The Complexity of Verifying Ground Tree Rewrite Systems

Stefan Göller
(University of Bremen)

joint work with Anthony Widjaja Lin
(Oxford University)

LICS 2011, Toronto

Model checking and equivalence checking

Model checking a class of structures $\mathcal{C}$ against a logic $\mathcal{L}$

INPUT: Structure $S \in \mathcal{C}$ + formula $\varphi \in \mathcal{L}$
QUESTION: $S \models \varphi$?

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$\equiv$-Equivalence checking between structures in $\mathcal{C}$

INPUT: Structures $S_1, S_2 \in \mathcal{C}$
QUESTION: $S_1 \equiv S_2$?

Pushdown and prefix-recognizable systems

States: All finite words (over some finite alphabet)

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  - Rewrite rules: $u \overset{a}{\hookrightarrow} v$ (u, v words)
  - Transitions: $uw \overset{a}{\rightarrow} vw$ for all words w.

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- **Prefix-recognizable systems:**
  - Rewriting rules: $L_1 \overset{a}{\hookrightarrow} L_2$ (L_1, L_2 regular word languages)
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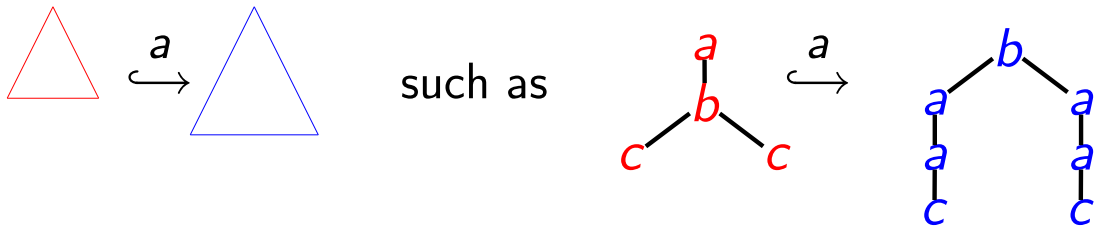
- **Ground tree rewrite systems (GTRS):**
  - Rewrite rules



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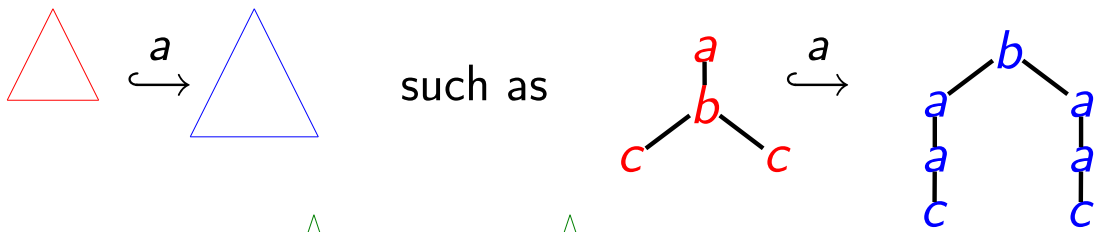

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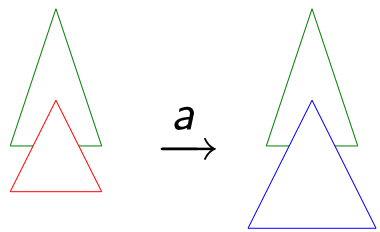

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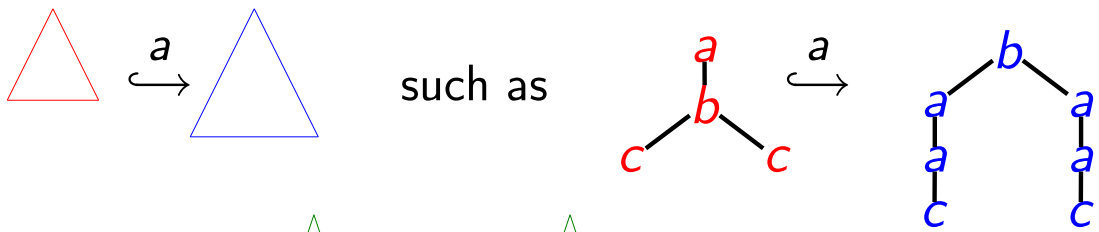

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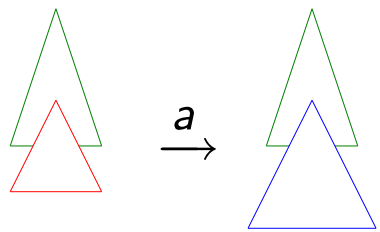

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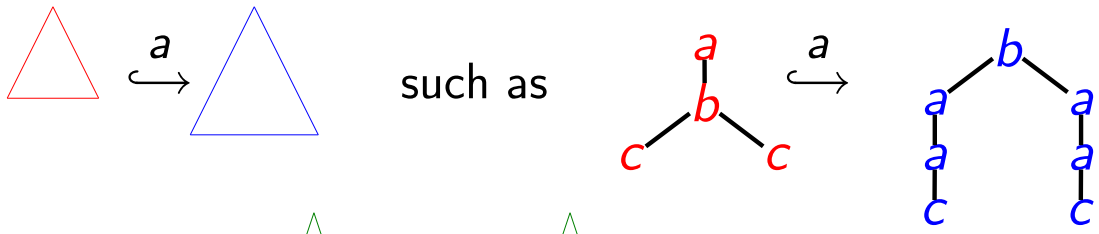

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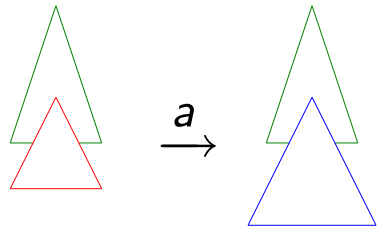

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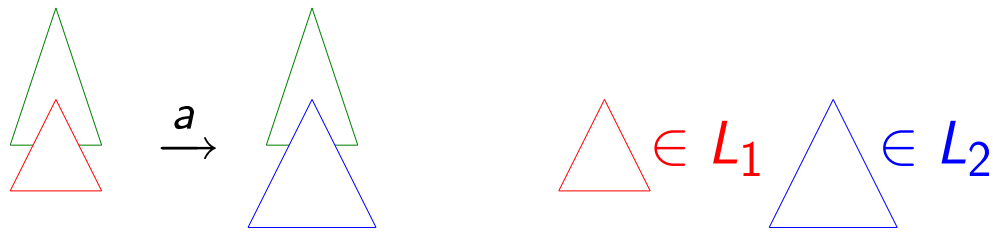

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- Main tool: Study model-checking problem of EF on **GTRS**.

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Formulas φ of the logic EF are given by the following grammar

$$\varphi \ ::= \quad \texttt{true} \quad | \quad \neg\varphi \quad | \quad \varphi \wedge \varphi \quad | \quad \mathsf{EX}_A\varphi \quad | \quad \mathsf{EF}\varphi,$$

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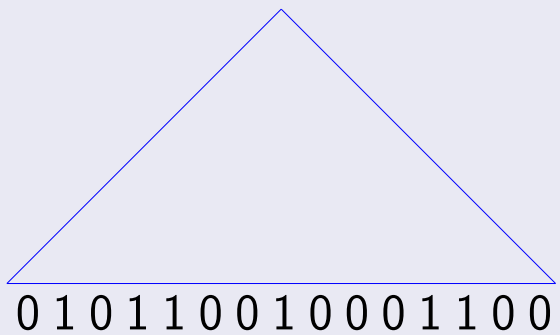

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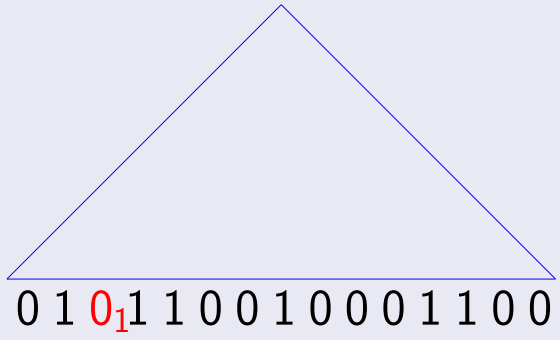

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EF **formula:**

EX_{a_1}

Rewrite rules:

$0 \overset{a_1}{\hookrightarrow} 0_1$ (and $1 \overset{a_1}{\hookrightarrow} 1_1$)

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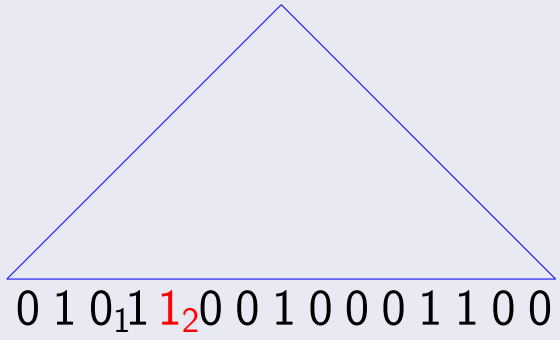

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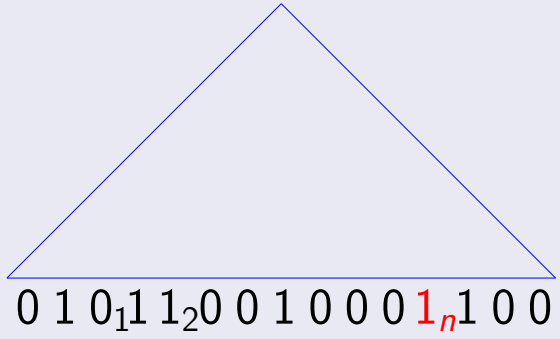

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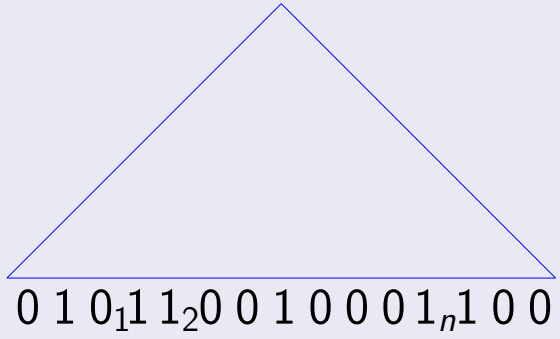

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$$\mathsf{EX}_{a_1}\mathsf{AX}_{a_2} \cdots \mathsf{AX}_{a_n}\mathsf{EF}\,\mathsf{EX}_{\mathsf{acc}}$$

Rewrite rules:

transitions of tree automaton accepting $[\![\psi]\!]$

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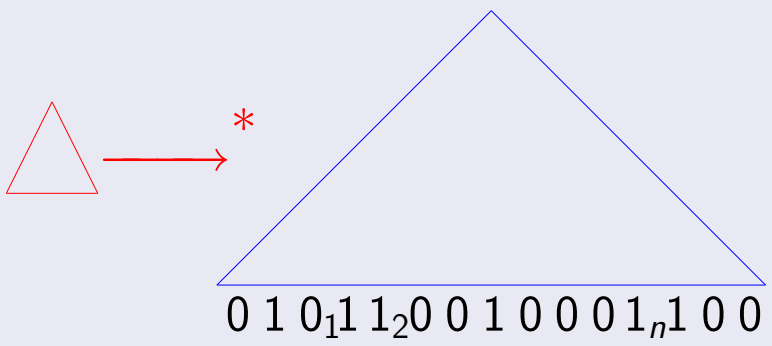

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  - Find the nonelementary border for EF.
  - **Theorem:** (Jančar, Kučera, Moller)
    Strong bisimilarity against finite systems is polytime-reducible to model checking EF_1.

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  How can one bound the index of $\simeq_i$?

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Our solution: Define relation $\equiv_i$ where $T_1 \equiv_i T_2$ iff T_1 and T_2 have the same number of subtrees of depth $\leq i \cdot \text{poly}(|G|)$ up to some threshold θ.

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Finally use $P^{NEXP} = \text{PSPACE}^{NEXP}$ (Hemaspaandra,Allender et al).

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Theorem

For a PA process P and a finite system F, one can decide in coNEXP *whether $P \sim F$.*

(gives a first elementary upper bound for this problem)

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Decidability and complexity

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Thanks for your attention