infty)^2 : \max_{a, \sigma} E \left[ \log \{ n(S_T - K)^+ \} - \frac{1}{2} \int_0^T a_t^2 dt \right] \geq R \right\}$$

- ▶ Objective of the firm

$$\max_{(K, n) \in A(R)} h(K, n)$$

# Model: Firm with *incomplete* information (I)

- ▶ There are different “types” of executives

# Model: Firm with *incomplete* information (I)

- ▶ There are different “types” of executives
  - ▶ Characterized by different values of  $\delta$

# Model: Firm with *incomplete* information (I)

- ▶ There are different “types” of executives
  - ▶ Characterized by different values of  $\delta$
  - ▶ Can be interpreted as “quality” of the executive or as level of commitment

# Model: Firm with *incomplete* information (I)

- ▶ There are different “types” of executives
  - ▶ Characterized by different values of  $\delta$
  - ▶ Can be interpreted as “quality” of the executive or as level of commitment
- ▶ The firm doesn’t know the value of  $\delta$



# Model: Firm with *incomplete* information (I)

- ▶ There are different “types” of executives
  - ▶ Characterized by different values of  $\delta$
  - ▶ Can be interpreted as “quality” of the executive or as level of commitment
- ▶ The firm doesn’t know the value of  $\delta$
- ▶ In particular, we assume

# Model: Firm with *incomplete* information (I)

- ▶ There are different “types” of executives
  - ▶ Characterized by different values of  $\delta$
  - ▶ Can be interpreted as “quality” of the executive or as level of commitment
- ▶ The firm doesn't know the value of  $\delta$
- ▶ In particular, we assume
  - ▶ the executive can have  $\delta_H$  with probability  $p_H$ , or  $\delta_L$  with  $\delta_H > \delta_L$

# Model: Firm with *incomplete* information (I)

- ▶ There are different “types” of executives
  - ▶ Characterized by different values of  $\delta$
  - ▶ Can be interpreted as “quality” of the executive or as level of commitment
- ▶ The firm doesn't know the value of  $\delta$
- ▶ In particular, we assume
  - ▶ the executive can have  $\delta_H$  with probability  $p_H$ , or  $\delta_L$  with  $\delta_H > \delta_L$
  - ▶ the firm knows the possible types and their distribution,

# Model: Firm with *incomplete* information (I)

- ▶ There are different “types” of executives
  - ▶ Characterized by different values of  $\delta$
  - ▶ Can be interpreted as “quality” of the executive or as level of commitment
- ▶ The firm doesn’t know the value of  $\delta$
- ▶ In particular, we assume
  - ▶ the executive can have  $\delta_H$  with probability  $p_H$ , or  $\delta_L$  with  $\delta_H > \delta_L$
  - ▶ the firm knows the possible types and their distribution,
  - ▶ the firm cannot tell the particular type of the executive it is negotiating with

# Model: Firm with *incomplete* information (I)

- ▶ There are different “types” of executives
  - ▶ Characterized by different values of  $\delta$
  - ▶ Can be interpreted as “quality” of the executive or as level of commitment
- ▶ The firm doesn’t know the value of  $\delta$
- ▶ In particular, we assume
  - ▶ the executive can have  $\delta_H$  with probability  $p_H$ , or  $\delta_L$  with  $\delta_H > \delta_L$
  - ▶ the firm knows the possible types and their distribution,
  - ▶ the firm cannot tell the particular type of the executive it is negotiating with
  - ▶ finally, different types have different reservation wages  $R_H, R_L$ , with  $R_H > R_L$

# Model: Firm with *incomplete* information (II)

- ▶ Objective function of the firm in several cases

## Model: Firm with *incomplete* information (II)

- ▶ Objective function of the firm in several cases

i) Menu of contracts, no exclusion,

$$\begin{aligned} h_M(K_H, K_L, n_H, n_L) &:= p_H \left( \lambda E^H[S_T] - n_H E^H[(S_T - K_H)^+] \right) \\ &\quad + p_L \left( \lambda E^L[S_T] - n_L E^L[(S_T - K_L)^+] \right) \end{aligned}$$

## Model: Firm with *incomplete* information (II)

► Objective function of the firm in several cases

i) Menu of contracts, no exclusion,

$$h_M(K_H, K_L, n_H, n_L) := p_H \left( \lambda E^H[S_T] - n_H E^H[(S_T - K_H)^+] \right) \\ + p_L \left( \lambda E^L[S_T] - n_L E^L[(S_T - K_L)^+] \right)$$

ii) Single contract, no exclusion,

$$h_S(K, n) := p_H \left( \lambda E^H[S_T] - n E^H[(S_T - K)^+] \right) \\ + p_L \left( \lambda E^L[S_T] - n E^L[(S_T - K)^+] \right)$$



## Model: Firm with *incomplete* information (II)

- Objective function of the firm in several cases

i) Menu of contracts, no exclusion,

$$h_M(K_H, K_L, n_H, n_L) := p_H \left( \lambda E^H[S_T] - n_H E^H[(S_T - K_H)^+] \right) \\ + p_L \left( \lambda E^L[S_T] - n_L E^L[(S_T - K_L)^+] \right)$$

ii) Single contract, no exclusion,

$$h_S(K, n) := p_H \left( \lambda E^H[S_T] - n E^H[(S_T - K)^+] \right) \\ + p_L \left( \lambda E^L[S_T] - n E^L[(S_T - K)^+] \right)$$

iii) Single contract, exclusion of low type

$$h_H(K, n) := \lambda E^H[S_T] - n E^H[(S_T - K)^+]$$

## Model: Firm with *incomplete* information (II)

- Objective function of the firm in several cases

i) Menu of contracts, no exclusion,

$$h_M(K_H, K_L, n_H, n_L) := p_H \left( \lambda E^H[S_T] - n_H E^H[(S_T - K_H)^+] \right) \\ + p_L \left( \lambda E^L[S_T] - n_L E^L[(S_T - K_L)^+] \right)$$

ii) Single contract, no exclusion,

$$h_S(K, n) := p_H \left( \lambda E^H[S_T] - n E^H[(S_T - K)^+] \right) \\ + p_L \left( \lambda E^L[S_T] - n E^L[(S_T - K)^+] \right)$$

iii) Single contract, exclusion of low type

$$h_H(K, n) := \lambda E^H[S_T] - n E^H[(S_T - K)^+]$$

iv) Single contract, exclusion of high type,

$$h_L(K, n) := \lambda E^L[S_T] - n E^L[(S_T - K)^+]$$

# Model: Firm with *incomplete* information (III)

- ▶ Denote

# Model: Firm with *incomplete* information (III)

► Denote

$$\begin{aligned}h_M^* &= \max_{\substack{\{(K_H, n_H) \in A(R_H), \notin A(R_L)\} \\ \{(K_L, n_L) \in A(R_L), \notin A(R_H)\}}} h_M(K_H, K_L, n_H, n_L) \\h_S^* &= \max_{\{(K, n) \in A(R_H) \cap A(R_L)\}} h_S(K, n) \\h_H^* &= \max_{\{(K, n) \in A(R_H), \notin A(R_L)\}} h_H(K, n) \\h_L^* &= \max_{\{(K, n) \in A(R_L), \notin A(R_H)\}} h_L(K, n).\end{aligned}$$

# Model: Firm with *incomplete* information (III)

## ► Denote

$$\begin{aligned}h_M^* &= \max_{\substack{\{(K_H, n_H) \in A(R_H), \notin A(R_L)\} \\ \{(K_L, n_L) \in A(R_L), \notin A(R_H)\}}} h_M(K_H, K_L, n_H, n_L) \\h_S^* &= \max_{\{(K, n) \in A(R_H) \cap A(R_L)\}} h_S(K, n) \\h_H^* &= \max_{\{(K, n) \in A(R_H), \notin A(R_L)\}} h_H(K, n) \\h_L^* &= \max_{\{(K, n) \in A(R_L), \notin A(R_H)\}} h_L(K, n).\end{aligned}$$

## ► Objective of the firm

# Model: Firm with *incomplete* information (III)

## ► Denote

$$\begin{aligned}h_M^* &= \max_{\substack{\{(K_H, n_H) \in A(R_H), \notin A(R_L)\} \\ \{(K_L, n_L) \in A(R_L), \notin A(R_H)\}}} h_M(K_H, K_L, n_H, n_L) \\h_S^* &= \max_{\{(K, n) \in A(R_H) \cap A(R_L)\}} h_S(K, n) \\h_H^* &= \max_{\{(K, n) \in A(R_H), \notin A(R_L)\}} h_H(K, n) \\h_L^* &= \max_{\{(K, n) \in A(R_L), \notin A(R_H)\}} h_L(K, n).\end{aligned}$$

## ► Objective of the firm

- find the pair  $(K^*, n^*)$ , or menu  $(K_H^*, K_L^*, n_H^*, n_L^*)$  that achieves

# Model: Firm with *incomplete* information (III)

## ► Denote

$$\begin{aligned}h_M^* &= \max_{\substack{\{(K_H, n_H) \in A(R_H), \notin A(R_L)\} \\ \{(K_L, n_L) \in A(R_L), \notin A(R_H)\}}} h_M(K_H, K_L, n_H, n_L) \\h_S^* &= \max_{\{(K, n) \in A(R_H) \cap A(R_L)\}} h_S(K, n) \\h_H^* &= \max_{\{(K, n) \in A(R_H), \notin A(R_L)\}} h_H(K, n) \\h_L^* &= \max_{\{(K, n) \in A(R_L), \notin A(R_H)\}} h_L(K, n).\end{aligned}$$

## ► Objective of the firm

- find the pair  $(K^*, n^*)$ , or menu  $(K_H^*, K_L^*, n_H^*, n_L^*)$  that achieves

$$\max(h_M^*, h_S^*, h_H^*, h_L^*)$$

# Model: Firm with *incomplete* information (IV)

- ▶ Two possible types of optimal results



# Model: Firm with *incomplete* information (IV)

- ▶ Two possible types of optimal results
  - i) Separating equilibrium, when the optimal contract is a menu, or it is a single pair  $(K^*, n^*)$  and  $(K^*, n^*) \in A(R_H), \notin A(R_L)$  or  $(K^*, n^*) \in A(R_L), \notin A(R_H)$

# Model: Firm with *incomplete* information (IV)

- ▶ Two possible types of optimal results
  - i) Separating equilibrium, when the optimal contract is a menu, or it is a single pair  $(K^*, n^*)$  and  $(K^*, n^*) \in A(R_H), \notin A(R_L)$  or  $(K^*, n^*) \in A(R_L), \notin A(R_H)$
  - ii) Pooling equilibrium, when the optimal contract is a single pair  $(K^*, n^*)$  and  $(K^*, n^*) \in A(R_H) \cap A(R_L)$

# Solution of the problem of the executive

- ▶ First consider the problem of the executive for a given compensation package (that is, a pair  $n, K$ )

# Solution of the problem of the executive

- ▶ First consider the problem of the executive for a given compensation package (that is, a pair  $n, K$ )
- ▶ Technically is like a consumption/portfolio allocation problem with complete markets

# Solution of the problem of the executive

- ▶ First consider the problem of the executive for a given compensation package (that is, a pair  $n, K$ )
- ▶ Technically is like a consumption/portfolio allocation problem with complete markets
  - ▶ Choice of optimal effort is like choice of consumption and choice of volatility like choice of optimal allocation in the risky security

# Solution of the problem of the executive

- ▶ First consider the problem of the executive for a given compensation package (that is, a pair  $n, K$ )
- ▶ Technically is like a consumption/portfolio allocation problem with complete markets
  - ▶ Choice of optimal effort is like choice of consumption and choice of volatility like choice of optimal allocation in the risky security
  - ▶ And we can use standard martingale -or convex duality- techniques

# Solution of the problem of the executive

- ▶ First consider the problem of the executive for a given compensation package (that is, a pair  $n, K$ )
- ▶ Technically is like a consumption/portfolio allocation problem with complete markets
  - ▶ Choice of optimal effort is like choice of consumption and choice of volatility like choice of optimal allocation in the risky security
  - ▶ And we can use standard martingale -or convex duality- techniques
- ▶ We derive analytic solutions for  $\hat{a}$  and  $\hat{\sigma}$

# Optimal strategy of the executive: Effort

- ▶ It is independent of  $n$  because of logarithmic utility



# Optimal strategy of the executive: Effort

- ▶ It is independent of  $n$  because of logarithmic utility
- ▶ Optimal effort increases with the strike price and decreases with the maturity of the option

# Optimal strategy of the executive: Effort

- ▶ It is independent of  $n$  because of logarithmic utility
- ▶ Optimal effort increases with the strike price and decreases with the maturity of the option
- ▶ Optimal effort as a function of  $\delta$  (type)

# Optimal strategy of the executive: Effort

- ▶ It is independent of  $n$  because of logarithmic utility
- ▶ Optimal effort increases with the strike price and decreases with the maturity of the option
- ▶ Optimal effort as a function of  $\delta$  (type)
  - ▶ If option is in-the-money the effort is increasing in  $\delta$

# Optimal strategy of the executive: Effort

- ▶ It is independent of  $n$  because of logarithmic utility
- ▶ Optimal effort increases with the strike price and decreases with the maturity of the option
- ▶ Optimal effort as a function of  $\delta$  (type)
  - ▶ If option is in-the-money the effort is increasing in  $\delta$
  - ▶ Opposite if out-of-the-money

# Optimal strategy of the executive: Effort

- ▶ It is independent of  $n$  because of logarithmic utility
- ▶ Optimal effort increases with the strike price and decreases with the maturity of the option
- ▶ Optimal effort as a function of  $\delta$  (type)
  - ▶ If option is in-the-money the effort is increasing in  $\delta$
  - ▶ Opposite if out-of-the-money
    - ▶ High-type executives prefer to choose higher volatility

# Optimal strategy of the executive: Effort

- ▶ It is independent of  $n$  because of logarithmic utility
- ▶ Optimal effort increases with the strike price and decreases with the maturity of the option
- ▶ Optimal effort as a function of  $\delta$  (type)
  - ▶ If option is in-the-money the effort is increasing in  $\delta$
  - ▶ Opposite if out-of-the-money
    - ▶ High-type executives prefer to choose higher volatility
- ▶ Expected effort is decreasing in  $\alpha$

# Optimal strategy of the executive: Volatility

- ▶ Optimal volatility is increasing in  $\alpha$

# Optimal strategy of the executive: Volatility

- ▶ Optimal volatility is increasing in  $\alpha$
- ▶ It is increasing in the type  $\delta$



# Optimal strategy of the executive: Volatility

- ▶ Optimal volatility is increasing in  $\alpha$
- ▶ It is increasing in the type  $\delta$ 
  - ▶ Higher quality executives can take more risk because they are more adept at fixing things through effort

# Optimal strategy of the executive: Volatility

- ▶ Optimal volatility is increasing in  $\alpha$
- ▶ It is increasing in the type  $\delta$ 
  - ▶ Higher quality executives can take more risk because they are more adept at fixing things through effort
- ▶ For short-term options, optimal risk and effort are (locally) negatively correlated