On the relative pricing of long maturity S&P 500 index options and CDX tranches

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November 2010

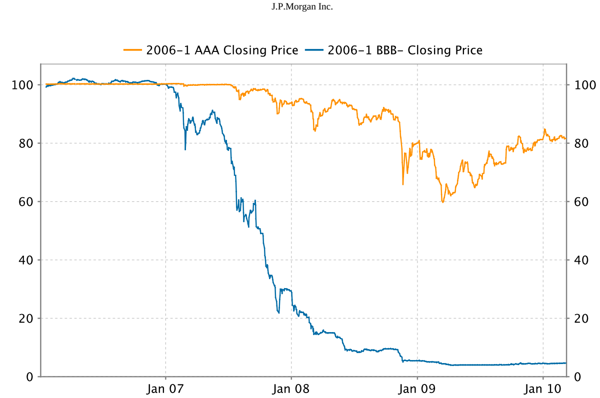
Securitized Credit Markets Crisis

- Pre-crisis saw large growth in securitized credit markets (CDO).
- Pooling and tranching used to create 'virtually risk-free' AAA securities, in response to high demand for highly rated securities.
- During the crisis **all** AAA markets were hit hard:
  - Home equity loan CDO prices fell (ABX.HE AAA $< 60\%$).
  - Super Senior (30-100) tranche spreads $>$ 100bps.
  - CMBX.AAA (super duper) $>$750bps.
- Raises several questions:
  - Q? Were ratings incorrect (ex-ante default probability higher than expected)?
  - Q? Are ratings sufficient statistics (risk $\neq$ expected loss)?
  - Q? Were AAA tranches mis-priced (relative to option prices)?
- Many other surprises:
  - Corporate Credit spreads widened (CDX-IG $>$ 200bps).
  - Cash-CDS basis **negative** (-200 bps for IG; -700bps for HY).
  - LIBOR-Treasury and LIBOR-OIS widened ($>$ 400bps).
  - Long term Swap spreads became negative (30 year swap over Treasury < -50 bps).
  - Defaults on the rise (Bear Stearns, Lehman).

Evidence from ABX markets

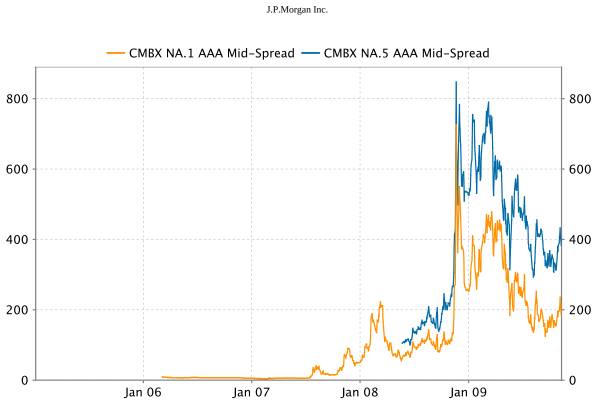
- ABX.HE (subprime) AAA and BBB spreads widened dramatically (prices dropped)

J.P.Morgan DataQuery



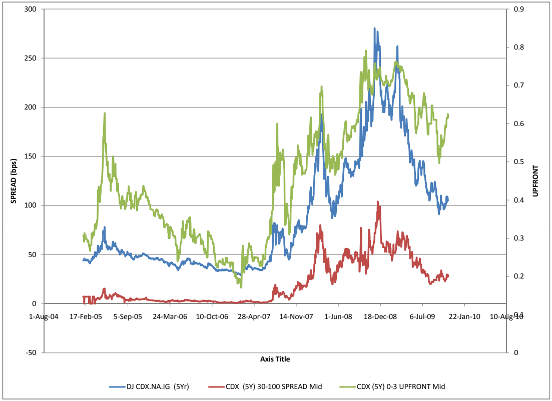
Evidence from CMBX markets

- CMBX (commercial real estate) AAA spreads widened even more dramatically



Corporate IG CDX Tranche spreads

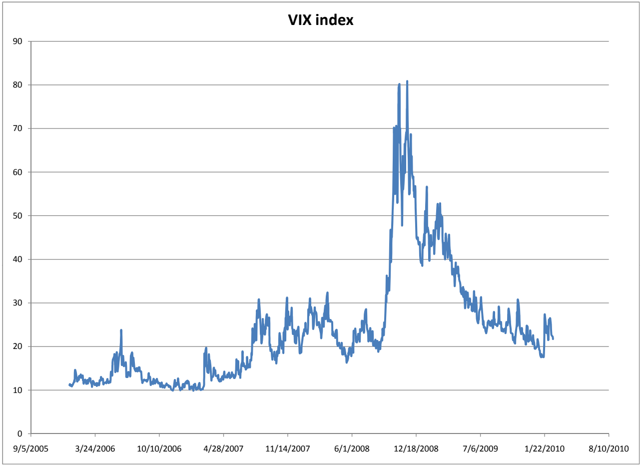
- The impact on tranche prices was dramatic



- Implied correlation on equity tranche hit $> 40\%$
- Correlation on Super-Senior tranches $> 100\%$(!) with standard recovery assumption
- Relative importance of expected loss in senior tranche versus in equity tranche indicates increased crash risk.

Evidence from S&P500 Option markets

- Implied volatility index widened dramatically: increased market and crash risk.



CDX Index & CDX Tranche Markets

- Credit Default Swaps (CDS)
  - Buyer of protection makes regular (quarterly) payments = CDS spread
  - Seller of protection makes buyer whole if underlying bond defaults
  - CDS spread $\approx$ corporate bond spread ($y - r_f$)
- CDX Investment Grade (IG) Index
  - portfolio of 125 IG credits
  - Buyer of protection makes regular payments on **remaining portfolio notional**
  - Seller of protection makes buyer whole at time of each bond default
  - CDX index spread $\approx$ weighted average of CDS spreads
- CDX (IG) Tranches written on same portfolio
  - Associated with standard attachment/detachment points (subordination levels):
    - 0-3% (Equity tranche)
    - 3-7% (Mezzanine tranche)
    - 7-10%
    - 10-15%
    - 15-30% (Senior tranche)
    - 30-100% (Super-senior tranche)
  - Buyer of protection makes regular payments on **remaining tranche notional**
  - Seller of protection makes buyer whole for each bond default which reduces tranche notional
- CDS, CDX index spreads determined from **marginal** default probabilities.
- CDX tranche spreads need entire **joint** distribution (correlation market).

Relation Between SP500 Index Option Prices and CDX Tranche Spreads

- Given the Arrow-Debreu (or state) prices for every date and every state of nature, one can determine the arbitrage-free price of any (derivative) security
- Given option prices across all strikes (and dates) of SP500 index options, one can back out the A/D prices
  - Breeden and Litzenberger (1978)
- Due to diversification effects of 125 firms composing CDX index, CF's associated with CDX tranche positions closely tied to overall market performance
  - ⇒ Identifying state prices from option prices should be useful for estimating tranche spreads
- In practice, strikes typically limited to (70% - 130%) of current index levels
- Can we extrapolate state prices from SP500 option prices to price credit derivatives?
  - Payoffs of most senior tranches associated with losses well below 70% of current levels
  - Need to extrapolate well beyond observable prices

Structural/Copula Models of Default

- Specify market (S&P500) value dynamics as:

$$\frac{dM}{M} = (r - \delta_M)\, dt + \sigma_M\, dz_M^Q$$

- Specify firm asset value dynamics via CAPM (market plus idiosyncratic risks):

$$\frac{dA_i}{A_i} = (r - \delta_i)\, dt + \beta_i \sigma_M\, dz_M^Q + \sigma_i\, dz_i^Q$$

Note: total variance is sum of market variance plus idiosyncratic variance

$$v_i^2 = (\beta_i \sigma_M)^2 + \sigma_i^2$$

- Default occurs if $A(t) \leq B$ for $t < T$
- From Black/Scholes/Merton, to determine CDS spread, only need to know v^2
  - To determine CDX index spread on 2 (or 125) identical firms, only need to know v^2
- Consider insurance contract ($\sim$ CDX tranches) that pays iff exactly 1 firm defaults
  - If $v^2 = (\beta\sigma_M)^2$, returns perfectly correlated: either zero firms or all firms will default
    - value of insurance on exactly one default is zero
  - If $v^2 > (\beta\sigma_M)^2$, returns are imperfectly correlated: a single default is possible
    - value of insurance on exactly one default is positive

Coval, Jurek and Stafford (CJS, 2009)

- Model Specification ($\sim$ standard copula with Option-implied market factor)
  - Estimate 5-year state prices using 5-year SP500 option prices ($\sim$ local vol model)
  - Specify idiosyncratic risk as Gaussian diffusion
  - Calibrate model to match the 5-year CDX index spread
    - Have only 5-year state prices; estimating PV[CF's] (0-5 years)

- Findings: Observed spreads on
  - equity tranche too high compared to model predictions
  - other tranches (except super-senior) too low compared to model predictions

	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%
data	1472	135	37	17	8	4
CJS	914	267	150	87	28	1

- Interpretation:
  - sellers of insurance on senior tranches naive:
    - focused on high credit ratings/low probability of payout
    - did not properly account for the level of systematic risk exposure

Our Approach

- Methodology:
  - Specify several (jump-diffusion-SV) structural model for both market (S&P500) and individual (CDX) firm dynamics.
  - Price options (closed-form) and tranches (Monte-carlo simulations).
  - Calibrate market dynamics to match **all maturities** and strikes of SP500 options.
  - Calibrate idiosyncratic dynamics to match **all maturities** of CDX index spreads.
  - Calibrate to beta and total variance (estimated from CRSP/Compustat for constituents of CDX index).
- Main Findings:
  - Spread on super-senior tranche too far out of the money to estimate using option prices
  - Taking Super Senior spreads as input, other tranche spreads well estimated by **any** model
- Interpretation:
  - sellers of insurance on senior tranches sophisticated:
    - Required fair (relative) compensation for risks involved
    - May have enjoyed the "window dressing" associated with highly rated securities ($\sim$ rating 'arbitrage').

A structural model for pricing long-dated S&P500 options

- The market model is the **S**tochastic **V**olatility **C**ommon **J**ump (**SVCJ**) model of Broadie, Chernov, Johannes (2009):

$$\begin{aligned}
\frac{dM_t}{M_t} &= (r-\delta)\,dt + \sqrt{V_t}\,dw_1^Q + (e^y - 1)\,dq - \bar{\mu}_y \lambda^Q dt + (e^{y_C} - 1)\,(dq_C - \lambda_C^Q\,dt) \\
dV_t &= \kappa_V(\bar{V} - V_t)dt + \sigma_V \sqrt{V_t}(\rho dw_1^Q + \sqrt{1-\rho^2}dw_2^Q) + y_V\,dq \\
d\delta_t &= \kappa_\delta(\bar{\delta} - \delta_t)\,dt + \sigma_\delta \sqrt{V_t}(\rho_1\,dw_1^Q + \rho_2\,dw_2^Q + \sqrt{1-\rho_1^2-\rho_2^2}\,dw_3^Q) + y_\delta\,dq.
\end{aligned}$$

- We add stochastic dividend yield (SV**D**CJ) to help fit long-dated options as well.
- The parameters of the model are calibrated to 5-year index option prices obtained from CJS.
- State variables are extracted given parameters from time-series of short maturity options (obtained from OptionMetrics).
- Advantage of using structural model: Arbitrage-free extrapolation into lower strikes (needed for senior tranches).

A structural model of individual firm's default

- Given market dynamics, we assume individual firm i dynamics:

$$\frac{dA_i(t)}{A_i(t)} + \delta_A\,dt - rdt = \beta_i\left(\sqrt{V_t}dw_1^Q + (e^y - 1)\;dq - \bar{\mu}_y\lambda^Q dt\right) + \sigma_i\,dw_i$$
$$+ (e^{y_C} - 1)\,(dq_C - \lambda_C^Q\,dt) + (e^{y_i} - 1)\,(dq_i - \lambda_i^Q\,dt).$$

- Note
  - β: exposure to market excess return (i.e., systematic diffusion **and** jumps).
  - dq_C: 'catastrophic' market wide jumps.
  - dq_i: idiosyncratic firm specific jumps.
  - dw_i: idiosyncratic diffusion risks.
- Default occurs the first time firm value falls below a default barrier B_i (Black (1976)):

$$\tau_i = \inf\{t\ :\ A_i(t) \leq B_i\}. \quad (1)$$

- Recovery upon default is a fraction $(1 - \ell)$ of the remaining asset value.

Pricing of the CDX index via Monte-Carlo

- The running spread on the CDX index is closely related to a weighted average of CDS spreads.
- Determined such that the present value of the **protection leg** ($V_{idx,prot}$) equals the PV of the **premium leg** ($V_{idx,prem}$):

$$
\begin{aligned}
V_{idx,prem}(S) &= S\,\mathrm{E}\left[\sum_{m=1}^{M} e^{-rt_m}\left(1-n(t_m)\right)\Delta + \int_{t_{m-1}}^{t_m} du\, e^{-ru}\left(u-t_{m-1}\right) dn_u\right] \\
V_{idx,prot} &= \mathrm{E}\left[\int_0^T e^{-rt}\, dL_t\right].
\end{aligned}
$$

- We have defined:
  - The (percentage) defaulted notional in the portfolio: $n(t) = \frac{1}{N}\sum_i \mathbf{1}_{\{\tau_i \le t\}}$,
  - The cumulative (percentage) loss in the portfolio: $L(t) = \frac{1}{N}\sum_i \mathbf{1}_{\{\tau_i \le t\}}(1-R_i(\tau_i))$

Pricing of the CDX Tranches via Monte-Carlo

- The tranche loss as a function of portfolio loss is

$$T_j(L(t)) \quad = \quad \max\left[L(t) - K_{j-1}, 0\right] - \max\left[L(t) - K_j, 0\right].$$

- The initial value of the protection leg on tranche-j is

$$Prot_j(0, T) \quad = \quad \mathrm{E}^Q\left[\int_0^T e^{-rt}\, dT_j(L(t))\right]$$

- For a tranche spread S_j, the initial value of the premium leg on tranche-j is

$$Prem_j(0, T) \quad = \quad S_j \mathrm{E}^Q\left[\sum_{m=1}^{M} e^{-rt_m}\int_{t_{m-1}}^{t_m} du\left(K_j - K_{j-1} - T_j(L(u))\right)\right].$$

- Appropriate modifications to the cash-flows
    - Equity tranche (upfront payment),
    - Super-senior tranche (recovery accounting).

Calibration of firms' asset value processes

- Calibrate 7 (unlevered) asset value parameters $(\beta, \sigma, B, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$ to match median CDX-series firm's:
  - Market beta
  - Idiosyncratic risk (estimated from rolling regressions for CDX series constituents using CRSP-Compustat)
  - Term structure of CDX spreads (1 to 5 year)
- Set jump size to -2 ($\sim$ jump to default).
- When present, calibrate catastrophic jump intensity to match super-senior ($\lambda_C < 1$ event per 1000 years).
- Set loss given default $1 - \ell$ to 40% ($\sim$ match historical average) in normal times.
- Set $1 - \ell = 20\%$ if catastrophe jump occurs ($\sim$ Altman et al.).
- Market volatility, jump-risk, dividend-yield all estimated from S&P500 option data in previous step.

Average tranche spreads predicted for pre-crisis period

- We report six tranche spreads averaged over the pre-crisis period Sep 04 - Sep 07:
  - The historical values;
  - Benchmark model: Catastrophic jumps calibrated to match the super-senior tranche; Idiosyncratic jumps and default boundary calibrated to match the 1 to 5 year CDX index.
  - $\lambda_C^Q = 0$: No catastrophic jumps; Idiosyncratic jumps and default boundary calibrated to match 1 to 5 year CDX index;
  - $\lambda_i^Q = 0$: Catastrophic jumps calibrated to match the super-senior tranche; No idiosyncratic jumps; Default boundary calibrated to match only the 5Y CDX index.
  - $\lambda_C^Q = 0$, $\lambda_i^Q = 0$: No catastrophic jumps; No idiosyncratic jumps; Default boundary calibrated to match only the 5Y CDX index;
  - The results reported by CJS

	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%	0-3% Upfrt
data	1472	135	37	17	8	4	0.34
benchmark	1449	113	25	13	8	4	0.33
$\lambda_C^Q = 0$	1669	133	21	6	1	0	0.40
$\lambda_i^Q = 0$	1077	206	70	32	12	4	0.22
$\lambda_C^Q = 0$, $\lambda_i^Q = 0$	1184	238	79	31	6	0	0.26
CJS	914	267	150	87	28	1	na
$\frac{\lvert CJS - Data \rvert}{\lvert Benchmark - Data \rvert}$	24.3	6	9.4	17.5	∞	∞	

Interpretation

- Errors are an order of magnitude smaller than those reported by CJS.
- However, model without jumps ($\lambda_C^Q = 0$, $\lambda_i^Q = 0$) generates similar predictions to CJS.
- Why? Problem is two-fold:
  - **Backloading** of defaults in standard diffusion model:

Average CDX index spreads for different models

	1 year	2 year	3 year	4 year	5 year
Data	13	20	28	36	45
Benchmark	13	20	28	36	45
$\lambda_C^Q = 0$	13	20	28	36	45
$\lambda_i^Q = 0$	6	7	16	29	45
($\lambda_C^Q = 0$, $\lambda_i^Q = 0$)	0	3	13	28	45

  - Idiosyncratic jumps generates a five-year loss distribution that is **more peaked** around the risk-neutral expected losses of 2.4%.
    (loss distribution with $\lambda_C^Q = 0$, $\lambda_i^Q = 0$ has std dev of 2.9%, whereas loss distribution with ($\lambda_i^Q > 0$, $\lambda_C^Q = 0$) has std dev of 1.7%).

More Generally....

- We claim that if:
  - Take **any** "reasonable" dynamic model of market returns to match SP500 option prices
  - Specify idiosyncratic dynamics as a diffusion process
  - Calibrate the model to match the 5-year CDX index
- Then model will generate:
  - **Short term credit spreads that are well below observed levels**
  - Tranche spreads similar to those found by CJS

	1 year	2 year	3 year	4 year	5 year
data	13	20	28	36	45
$E^Q[\#def]$	0.27	0.83	1.75	3.00	4.69
our model	0	3	13	28	45
SVCJ	0	3	14	29	45
Heston	0	2	12	28	45
$E^Q[\#def]$	0.01	0.13	0.81	2.33	4.69

More Generally....

- We claim that if:
  - Take **any** "reasonable" dynamic model of market returns to match SP500 option prices
  - Specify idiosyncratic dynamics as a diffusion process
  - Calibrate the model to match the 5-year CDX index
- Then model will generate:
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  - **Tranche spreads similar to those found by CJS**

	0-3% Upfrt	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%
data	0.34	1472	135	37	17	8	4
our model	0.26	1184	238	79	31	6	0
SVCJ	0.22	1078	243	96	44	11	0
Heston	0.23	1097	230	83	39	10	0
CJS	na	914	267	150	87	28	1

Intuition for Findings

- Diffusion-based structural models can't explain short maturity spreads for IG debt
  - Some level of jumps captured in market dynamics implied from options
  - However, most risk at individual firm level is idiosyncratic
    - Need to specify idiosyncratic dynamics with jumps to capture short term spreads
- By calibrating model to 5Y CDX index, all models agree on 5Y expected loss
- By calibrating model to observed term structure of spreads, defaults occur earlier
  - eliminate "backloading" of defaults
  - crucial for pricing equity tranche spreads
    - first default associated with $\approx 16\%$ drop in insurance premium payments
    - timing of defaults so crucial that equity tranche typically priced with an up-front premium
  - Agents willing to pay more initially if future payments expected to drop more quickly
  - "Backloading" biases equity tranche spreads downward
  - Downward bias on equity tranche generates an upward bias on senior tranches
- In addition, calibrating model to short maturity spreads increases proportion of idiosyncratic risk to systematic risk
  - Tends to make loss distribution more peaked
  - Also tends to increase spreads on equity tranche/decrease spreads on senior tranches

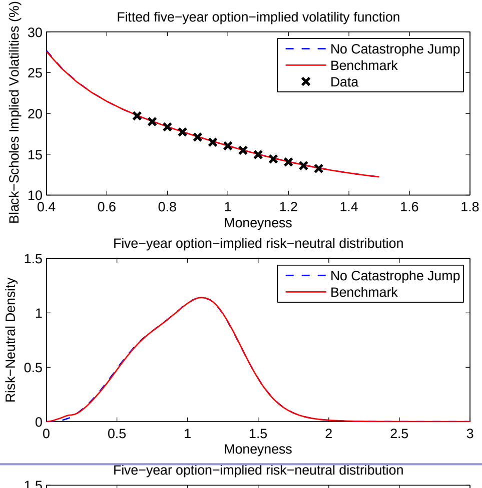
Calibrating Model to Term Structure of CDX Index Spreads

- When models are calibrated to match short term credit spreads, the results of CJS disappear, and sometimes are even reversed!!
- Predicted super-senior tranche spreads ≈ 0

	0-3% Upfrt	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%
data	0.34	1472	135	37	17	8	4
our model	0.40	1669	133	21	6	1	0
SVCJ	0.35	1505	166	45	19	4	0
Heston	0.34	1500	157	42	18	5	0

Calibrating Model to Term Structure of CDX Index Spreads and SS Spread

- However, can add a "catastrophic jump" to market dynamics
  - Rietz (1988), Barro (2006)
  - has negligible impact on observed option prices
  - has large impact on SS spreads.



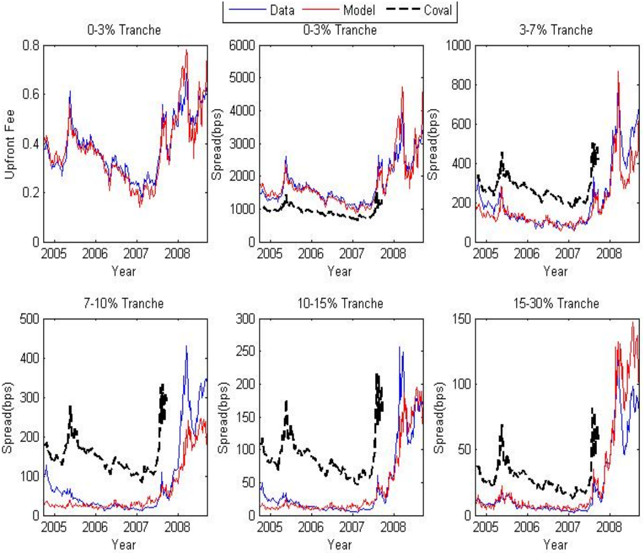
Calibrating Model to Term Structure of CDX Index Spreads and SS Spread

- However, can add a "catastrophic jump" to market dynamics
  - Rietz (1988), Barro (2006)
  - has negligible impact on observed option prices
  - has large impact on SS spreads.
  - Can improve fit further by taking tranche spreads in-sample
    - Mortensen (2006), Longstaff and Rajan (2008), Eckner (2009)

	0-3% Upfrt	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%
data	0.34	1472	135	37	17	8	4
our model	0.33	1449	113	25	13	8	4
SVCJ	0.30	1330	138	47	26	12	4
Heston	0.29	1301	142	46	24	12	4
CJS	na	914	267	150	87	28	1

Time Series Performance

- Model fits data well, both pre-crisis and crisis periods



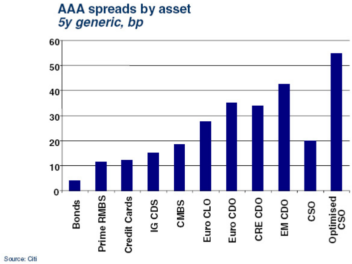
Conclusion

- CF's associated with CDX tranche spreads occur throughout 5 year horizon
  - need dynamic model of market and idiosyncratic dynamics to price consistently
- Market dynamics (mostly) extracted from option prices
- idiosyncratic dynamics extracted from term structure of credit spreads
  - need idiosyncratic jumps to explain short maturity spreads
- without these jumps:
  - default events are "backloaded"
  - ratio of idiosyncratic to market risk is off
    - CDX equity tranche spreads biased downward
    - CDX senior tranche spreads biased upward
- Super senior tranche spreads cannot be estimated via extrapolation
  - Instead, need to take them as input
  - Other tranche spreads well-predicted by any model that also matches option prices, CDS spreads
- Calibrating model to term structure of credit spreads imposes more structure/ less freedom
  - We used "HJM approach"
    - More consistently, can add state variables driving idiosyncratic jump processes

Are senior tranches priced inefficiently by naive investors?

- Investors care only about expected losses ($\sim$ ratings) and not about covariance (ironic since they trade in correlation markets!).

$\Rightarrow$ Spreads across AAA assets should be equalized. Are they?



$\Rightarrow$ All spreads should converge to **Physical** measure expected loss.

- We observe large risk-premium across the board ($\lambda^Q/\lambda^P > 6$.)
- Large time-variation in that risk-premium.

$\Rightarrow$ Time-variation in spreads should be similar to that of rating changes (smoother?).

- Evidence seems inconsistent with marginal price setters caring only about expected loss ($\sim$ ratings).

What drives differences between structured AAA spreads?

- 'Reaching for yield' by rating constrained investors who **want** to take more risk because their incentives (limited liability) and **can** because ratings simply do not reflect risk and/or expected loss.
- Taking more risk by loading on systematic risk was the name of the game (agency conflicts).
- Possible that excess 'liquidity'/leverage lead to spreads being 'too' narrow **in all** markets, but little evidence that markets were ex-ante mis-priced on a relative basis.
- Ex-post (during the crisis) other issues, such as availability of collateral and funding costs, seem more relevant to explain cross-section of spreads across markets.
- Indeed, how to explain negative and persistent:
  - swap spreads?
  - cds basis?