LICS 2011
Toronto, Canada

A decidable two-way logic on data words

Diego Figueira
U. Warsaw & U. Edinburgh

A data word	a	b	a	c	b	b	a	c	b	c	a
	3	1	7	7	1	3	1	6	5	1	5

A data word

$$\begin{array}{ccccccccccc} a & b & a & c & b & b & a & c & b & c & a \\ 3 & 1 & 7 & 7 & 1 & 3 & 1 & 6 & 5 & 1 & 5 \end{array} \in (\mathbb{A} \times \mathbb{D})^*$$

...word over a finite alphabet infinite domain

A data word

$$\begin{array}{ccccccccccc} a & b & a & c & b & b & a & c & b & c & a \\ 3 & 1 & 7 & 7 & 1 & 3 & 1 & 6 & 5 & 1 & 5 \end{array} \in (\mathbb{A} \times \mathbb{D})^*$$

...word over a finite alphabet infinite domain

Logics / automata for data words

high complexity / undecidable
or
very limited expressive power

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

Properties

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Properties

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Properties

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

Properties

$\boxed{\begin{matrix} a \\ 3 \end{matrix}}$	b	a	c	b	c	a	c	b	c	b
	1	7	7	3	3	1	6	7	1	1

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Properties

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1

b	a	b
d'	d	d'

$=$

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

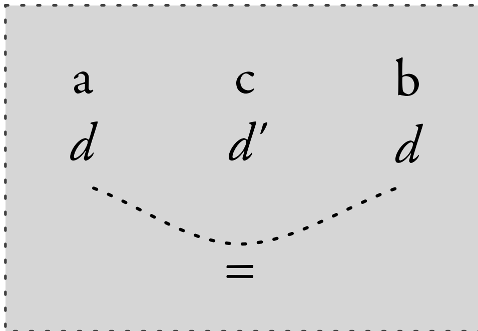
(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Properties

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1



(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Properties

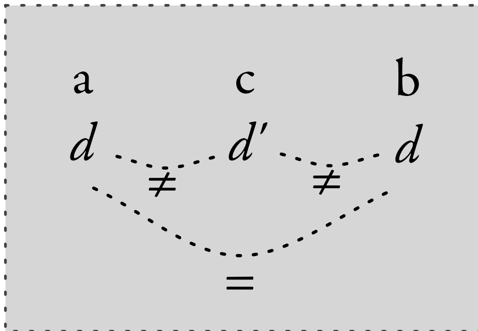
(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1



(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Properties

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

Properties

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Properties

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1

but... logics that can express these properties are undecidable

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

Properties

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1

but... logics that can express these properties are undecidable

(a)+(e) : non Primitive Recursive hard

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Properties

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1

but... logics that can express these properties are undecidable

(a)+(e) : non Primitive Recursive hard

(a) + (f) : non Primitive Recursive hard

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

Properties

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1

but... logics that can express these properties are undecidable

(a)+(e) : non Primitive Recursive hard

(a) + (f) : non Primitive Recursive hard

(a) + (c) + (e) : undecidable

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

Properties

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1

but... logics that can express these properties are undecidable

(a)+(e) : non Primitive Recursive hard

(a) + (f) : non Primitive Recursive hard

(a) + (c) + (e) : undecidable

(a) + (c) + (f) : undecidable

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

Properties

a	b	a	c	b	c	a	c	b	c	b
3	1	7	7	3	3	1	6	7	1	1

but... logics that can express these properties are undecidable

(a)+(e) : non Primitive Recursive hard

(a) + (f) : non Primitive Recursive hard

(a) + (c) + (e) : undecidable

(a) + (c) + (f) : undecidable

(a) + (c) + (h) : Petri Net reachability hard

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Logics for data words

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

$\mathrm{FO}^2(<, +1, \sim)$ [Bojańczyk et al.]

decidable, PN-reach hard

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Logics for data words

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

$\mathrm{FO}^2(<, +1, \sim)$ [Bojańczyk et al.]

decidable, PN-reach hard

$\mathrm{FO}^2(<, \sim)$

NExpTime-c

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Logics for data words

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

$\mathrm{FO}^2(<, +1, \sim)$ [Bojańczyk et al.]

decidable, PN-reach hard

$\mathrm{FO}^2(<, \sim)$

NExpTime-c

$\mathrm{LTL}^{\downarrow}(F, U, X)$ [Demri, Lazić]

decidable, non-PR hard

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Logics for data words

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

$\mathrm{FO}^2(<, +1, \sim)$ [Bojańczyk et al.]

decidable, PN-reach hard

$\mathrm{FO}^2(<, \sim)$

NExpTime-c

$\mathrm{LTL}^{\downarrow}(F, U, X)$ [Demri, Lazić]

decidable, non-PR hard

$\mathrm{LTL}^{\downarrow}(F)$ [F, Segoufin]

decidable, non-PR hard

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Logics for data words

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

$\mathrm{FO}^2(<, +1, \sim)$ [Bojańczyk et al.]

decidable, PN-reach hard

$\mathrm{FO}^2(<, \sim)$

NExpTime-c

$\mathrm{LTL}^{\downarrow}(F, U, X)$ [Demri, Lazić]

decidable, non-PR hard

$\mathrm{LTL}^{\downarrow}(F)$ [F, Segoufin]

decidable, non-PR hard

$\mathrm{LTL}^{\downarrow}(F, F^{-1})$ [F, Segoufin]

undecidable

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

XPath *on data words*

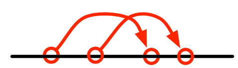
node expressions

$\varphi, \psi \ ::= \ \mathbf{a} \mid \neg\varphi \mid \varphi \wedge \psi \mid \alpha{=}\beta \mid \alpha{\neq}\beta \mid \alpha \qquad \mathbf{a} \in \mathcal{A}$

denote sets of positions

path expressions

$\alpha, \beta \ ::= \ \varepsilon \mid \alpha\, \beta \mid \alpha[\varphi] \mid o$



denote binary relations

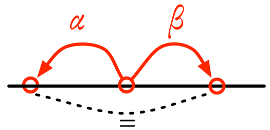
$$\boldsymbol{o} \in \{ \rightarrow, \ \rightarrow^{+}, \ \rightarrow^{*}, \ \leftarrow, \ {}^{+}\!\!\leftarrow, \ {}^{*}\!\!\leftarrow \}$$

XPath *on data words*

node expressions

$\varphi, \psi ::= \mathbf{a} \mid \neg\varphi \mid \varphi \wedge \psi \mid \alpha{=}\beta \mid \alpha{\neq}\beta \mid \alpha \qquad \mathbf{a} \in \mathbb{A}$

denote sets of positions



path expressions

$\alpha, \beta ::= \varepsilon \mid \alpha\,\beta \mid \alpha[\varphi] \mid o$

denote binary relations

$o \in \{ \rightarrow, \rightarrow^{+}, \rightarrow^{*}, \leftarrow, {}^{+}\!\!\leftarrow, {}^{*}\!\!\leftarrow \}$

XPath *on data words*

we note "XPath($\rightarrow, \rightarrow^*, {}^*\!\leftarrow$)"

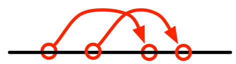
node expressions

$\varphi, \psi \ ::= \ \mathbf{a} \mid \neg\varphi \mid \varphi \wedge \psi \mid \alpha{=}\beta \mid \alpha{\neq}\beta \mid \alpha \qquad \mathbf{a} \in A$

denote sets of positions

path expressions

$\alpha, \beta \ ::= \ \varepsilon \mid \alpha\, \beta \mid \alpha[\varphi] \mid o$



denote binary relations

$$o \in \{ \rightarrow, \rightarrow^+, \rightarrow^*, \leftarrow, {}^+\!\leftarrow, {}^*\!\leftarrow \}$$

(a) For every **a** there is a future **b** with the same dv.

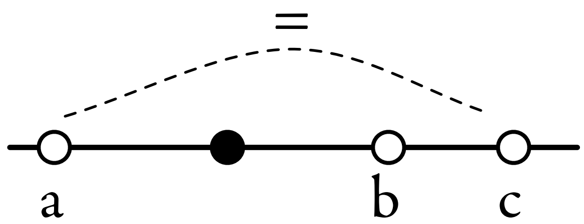
(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

E.g.

$$[\ [a]^{*}\!\leftarrow\ =\ \rightarrow^{*}[b] \rightarrow^{*}[c]\ ]$$



(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

E.g.

$$[\ [a]^{*}\!\leftarrow\ =\ \rightarrow^{*} [b] \rightarrow^{*} [c]\ ]$$

(a) for every position,

$$a \quad \Rightarrow \quad \varepsilon = \rightarrow^{*} [b]$$

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

E.g.

$$[\ [a]^{*}\!\leftarrow\ =\ \rightarrow^{*}[b]\rightarrow^{*}[c]\ ]$$

(a)

$$\neg \rightarrow^{*} [\ \neg(\ a \ \Rightarrow\ \varepsilon = \rightarrow^{*} [b]\ )\ ]$$

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

(a) For every **a** there is a future **b** with the same dv.

(b) Only one dv occurs under the label **a**.

(c) For every **a** there is a previous **b** with the same data value.

(d) Every **a** occurs in between two positions labeled with **b** with the same dv.

E.g.

$$[\ [a]^{*}\!\!\leftarrow\ =\ \rightarrow^{*}[b]\rightarrow^{*}[c]\ ]$$

(a)

$$\neg \rightarrow^{*} [\ \neg(\ a \ \Rightarrow\ \varepsilon = \rightarrow^{*} [b]\ )\ ]$$

(c) with XPath($^{*}\!\!\leftarrow$)

(d) with XPath($\rightarrow^{*}$, $^{*}\!\!\leftarrow$)

XPath($\rightarrow^{*}$, $^{*}\!\!\leftarrow$) cannot express (f), (g)

(b) with XPath($\rightarrow^{*}$)

(e) Between any **a** and **b** with the same dv there is exactly one **c**.

(f) For every **a** there is a future **b** with the same dv such that in between there is a **c** with a different dv.

(g) There are at least two elements in the word.

(h) The next element of an **a** is always a **b**.

Known results

XPath($\rightarrow^{+}, {}^{+}\!\!\leftarrow$) undecidable [F,Segoufin '10]

Known results

$\mathrm{XPath}(\rightarrow^{+}, {}^{+}\!\!\leftarrow)$ undecidable [F,Segoufin '10]

$\mathrm{XPath}(\rightarrow^{+}, {}^{*}\!\!\leftarrow)$ undecidable

Known results

XPath($\rightarrow^{+}, {}^{+}\!\leftarrow$) undecidable [F,Segoufin '10]

XPath($\rightarrow^{+}, {}^{*}\!\leftarrow$) undecidable

XPath($\rightarrow, \rightarrow^{*}, {}^{*}\!\leftarrow$) undecidable

Known results

XPath($\rightarrow^{+}$, ${}^{+}\!\leftarrow$)	undecidable	[F,Segoufin '10]
XPath($\rightarrow^{+}$, ${}^{*}\!\leftarrow$)	undecidable	
XPath($\rightarrow$, $\rightarrow^{*}$, ${}^{*}\!\leftarrow$)	undecidable	
XPath($\rightarrow^{+}$)	decidable in non-PR time	

Known results

$\mathrm{XPath}(\rightarrow^{+}, {}^{+}\!\leftarrow)$ undecidable [F,Segoufin '10]

$\mathrm{XPath}(\rightarrow^{+}, {}^{*}\!\leftarrow)$ undecidable

$\mathrm{XPath}(\rightarrow, \rightarrow^{*}, {}^{*}\!\leftarrow)$ undecidable

$\mathrm{XPath}(\rightarrow^{+})$ decidable in non-PR time

Our result

$\mathrm{XPath}(\rightarrow^{*}, {}^{*}\!\leftarrow)$

Known results

$\mathrm{XPath}(\rightarrow^{+}, {}^{+}\!\leftarrow)$	undecidable	[F,Segoufin '10]
$\mathrm{XPath}(\rightarrow^{+}, {}^{*}\!\leftarrow)$	undecidable	
$\mathrm{XPath}(\rightarrow, \rightarrow^{*}, {}^{*}\!\leftarrow)$	undecidable	
$\mathrm{XPath}(\rightarrow^{+})$	decidable in non-PR time	

Our result

$\mathrm{XPath}(\rightarrow^{*}, {}^{*}\!\leftarrow)$	decidable in 2ExpSpace (or ExpSpace)

Proof idea

$\phi :$
- there is only one dv under a **c**
- for every **a**, there is a **b** accessible via a **c** with the same dv
- there is a **c** with the same dv as the current position

e.g.	a	b	b	a	b	c	a	b	c	c	a	a	b	c	b
	1	2	4	4	3	1	5	1	1	1	4	4	5	1	4

$\models \phi$

Proof idea

$$\phi : \begin{cases} \text{there is only one dv under a } \mathbf{c} \\ \text{for every } \mathbf{a}\text{, there is a } \mathbf{b} \text{ accessible via a } \mathbf{c} \text{ with the same dv} \\ \text{there is a } \mathbf{c} \text{ with the same dv as the current position} \end{cases}$$

e.g.

a	b	b	a	b	c	a	b	c	c	a	a	b	c	b	$\models \phi$
1	2	4	4	3	1	5	1	1	1	4	4	5	1	4	

mosaic

- a label
- a data value
- for every dv d, all the paths that can access d

restricted to subpaths of $\phi = \{ \rightarrow^*[c]\rightarrow^*[b], \rightarrow^*[c], \rightarrow^*[b] \}$

Proof idea

ϕ :
- there is only one dv under a **c**
- for every **a**, there is a **b** accessible via a **c** with the same dv
- there is a **c** with the same dv as the current position

e.g.

$$\begin{array}{ccccccccccccccc} a & b & b & \boxed{a} & b & c & a & b & c & c & a & a & b & c & b \\ 1 & 2 & 4 & \boxed{4} & 3 & 1 & 5 & 1 & 1 & 1 & 4 & 4 & 5 & 1 & 4 \end{array} \models \phi$$

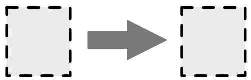
a	$1 : \{ \rightarrow^*[c]\rightarrow^*[b], \rightarrow^*[b], \rightarrow^*[c] \}$
4	$2 : \{ \ \}$
	$3 : \{ \rightarrow^*[b] \}$
	$\vdots$

mosaic
- a label
- a data value
- for every dv *d*, all the paths that can access *d*

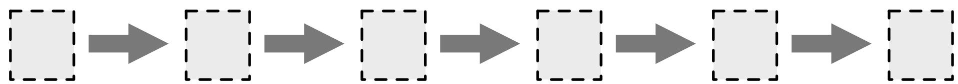
restricted to subpaths of $\phi = \{ \rightarrow^*[c]\rightarrow^*[b], \rightarrow^*[c], \rightarrow^*[b] \}$

Proof idea

ϕ : {
- there is only one dv under a **c**
- for every **a**, there is a **b** accessible via a **c** with the same dv
- there is a **c** with the same dv as the current position

a relation  if they can abstract consecutive positions in a word

$\leadsto$ An infinite transition system TS(ϕ) over mosaics.

ϕ is satisfiable $\Leftrightarrow$ there is 

in TS(ϕ) s.t. the first and last elements have certain conditions

Proof idea

ϕ : $\left\{\begin{array}{l}\text{there is only one dv under a } \mathbf{c} \\ \text{for every } \mathbf{a}\text{, there is a } \mathbf{b} \text{ accessible via a } \mathbf{c} \text{ with the same dv} \\ \text{there is a } \mathbf{c} \text{ with the same dv as the current position}\end{array}\right.$

e.g.

a	b	b	a	b	c	a	b	c	c	a	a	b	c	b
1	2	4	4	3	1	5	1	1	1	4	4	5	1	4

$\models \phi$

We can add a dv that simulates 4

Proof idea

$$\phi : \begin{cases} \text{there is only one dv under a } \textbf{c} \\ \text{for every } \textbf{a}\text{, there is a } \textbf{b} \text{ accessible via a } \textbf{c} \text{ with the same dv} \\ \text{there is a } \textbf{c} \text{ with the same dv as the current position} \end{cases}$$

e.g.	a	b	bbaab	c	a	b	c	c	aaaab	c	bb	$\models \phi$
	1	2	49493	1	5	1	1	1	49495	1	49	

We can add a dv that simulates 4

Proof idea

$$\phi : \begin{cases} \text{there is only one dv under a } \mathbf{c} \\ \text{for every } \mathbf{a}\text{, there is a } \mathbf{b} \text{ accessible via a } \mathbf{c} \text{ with the same dv} \\ \text{there is a } \mathbf{c} \text{ with the same dv as the current position} \end{cases}$$

e.g.

$$\begin{array}{cccccccccccc} \mathrm{a} & \mathrm{b} & \mathrm{bbaab} & \mathrm{c} & \mathrm{a} & \mathrm{b} & \mathrm{c} & \mathrm{c} & \mathrm{aaaab} & \mathrm{c} & \mathrm{bb} & \\ 1 & 2 & 49493 & 1 & 5 & 1 & 1 & 1 & 49495 & 1 & 49 & \models \phi \end{array}$$

We can add a dv that simulates 4

...but we cannot simulate 1.

Proof idea

ϕ : $\begin{cases} \text{there is only one dv under a } \mathbf{c} \\ \text{for every } \mathbf{a}\text{, there is a } \mathbf{b} \text{ accessible via a } \mathbf{c} \text{ with the same dv} \\ \text{there is a } \mathbf{c} \text{ with the same dv as the current position} \end{cases}$

e.g.

a	b	b b a a b	c	a	b	c	c	a a a a b	c	b b
1	2	4 9 4 9 3	1	5	1	1	1	4 9 4 9 5	1	4 9

$\models \phi$

We can add a dv that simulates $\boxed{4}$ $\longrightarrow$ flexible value

...but we cannot simulate $\boxed{1}$. $\longrightarrow$ rigid value

satisfaction of ϕ is closed under simulation of flexible values

rigid value : from some position it is the only dv accessed with a path α

Proof idea

ϕ :
- there is only one dv under a **c**
- for every **a**, there is a **b** accessible via a **c** with the same dv
- there is a **c** with the same dv as the current position

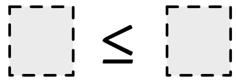
adding simulated values of flexible values: ≤

$\boxed{} \leq \boxed{}$

Proof idea

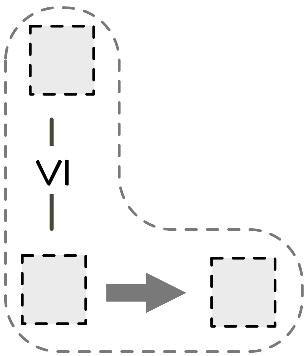
ϕ :
- there is only one dv under a **c**
- for every **a**, there is a **b** accessible via a **c** with the same dv
- there is a **c** with the same dv as the current position

adding simulated values of flexible values: ≤



A monotonicity property:

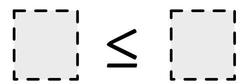
∀



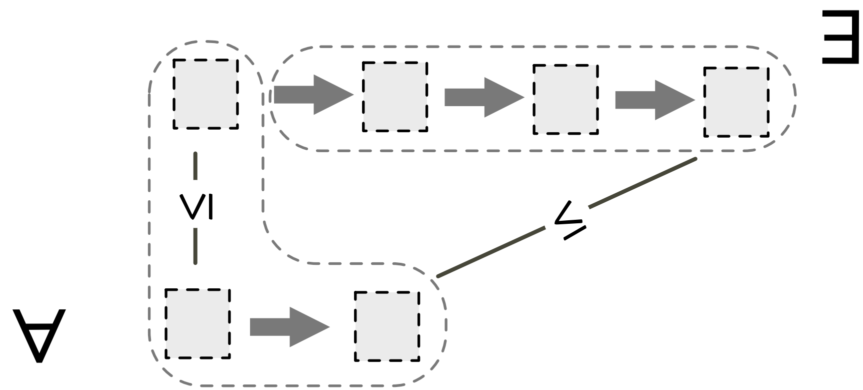
Proof idea

ϕ : { there is only one dv under a **c**
for every **a**, there is a **b** accessible via a **c** with the same dv
there is a **c** with the same dv as the current position }

adding simulated values of flexible values: ≤



A monotonicity property:



Proof idea

$$\phi : \begin{cases} \text{there is only one dv under a } \mathbf{c} \\ \text{for every } \mathbf{a}\text{, there is a } \mathbf{b} \text{ accessible via a } \mathbf{c} \text{ with the same dv} \\ \text{there is a } \mathbf{c} \text{ with the same dv as the current position} \end{cases}$$

adding simulated values of flexible values: ≤ $\boxed{} \leq \boxed{}$

we only need to consider ≤-minimal mosaics

there are boundedly many (since there are boundedly many rigid values)

we reduce to a derivation problem for a *finite* transition system

Future work XPath on data trees

Decidability/complexity of...

$\mathrm{XPath}(\downarrow,\downarrow_*,\rightarrow^*,{}^*\!\leftarrow)$?

(XPath($\downarrow,\downarrow_*$) in ExpTime)

[F '09]

$\mathrm{XPath}(\downarrow,\downarrow_*,\uparrow^*,\rightarrow^*,{}^*\!\leftarrow)$?

$\mathrm{XPath}(\downarrow,\downarrow_*,\uparrow,\uparrow^*,\rightarrow^*,{}^*\!\leftarrow)$?

(XPath($\downarrow,\downarrow_*,\uparrow,\uparrow^*$) decidable)

[F,Segoufin '11]

Future work XPath on data trees

Decidability/complexity of...

XPath($\downarrow, \downarrow_*, \rightarrow^*, {}^*\!\leftarrow$) ?

XPath($\downarrow, \downarrow_*, \uparrow^*, \rightarrow^*, {}^*\!\leftarrow$) ?

(XPath($\downarrow, \downarrow_*$) in ExpTime)

[F '09]

XPath($\downarrow, \downarrow_*, \uparrow, \uparrow^*, \rightarrow^*, {}^*\!\leftarrow$) ?

(XPath($\downarrow, \downarrow_*, \uparrow, \uparrow^*$) decidable)

[F,Segoufin '11]

thank you