Zero discord

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August 2009

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$\boxed{\rho_i}$

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Quantum measurements on a bipartite system

A classical bipartite system

Alice

$p_1^A..p_n^A$

Bob

$p_1^B..p_n^B$

Entropy: $H(A) = \sum_i p_i^A log_2 p_i^A$, $H(A,B) = \sum_i p_i^{A,B} log_2 p_i^{A,B}$

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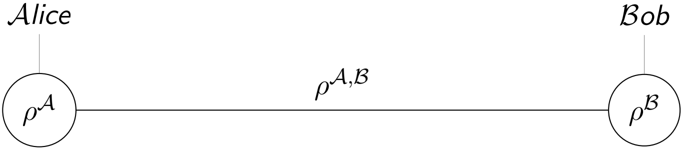

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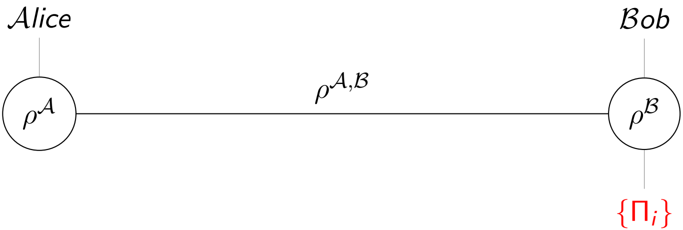

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Conditional Entropy: $H(\mathcal{A}|\mathcal{B}_{\{\Pi_i^{\mathcal{B}}\}}) = \sum_i p(\Pi_i^{\mathcal{B}}) H(A|\Pi_i^{\mathcal{B}})$

Mutual information

Classical Mutual information, two equivalent expressions

$$\mathcal{I}(A:B) = H(A) + H(B) - H(A,B) \tag{1}$$

$$\mathfrak{J}(A:B) = H(A) - H(A|B) \tag{2}$$

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$$\mathfrak{J}(\mathcal{A}:\mathcal{B})_{\{\Pi_j^{\mathcal{B}}\}} = H(\mathcal{A}) - H(\mathcal{A}|\{\Pi_j^{\mathcal{B}}\}) \tag{3}$$

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Discord [1]

$$\boxed{\delta(\mathcal{A}:\mathcal{B})_{\{\Pi_j^{\mathcal{B}}\}} = \mathcal{I} - \mathfrak{J}(\mathcal{A}:\mathcal{B})_{\{\Pi_j^{\mathcal{B}}\}}}$$

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Discord

- Discord is the difference in the MI before and after a measurement
- The expression (3) can be used to measure classical correlation[2]
- Usually some sort of optimization over all possible local measurements is used to get the Discord

$$\delta(\mathcal{A}:\mathcal{B})_{\{\Pi_j^{\mathcal{B}}\}} = min_{\{\Pi_j^{\mathcal{B}}\}}[H(\mathcal{B}) - H(\mathcal{A},\mathcal{B}) + H(\mathcal{A}|\{\Pi_j^{\mathcal{B}}\})] \quad (4)$$

- Separable systems can have non zero discord.
- Discord is not symmetric

[2] L. Henderson and V. Vedral, J Phys 34 (2001).

Examples of discord

1. $\boxed{\frac{1}{2}[\left|00\right\rangle\left\langle 00\right| + \left|1+\right\rangle\left\langle 1+\right|]}$

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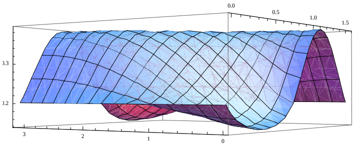
2. $\boxed{\frac{1}{2}[|00\rangle\langle 00| + |++\rangle\langle ++|]}$ $\delta(\mathcal{A}:\mathcal{B}) = \delta(\mathcal{B}:\mathcal{A}) \approx 0.14$

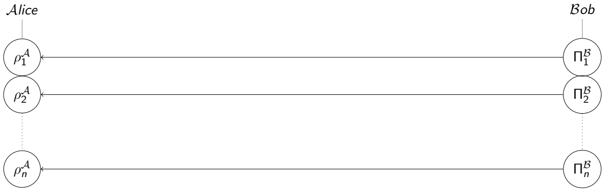


Using discord

- Measure of classical correlations.
- Quantum Maxwell's demons - Zurek (2003), Oppenheim, Horodecki, Horodecki, and Horodecki (2002).
- Correlations in DQC1 - Datta,Shaji and Caves (2008)
- The most interesting results involve zero discord.
- Zero discord defines a Pointer basis.
- CP evolution - Jordan, Shaji and Sudarshan (2007)

A zero discord systems can be as a sum of (orthogonal) systems of the type

$$\rho_{\mathcal{A},\mathcal{B}} = \sum_j a_j \rho_j^{\mathcal{A}} \otimes \Pi^{\mathcal{B}} \tag{5}$$



Zero discord between a system and the environment is essential for CP evolution.[3]

[3] Shabani and Lidar PRL 102 (2009)

Calculating zero discord

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- Calculating discord requires optimization. **It's hard work**
- The marginals are diagonal in the zero discord basis.

$$\rho^{\mathcal{B}} = \begin{pmatrix} a_1 & 0 & \dots & 0 \\ 0 & a_2 & \dots & 0 \\ . & \dots & \dots & 0 \\ \vdots & & a_j & \\ 0 & \dots & \dots & a_d \end{pmatrix} \tag{6}$$

Calculating zero discord

Given a density matrix $\rho^{\mathcal{AB}}$

1. Find the marginal $\rho^{\mathcal{B}} = tr_{\mathcal{A}}\rho^{\mathcal{AB}}$
2. Find the eigenstates $|j\rangle^{\mathcal{B}}$
3. Use the eigenstates to build a projector basis $\Pi_j^{\mathcal{B}} = |j\rangle^{\mathcal{B}} \langle j|$
4. Calculate the discord in this basis
   $\delta(\mathcal{A}:\mathcal{B})_{\{\Pi_j^{\mathcal{B}}\}} = H(\mathcal{B}) - H(\mathcal{A},\mathcal{B}) + H(\mathcal{A}|\{\Pi_j^{\mathcal{B}}\})$
5. If and only if $\delta(\mathcal{A}:\mathcal{B})_{\Pi_j^{\mathcal{B}}} = 0$, the state is a zero discord state.

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Does not work for maximally mixed marginals :(

Locally distinguishable states

Non-locality without entanglement[4]

[4] Bennett et al PRA 59 (1999)

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Examples:

State	Discord	Distinguishability
Equal mixture of locally orthogonal states	DZero	Local
Equal mixture of the 9 Bennett states	DZero	Non local
$\lvert 00\rangle\langle 00\rvert$ and $\lvert ++\rangle\langle ++\rvert$	Non zero	Impossible
A singlet and a $\lvert 11\rangle$ state	Non zero	Local

[4] Bennett et al PRA 59 (1999)

Conclusions

1. Discord is a measure of quantum correlations.
2. Zero discord can be verified easily using the local diagonal basis.
3. There is no simple way to define distinguishability using discord.
4. Completely mixed marginals are strange.

The end

Questions?