
Semantics for Higher-Order Quantum Computation via Geometry of Interaction

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Quantum Lambda Calculus

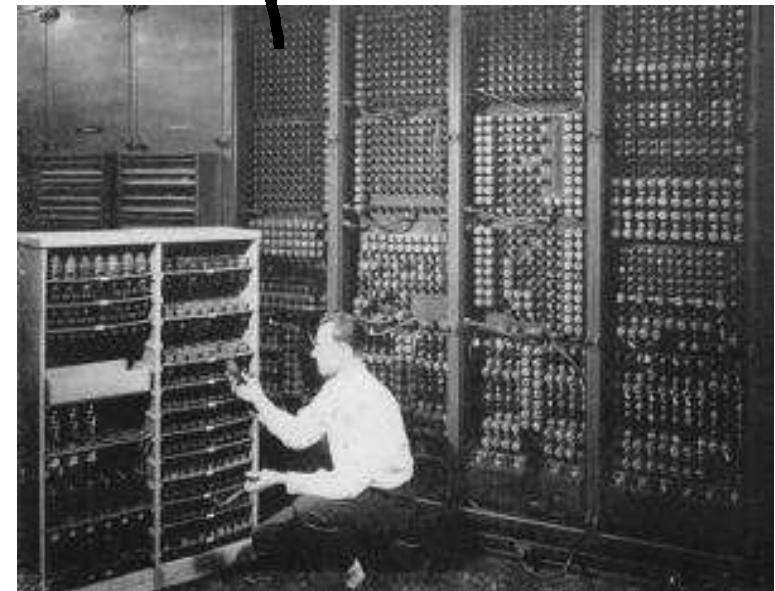
Quantum computation

+ Higher order computation

- quantum state preparation
- unitary transformation
- measurement
- programs as first-class objects
- Shor's algorithm, Deutsch-Jozsa algorithm, Grover's algorithm ...
- data types, modularity, security, optimisation ...

Classical Control and Quantum Data (Selinger and Valiron)

Quantum data



Classical control

Linearity

Type $A := \text{bit} \mid n\text{-qbit} \mid A \multimap A \mid !A \mid \dots$

- No-cloning of quantum bits:

$$\begin{aligned} x : n\text{-qbit} &\not\vdash \langle x, x \rangle : n\text{-qbit} \otimes n\text{-qbit} \\ x : !A &\vdash \langle x, x \rangle : !A \otimes !A \end{aligned}$$

- $M : !A \iff$ We know how to create $M : A$.

- Implicit linearity tracking

Operational Semantics

- Call-by-value evaluation
- Probabilistic branching

The evaluation result of $M : \text{bit}$

$$M \Downarrow (p, q)$$

β -reduction

$$(\lambda x^A.M)V \rightarrow_1 M[V/x]$$

Unitary transformation

$$U \text{ new}_\rho \rightarrow_1 \text{new}_{U\rho U^\dagger}$$

Measurement

$$\begin{array}{ccc} \text{meas}^1 \text{ new}_\rho & \xrightarrow{p} & \text{true} \\ & \xrightarrow{q} & \text{false} \end{array}$$

$$(p = \langle 0 | \rho | 0 \rangle, q = \langle 1 | \rho | 1 \rangle)$$

Operational Semantics

- Call-by-value evaluation
- Probabilistic branching

β -reduction

$$(\lambda x^A.M)V \rightarrow_1$$

Unitary transformation

$$U \text{ new}_\rho \rightarrow_1$$

Measurement

$$\text{meas}^1 \text{ new}_\rho$$

true

The evaluation result of $M : \text{bit}$

$$M \Downarrow (p, q)$$

$$M[V/x]$$

$$\text{new}_{U\rho U^\dagger}$$

false

true

false

$$(p = \langle 0 | \rho | 0 \rangle, q = \langle 1 | \rho | 1 \rangle)$$

p

q

Example

$$(\lambda x^{\text{bit}}. \text{not } x) \left(\text{meas} \left(\frac{1}{\sqrt{3}} |\text{true}\rangle + \sqrt{\frac{2}{3}} |\text{false}\rangle \right) \right)$$

Example

$$(\lambda x^{\text{bit}}. \text{not } x) \left(\text{meas} \left(\frac{1}{\sqrt{3}} |\text{true}\rangle + \sqrt{\frac{2}{3}} |\text{false}\rangle \right) \right)$$

$\frac{1}{3}$

↗

$(\lambda x^{\text{bit}}. \text{not } x) \text{ true}$

Example

$$(\lambda x^{\text{bit}}. \text{not } x) \left(\text{meas} \left(\frac{1}{\sqrt{3}} |\text{true}\rangle + \sqrt{\frac{2}{3}} |\text{false}\rangle \right) \right)$$

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$(\lambda x^{\text{bit}}. \text{not } x) \text{ true}$

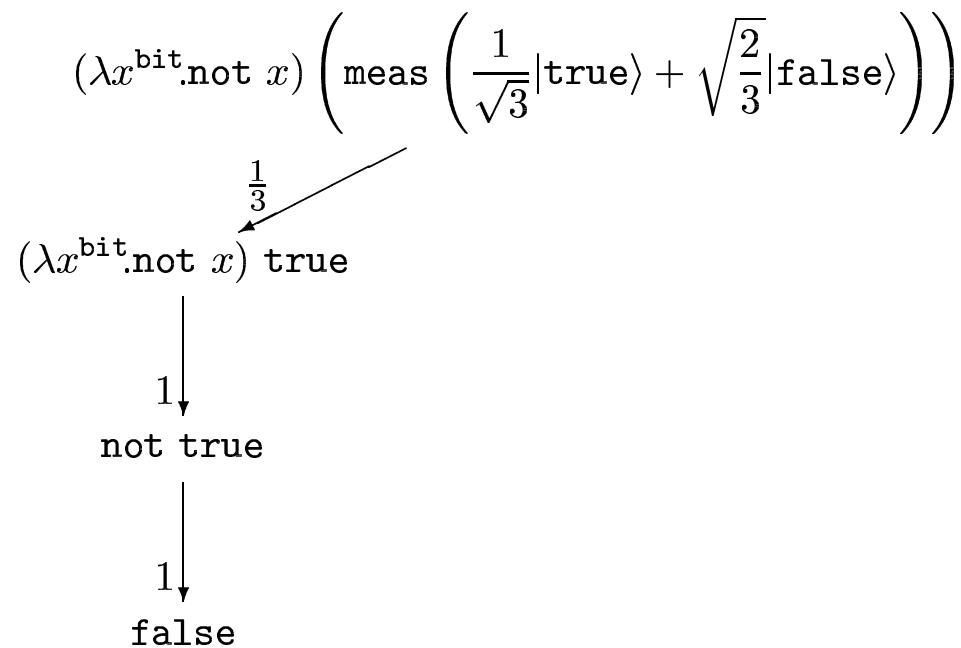
↓

1

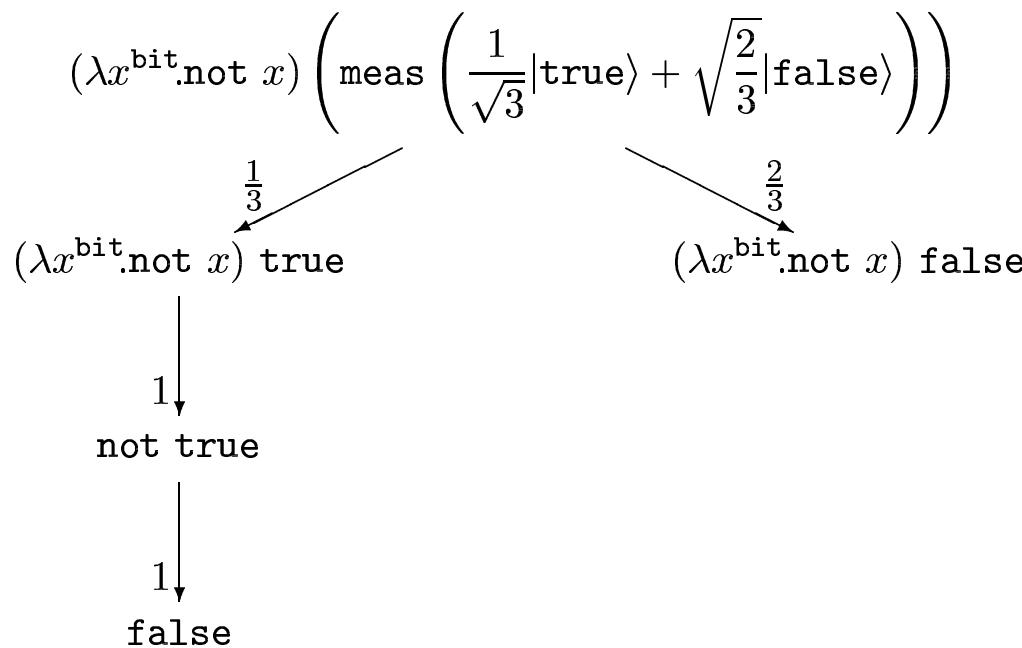
not true

```
graph TD; A["(\lambda x^{\text{bit}}. \text{not } x) \left( \text{meas} \left( \frac{1}{\sqrt{3}} |\text{true}\rangle + \sqrt{\frac{2}{3}} |\text{false}\rangle \right) \right)"] -- "1/3" --> B["(\lambda x^{\text{bit}}. \text{not } x) \text{ true}"]; B -- "1" --> C["not true"];
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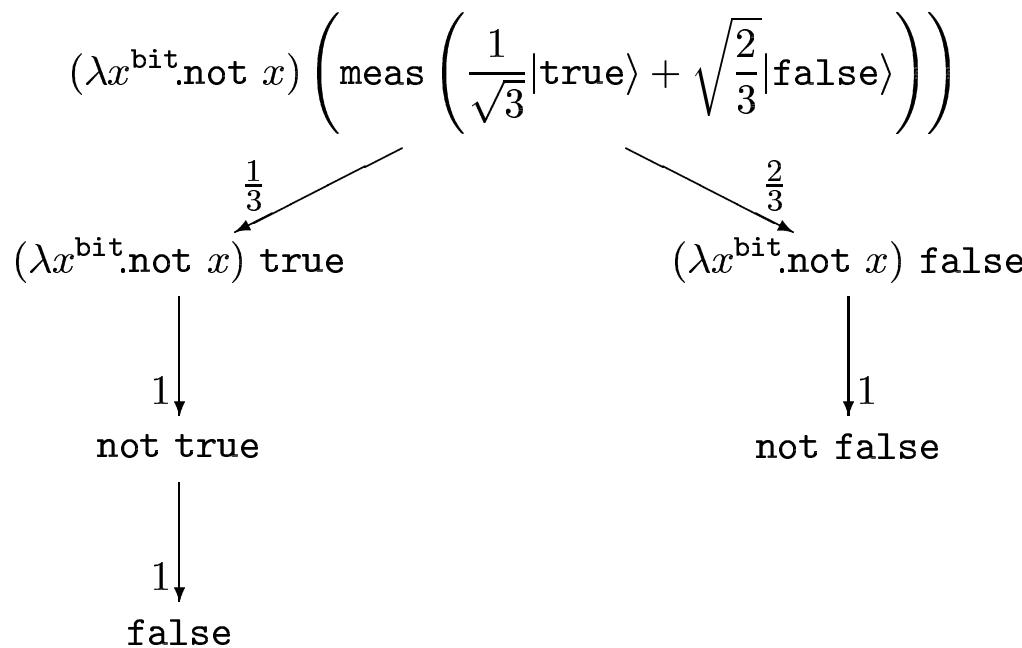
Example



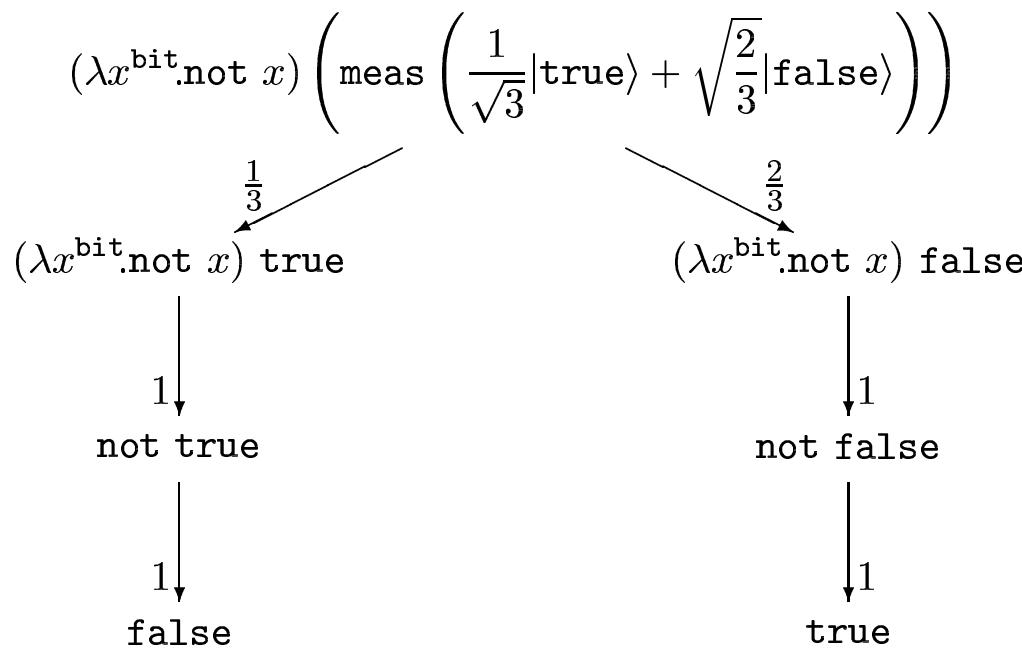
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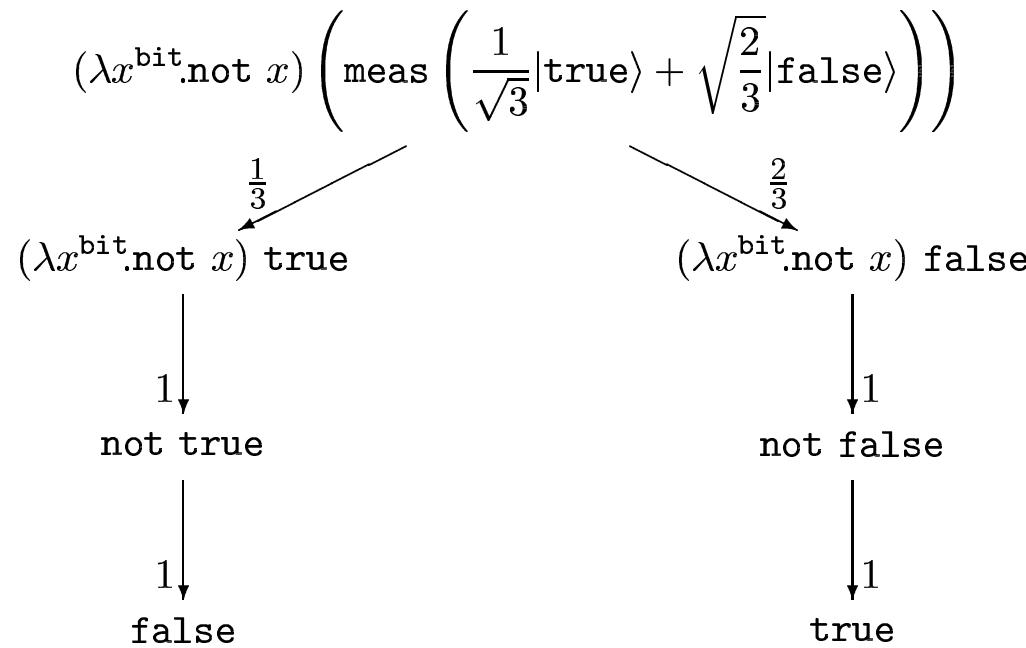
Example



Example



Example



$$(\lambda x^{\text{bit}}. \text{not } x) \left(\text{meas} \left(\frac{1}{\sqrt{3}} |\text{true}\rangle + \sqrt{\frac{2}{3}} |\text{false}\rangle \right) \right) \Downarrow \left(\frac{2}{3}, \frac{1}{3} \right)$$

Contribution

- Existing work by Selinger and Valiron
A fully abstract model for pure linear fragment
of SV-quantum lambda calculus.
- We construct an adequate denotational semantics for full fragment
 - !-operator
 - recursion(with several modifications to the syntax of SV-quantum lambda calculus)

Without
– !-operator
– recursion

Adequacy $M \Downarrow (p, q) \iff \text{prob}(\llbracket M \rrbracket(\text{test})) = (p, q)$

Categorical Techniques in our Construction

TSMC \longrightarrow Gol \longrightarrow realizability \longrightarrow a strong monad



Coalgebraic trace [Jacobs],
categorical generalisation of automata

A weak linear category for duplication (Selinger and Valiron)

- A symmetric monoidal category \mathcal{C} with an idempotent strong monoidal linear exponential comonad
- A strong monad T on \mathcal{C}
- A Kleisli exponential $\mathcal{C}(X \otimes Y, TZ) \cong \mathcal{C}(X, Y \multimap TZ)$

Categorical Techniques in our Construction

TSMC \longrightarrow Gol \longrightarrow realizability \longrightarrow a strong monad



Coalgebraic trace [Jacobs],
categorical generalisation of automata

A (degenerated) linear category + A strong monad



Quantum stuff.

Categorical Techniques in our Construction

TSMC \longrightarrow **Gol** \longrightarrow realizability \longrightarrow a strong monad



Coalgebraic trace [Jacobs],
categorical generalisation of automata

A (degenerated) linear category + A strong monad

+

Quantum stuff.

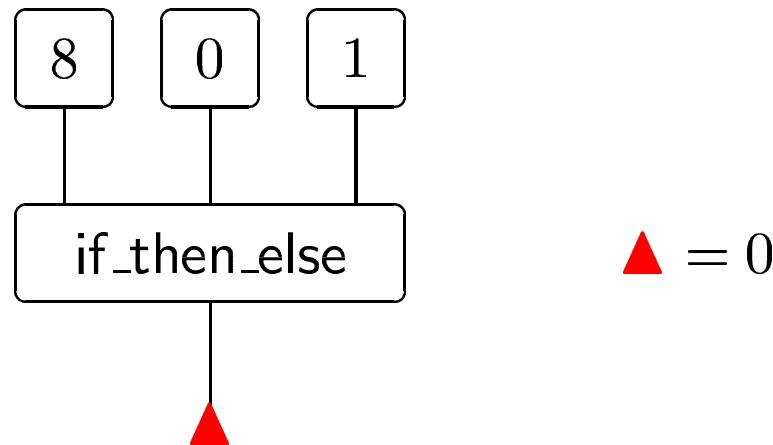
A Review of Categorical Geometry of Interaction

- A semantics of linear logic and cut elimination process [Girard]
- The categorical essence of G₀I interpretation consists of:
 - a traced symmetric monoidal category
 - **Int**-construction [Abramsky, Haghverdi, Scott]

G₀I interprets higher order computation in terms of low level computation:

- $X \multimap Y \cong X^* \otimes Y$
- $!X \approx X \otimes X \otimes X \otimes \dots$

Gol Interpretation of Programs

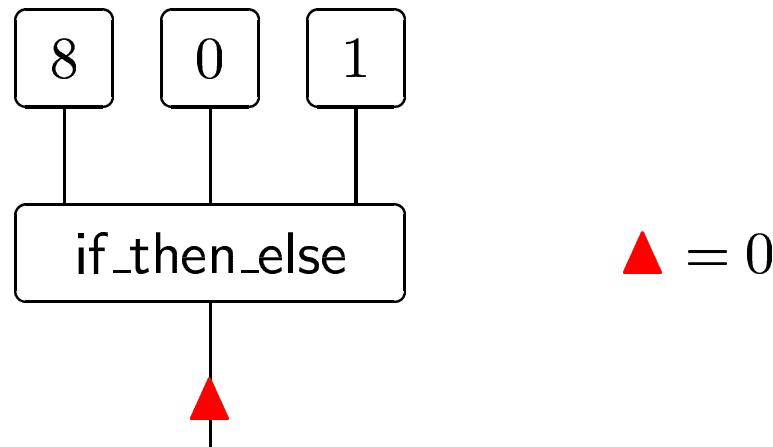


Tokens

- $t = * \mid \text{left}(t) \mid \text{right}(t) \mid \dots$
- $n \in \mathbb{N}$

(N.B. This presentation of Gol is different from the one in our paper.)

Gol Interpretation of Programs

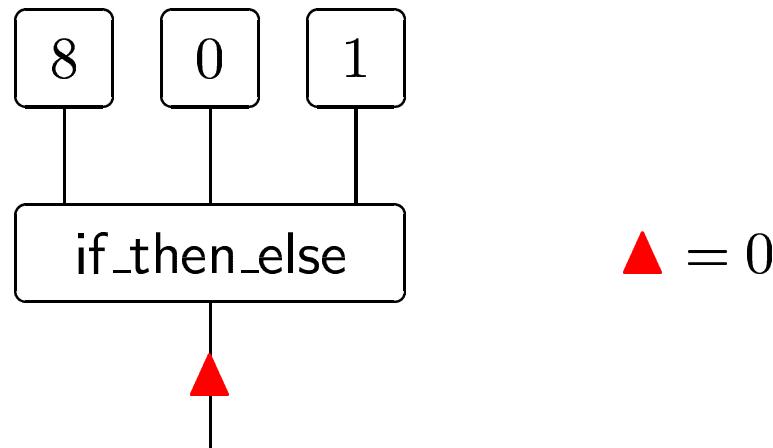


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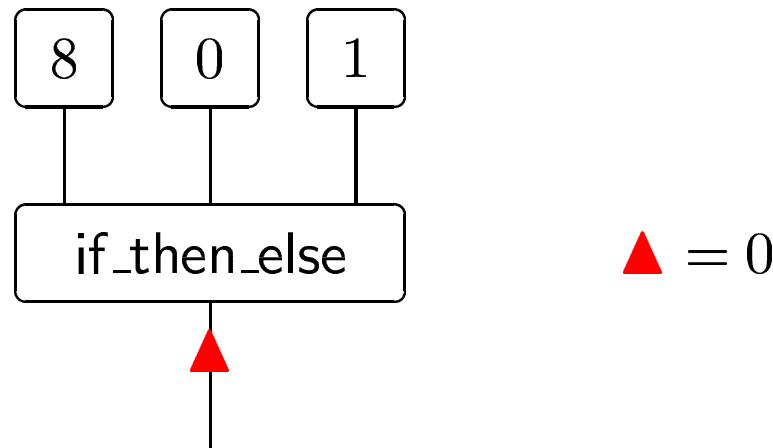


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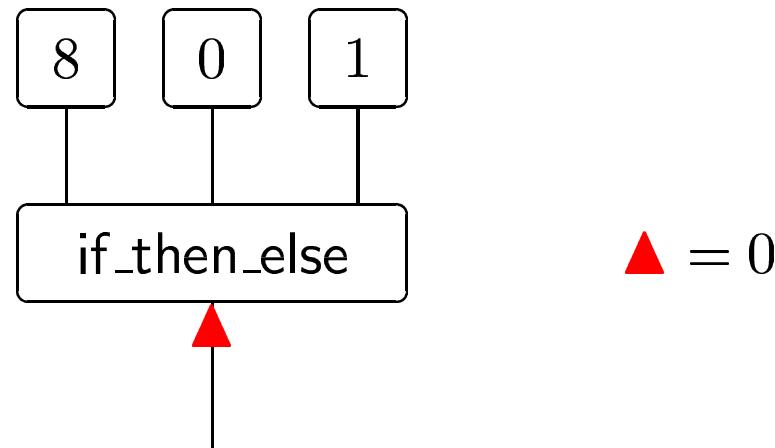
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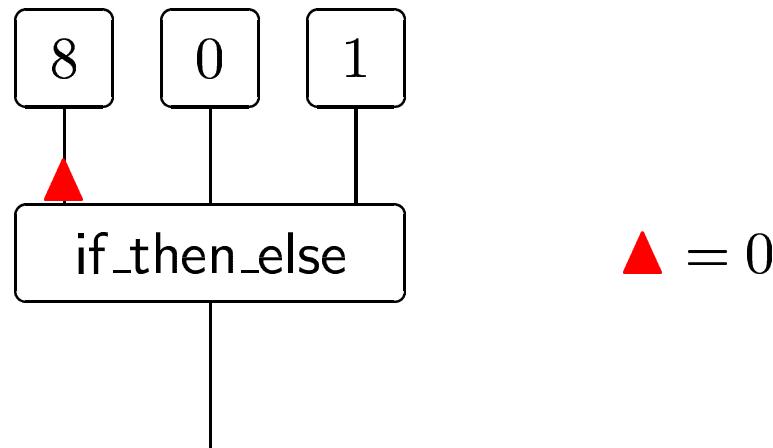


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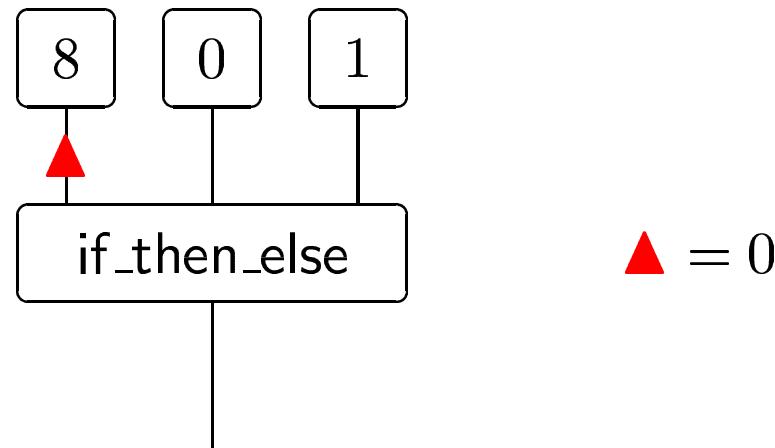


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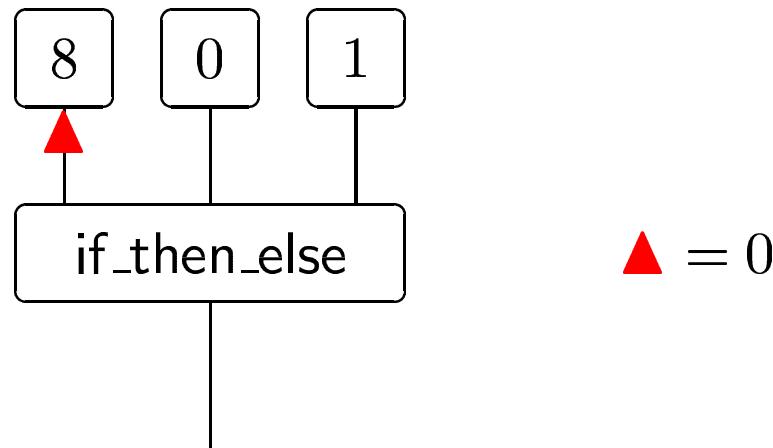
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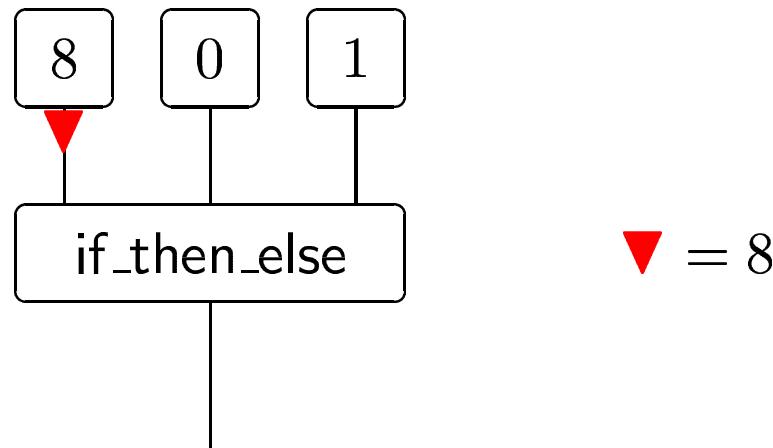
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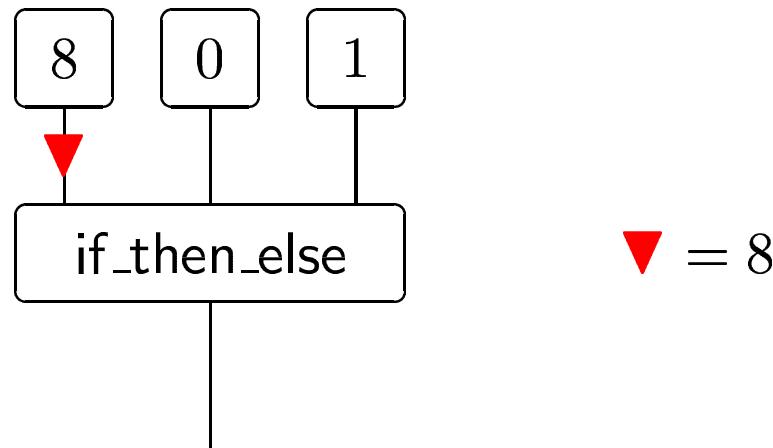


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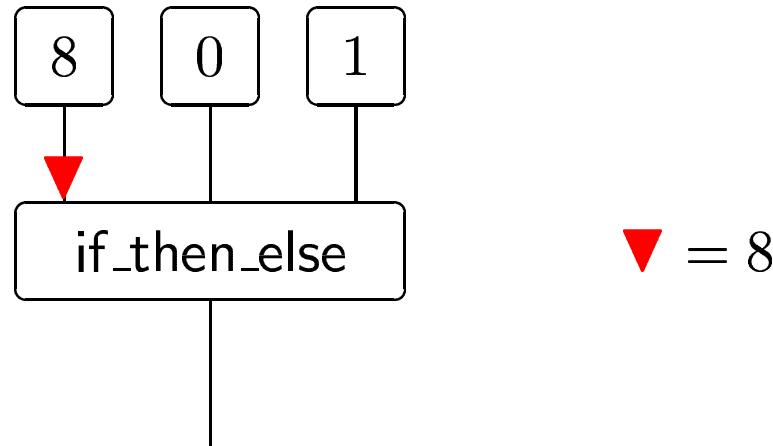


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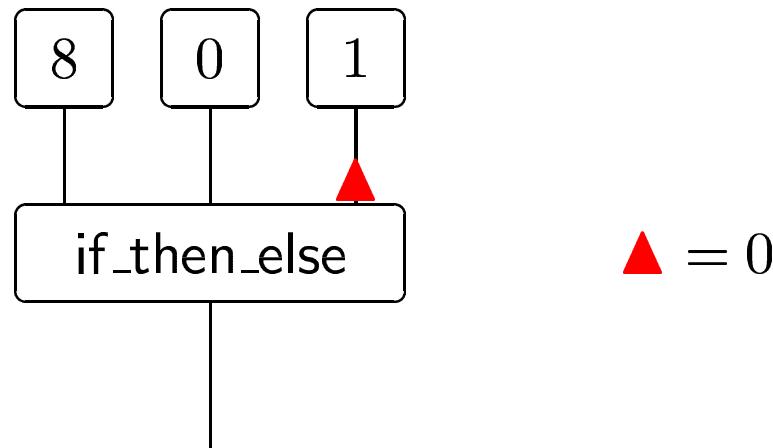


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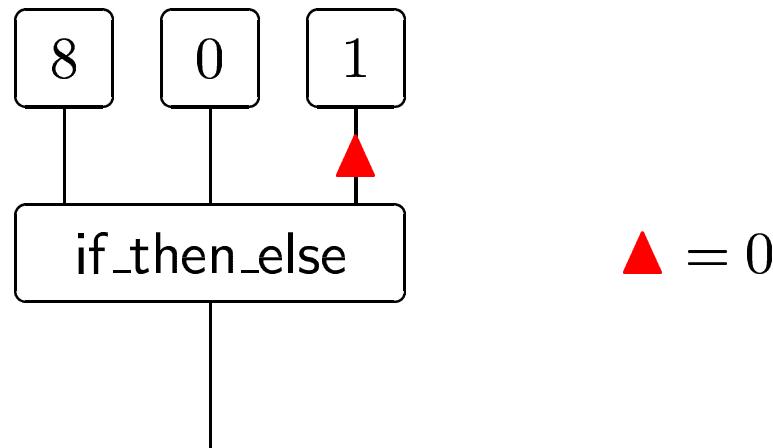


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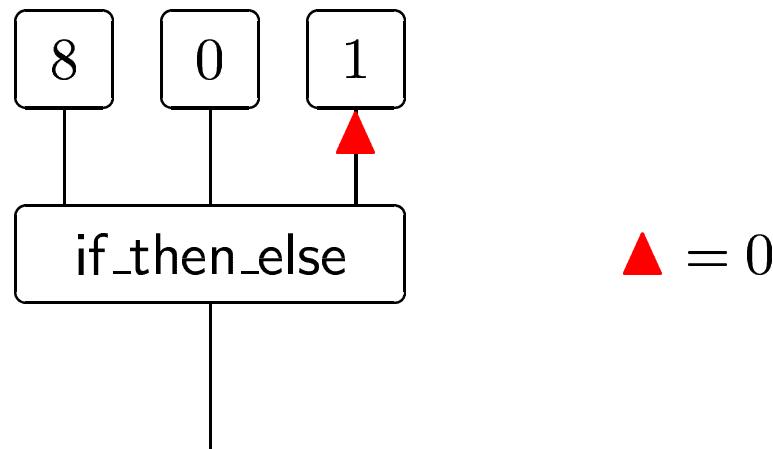


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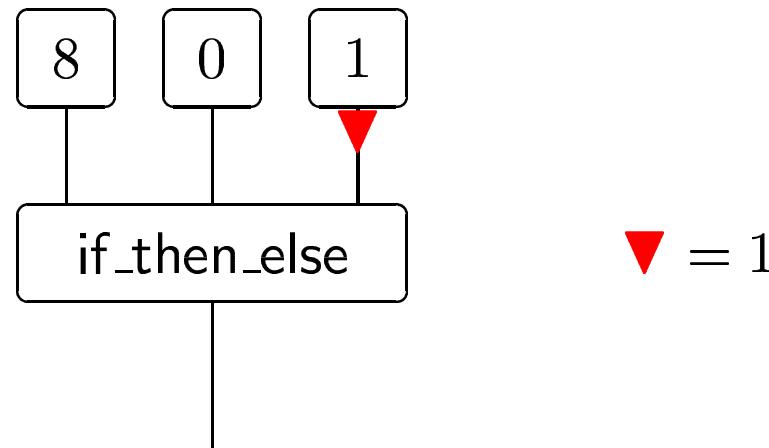


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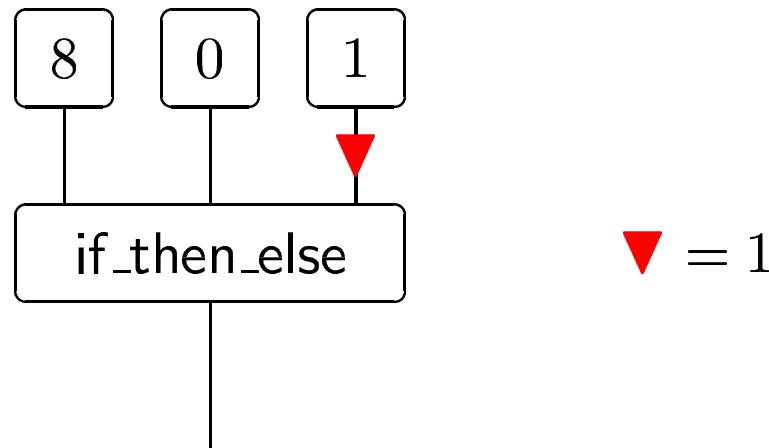


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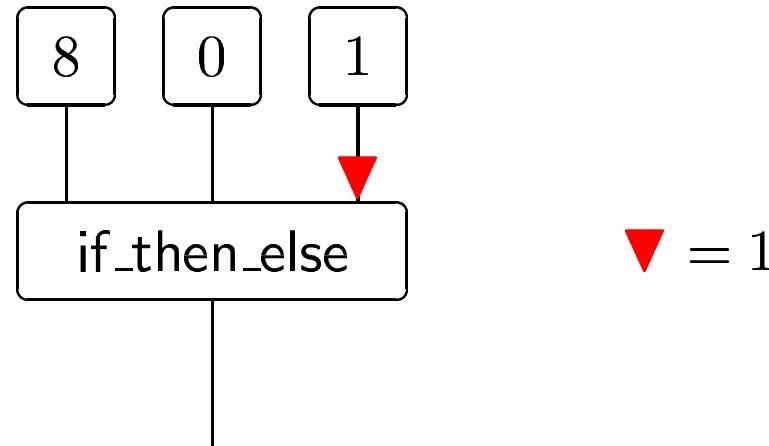


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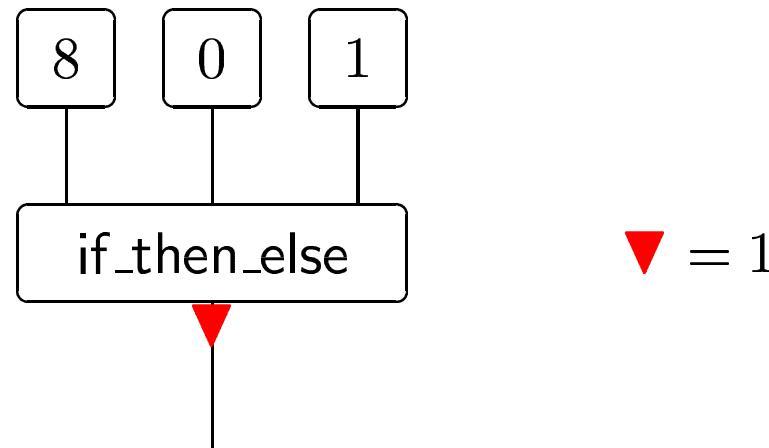


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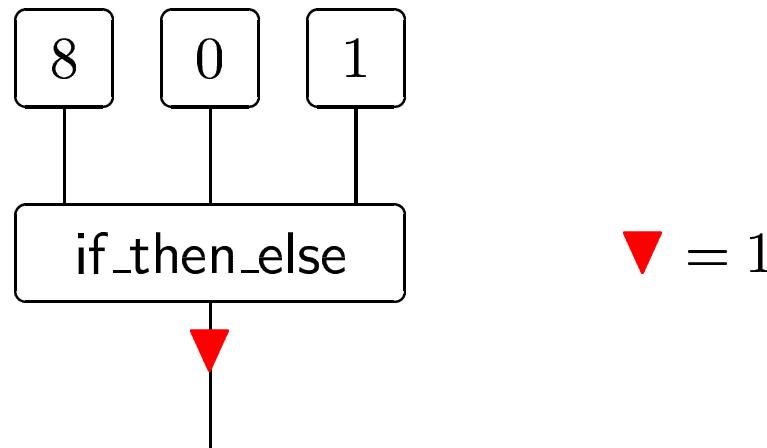


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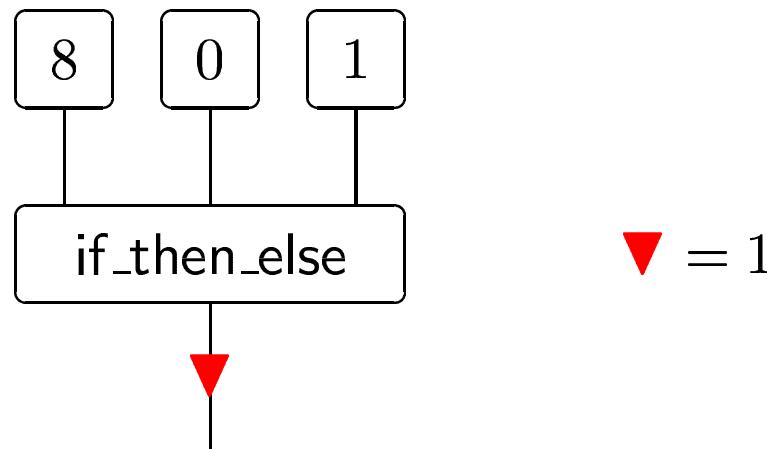


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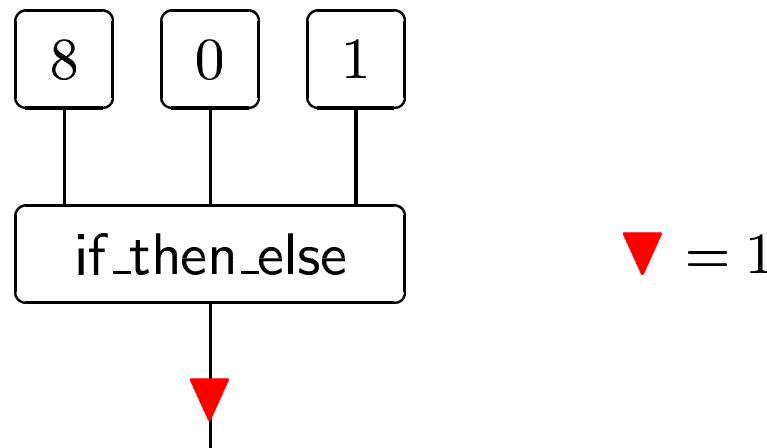


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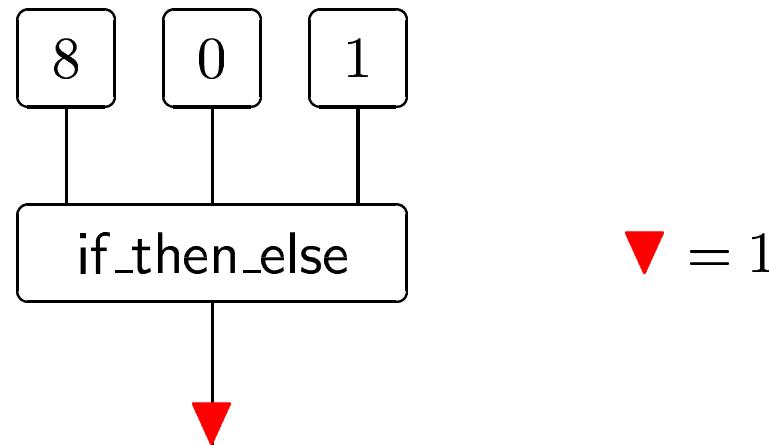


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Gol Interpretation of Programs



$$\nabla = 1$$

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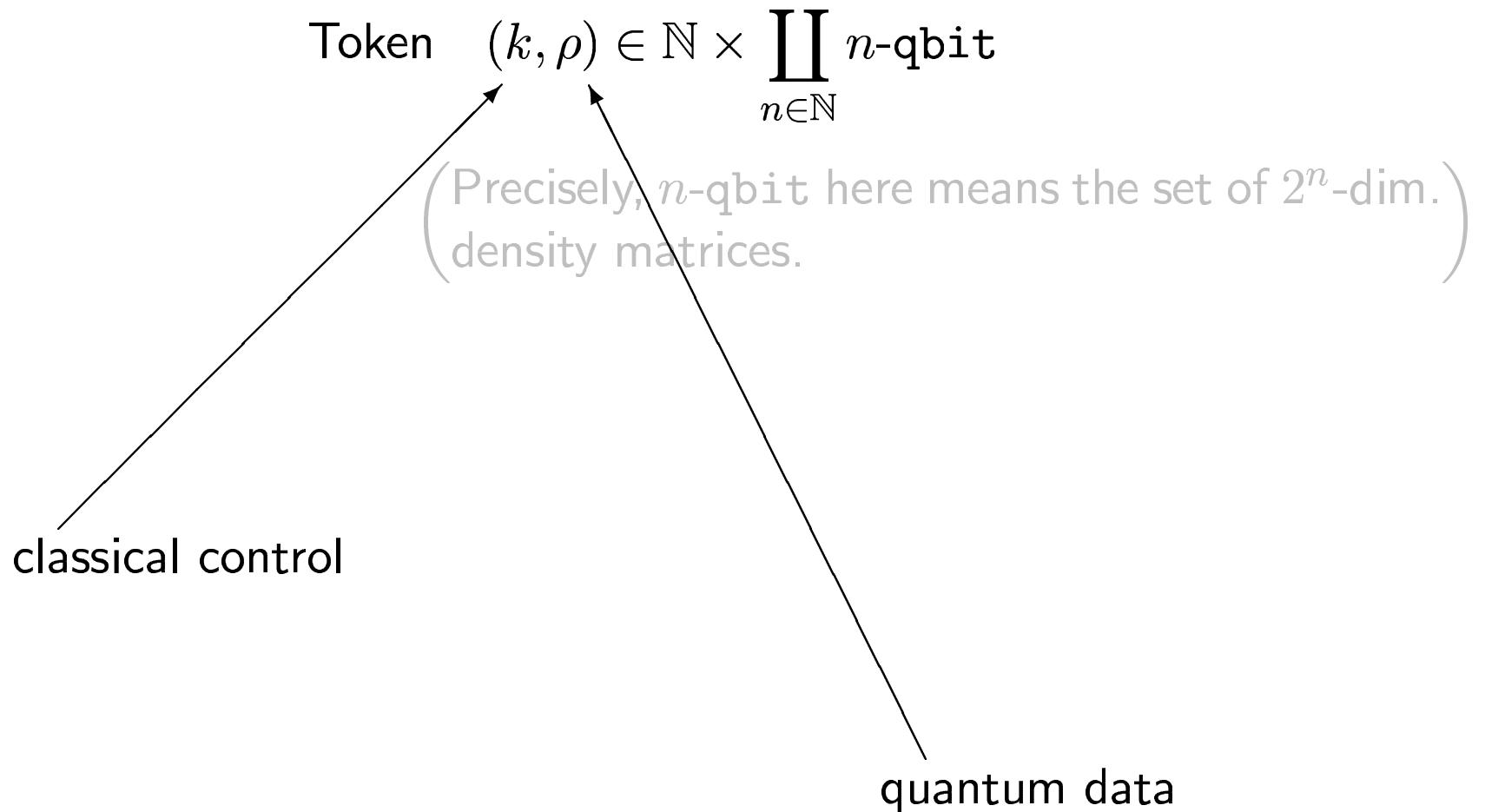
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Quantum Data and Classical Control again

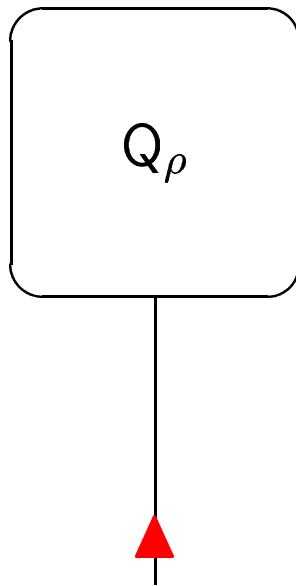
Token $(k, \rho) \in \mathbb{N} \times \coprod_{n \in \mathbb{N}} n\text{-qbit}$

$\left(\begin{array}{l} \text{Precisely, } n\text{-qbit here means the set of } 2^n\text{-dim.} \\ \text{density matrices.} \end{array} \right)$

Quantum Data and Classical Control again

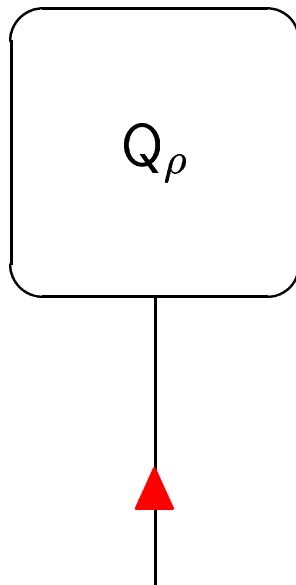


Quantum State Preparation



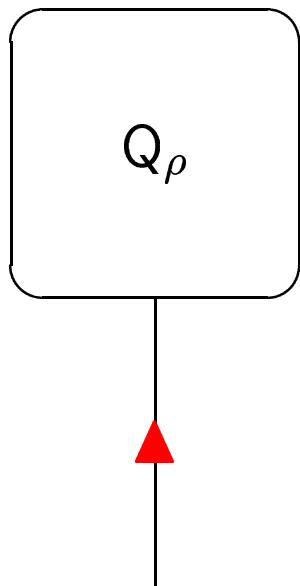
$$\blacktriangle = (k, 1)$$

Quantum State Preparation



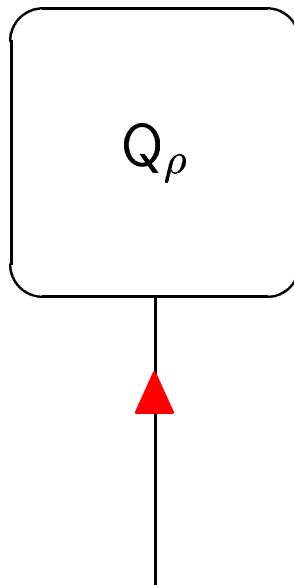
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Quantum State Preparation



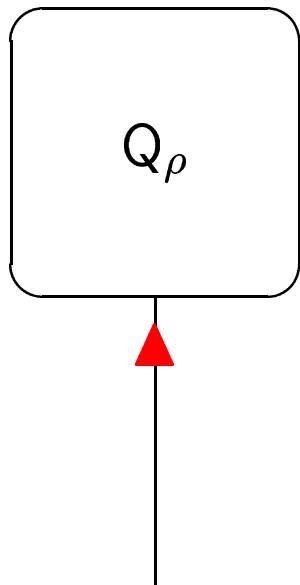
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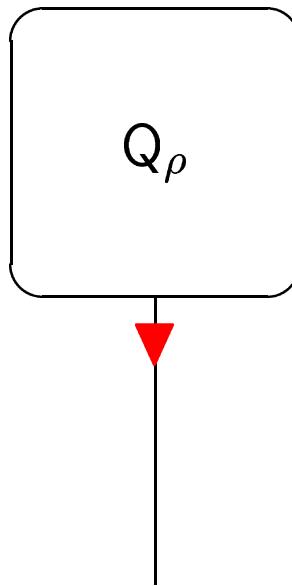
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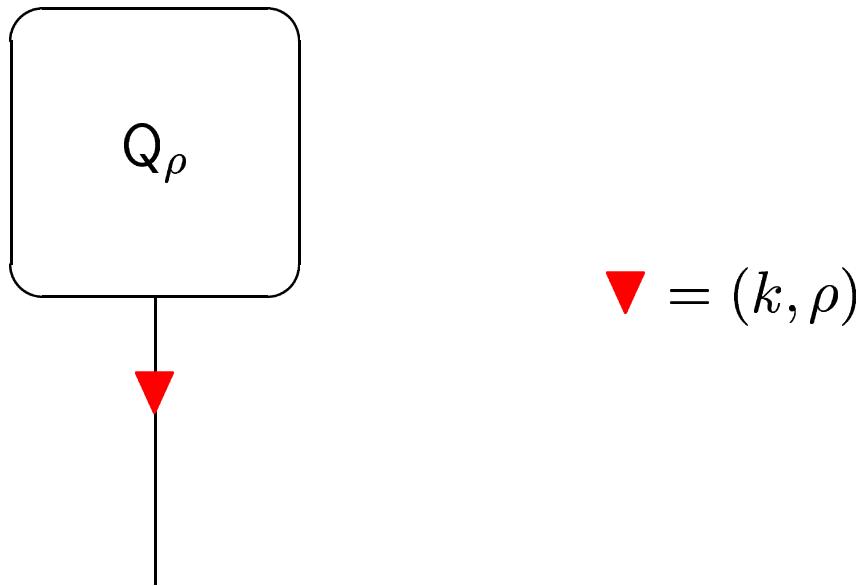
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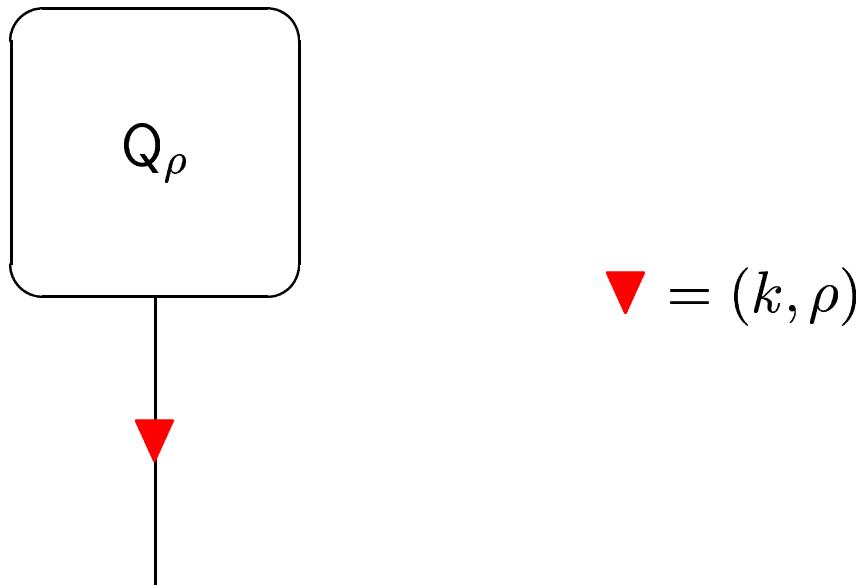


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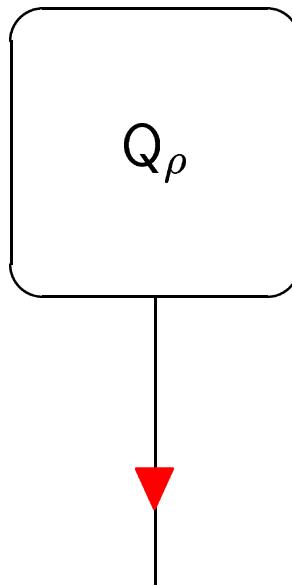
Quantum State Preparation



Quantum State Preparation

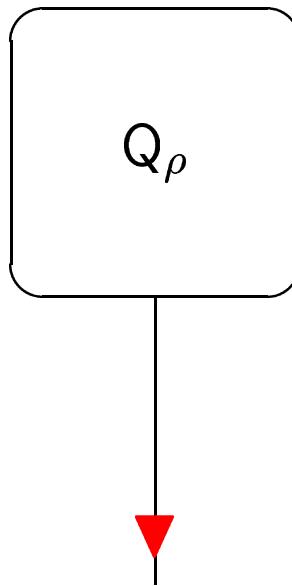


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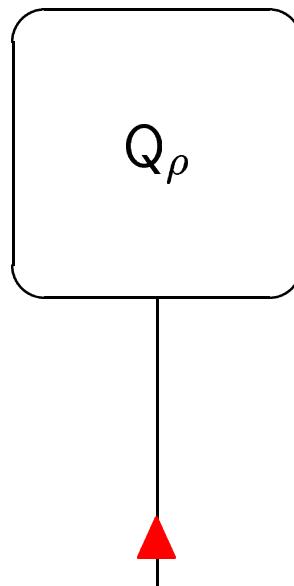
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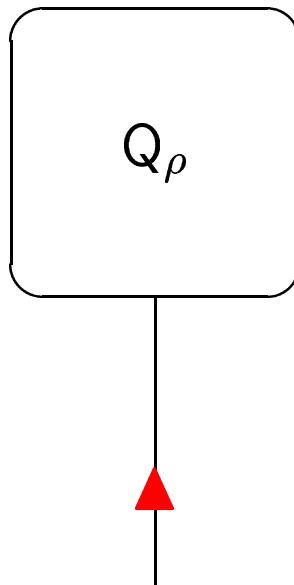
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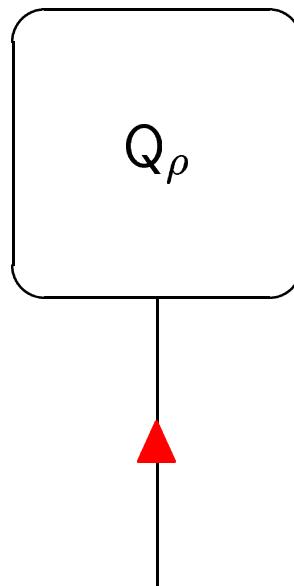
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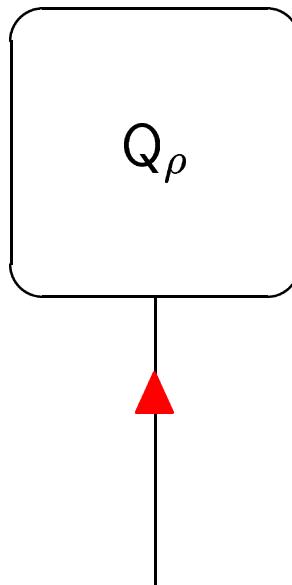
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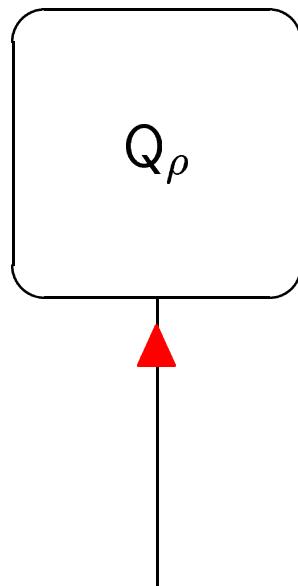
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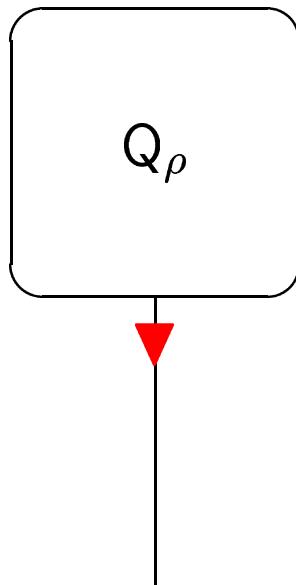
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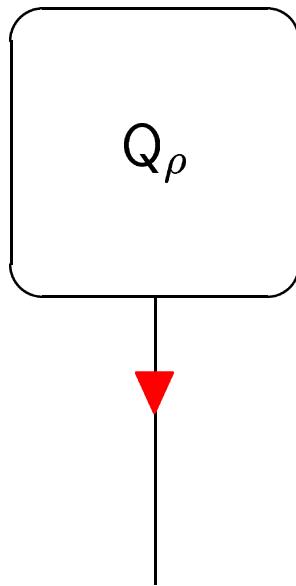
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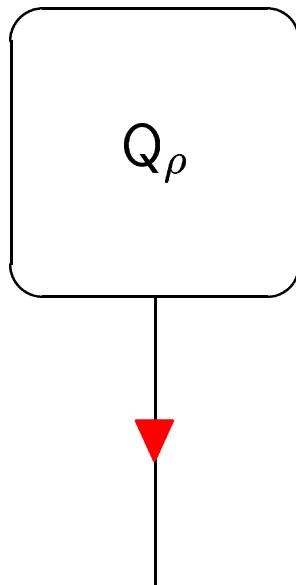
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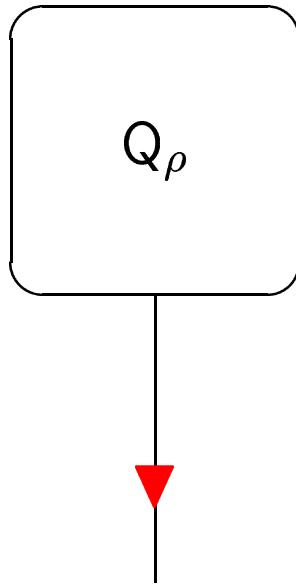
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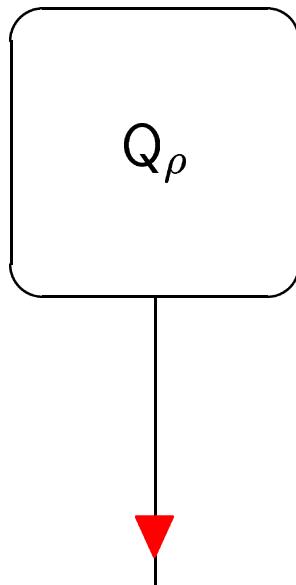
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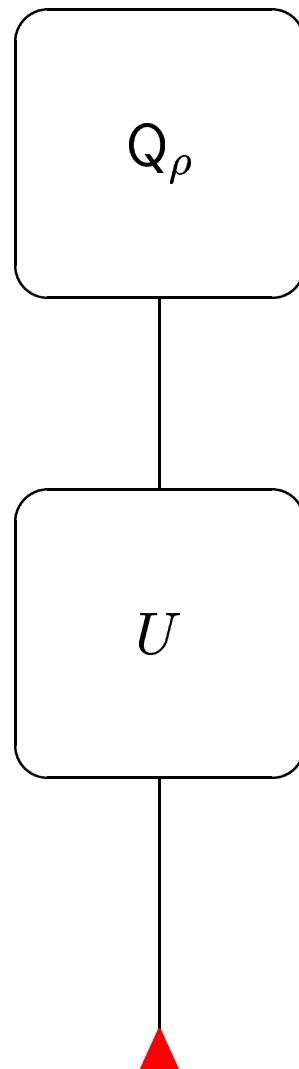
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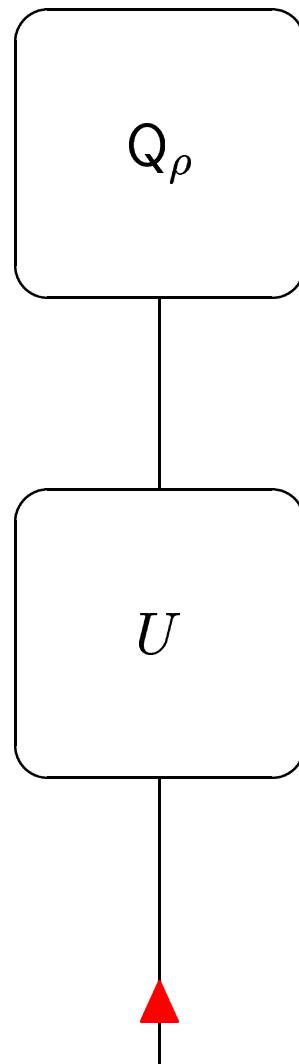
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Unitary Transformation



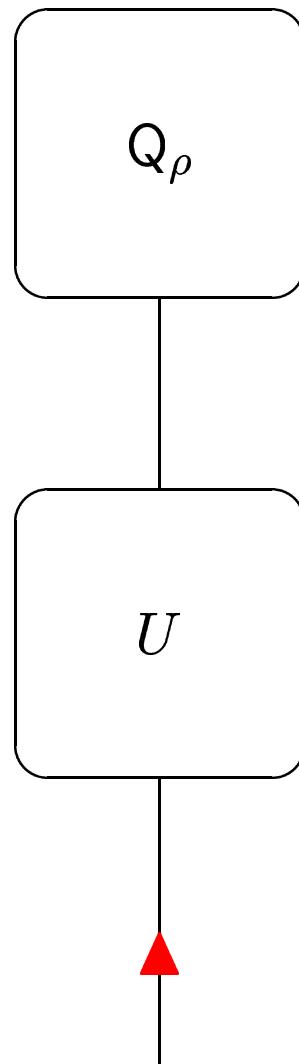
$$\blacktriangle = (k, \quad \sigma)$$

Unitary Transformation



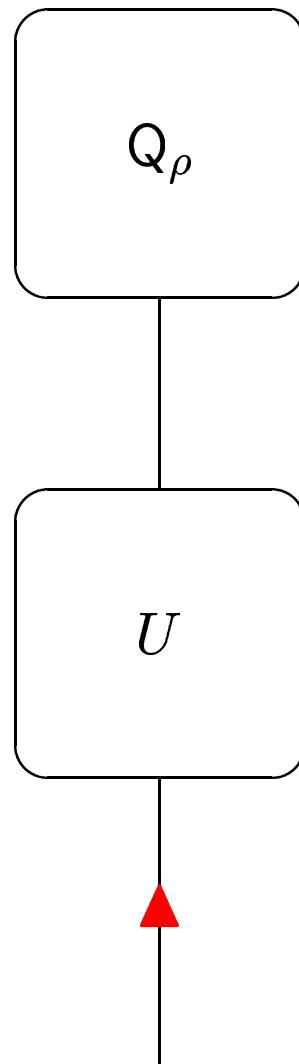
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Unitary Transformation



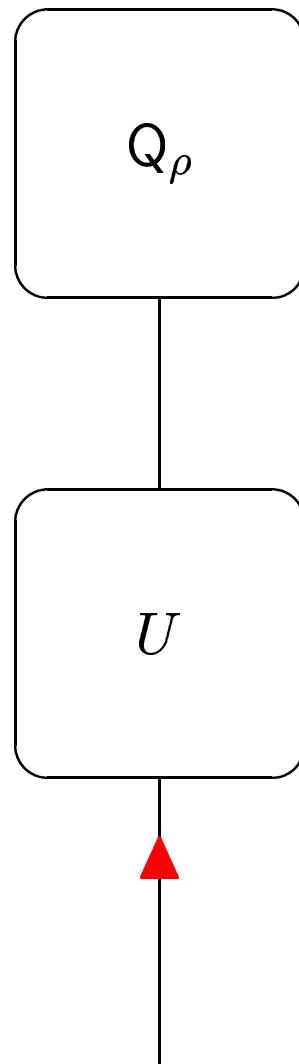
$$\blacktriangle = (k, \quad \quad \quad \sigma)$$

Unitary Transformation



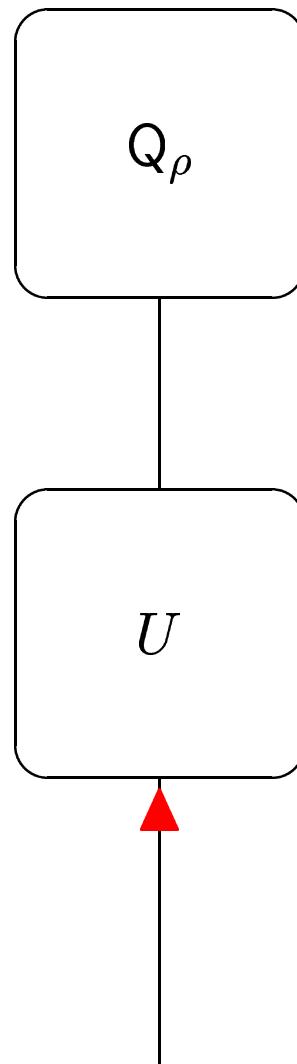
▲ = (k, σ)

Unitary Transformation



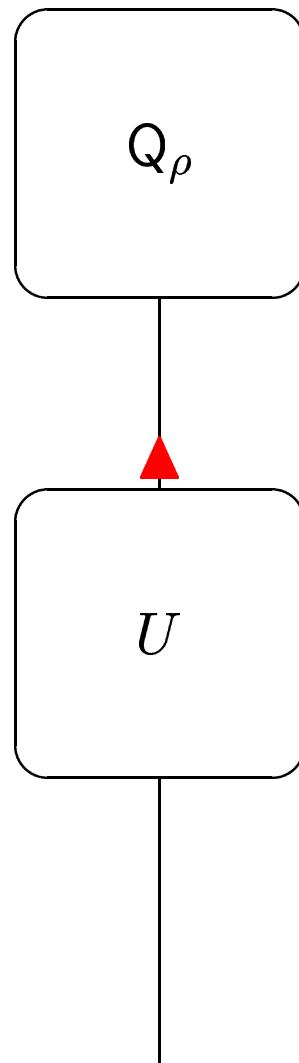
$$\blacktriangle = (k, \quad \quad \quad \sigma)$$

Unitary Transformation



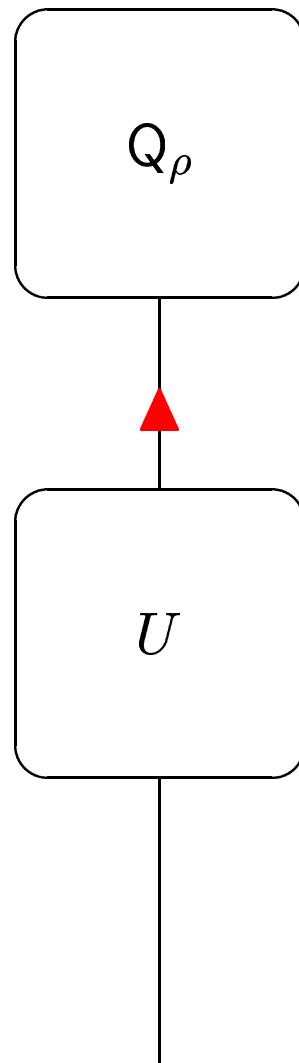
$$\blacktriangle = (k, \quad \quad \quad \sigma)$$

Unitary Transformation



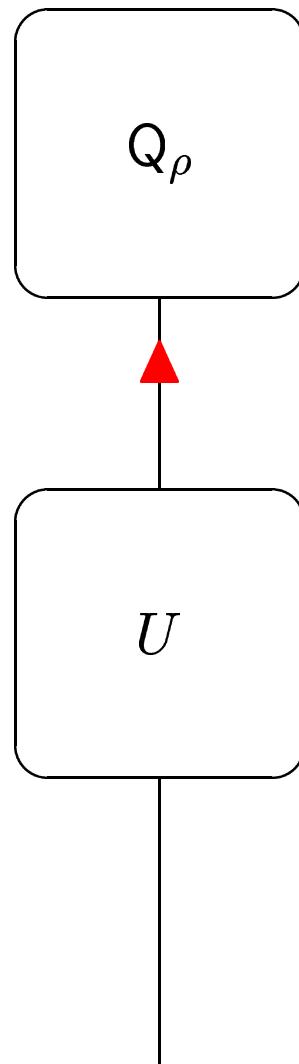
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Unitary Transformation



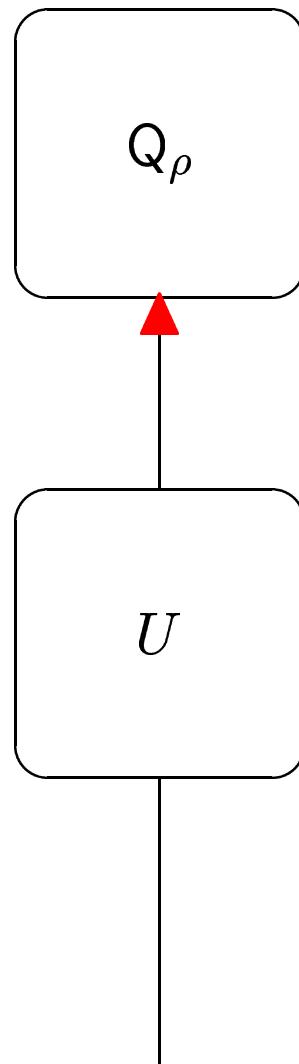
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Unitary Transformation



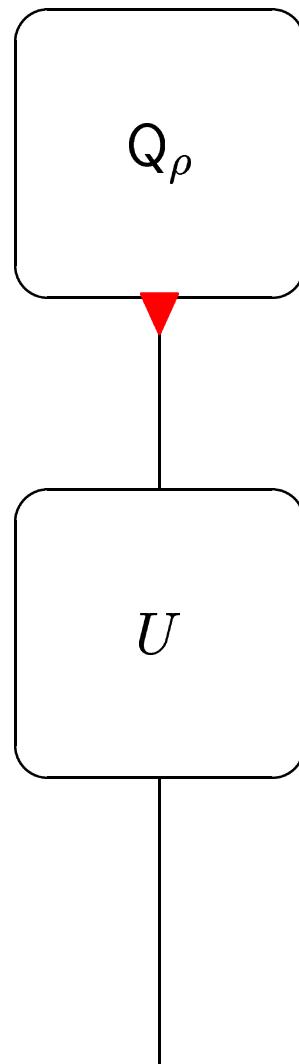
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Unitary Transformation



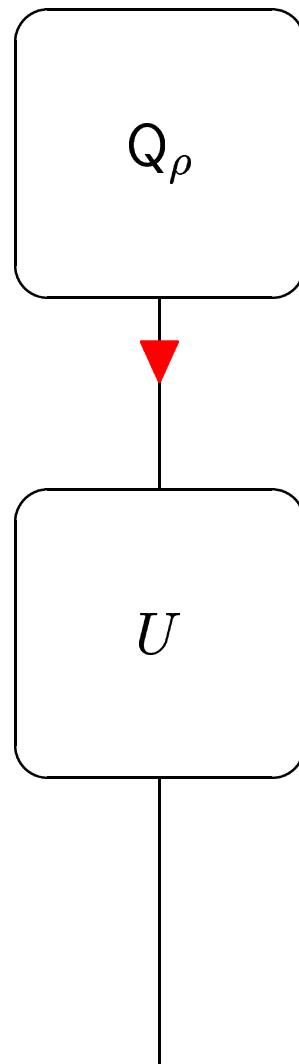
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Unitary Transformation



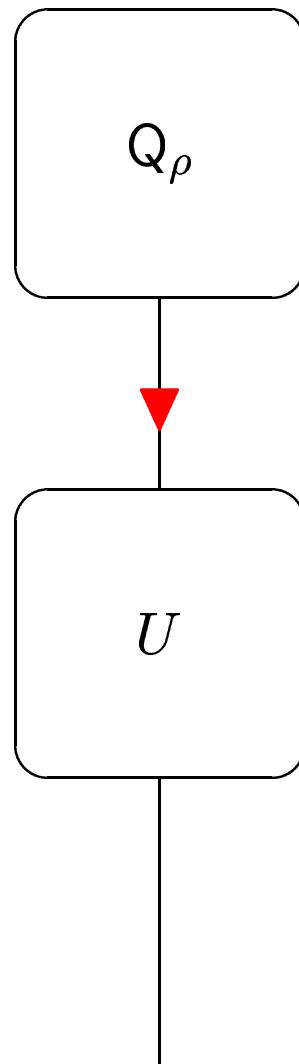
$$\blacktriangledown = (k, \rho \otimes \sigma)$$

Unitary Transformation



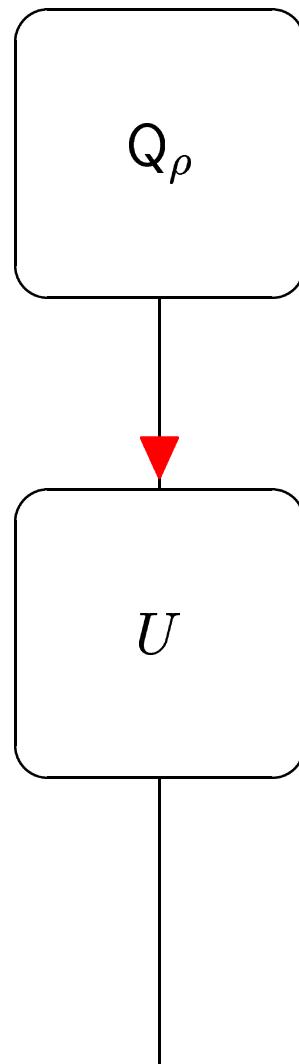
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Unitary Transformation



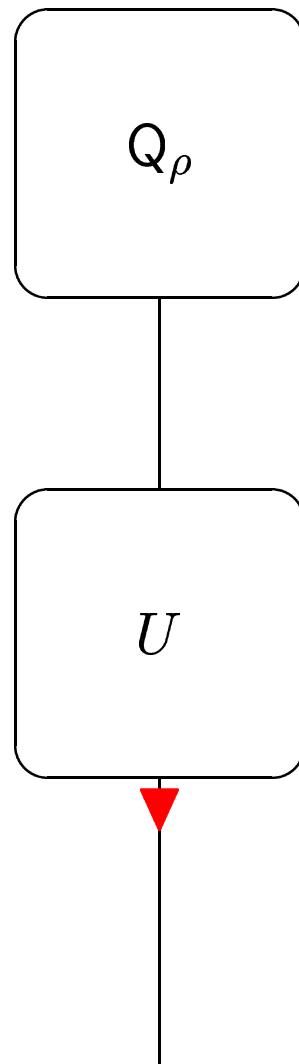
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Unitary Transformation



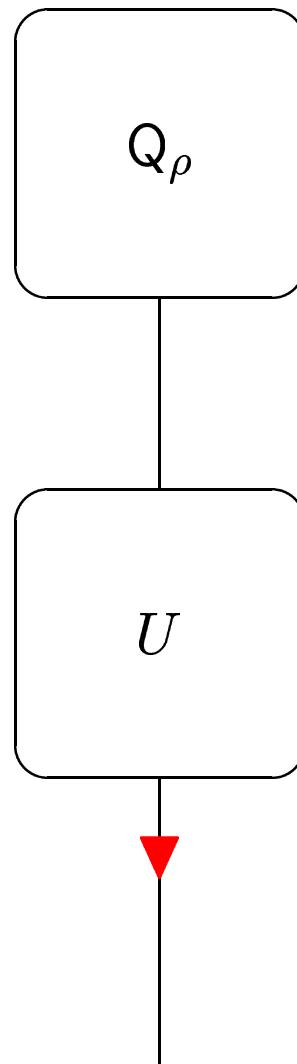
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Unitary Transformation



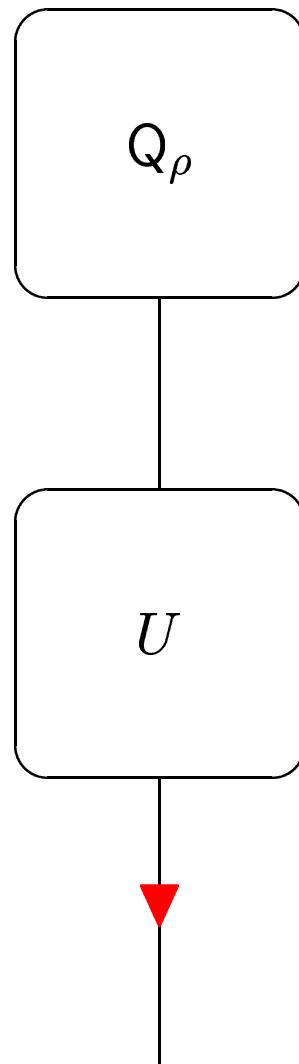
$$\blacktriangledown = (k, U\rho U^\dagger \otimes \sigma)$$

Unitary Transformation



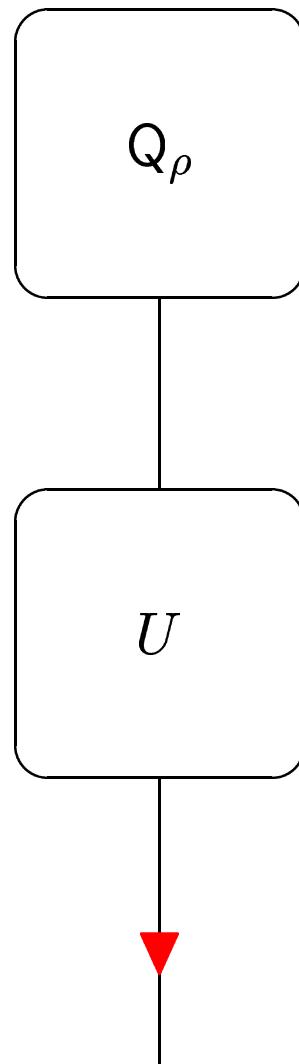
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Unitary Transformation



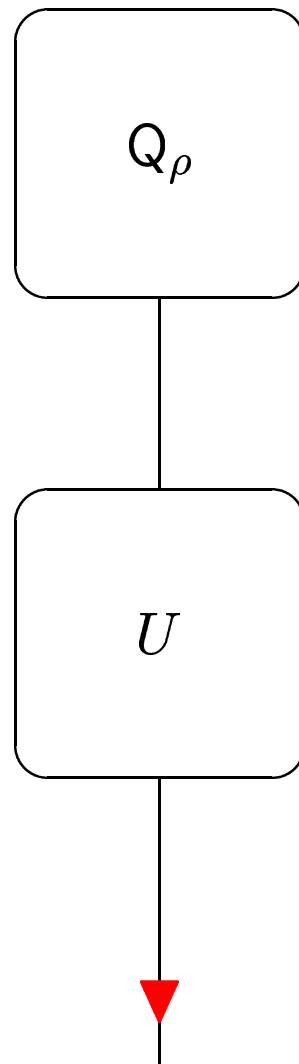
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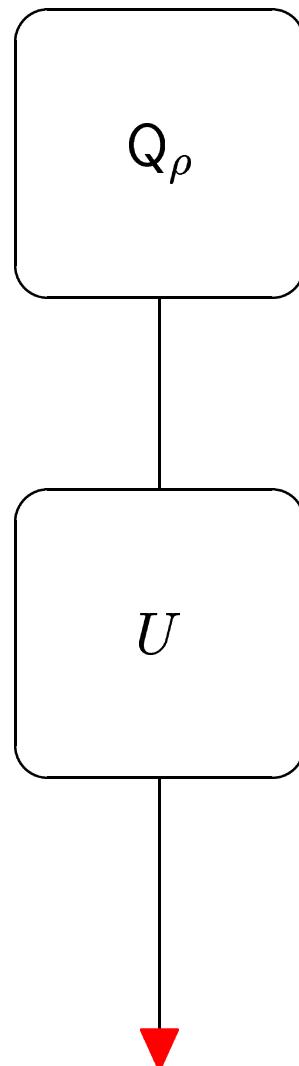
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Unitary Transformation



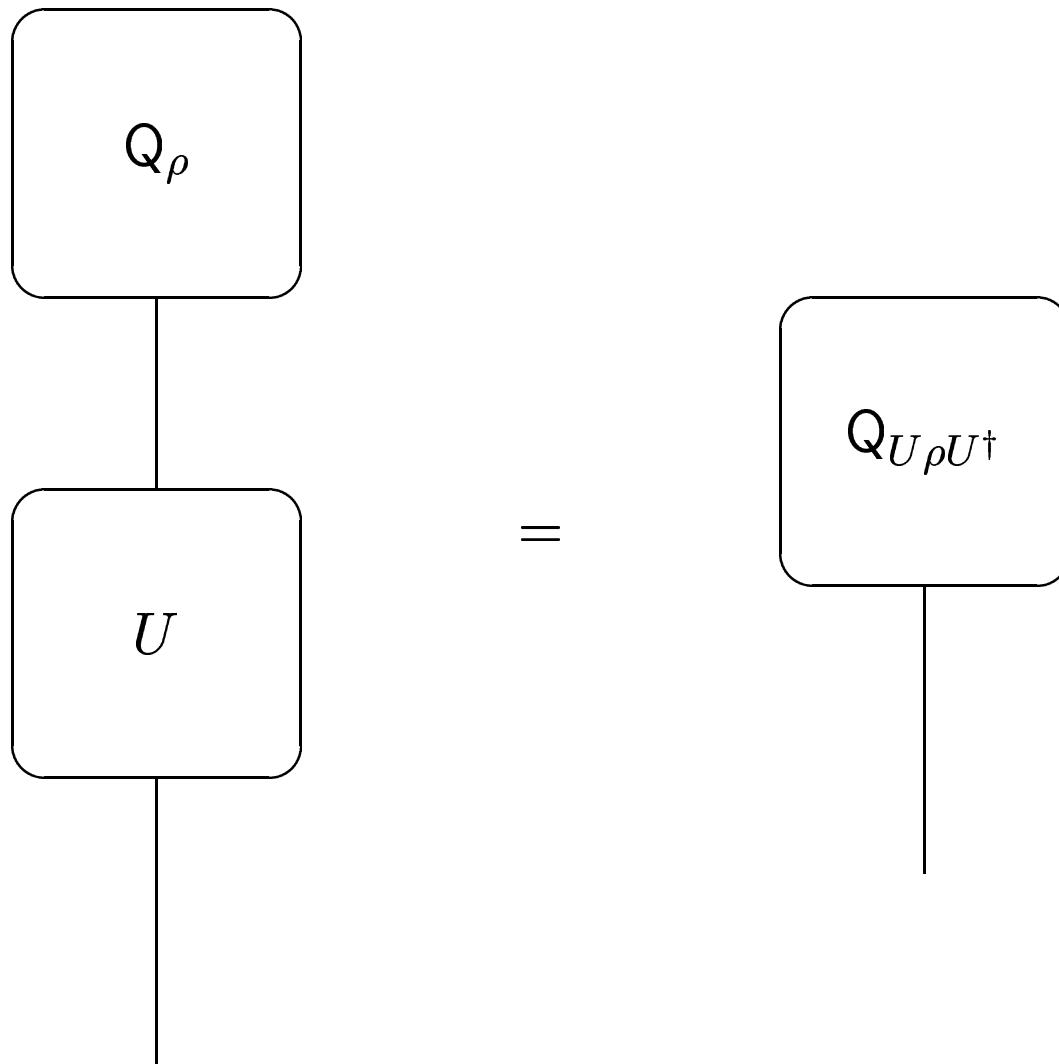
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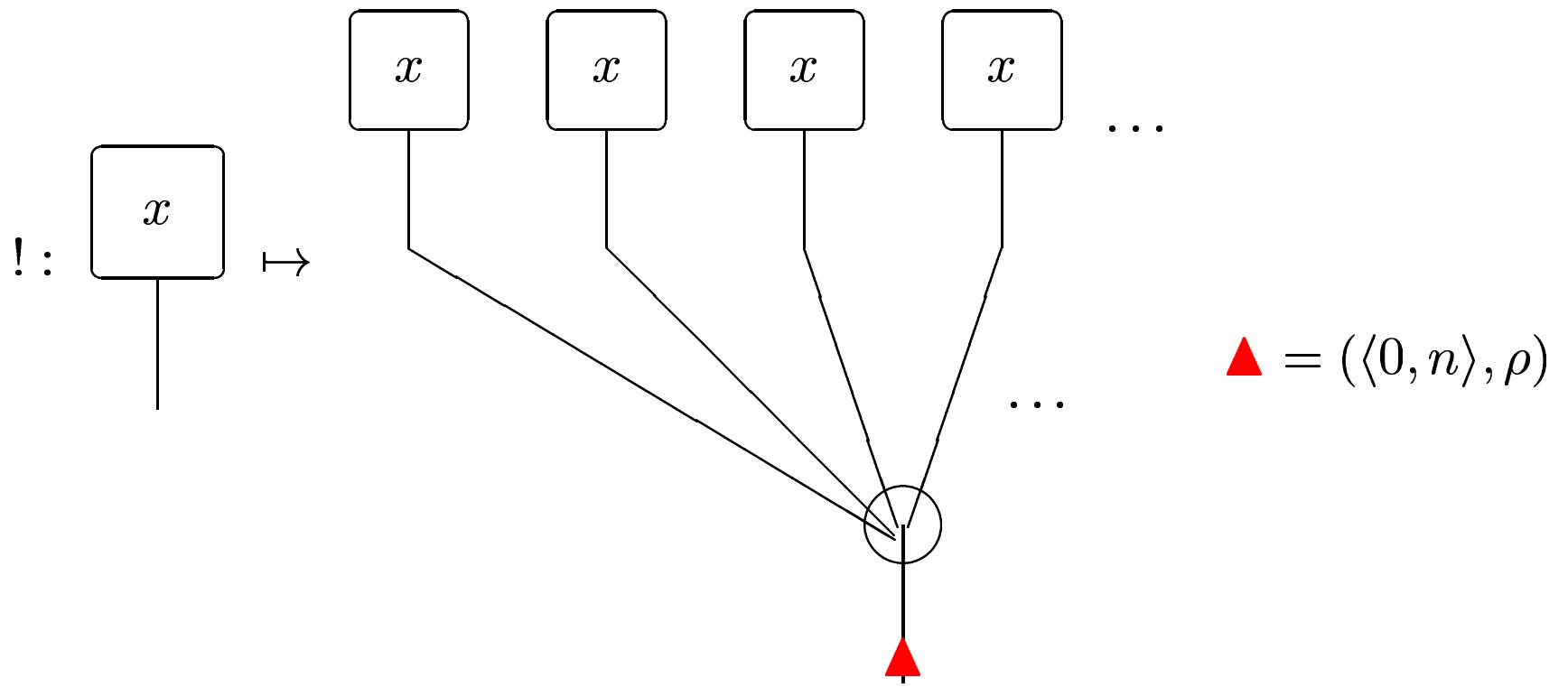


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Unitary Transformation



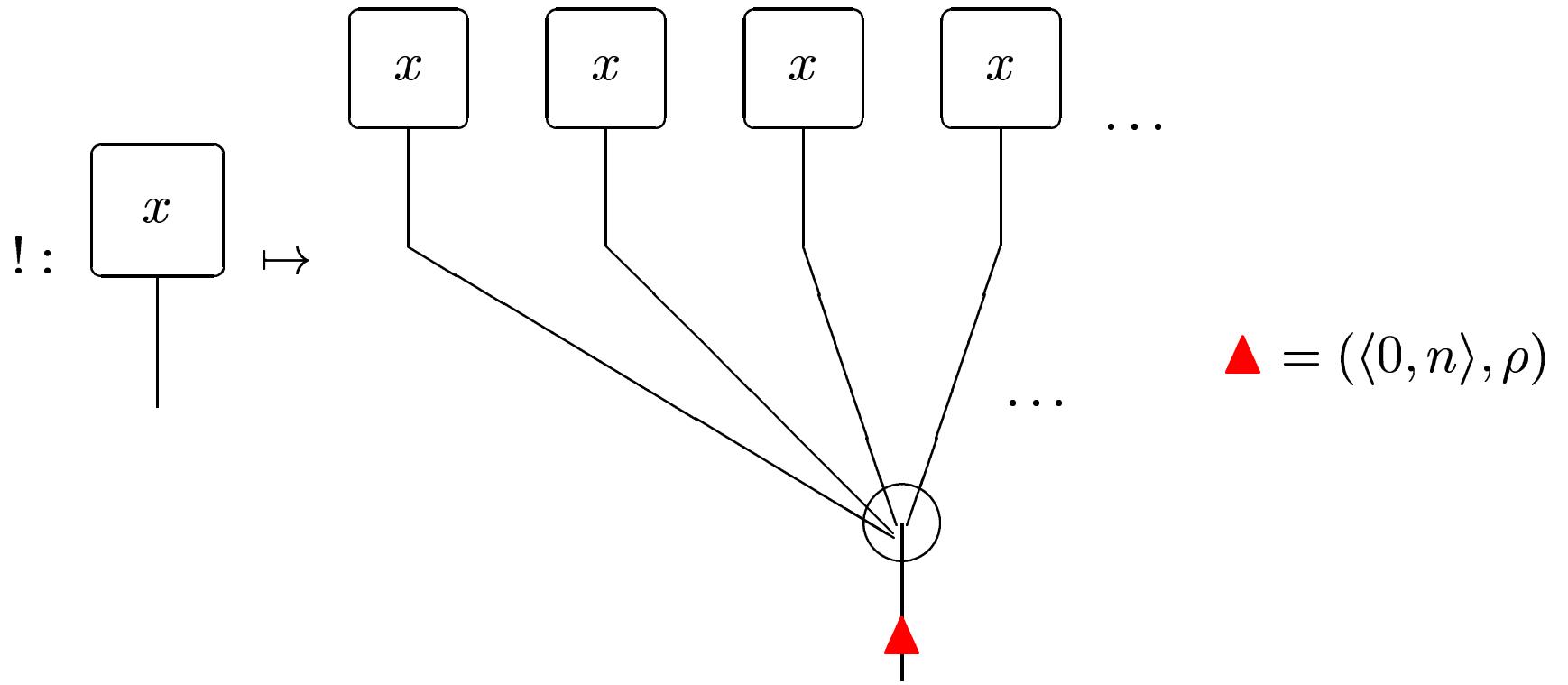
!-operator



where

$$\langle -, - \rangle : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N}$$

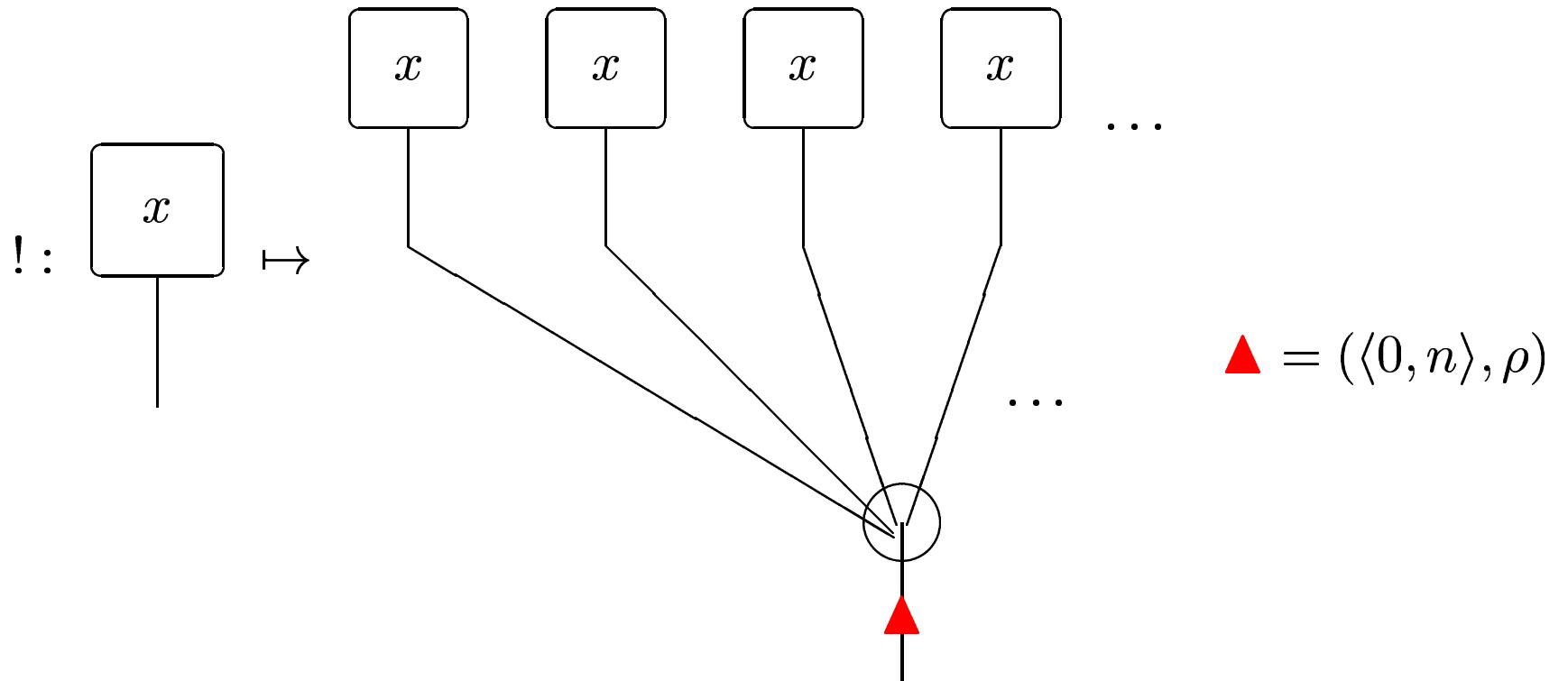
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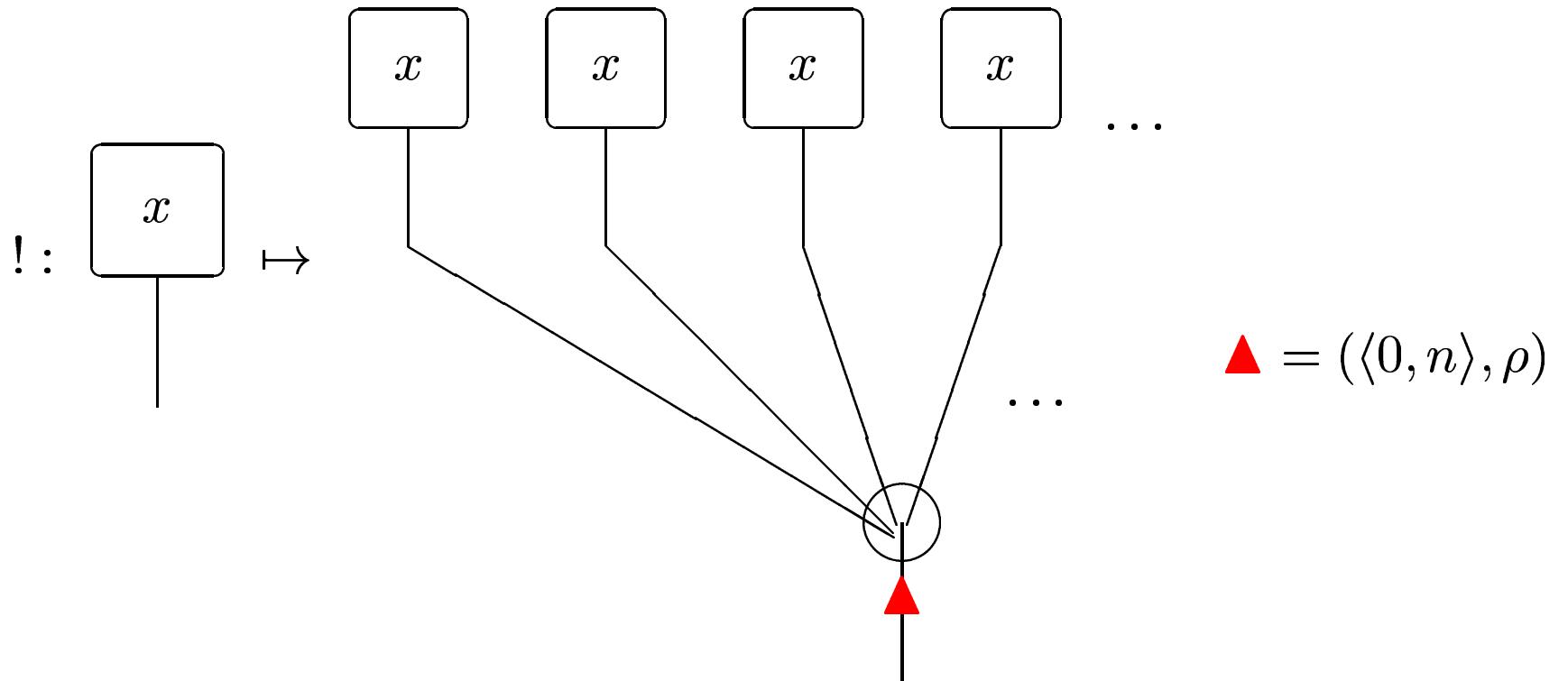
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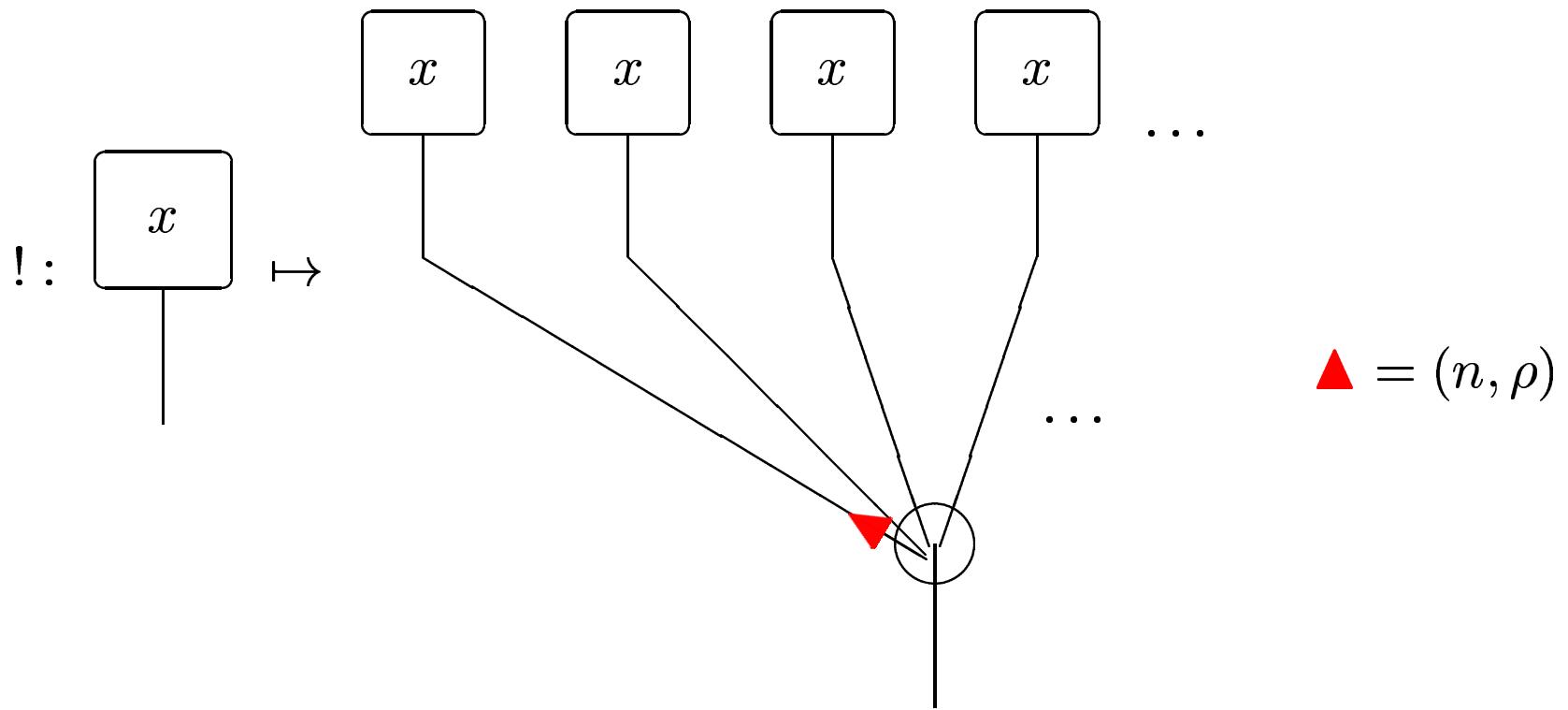
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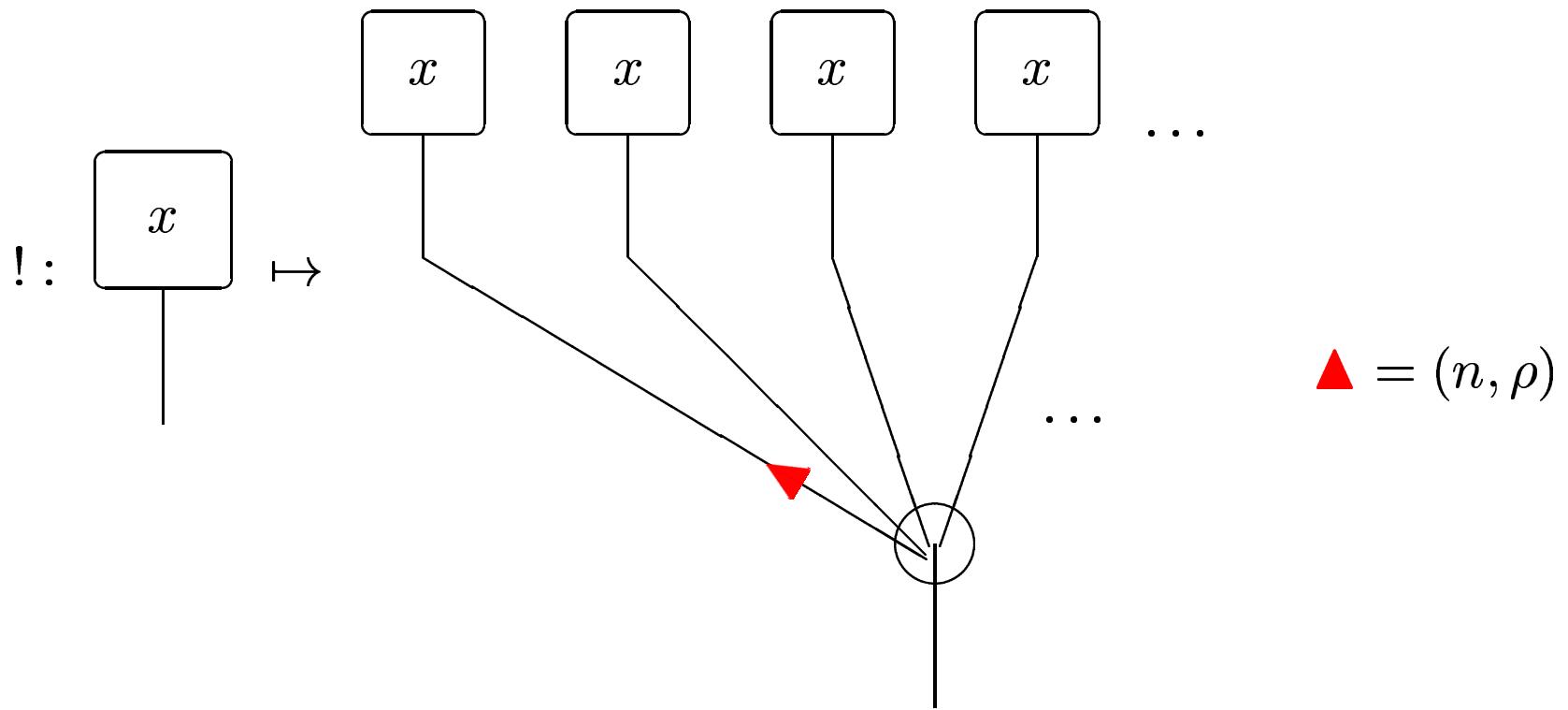
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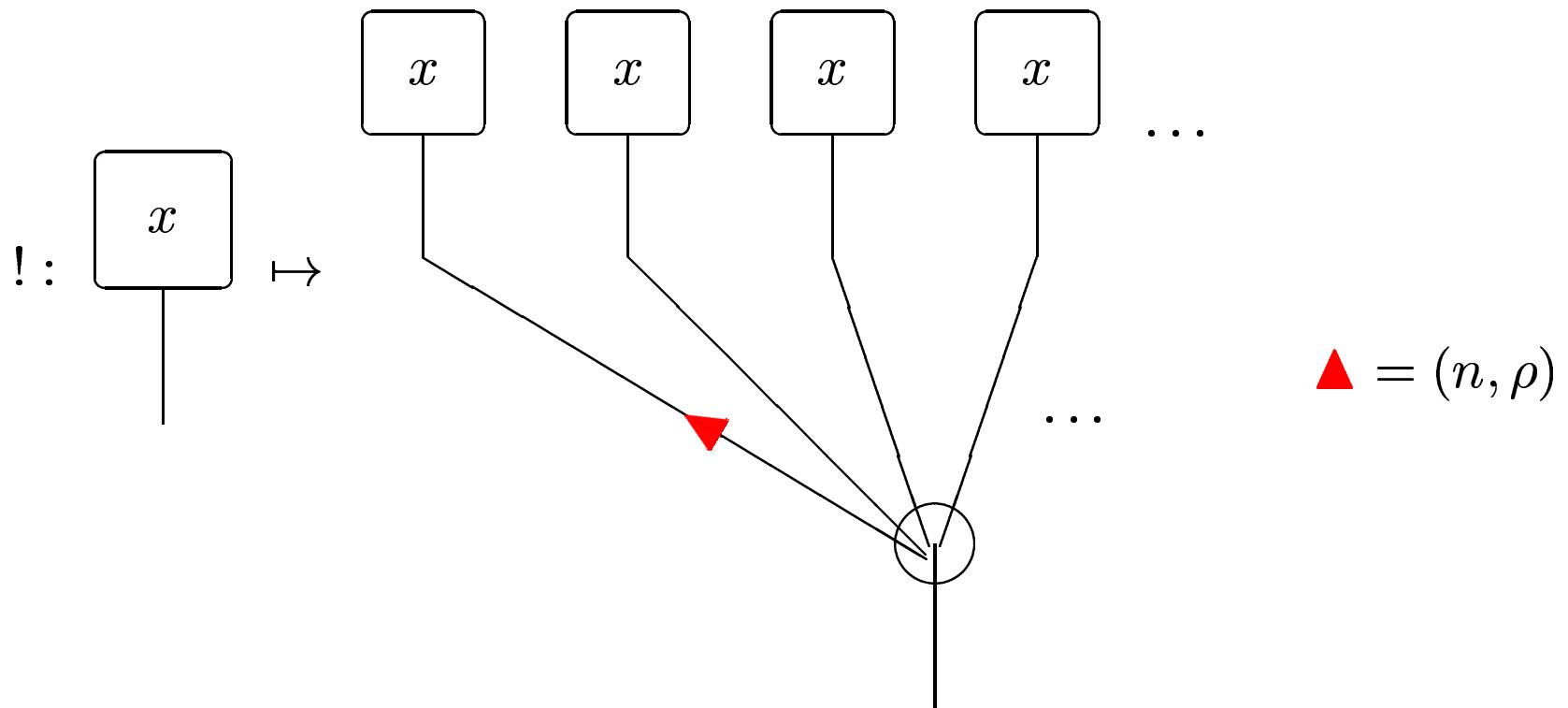
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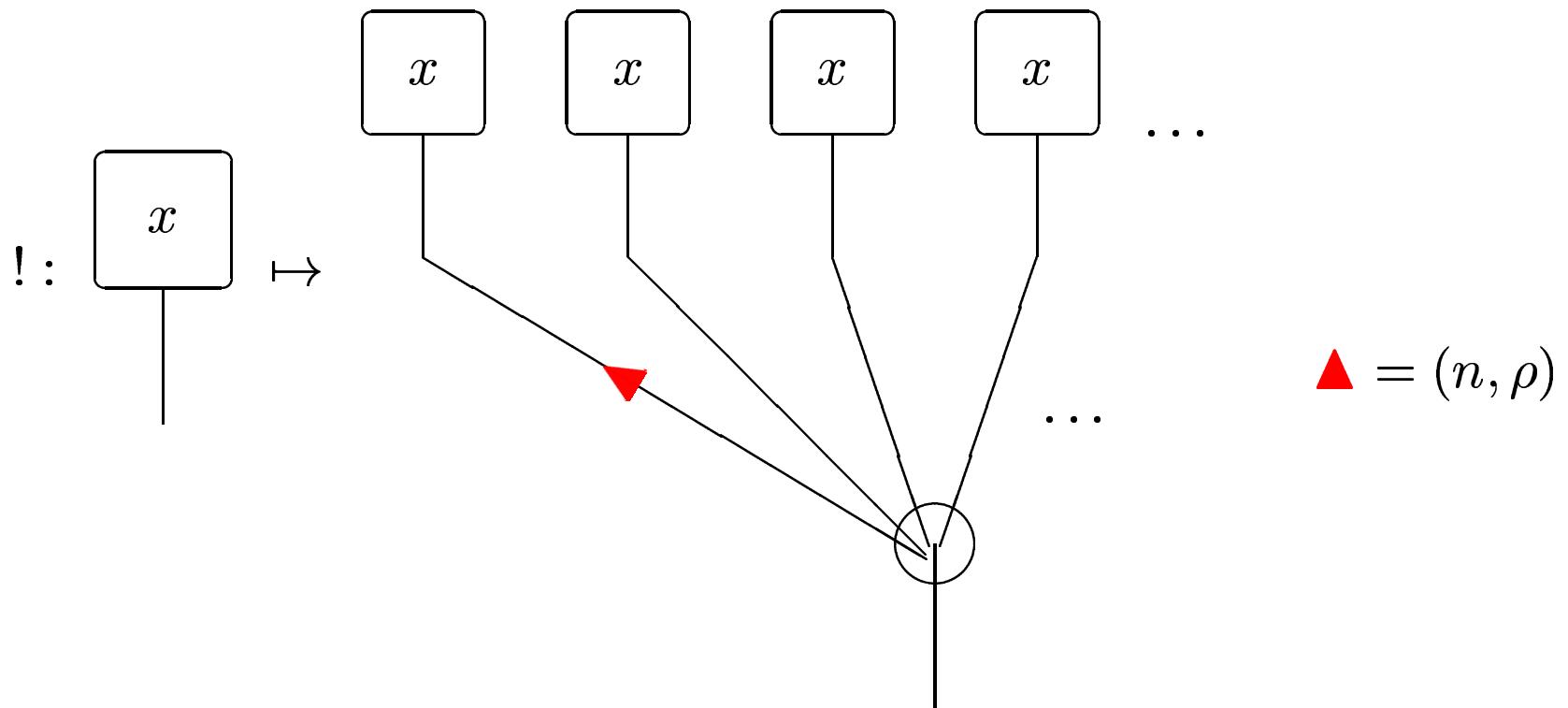
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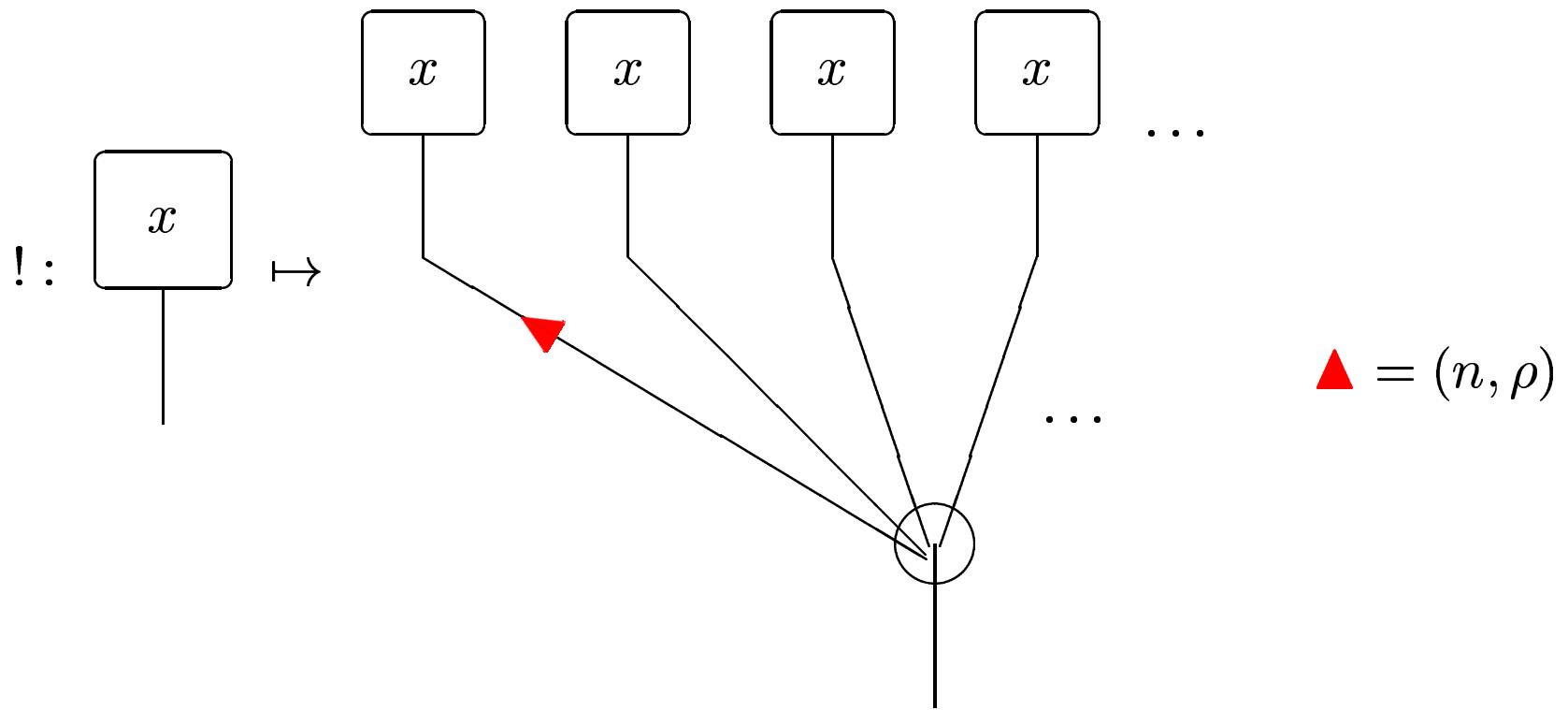
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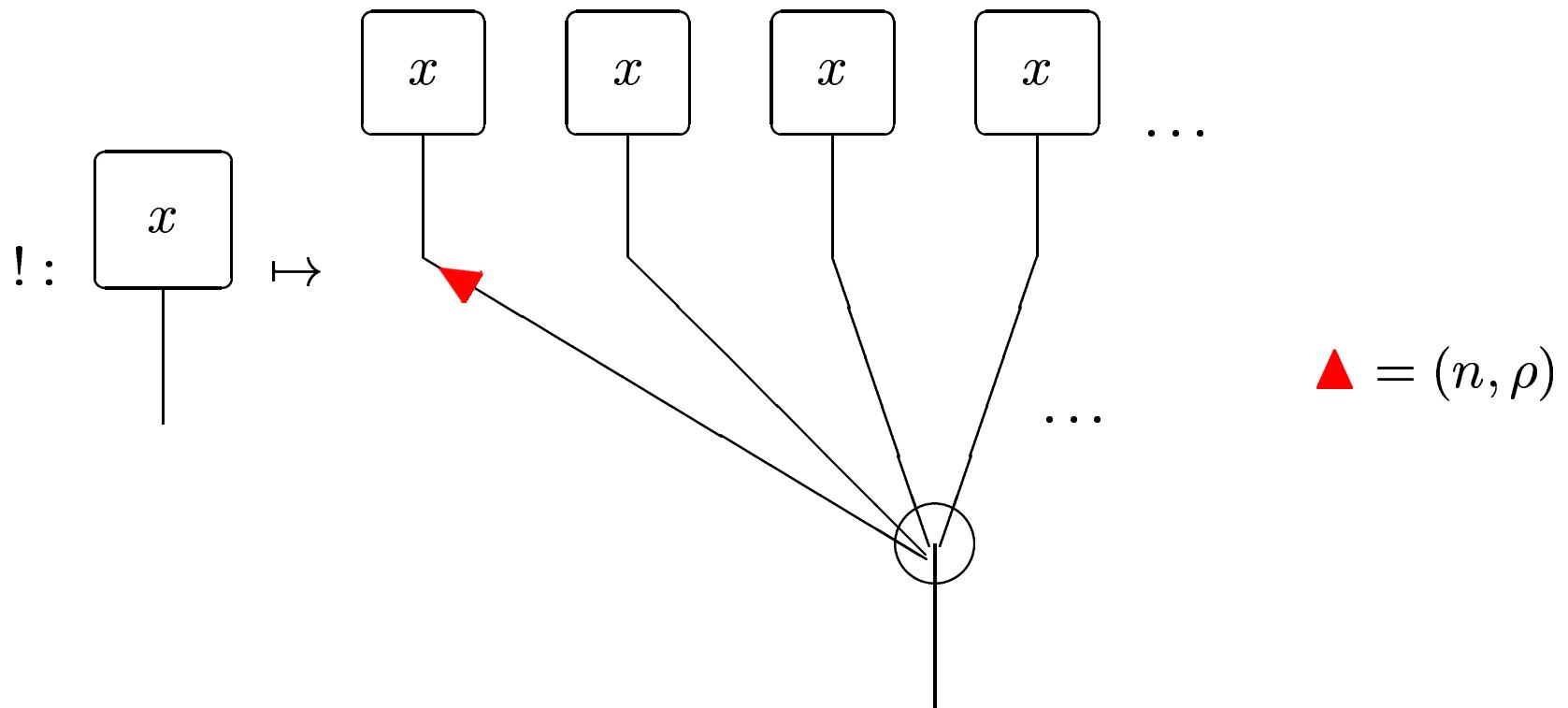
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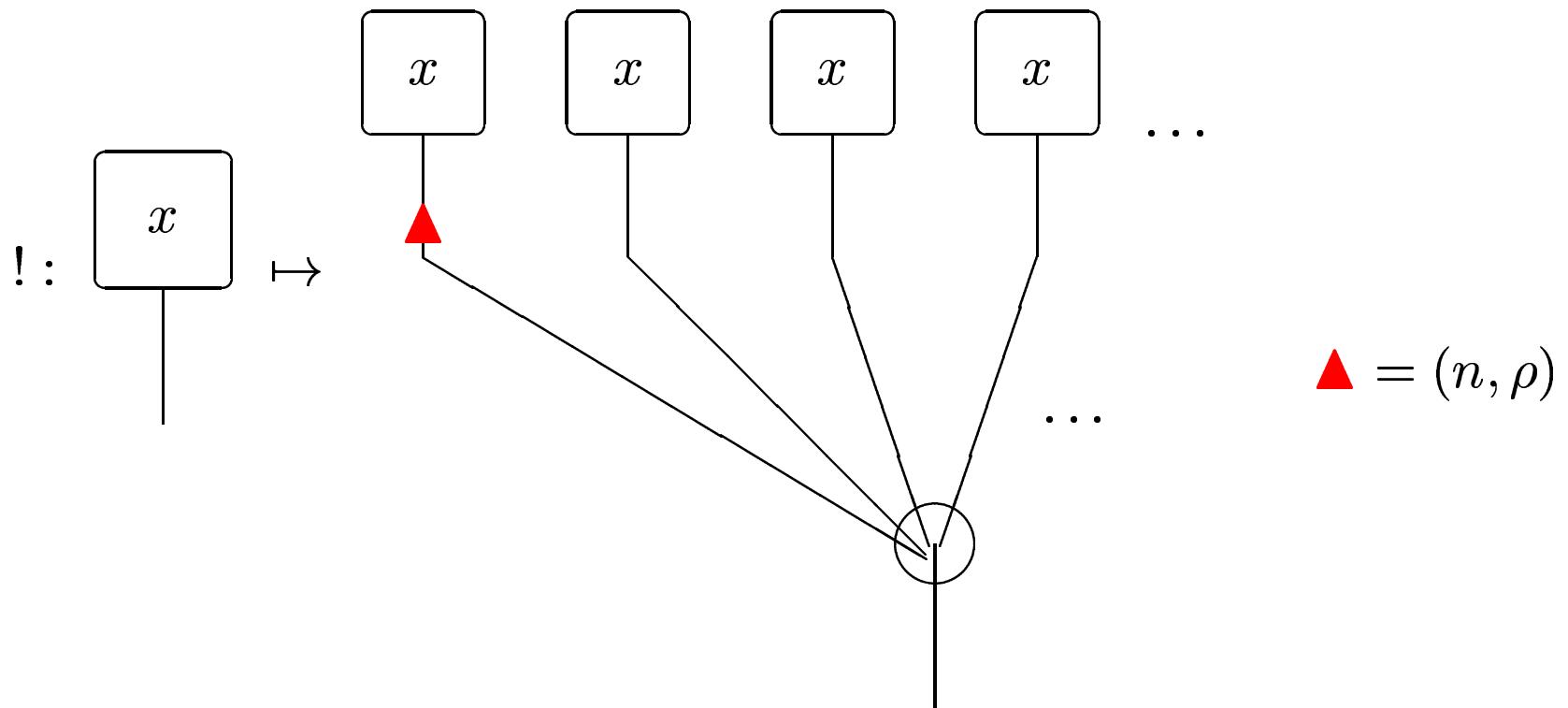
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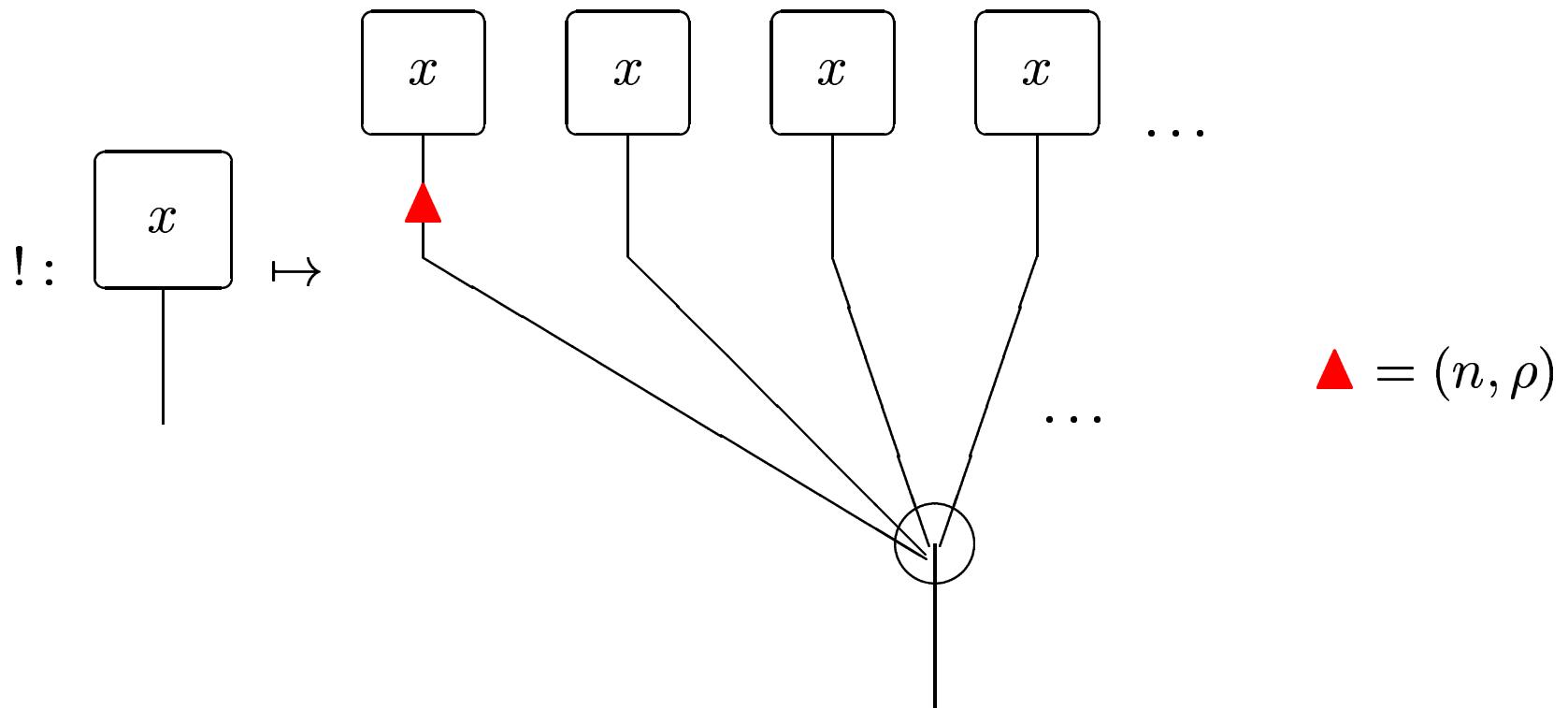
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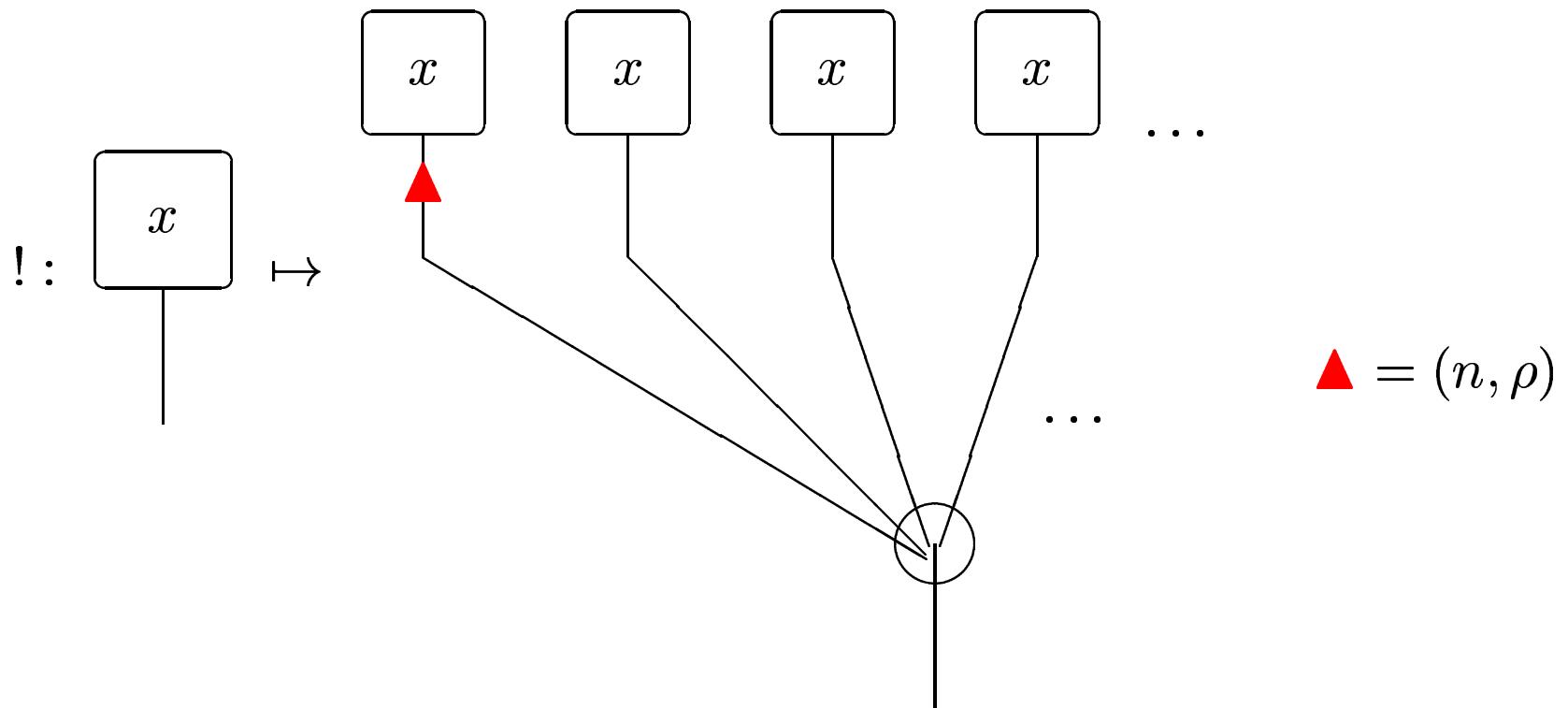
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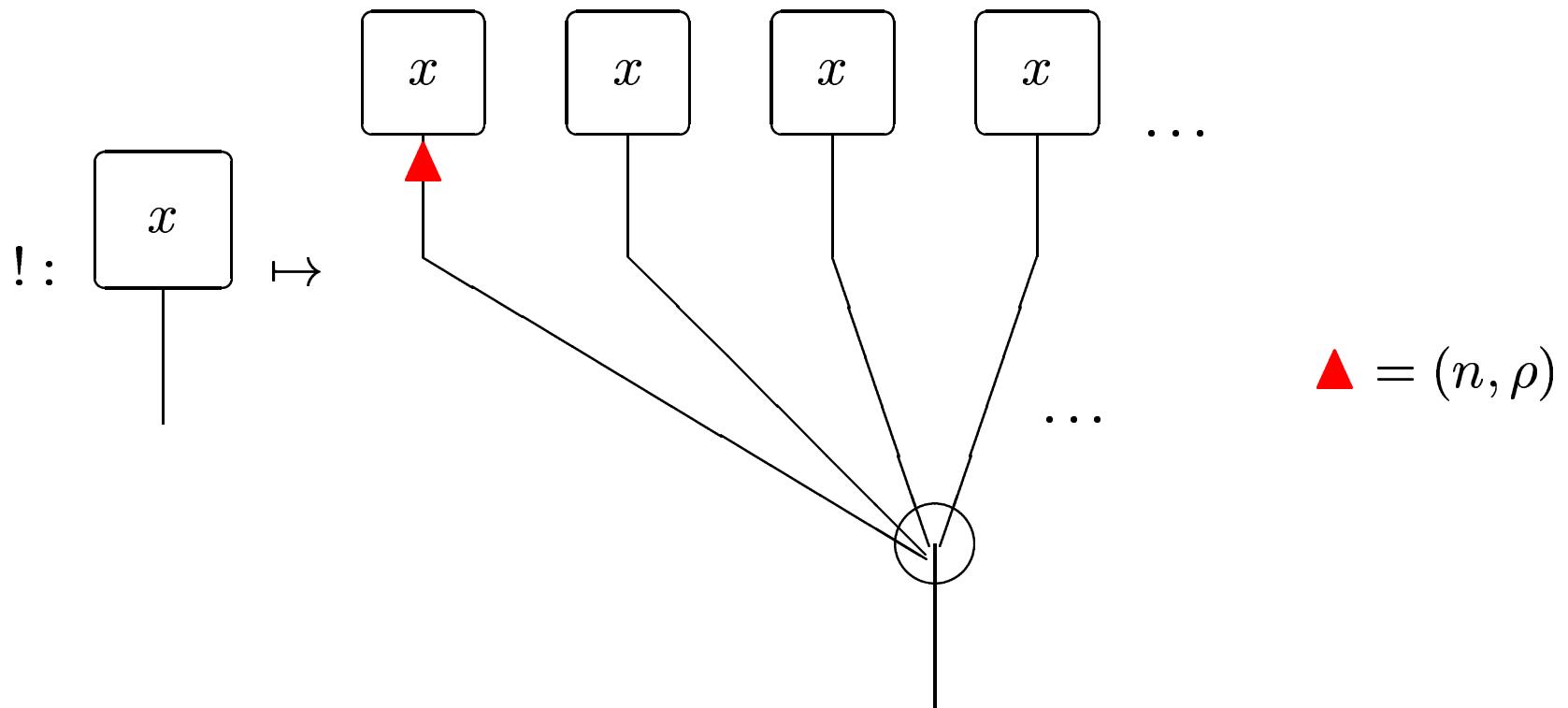
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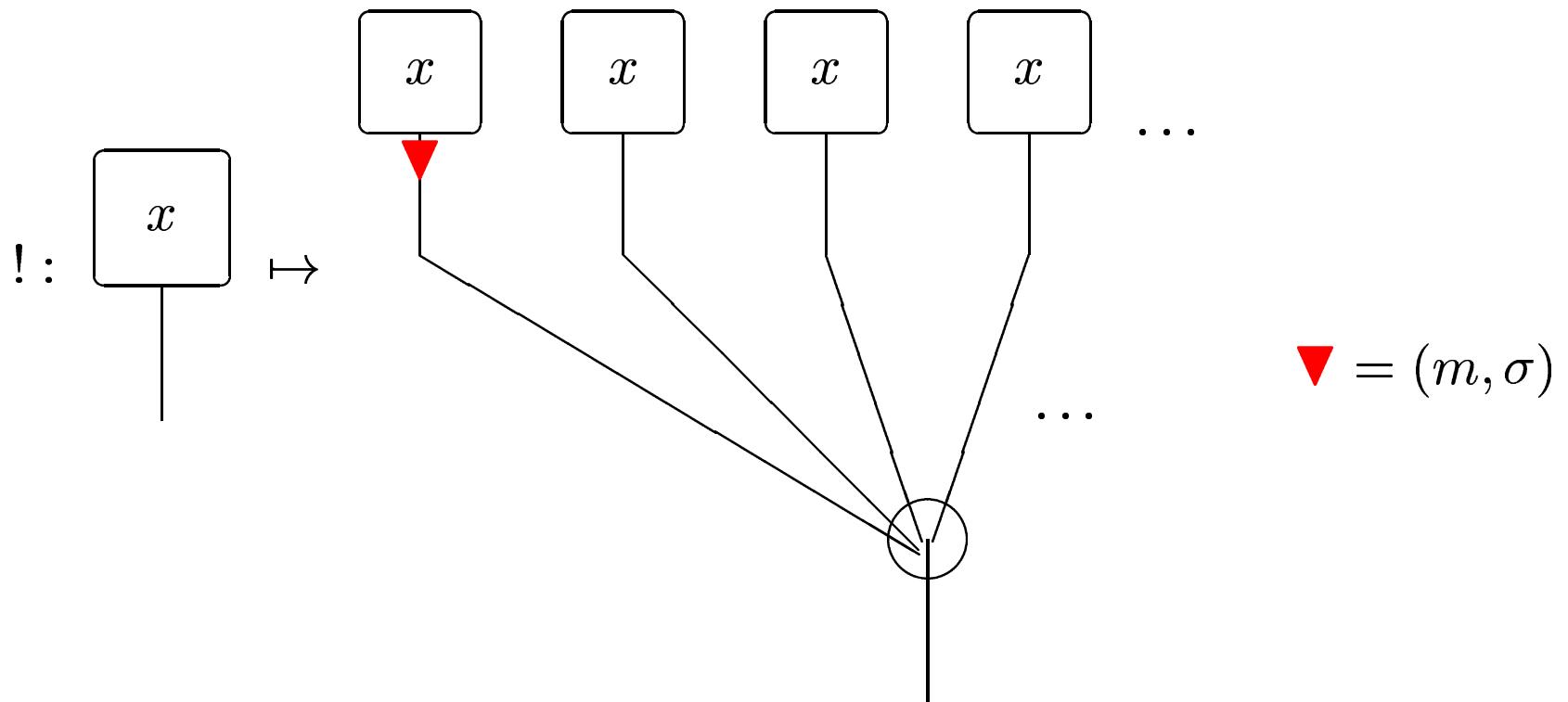
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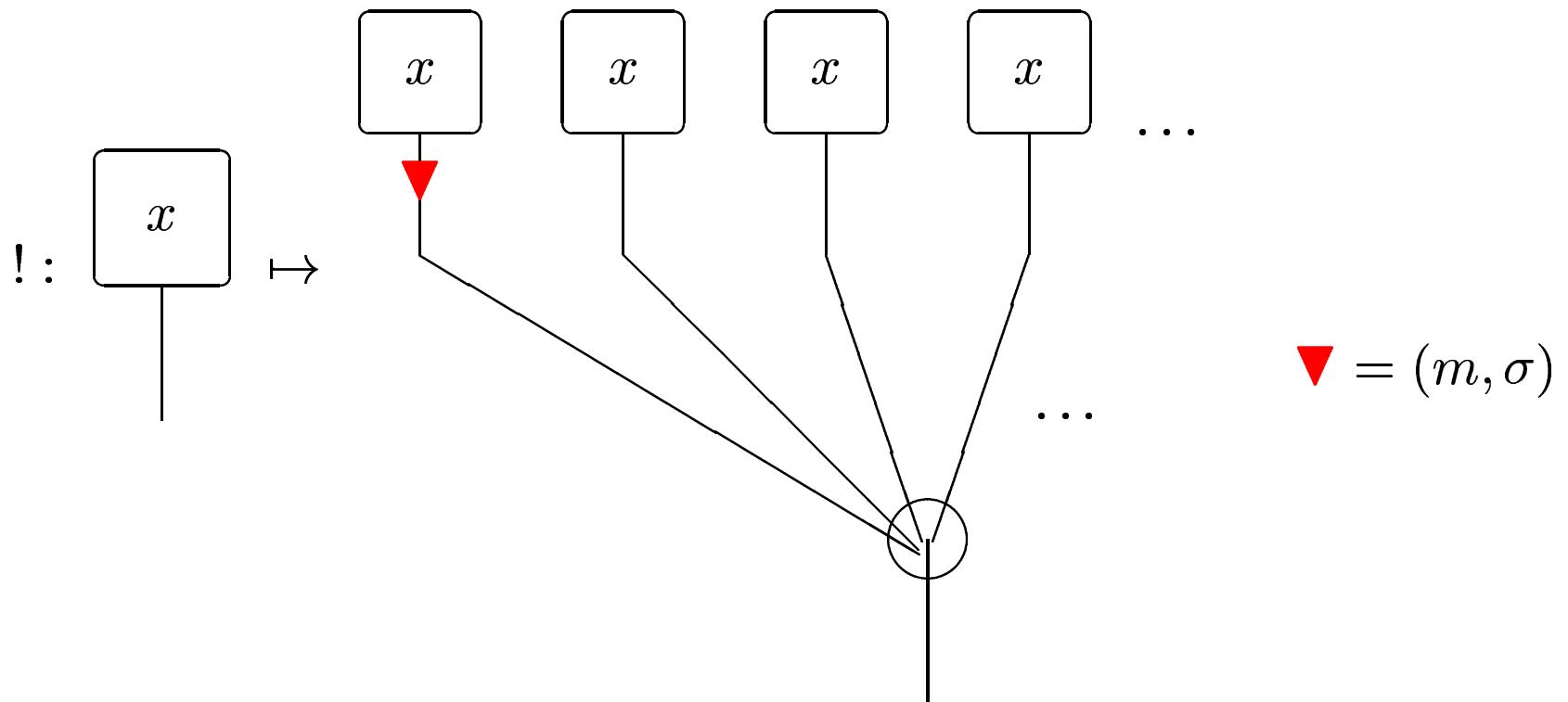
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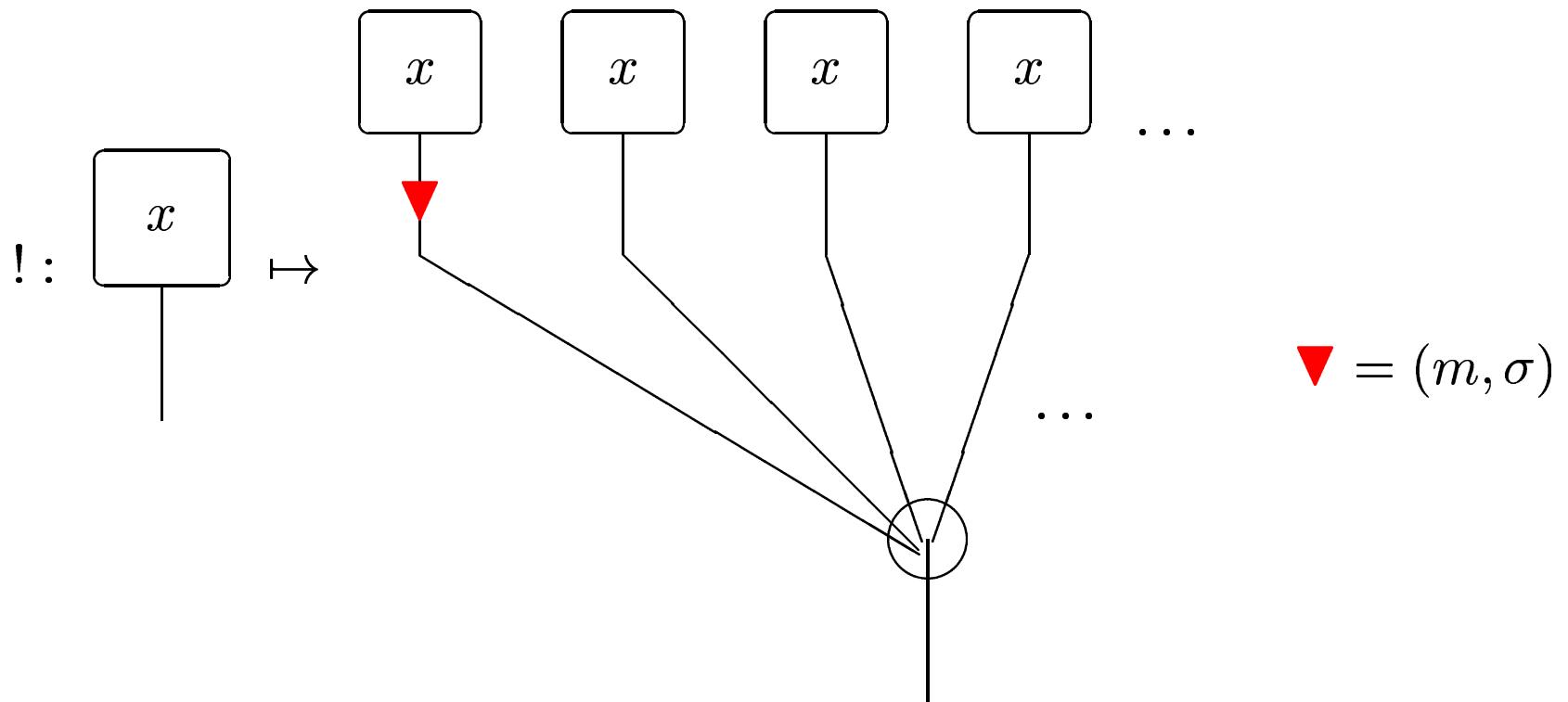
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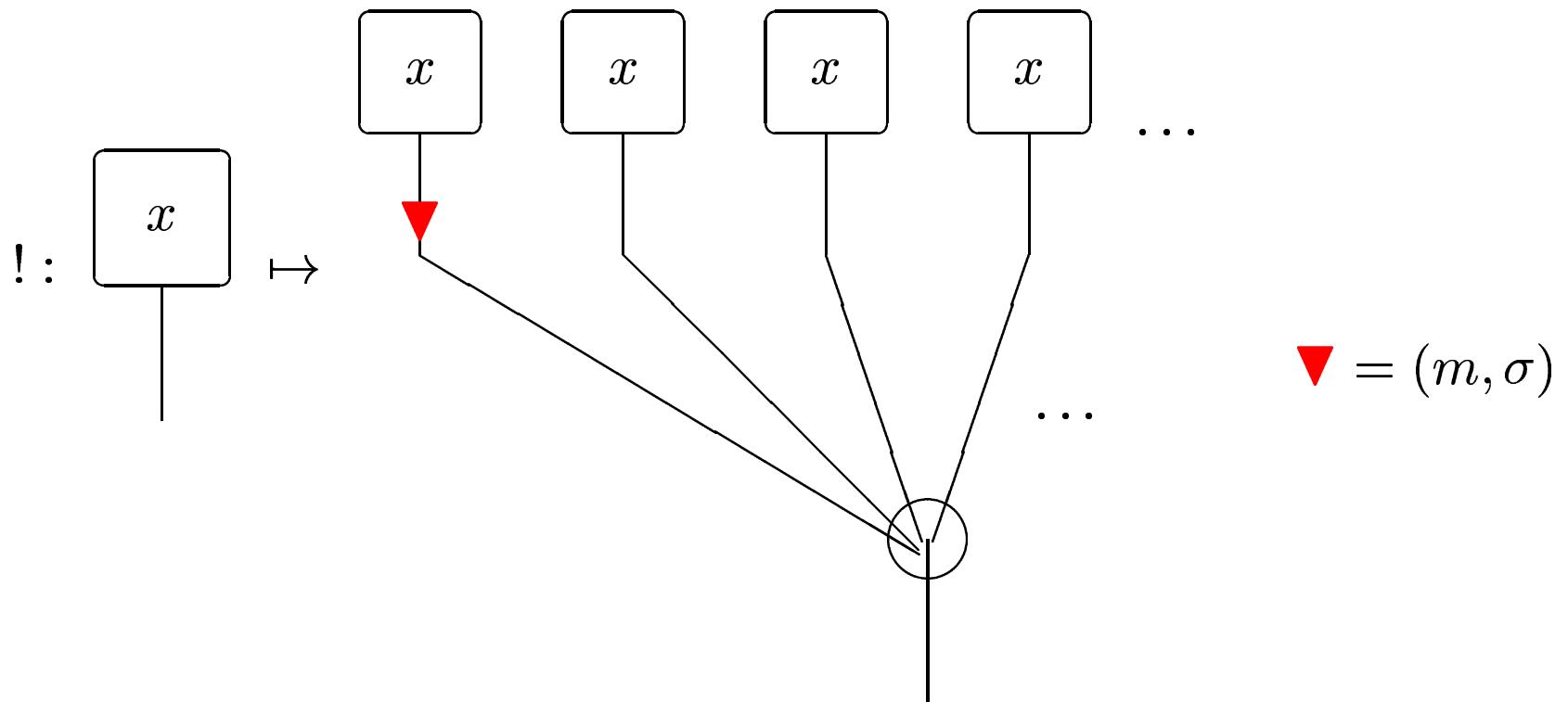
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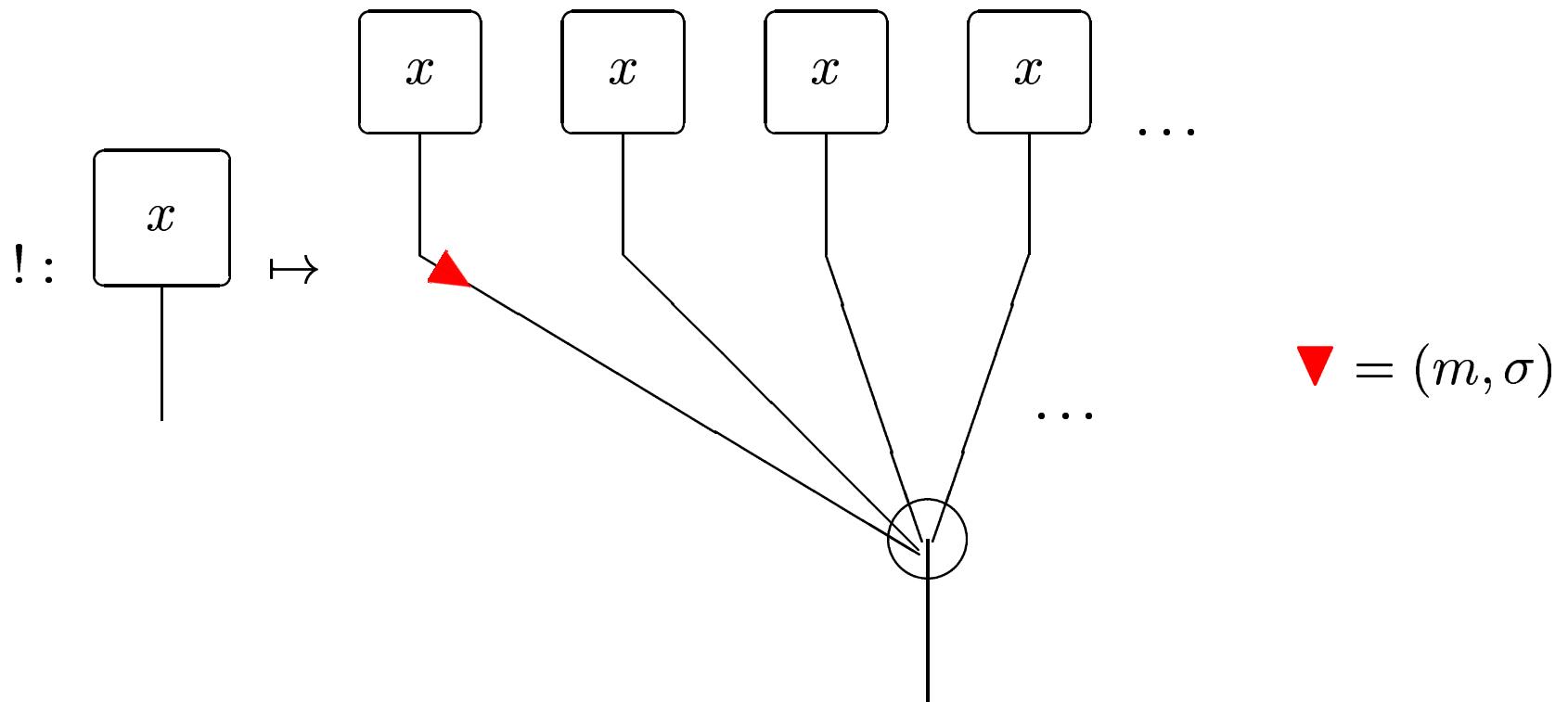
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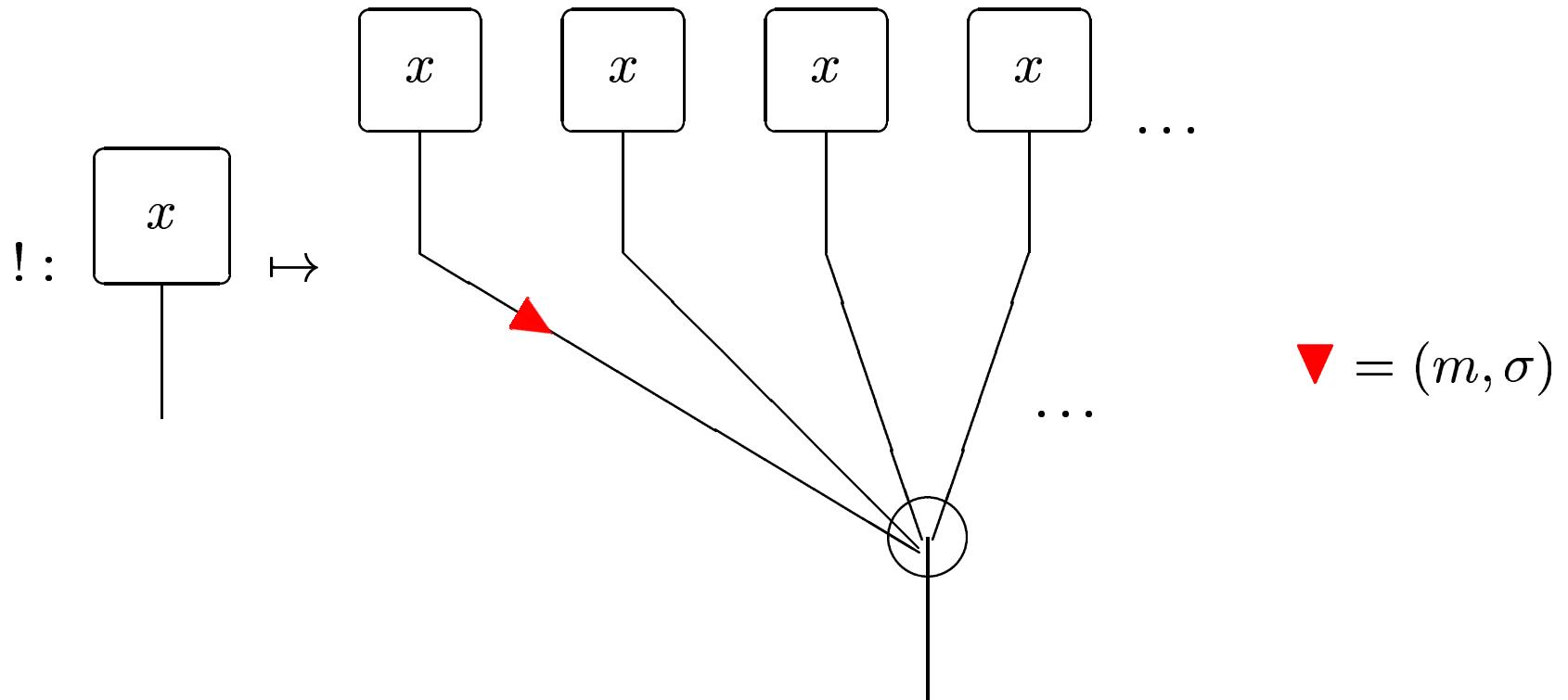
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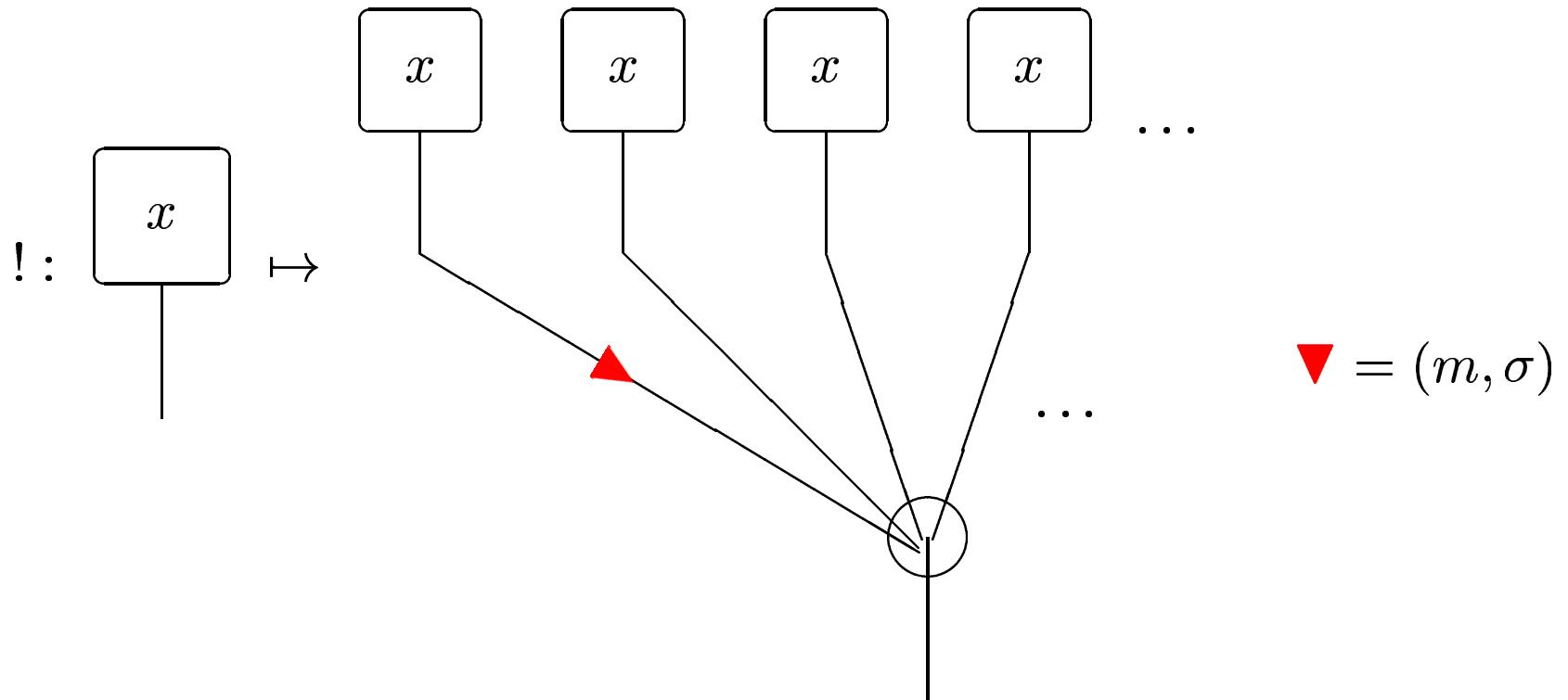
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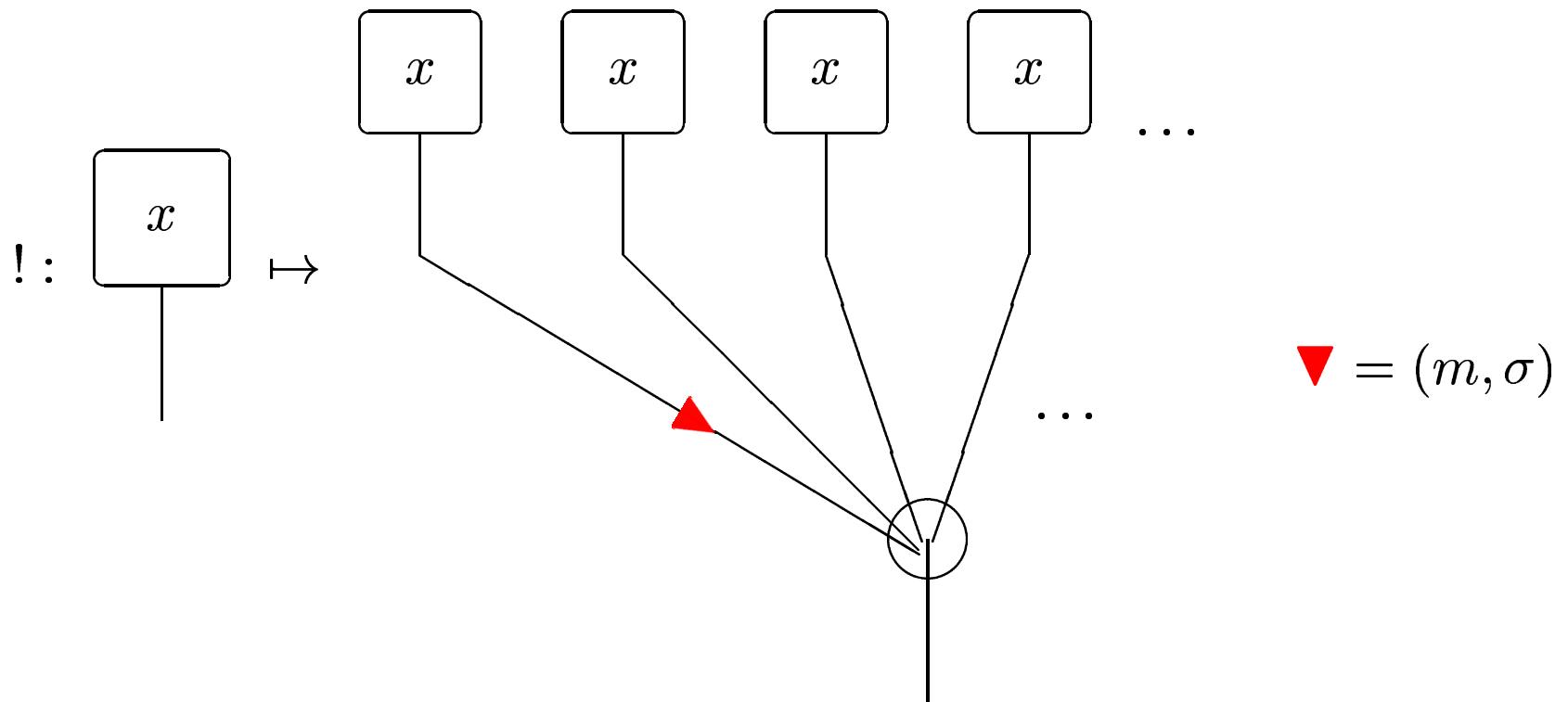
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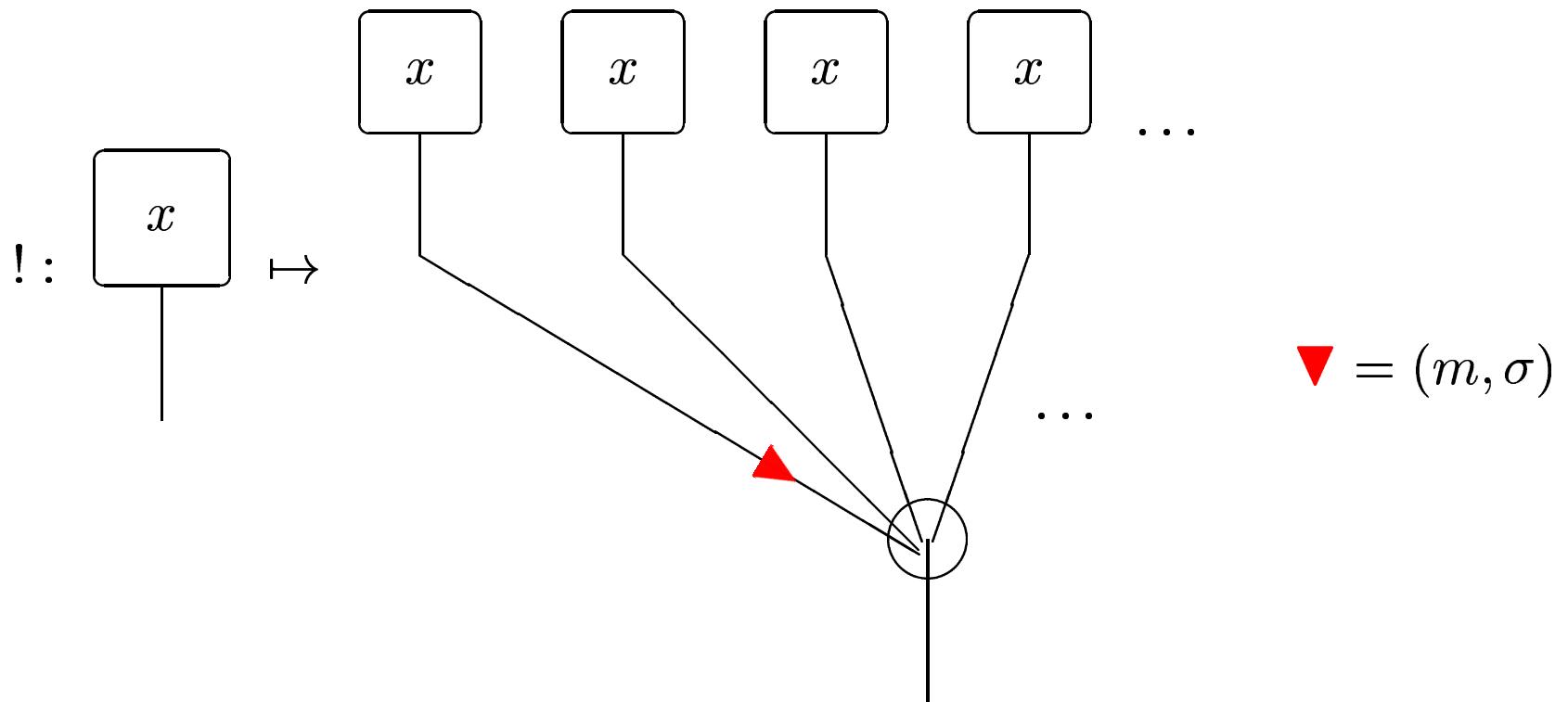
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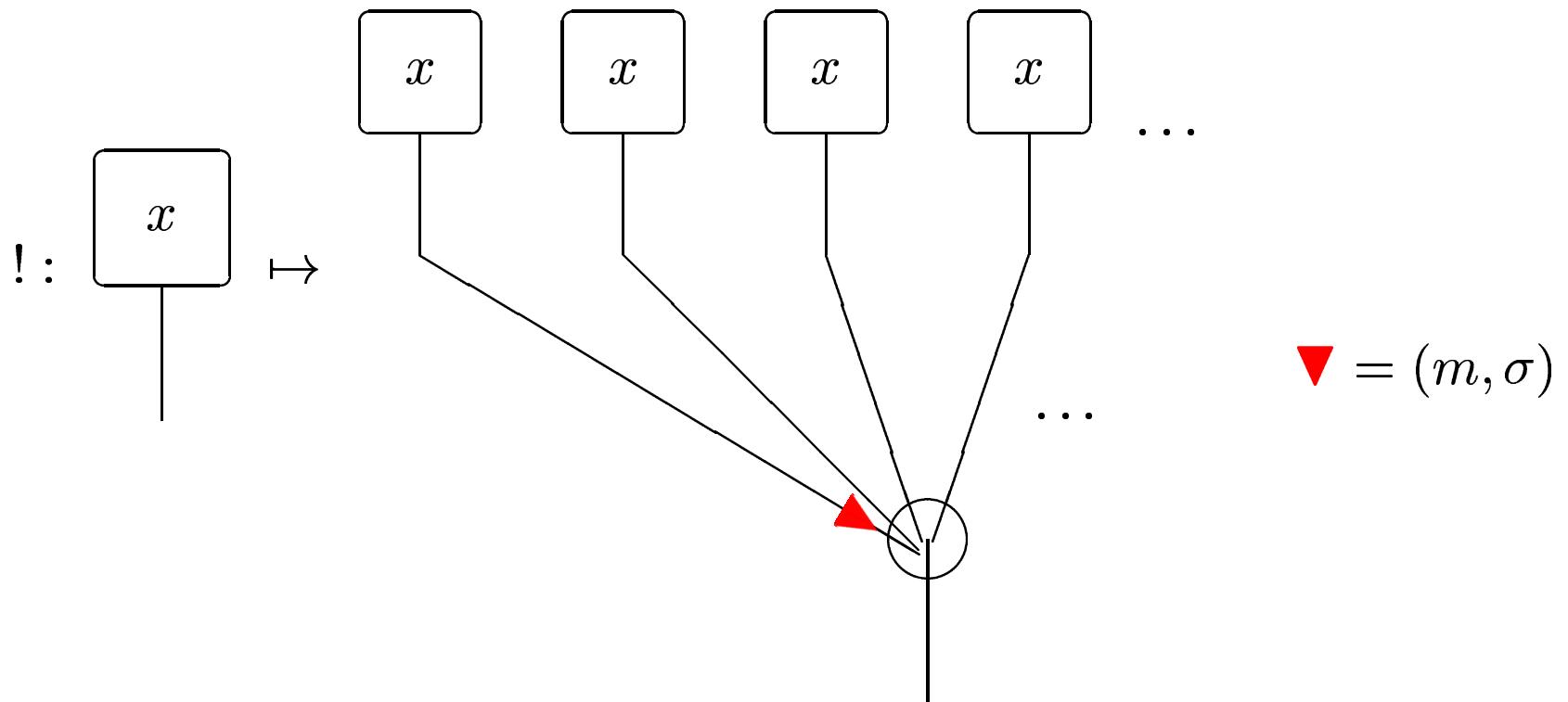
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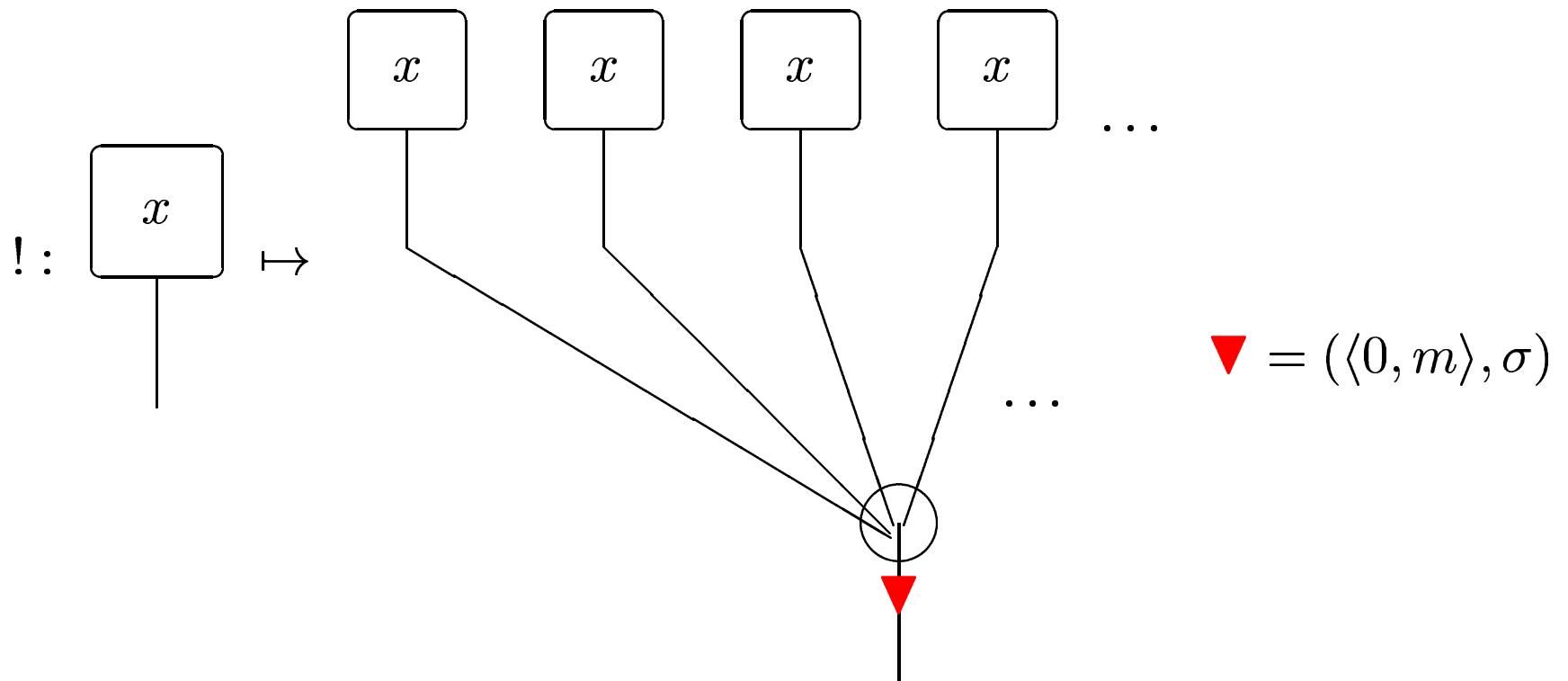
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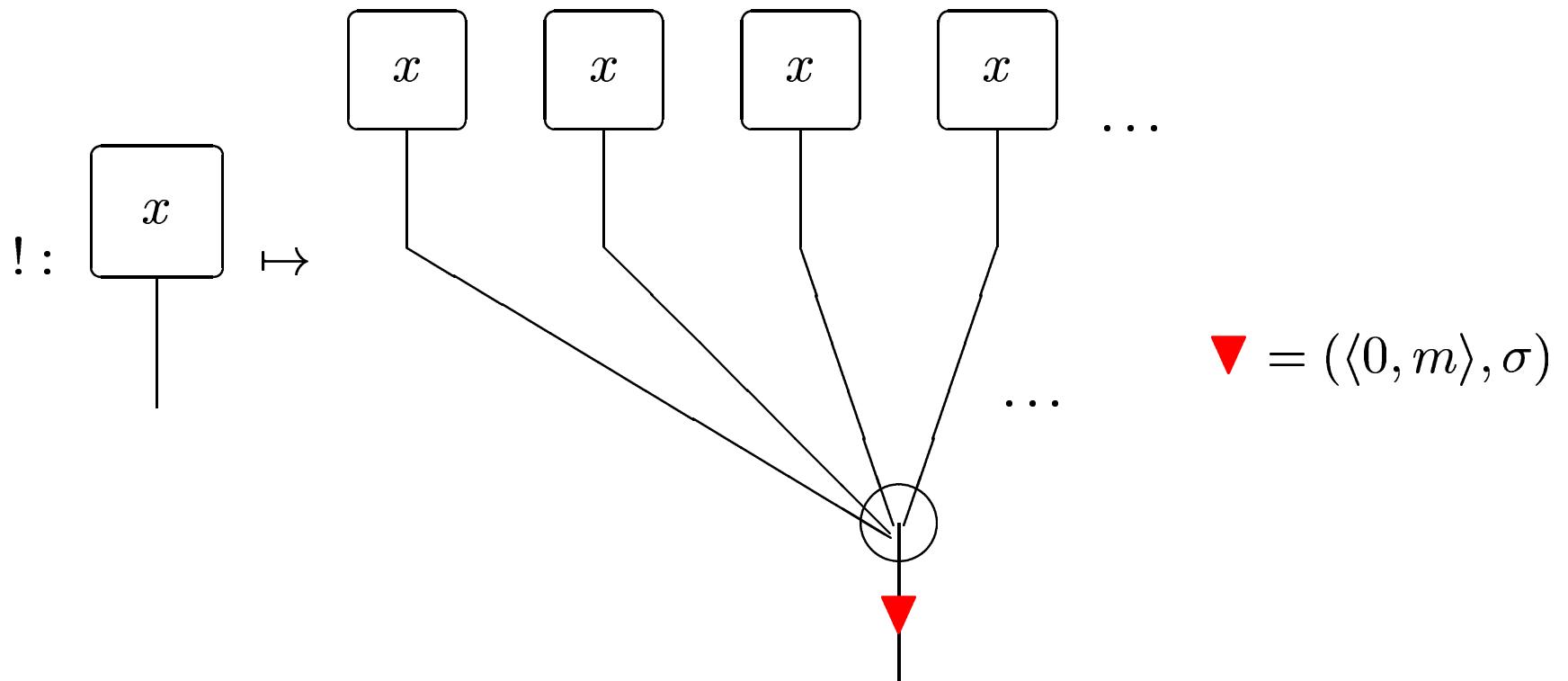
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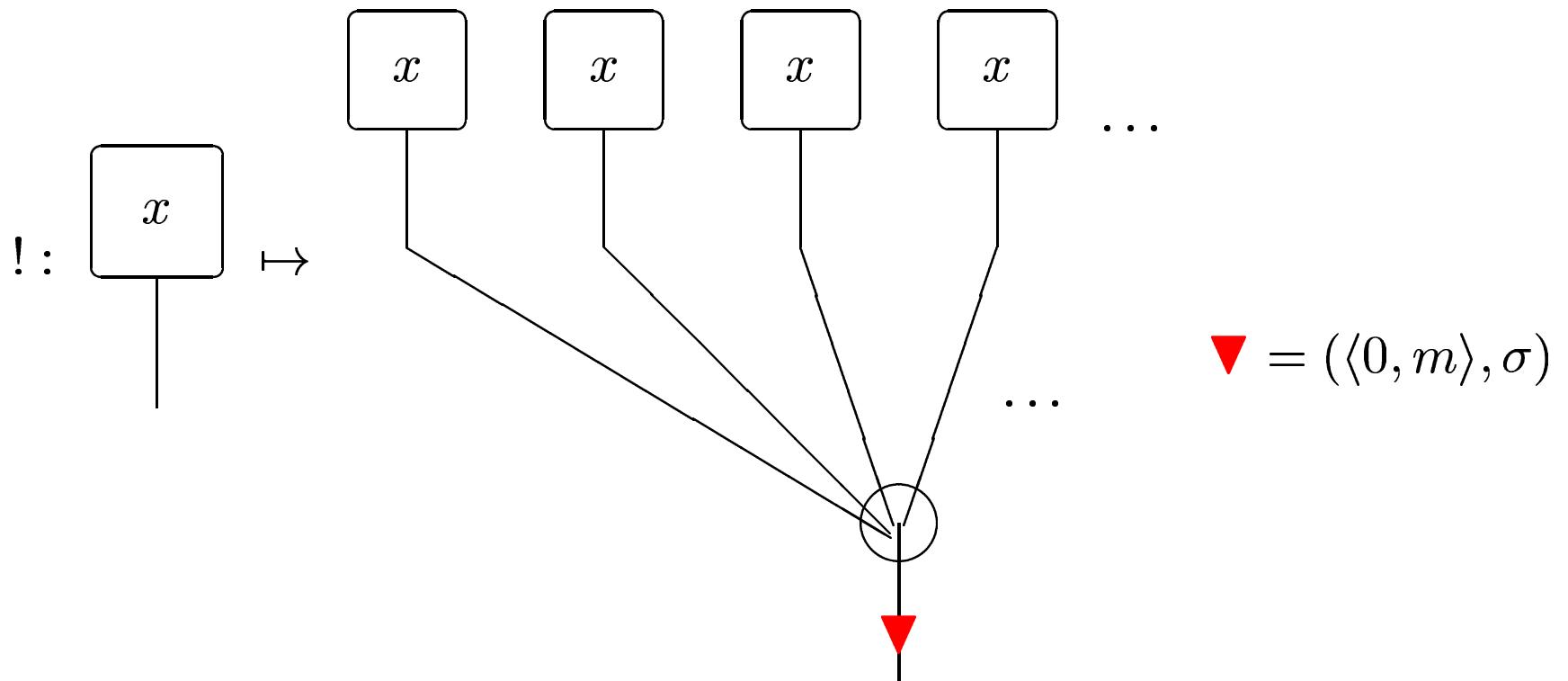
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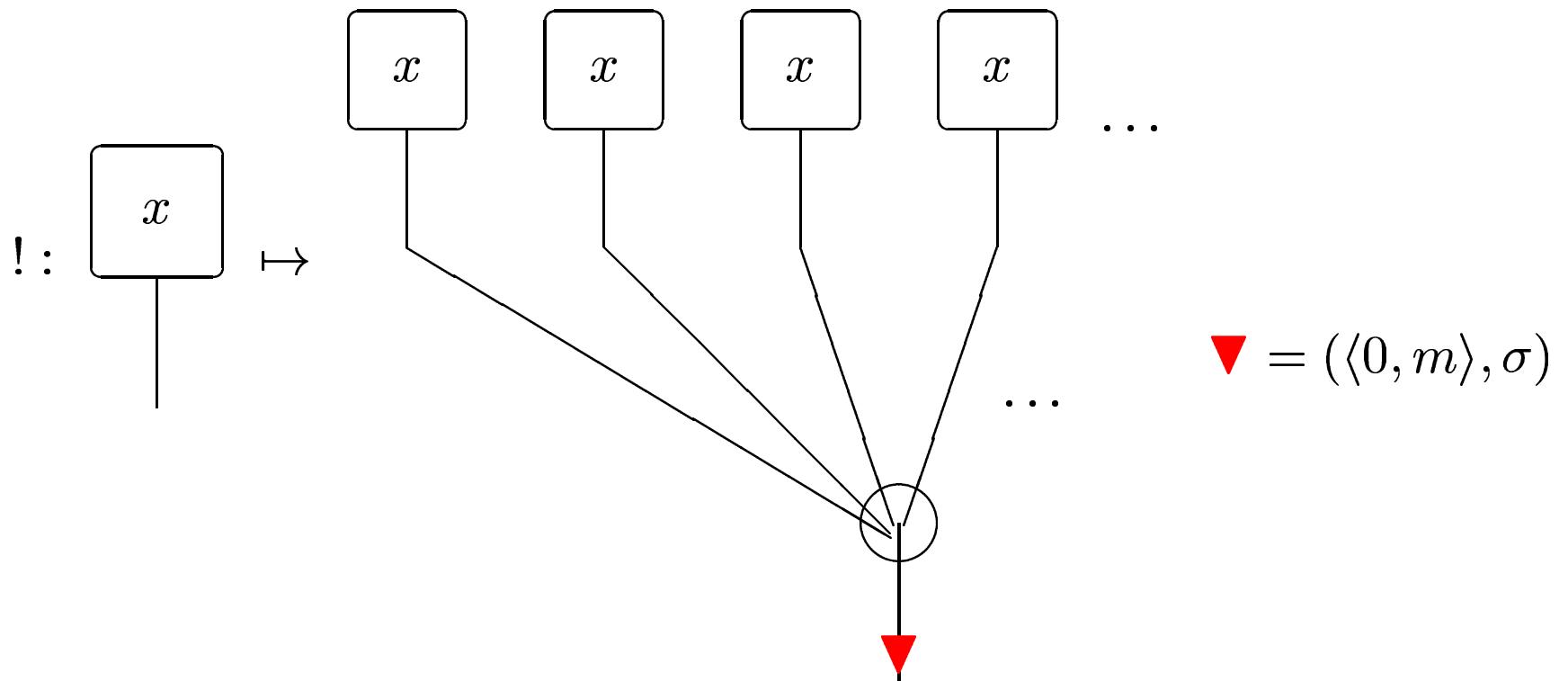
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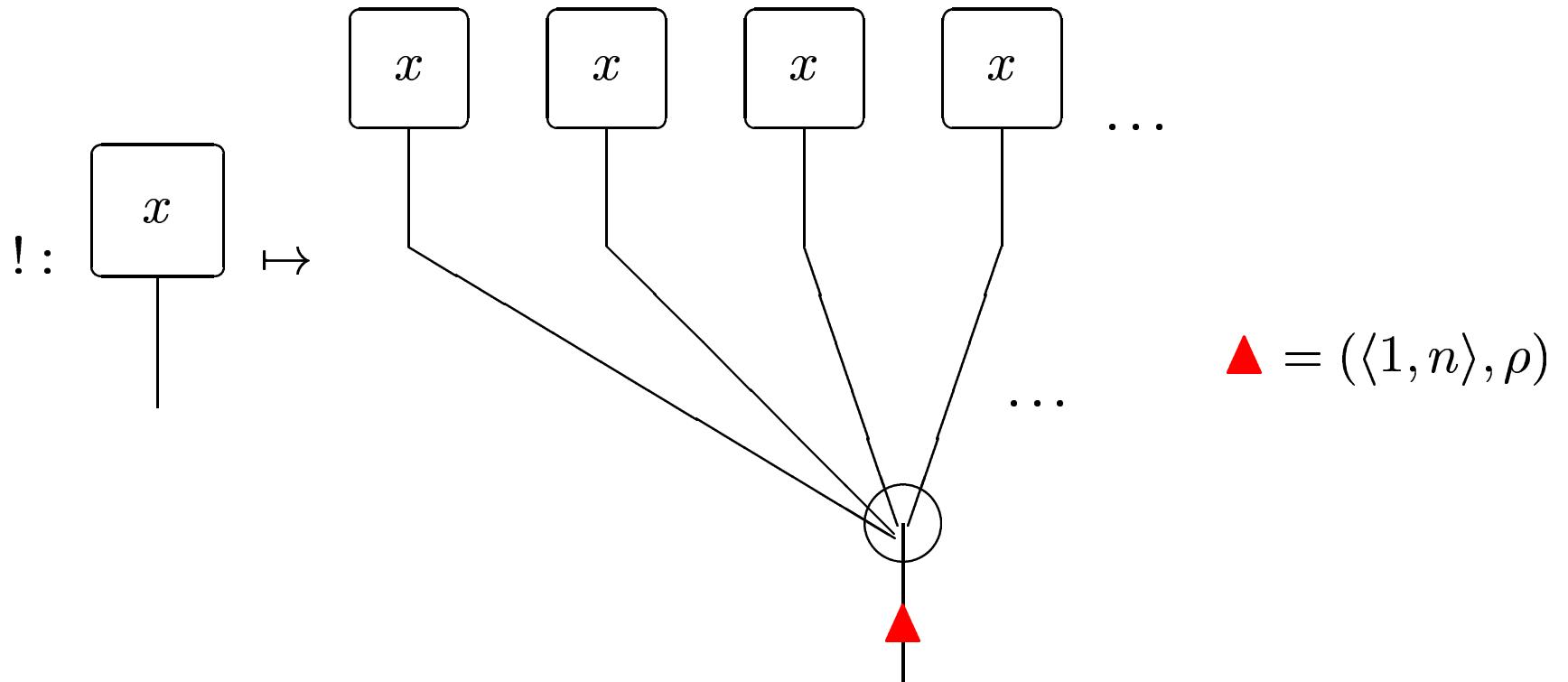
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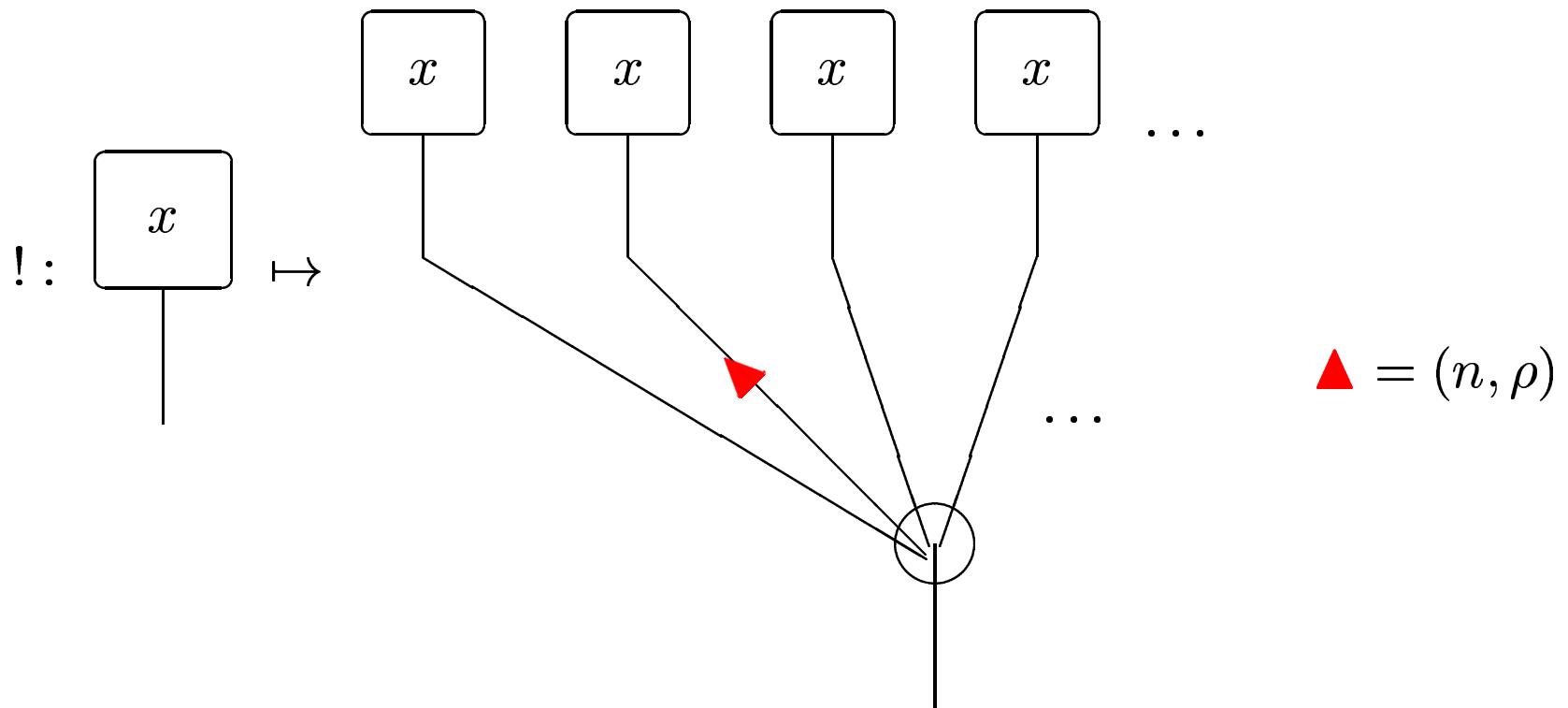
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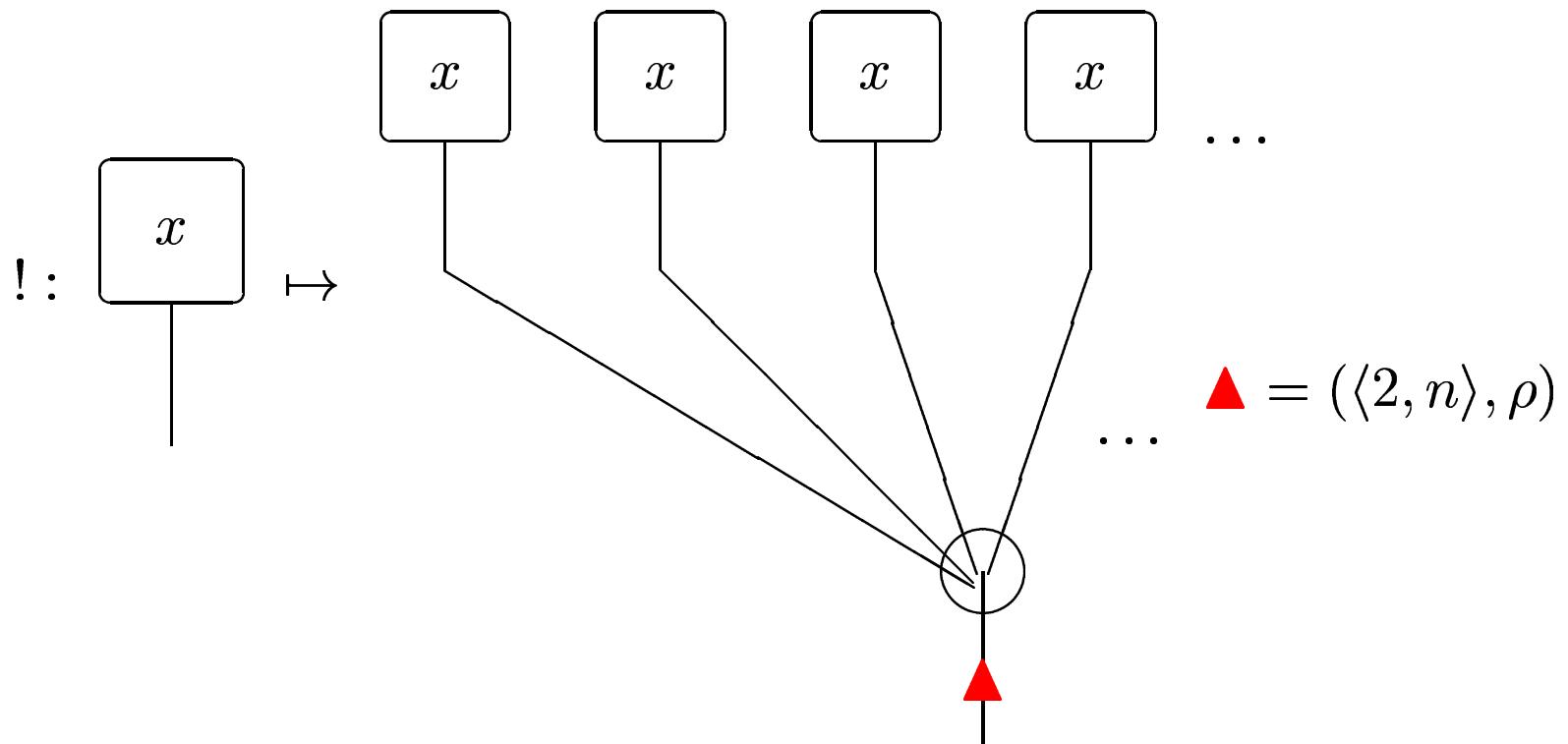
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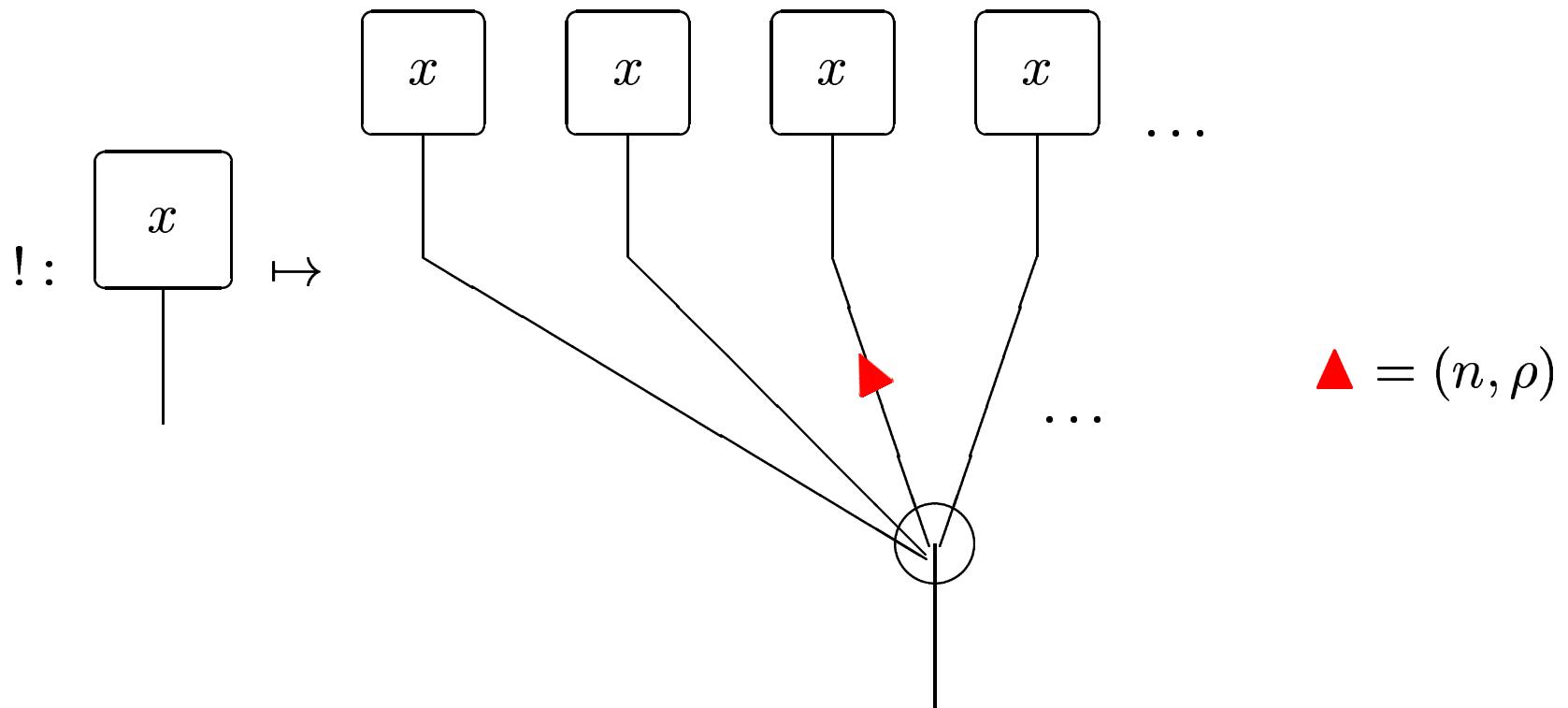
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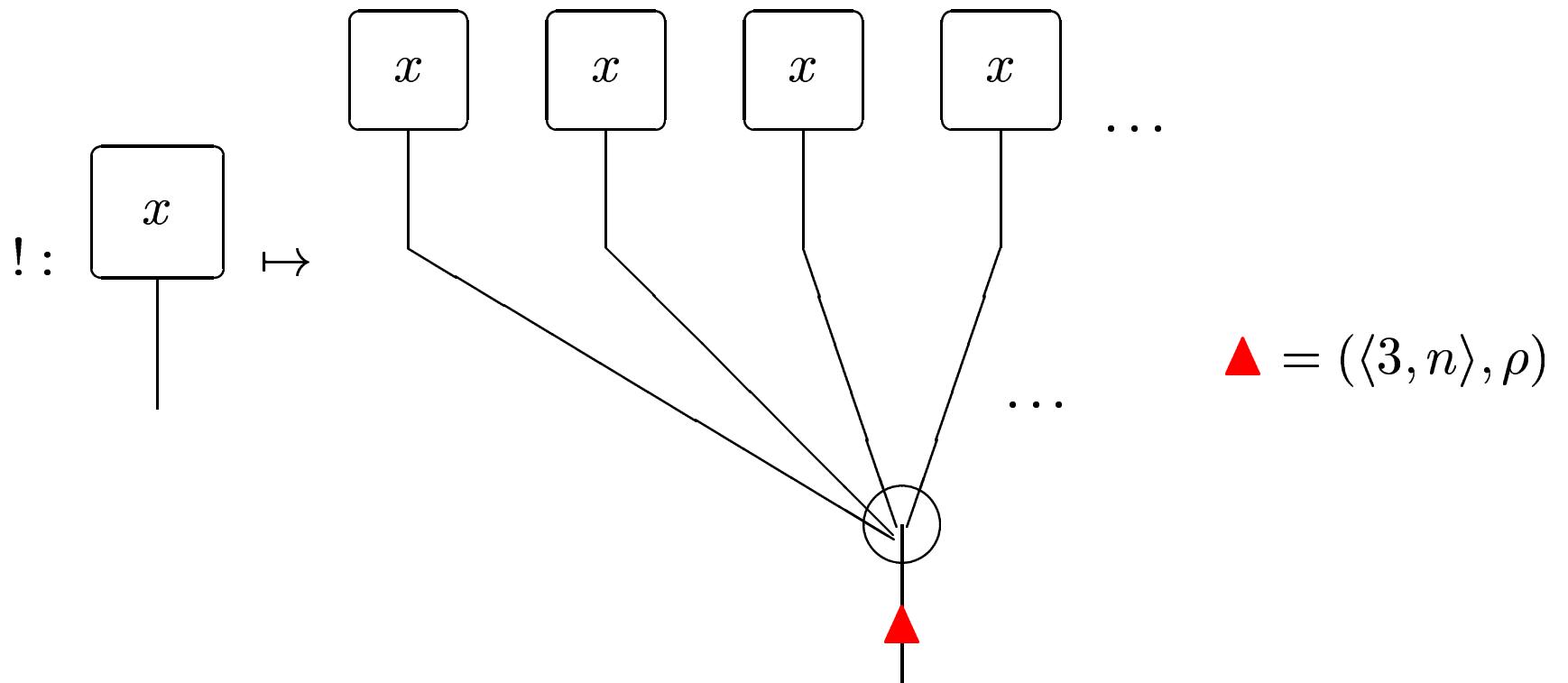
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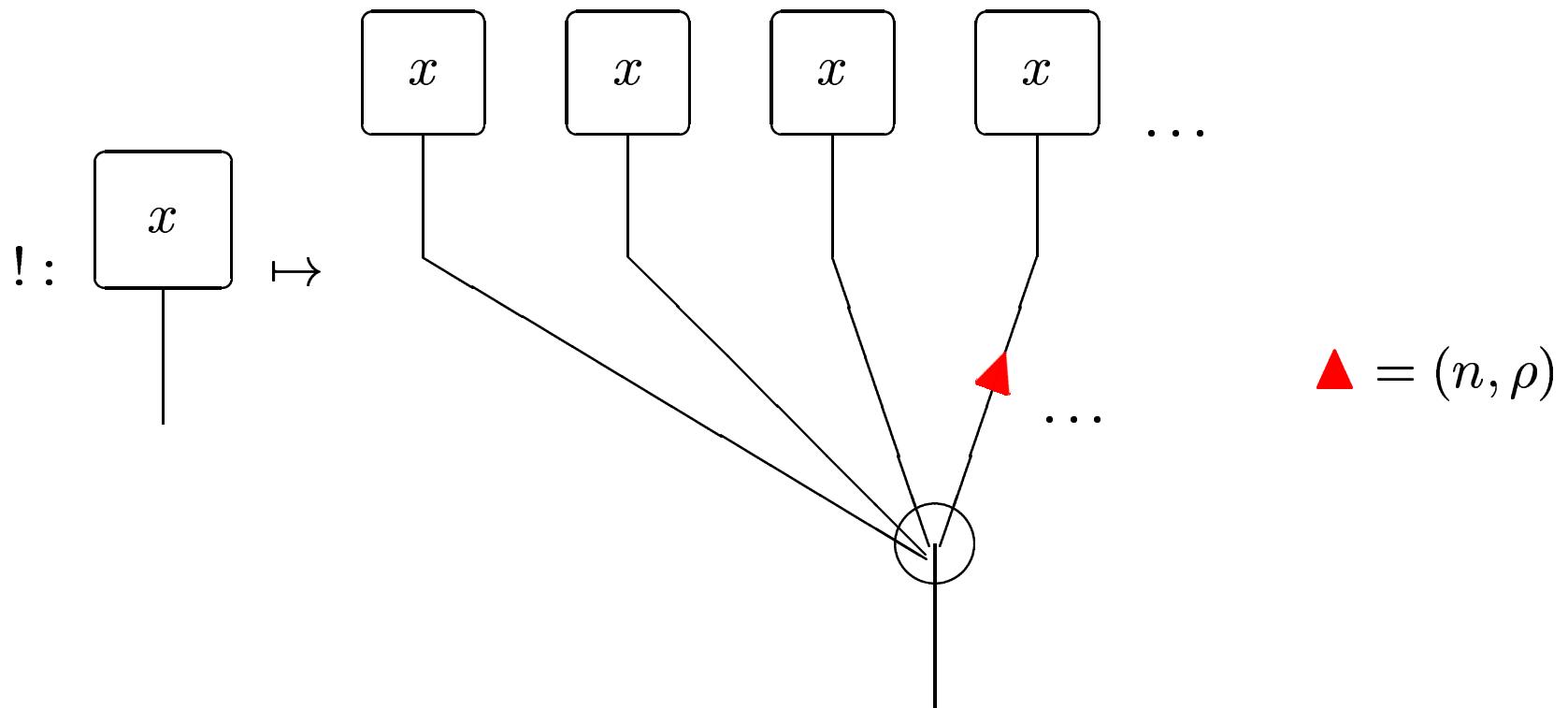
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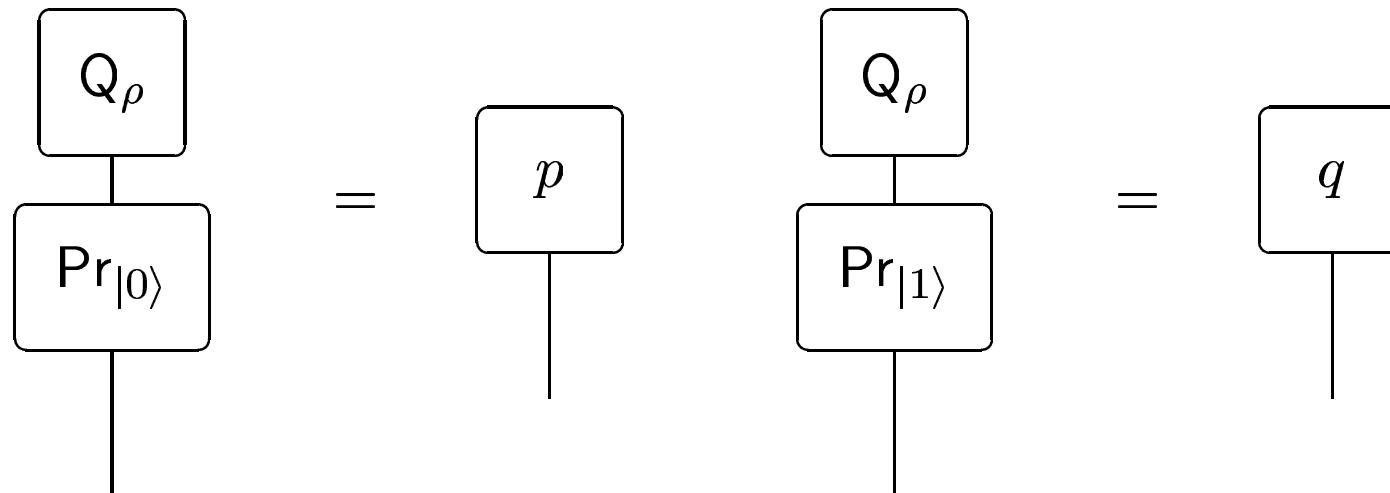
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where

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Measurement



when

\xrightarrow{p} true
meas¹ new _{ρ}
 \xrightarrow{q} false

$$(p = \langle 0 | \rho | 0 \rangle, q = \langle 1 | \rho | 1 \rangle)$$

Categorical Detail (1)

The categorical essence of GoI interpretation consists of:

- a traced symmetric monoidal category,
- **Int**-construction. [Abramsky, Haghverdi, Scott]

E.g. **Pinj** **Pfn** **Rel** **SRel**...

Our GoI is based on a TSMC **Set** $_{\mathcal{Q}}$ that is an “infinite version” of the category of finite dimensional vector spaces and superoperators (Selinger).

We choose a reflective object $U \in \mathbf{Set}_{\mathcal{Q}}$ with a certain data:

$$U \otimes U \triangleleft U, \quad \mathbb{N} \cdot U \triangleleft U, \dots$$

Prop. $\mathbf{Int}(\mathbf{Set}_{\mathcal{Q}})(\mathbf{I}, (U, U))$ is a (affine) linear combinatory algebra.

Categorical Detail (2)

Let $A_{\mathcal{Q}}$ be the obtained affine LCA.

Prop. $\mathbf{Per}(A_{\mathcal{Q}})$ with a monad T_R is a weak linear category for duplication.

$$T_R := (- \multimap R) \multimap R$$

Informally, R is a set of infinite binary tree whose edges are labelled by $p \in [0, 1]$:

$$\begin{aligned} R &\cong (D \times R) \times (D \times R) \\ D &\sim [0, 1] \end{aligned}$$

We interpret terms in the Kleisli category $\mathbf{Per}(A_{\mathcal{Q}})_{T_R}$:

$$\frac{\Gamma \vdash M : A}{[\![M]\!] : [\![\Gamma]\!] \rightarrow ([\![A]\!] \multimap R) \multimap R}$$

Denotation of Terms at bit

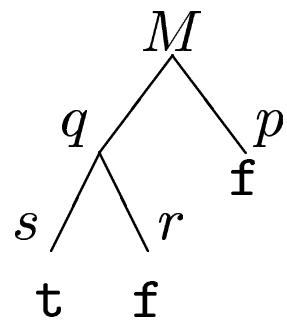
We define test : $\llbracket \text{bit} \rrbracket \multimap R$ by

$$\text{test(true)} = \begin{array}{c} 1 \\ / \quad \backslash \\ \triangle(0) \quad \triangle(0) \end{array}$$

$$\text{test(false)} = \begin{array}{c} 0 \\ / \quad \backslash \\ \triangle(0) \quad \triangle(0) \end{array}$$

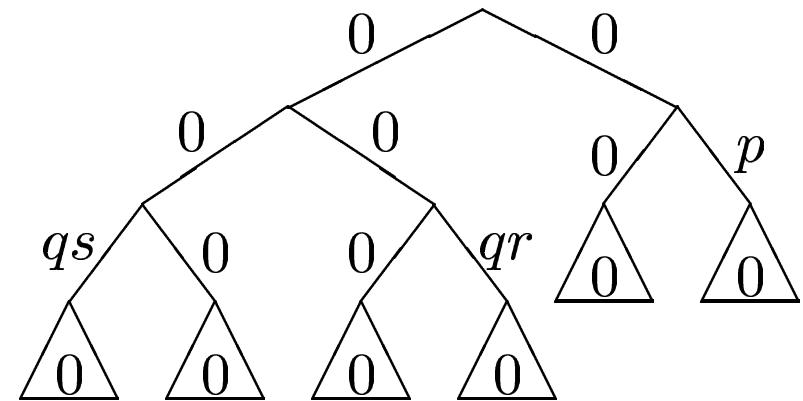
$\llbracket M \rrbracket(\text{test})$ has enough information of the evaluation tree of $M : \text{bit}$

E.g.



\Rightarrow

$\llbracket M \rrbracket(\text{test}) =$



Adequacy

Thm. For a closed $M : \text{bit}$,

$$M \Downarrow \left(\sum_{\substack{p \\ p \wedge_0 \in \llbracket M \rrbracket(\text{test})}} p, \sum_{\substack{q \\ 0 \wedge_q q \in \llbracket M \rrbracket(\text{test})}} q \right)$$

(A proof is by means of logical relation and finite approximation of terms.)

Cor. The term model up to observational equivalence is a weak linear category for duplication.

Conclusion and Future Work

We construct an adequate denotational semantics

- Classical control and quantum data
- Gol, realizability

Future work:

- Extraction of abstract machine
- Comparison with other quantum lambda calculi
- Extension to polymorphism

Thank you.