Reversibility of Time-Series

Mapleridge Capital Corporation

FWIPW10

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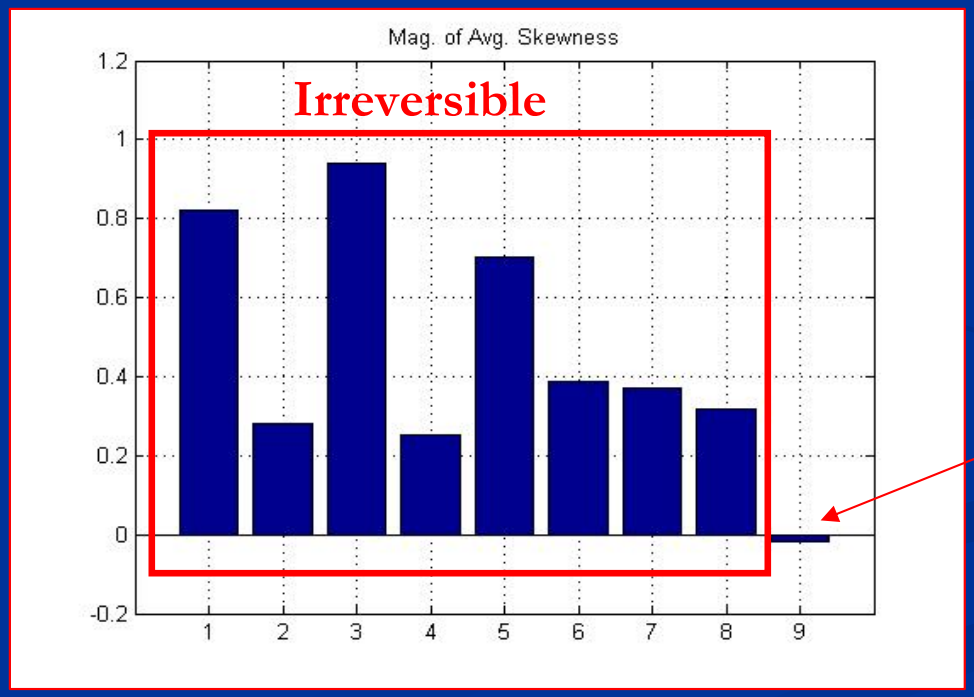
Objectives and Approaches

- **Part 1:** Determine if a time-series is reversible.
  - Skewness of log-increments (Returns)
  - Higher-Order Cumulants
  - Kernel-based Canonical Analysis
- **Part 2:** Determine the direction of time in financial datasets.
  - Skewness of Changes in Volatility
  - Multi-resolution Detection of Local Time-Asymmetry
  - Ratio of Market Activities
  - Causality in Volatilities and Market Activities
- Determine the time-reversibility of the GARCH model.

Part 1: Determine if a time-series is reversible

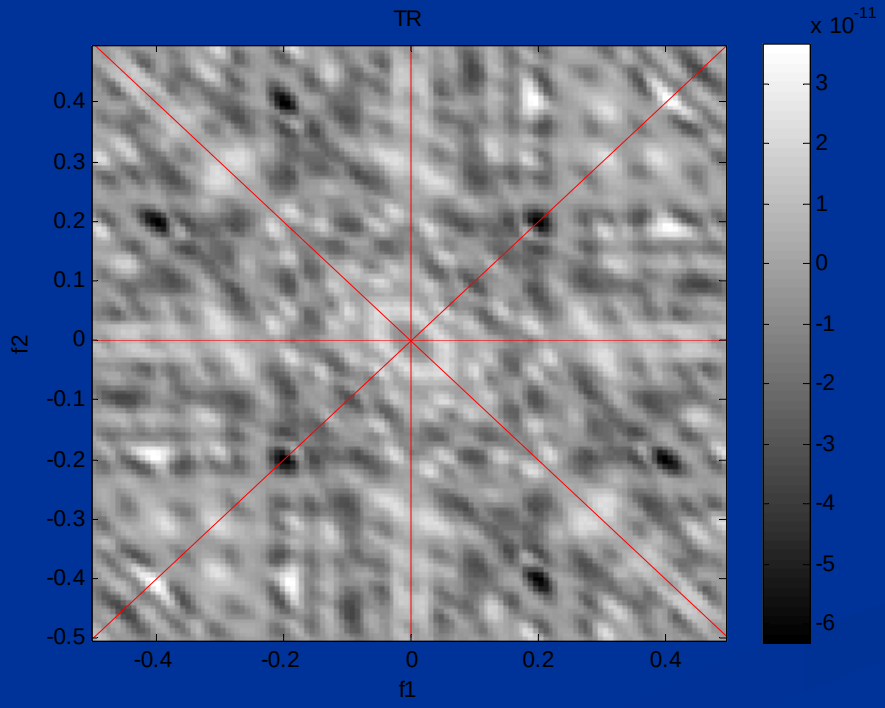
Skewness of log-increments

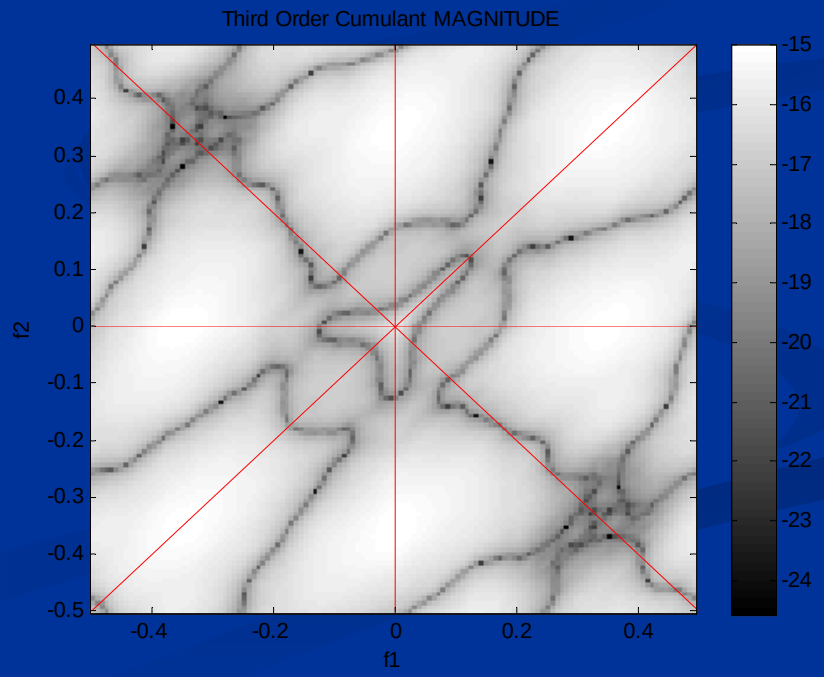
- Perform well for detecting Time-Reversibility
- Time irreversible series tends to have strong skewness.



Higher Order Statistics

- A time-series is time-reversible iff all high-order cumulants are time-symmetric;
- Not useful for determining the arrow of time.





Kernel-Based Canonical Analysis

- Decomposing the joint density f_h of (X_t, X_{t-h})

$$f_h(x_t, x_{t-h}) = f(x_t)f(x_{t-h})[1 + \sum_{i=1}^{\infty} \lambda_i \varphi_i(x_t)\psi_i(x_{t-h})]$$

- λ_i are the *canonical correlations* where $\lambda_1 \geqslant \lambda_2 \geqslant ... \geqslant 0$
- $\varphi_i(x_t), \psi_i(x_{t-h})$ are the *canonical directions* that satisfy some orthogonality conditions

Joint Density Estimator

- unknown joint density $f \Rightarrow$ define a kernel-based estimator $\hat{f}$

1. Unconstrained

$$\hat{f}_N(x,y) = \frac{1}{N}\sum_{1}^{N}\frac{1}{h^2}K(\frac{X_n - x}{h})K(\frac{Y_n - y}{h})$$

2. Constrained by reversibility condition
$f_h(x_t, x_{t-h}) = f_h(x_{t-h}, x_t)$

$$\hat{f}_N^R(x,y) = \frac{1}{2Nh^2}\sum_{1}^{N}K(\frac{X_n - x}{h})K(\frac{Y_n - y}{h}) + K(\frac{X_n - y}{h})K(\frac{Y_n - x}{h})$$

Canonical Correlation and Direction Estimators

- Select a Gaussian kernel K with bandwidth h
- Using the kernel estimators of the density functions $\hat{f}_N$ and $\hat{f}_N^R$ we solve the spectral problem

$$\frac{1}{2}(T^* + T)\hat{\varphi}^R = \hat{\lambda}^R \hat{\varphi}^R \Rightarrow \hat{\lambda}^R, \hat{\varphi}^R \quad (1)$$

$$T\hat{\varphi} = \hat{\lambda}\hat{\varphi} \Rightarrow \hat{\lambda}, \hat{\varphi} \quad (2)$$

$$T\varphi(y) = \int \varphi(x) \frac{\hat{f}_N(x,y)}{\hat{f}_N(.,y)} dx \quad T^*\varphi(x) = \int \varphi(y) \frac{\hat{f}_N(x,y)}{\hat{f}_N(x,.)} dy$$

- To prove *time reversibility* we want the correlation and direction estimators of the two cases to be close to each other
- By studying the correlations of 2 time-reversed series we might be able to determine *time directionality*

Part 2:
Determine the direction of time in financial datasets

Skewness of Changes in Volatility

- The rise and fall of volatilities in market prices tends to be asymmetric. I.e. rises quickly and decays slowly. This positively skews the distribution of changes in volatility.
- Detection rule is simple in this case, if the skewness of the change in volatility is positive, then the time-series is flowing forward.

Decision Rule: $$\sum_{k=1}^{3} skew(x_k) > 0$$

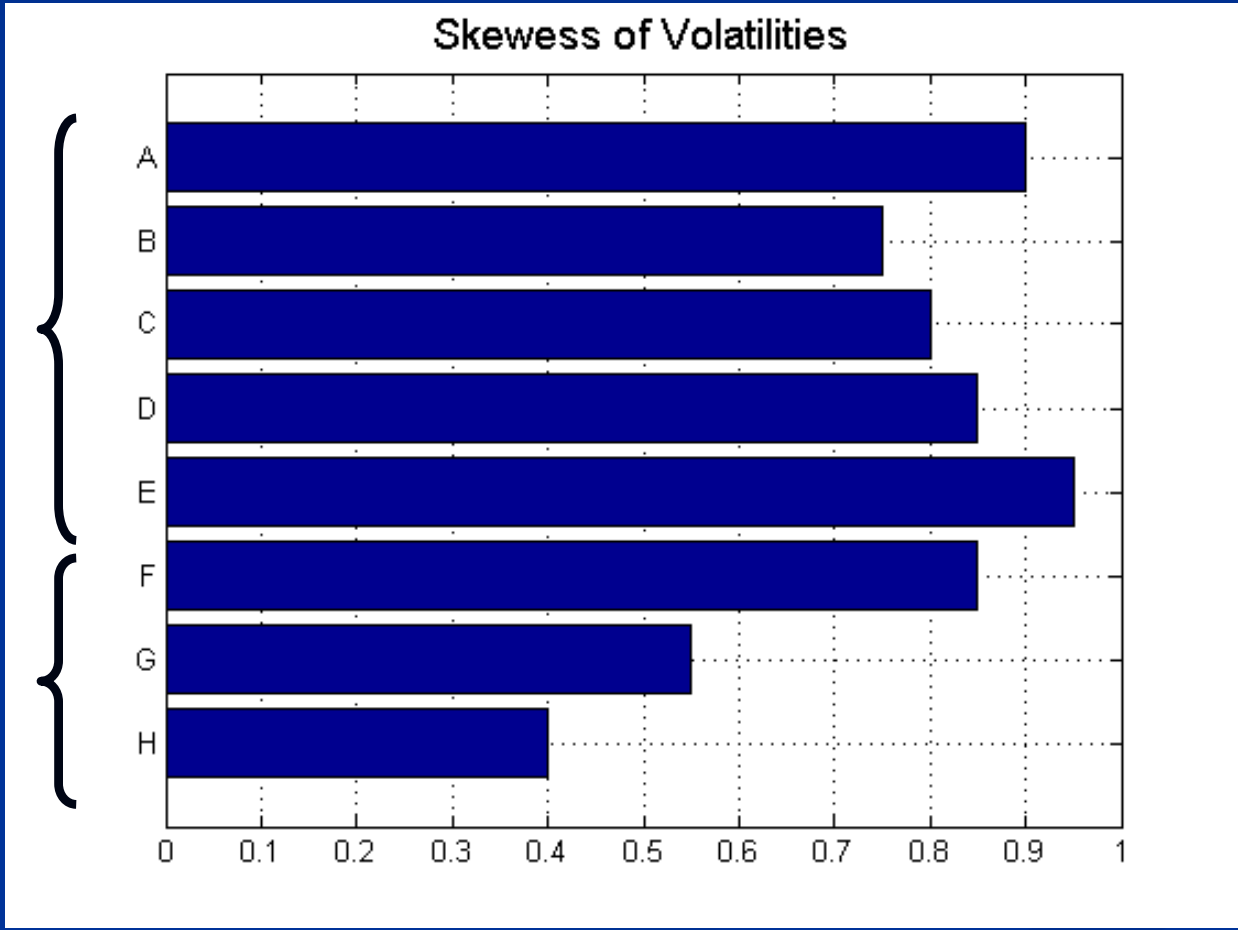
Where: $$x_k = \ln \frac{r_i + \varepsilon}{r_{i-k} + \varepsilon}$$ **and** $$r_i = h_i - l_i$$

Skewness of Changes in Volatility

Overall Performance

Future Contracts

Equities

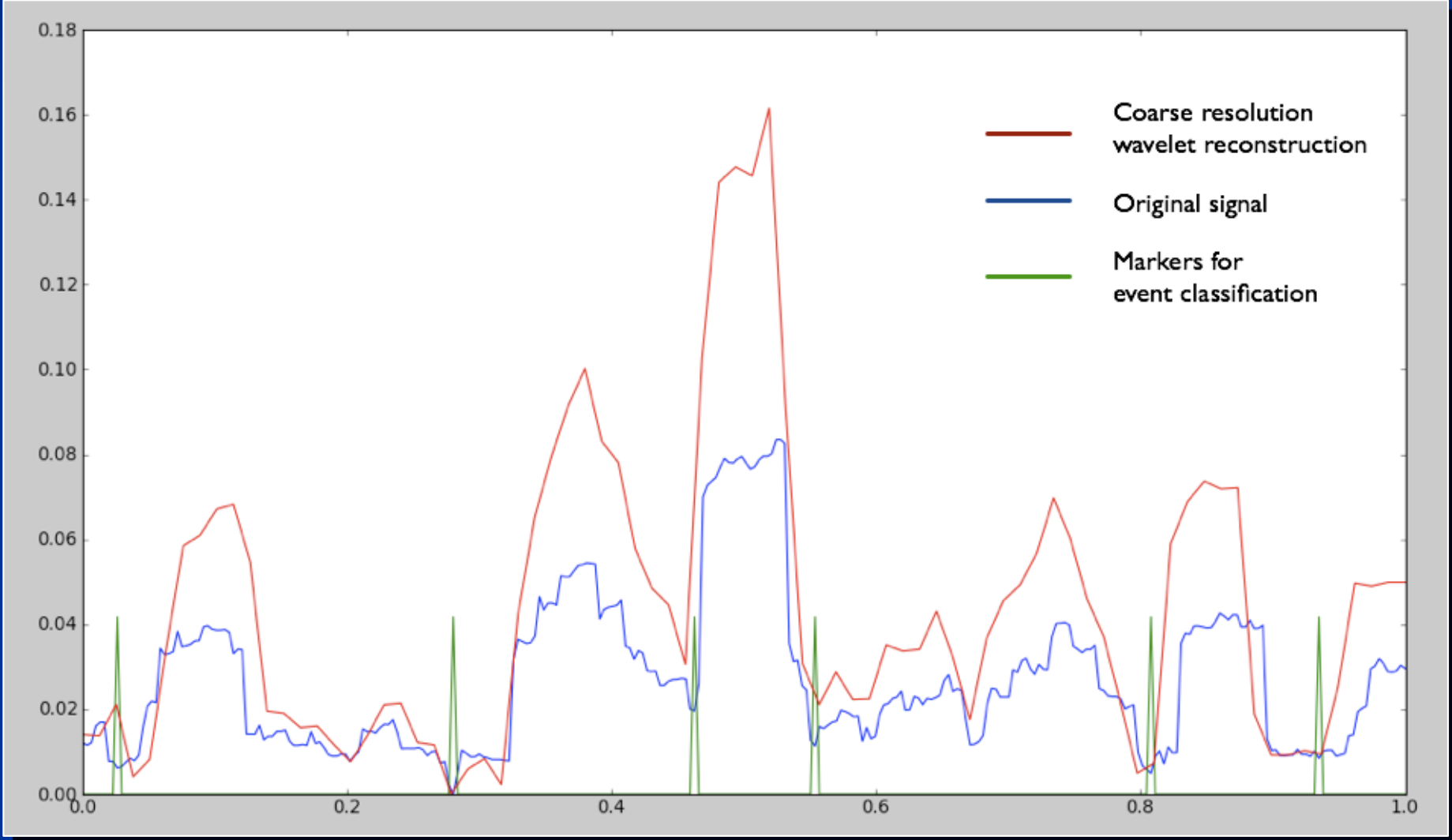


Multi-resolution Detection of Local Time-Asymmetry

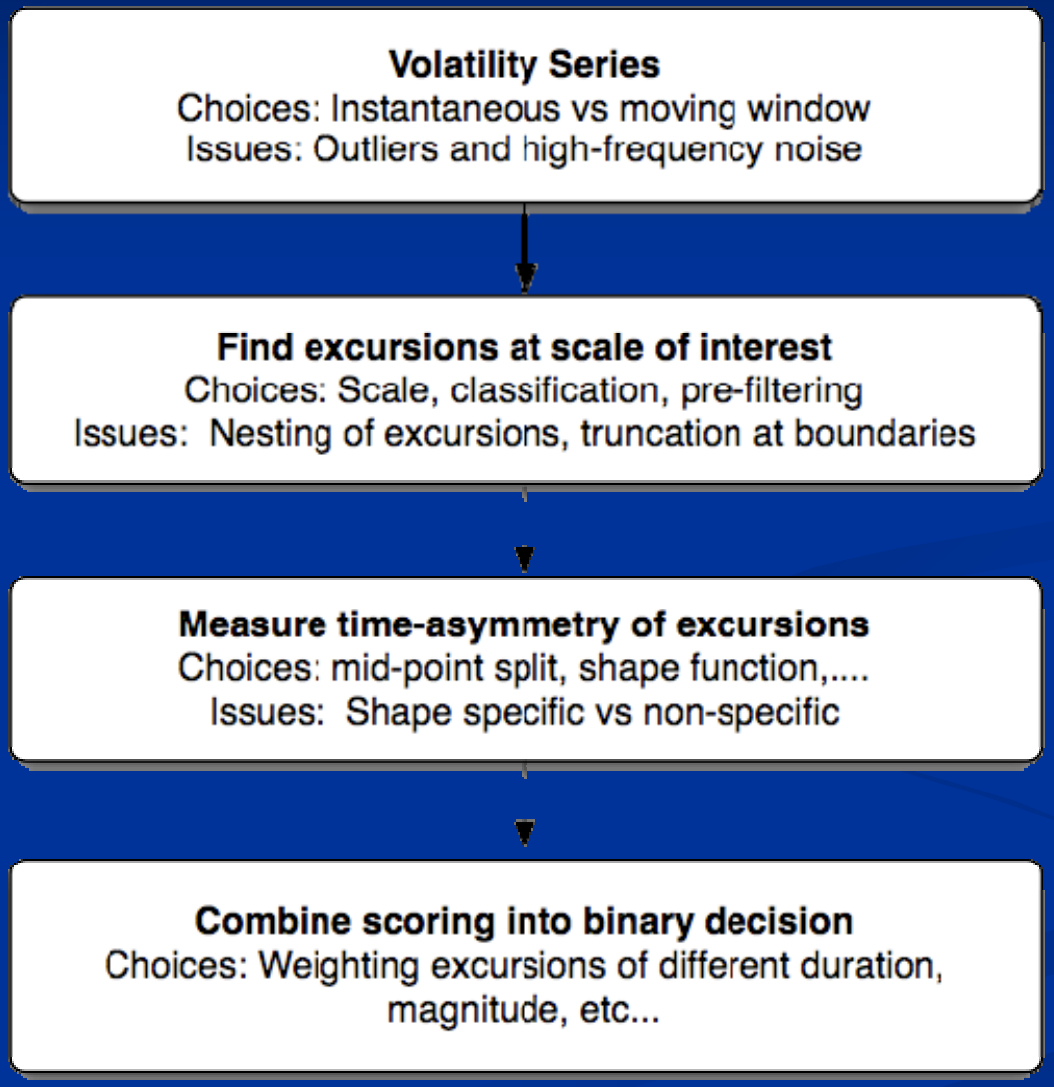
- **General** idea: Conjecture time-asymmetry is due to market processes with a specific signature in time-series data or its transforms.
- **Specific** idea: Look for spike + tail pattern in volatility excursions.
- **Follow-on** idea: Ask how "scale-invariant" this signature is.
- **Method** applied: A basic scale-specific pattern detection/local asymmetry test on suitably filtered data

Classifying Volatility Excursions

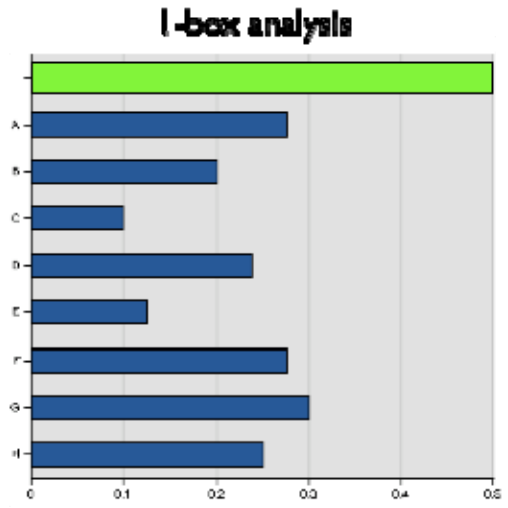
Example: 5-box analysis

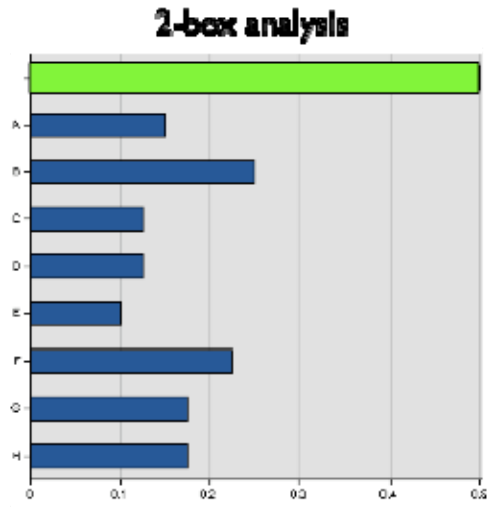


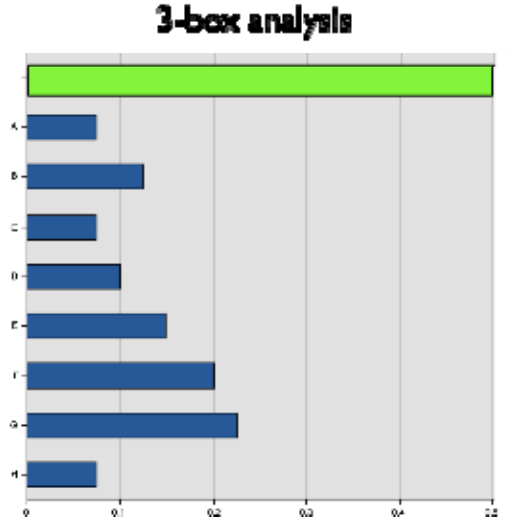
Algorithm Block Diagram

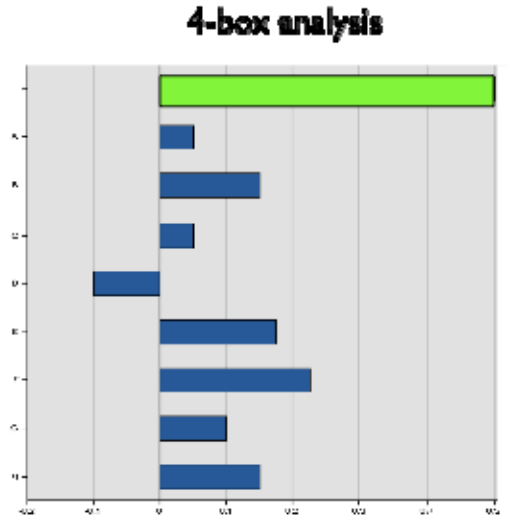


Results









Next Steps

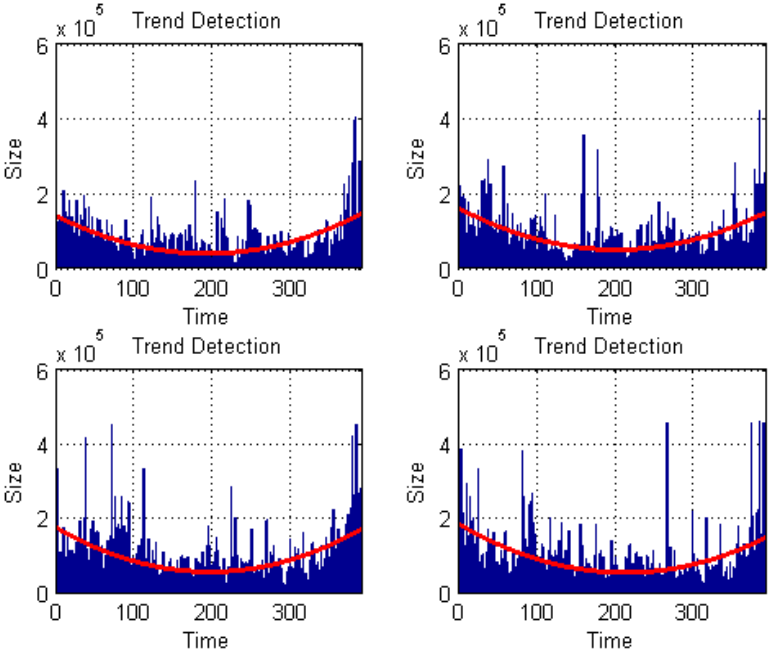
- Large “outlier” spikes in marginal performance sets - easily remedied.
- If volatility is (at least partially) driven by different shocks of new information, effect of large/longer shocks should be removed from “nested” shock.
- Same reasoning suggests known long-term trends should be removed in deciding asymmetry quesiton for shorter excursions.
- Allowing different decision parameters to vary with market type may reveal heterogeneity in volatility behavior.

Ratio of market activities

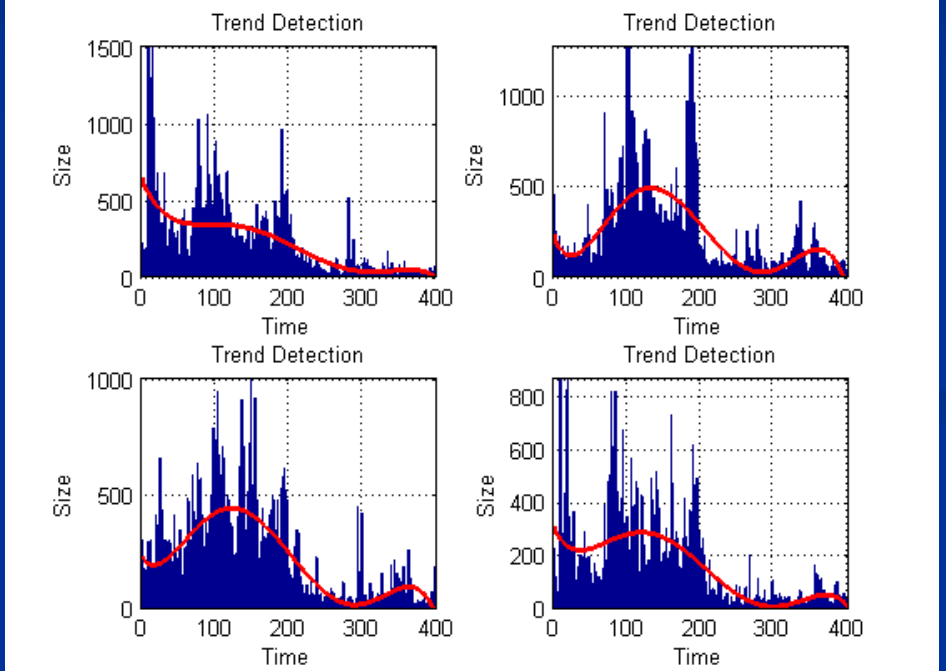
- Focuses on **traded volume**.
- Market activities tends to be heaviest shortly after the market opening and shortly before the market closing.
- **Idea:** is to test whether there is a discernible pattern in the detrended market activities that can provide the direction of time.

Ratio of market activities

Equity



Energy Futures

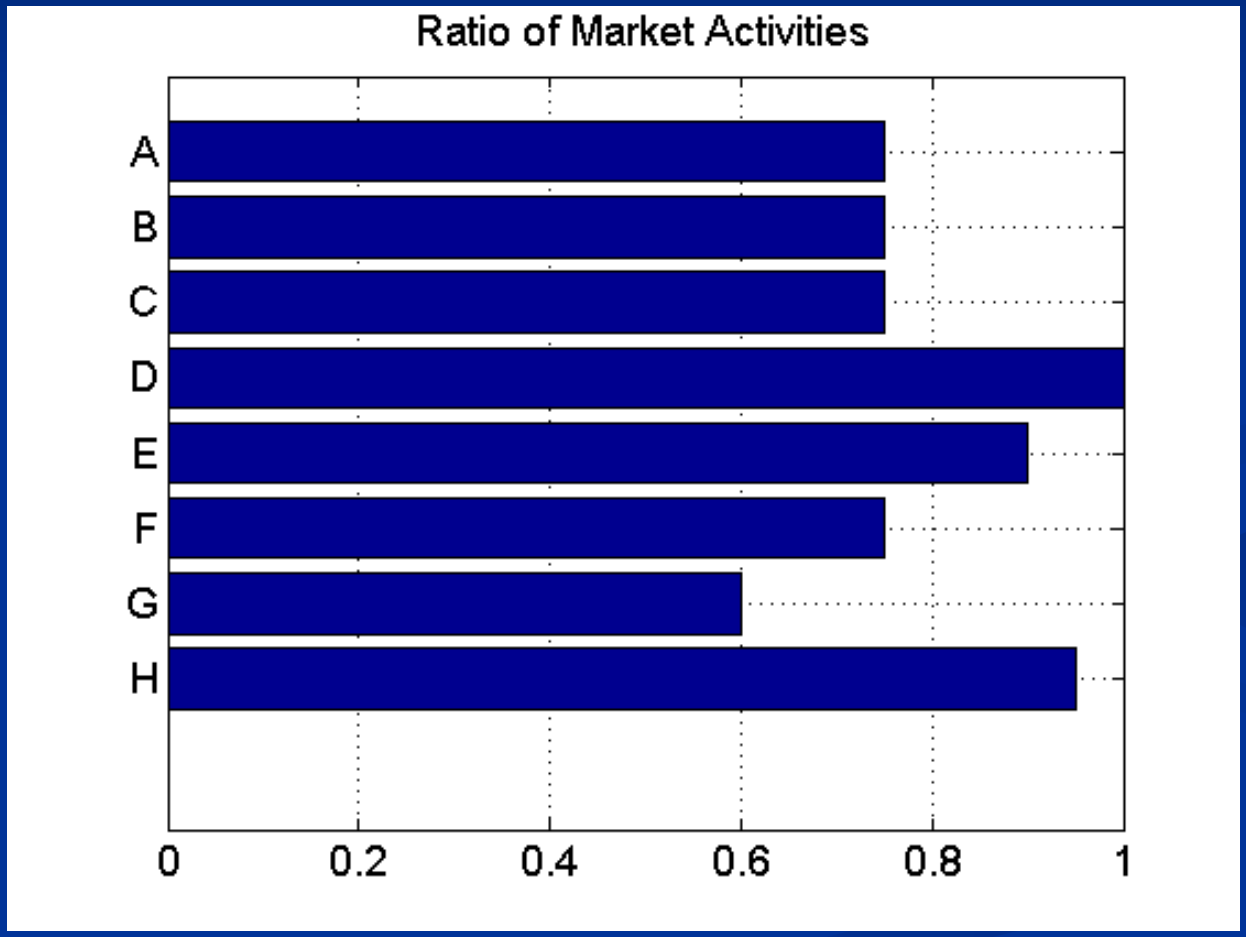


Decision Rule:

$$\operatorname{sgn}\left(\ln\frac{v_o}{v_c}\right)_{Test} \bullet \operatorname{sgn}\left(\ln\frac{v_o}{v_c}\right)_{Training} > 0$$

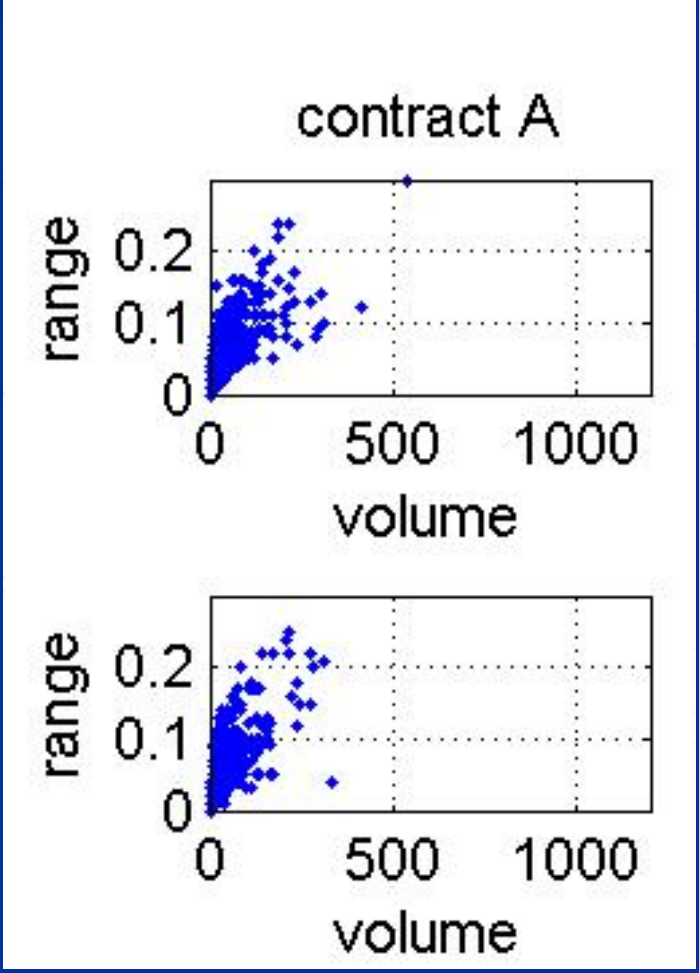
Where v_o and v_c are the variance of the detrended traded volume during market opening and closing.

Ratio of market activities

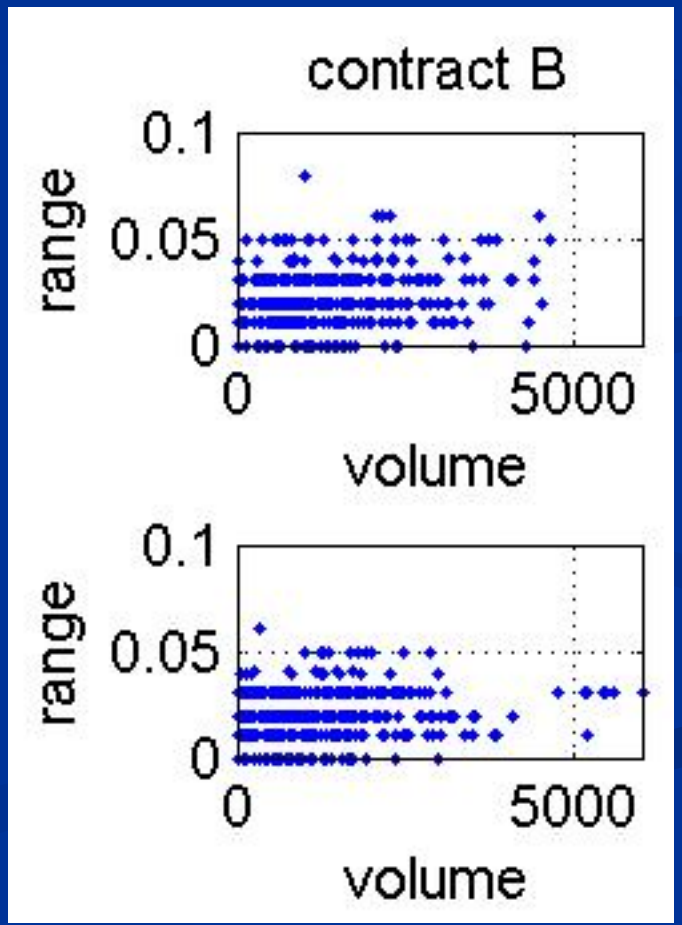


Causality Between Volatilities and Market Activities

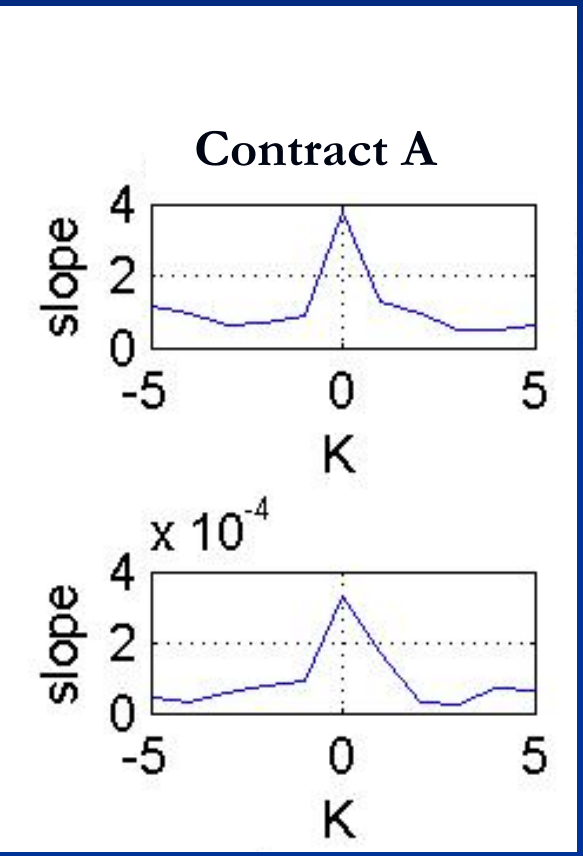
Relationship between volatility and volumes



Relationship between range(t) and volume(t+K), here K=1



Relationship between robust regression slope and time shift K



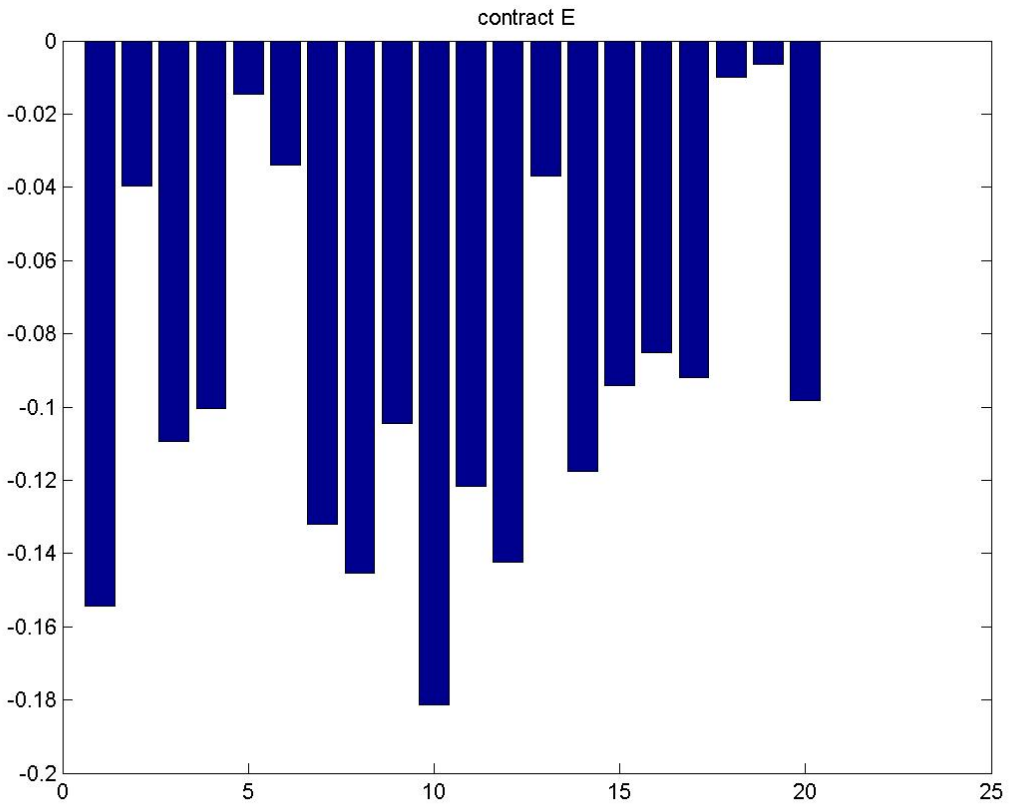
We proposed two testing method here to test the relationship between asymmetry and time orientation.

- First, we find the integration of positive and negative side and compare who is bigger.
- Second, we only compare the slope when K=1 and K=-1

Decision Rule

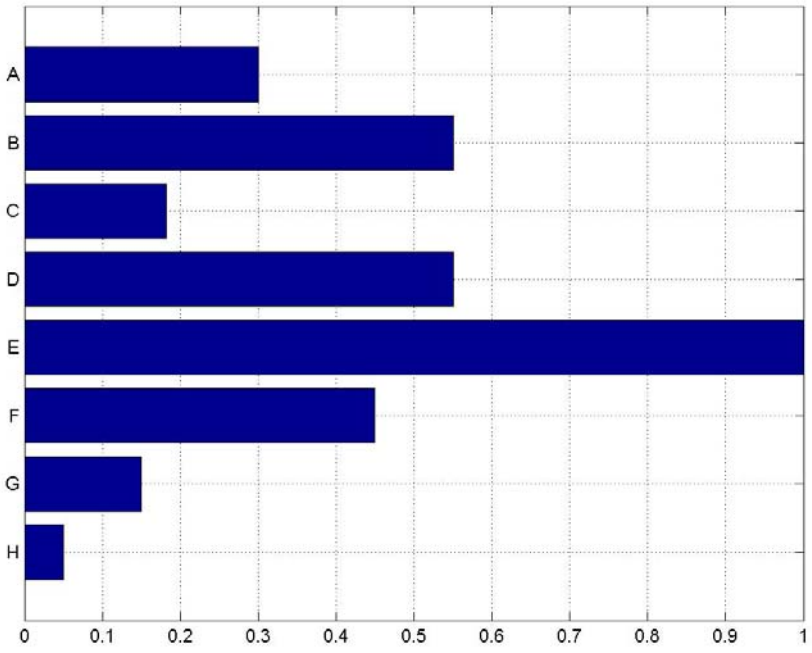
- Decision rules for the first method:
- Determine the majority of the log value is bigger or smaller than 0. (suppose it is smaller)
- Determine the log value of left integration divided by right integration. If it is bigger than 0, then we say the time series is flipped.

Sample Result

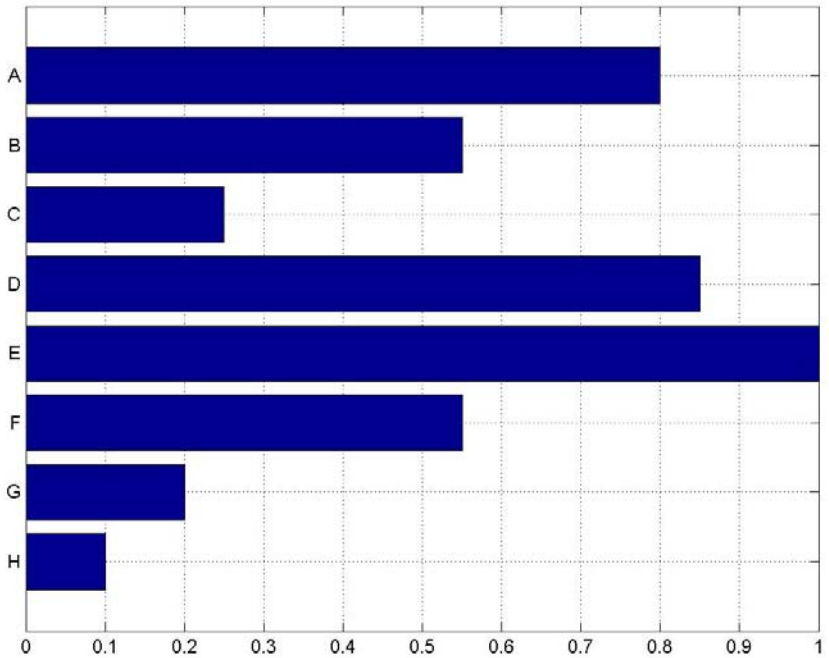


$$\ln\left(\frac{A_L}{A_R}\right)$$

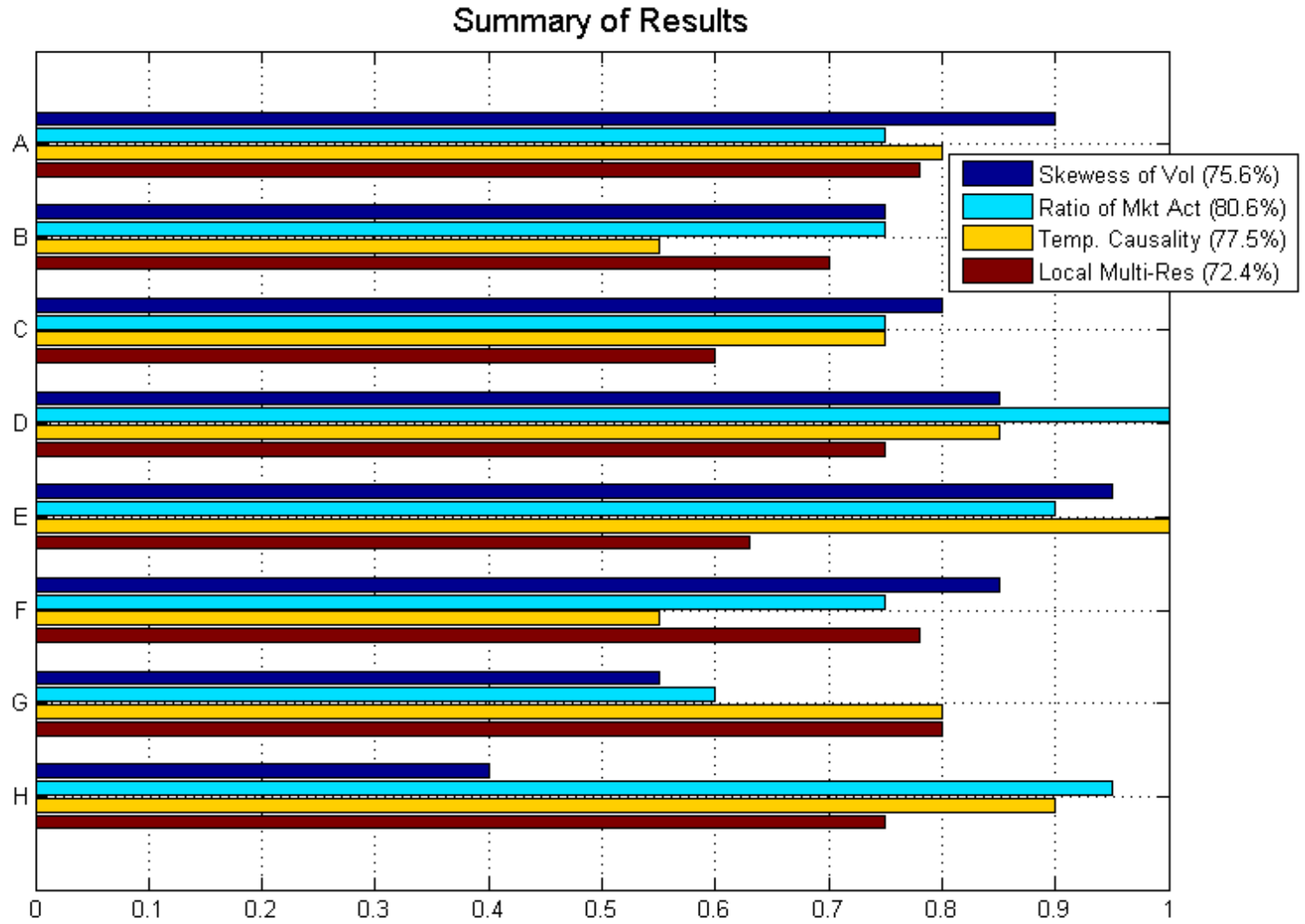
The ratio of left integration being smaller than right integration



The ratio of the slope at K=1 is greater than at K=-1



Summary of Results



Conclusion

- Developed / discovered three approaches for determining time-reversibility;
- Four approaches for determining the directions of time;
- Gain further understanding the fine structures in financial time-series.