Flow Analysis and Design: Issues and Challenges in CFD (Accuracy, Reliability, and Uncertainty)

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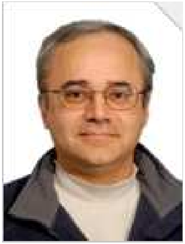
Fields Institute Optimization Seminar
Toronto, November 3rd , 2009

Outline

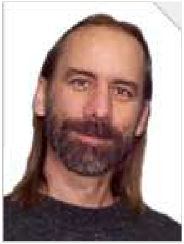
GRMIAO: leaders in CFD since 1984

R. Camarero

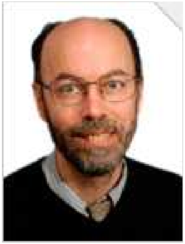


A. Garon

F. Guibault



B Ozell



D Pelletier

J-Y Trepanier

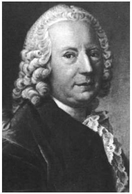
In the beginning ... the fluid

Héraclite
(536-470 AD)
Everything flows

Archimède
(287 - 212 AD)
Eureka

De Vinci
De moto dell'acqua

Newton
$\boldsymbol{F} = m\boldsymbol{a}$



D. Bernouilli
(1700-1782)
$p + \frac{1}{2}U^2 = const$

Euler
(1785 - 1836)

Navier
(1707 - 1783)

Stokes
(1818 - 1903)

Optimization problem

- Find design parameters α^* such as

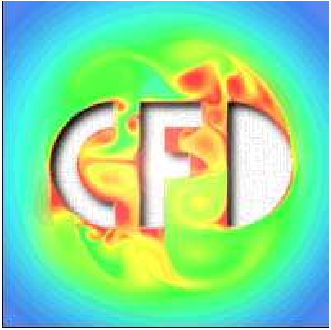
$$\mathcal{J}(\boldsymbol{U}(\alpha^*), \alpha^*) \leq \min_{\alpha} \mathcal{J}(\boldsymbol{U}(\alpha), \alpha)$$
$$\text{with } \boldsymbol{G}(\boldsymbol{U}, \alpha) = 0$$

- $\boldsymbol{G}$ = Navier-Stokes Equations

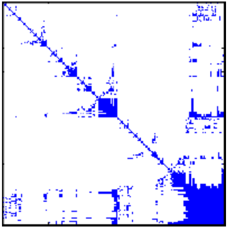
$$\begin{aligned} \text{Continuity:} \quad & \nabla \cdot \boldsymbol{u} = 0 \\ \text{Momentum:} \quad & \rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \nabla \cdot \boldsymbol{\tau}(\boldsymbol{u}) + \boldsymbol{f} \end{aligned}$$

$$\text{with } \boldsymbol{\tau}(\boldsymbol{u}) = \mu \left(\nabla \boldsymbol{u} + \nabla^T \boldsymbol{u} \right)$$

CFD: What is it?



CFD: Art of replacing fluid flows PDEs (impossible to solve) by a huge easier to solve $\boldsymbol{Ax} = \boldsymbol{b}$.



CFD : Where does it stand?

Analyses and design proceed by:

- Experimentation, intuition and empiricism
  - Wind tunnel
  - Collection of Measurements
  - The Wright brothers
- Development of simplified analytical models
  - Closed form solutions
  - Explanation, insight
  - BUT usually very simplified approximations of reality
- CFD = Applied Mathematics + Computing + Engineering Science
  - Almost no simplification
  - BUT mathematical model of physics Is critical (turbulence ...)
  - solution = set of discrete values

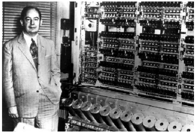
CFD was painful and slow!! Who?

- 1910 Richardson: Human computors
  - in 1910: 2000 ops/week
  - in 2009: 10^9 ops/sec

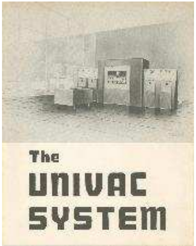
- 1933 Thom : first CFD computation for a cylinder
- 1953 Kawaguti: Mechanical calculator
  - Navier-Stokes flow around a cylindre
  - 20 hours / week for 18 months

Trigger events, When ?

Von Neumann



Eniac



univac

Cray X-MP 1983

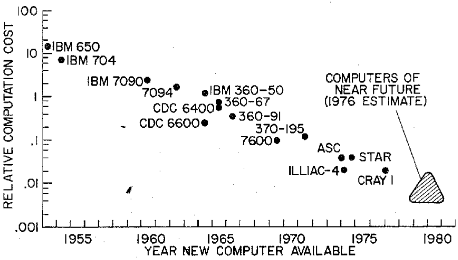
Why CFD ?

CFD because of its cost/benefit ratio.

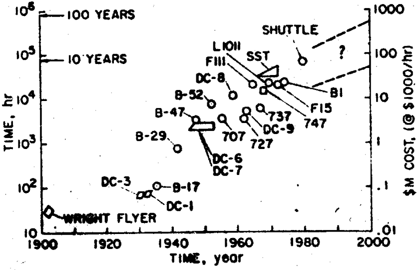
- free from physical limitations of experiments
- free from simplifications in analytical and empirical models
- applicable where measurements are impossible to make
- provides all information everywhere

CFD makes its own room

Computing costs drop, cost of experiments explodes



(a) Computing cost



(b) Cost of Experiment

1975 Dean Chapman (NASA): CFD spells the end of wind tunnels

Outline

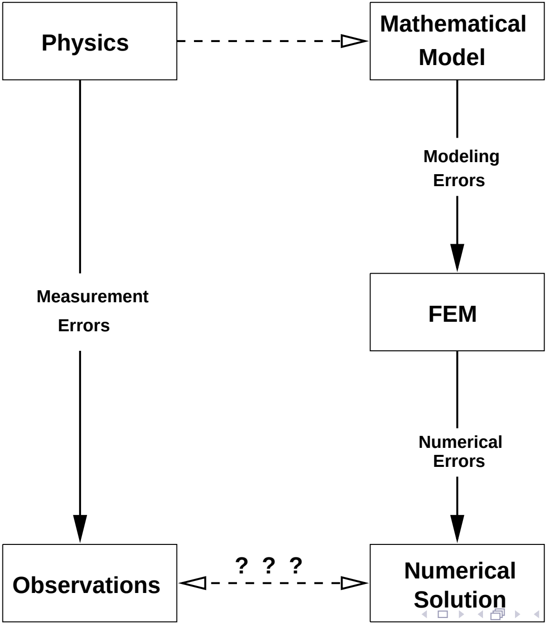
Accuracy; a Never Ending Challenge?

D. Mavriplis, *Unstructured-Mesh Discretizations and Solvers for Computational Aerodynamics*, AIAA Journal , Vol 46, No 6, pp. 1281-1298, June 2008.

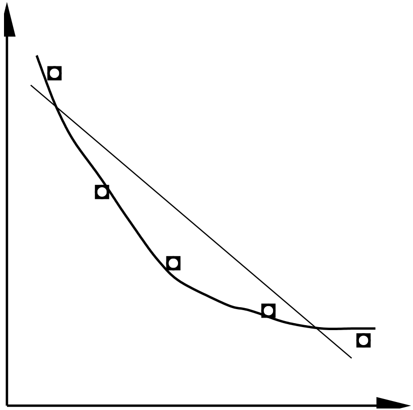
- Anisotropic grids: 1, 3, 9, and 72 $\times$ 10^6 points,
- Drag converges to 2^{nd} order, but
- 65 $\times$ 10^6 point grid with isotropic surface mesh
- Mach = 0.75: $C_D = 0.0280 \longrightarrow 0.0255$ but $C_{D_{exp}} = 0.0270$),
- Mesh Resolution is most important factor,
- Issues: Accuracy, Reliability, Uncertainty

Accuracy: How to get there? How to make sure we are there?

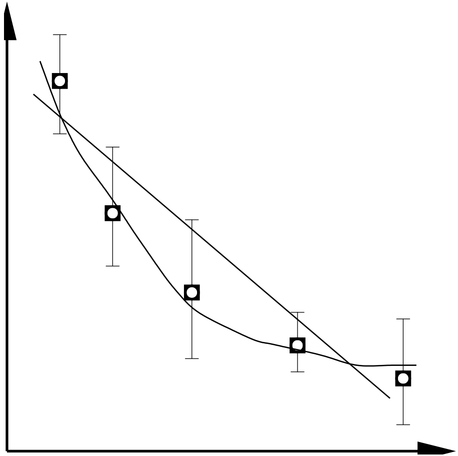
Modeling and Simulation - 1



Modeling and Simulation - 2



Modeling and Simulation - 3



From PDEs to **Ax = b**

- Weak forms of the equations
- Streamline Upwind/Petrov-Galerkin stabilized formulations
- RANSE : Velocity-pressure formulation
- Newton's linearization and sparse direct solver
- Equations solved in a partly segregated manner
- Taylor-Hood element ($P_2 - P_1$) : formal orders of accuracy

$$\begin{array}{lcl} u, v, \mathcal{K}, \mathcal{E} & : & ||.||_{L2} \equiv O(h^3) \text{ and } ||.||_{H1} \equiv O(h^2) \\ p & : & ||.||_{L2} \equiv O(h^2) \text{ and } ||.||_{H1} \equiv O(h) \end{array}$$

→ Adaptive grids: from **Ax = b** to Accuracy

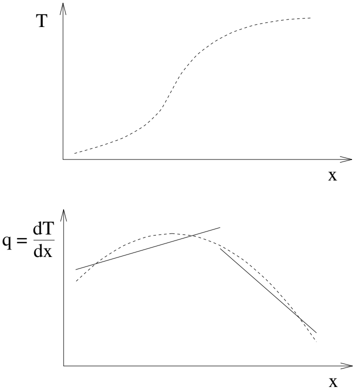
- Zhu-Zienkiewicz error estimator
- Mesh size obtained based on the convergence rate of the FEM and the principle of equi-distribution of the error
- Advancing front mesh generator
- Adaptation based on error estimates for $||\mathbf{u}||_{H^1}$, $||\mathcal{K}||_{eqv}$, $||\mathcal{E}||_{eqv}$, $||\mu_t||_{eqv}$

→ Zhu-Zienkiewicz error estimator

- Nodal-based Least-squares derivative recovery technique
- Measure of the error : difference between a post-processed field and a discontinuous FE field
- An example :

$$
\begin{aligned}
||p||_{H^1} &= \sqrt{\int_\Omega \nabla p \cdot \nabla p \; d\Omega} \\
||e_p||_{H^1}^{\text{exa}} &= \sqrt{\int_\Omega (\nabla p_{\text{exa}} - \nabla p_h) \cdot (\nabla p_{\text{exa}} - \nabla p_h) \; d\Omega} \\
||e_p||_{H^1}^{ZZ} &= \sqrt{\int_\Omega (\nabla p_{ZZ} - \nabla p_h) \cdot (\nabla p_{ZZ} - \nabla p_h) \; d\Omega}
\end{aligned}
$$

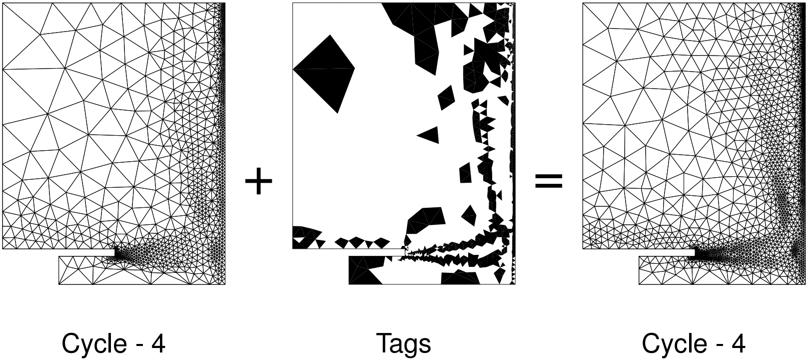
Projection Error Estimator



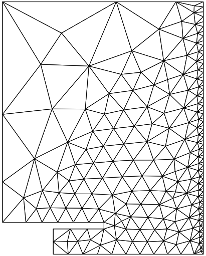
$$e^2 = \int (q_{ex} - q_h)^2 dx$$

$$e^2 \simeq E^2 = \int (q^* - q_h)^2 dx$$

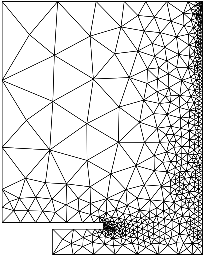
Grid Adaption Process



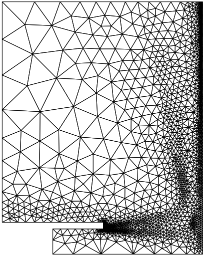
Adaptive Grids Sequence



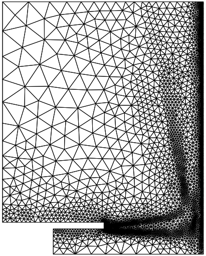
Cycle - 0



Cycle - 2

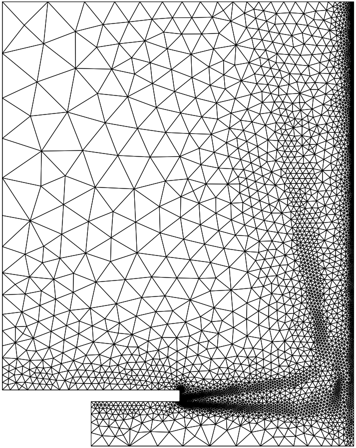


Cycle - 4



Cycle - 6

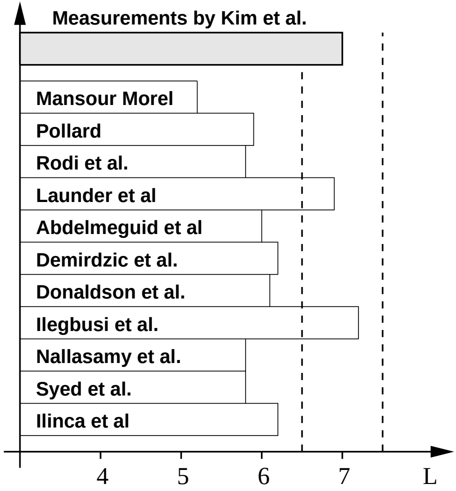
Final Mesh



Cycle - 6

How do we know if we are there ?

VnV: Modeling and Simulation - 4



Outline

Verification and Validation

- In an English thesaurus **Verification** and **Validation** are synonymous.
- In CFD the words have acquired accepted **technical** meanings.
- The same word can have different **Technical** meanings in different contexts.

In mechanical and aerospace engineering:

- **Verification**: *Are we solving these equations **right**?*
- **Validation**: *Are we solving the **right** equations for this problem?*

Verification

Verification is a mathematical activity

- Mathematics
- Numerical methods

Are we doing good numerical analysis for solving the differential equations at hand?

- Is the scheme/code O(δx^2)? (Code Verification)
- Is it O(δx^2) on this problem? (Simulation Verification)

Validation

Validation means having

- The proper physics.
- The proper science.
- An appropriate engineering model.

Are we doing good engineering modeling for the problem at hand?

Requirements:

- detailed measurements
- quality measurements
- quality predictions

Verification and Validation

Verification: *Are we solving the equations right?*

- 2 steps
- (1) Code Verification: MMS = true error, grid refinement study
- (2) Simulation Verification: Error estimator, grid refinement study

Validation: *Are we solving the right equations?*

- Code has been verified,
- Simulation has been verified
- Compare to quality data

Outline

The Method of Manufactured solution - 2

G. Polya: *Only a fool starts at the beginning; the wise starts at the end.*

Method of manufactured solutions:

- Pick a non-trivial continuum solution.
- Substitute in PDE (Navier-Stokes, Darcy, etc.)
- Determine source term $Q(t, x, y)$ for balance.
- Implement in solver.
- Perform grid refinement study.
- MMS = Code Accuracy and Reliablility

The Method of Manufactured solution - 2

We can pick the solution *before we specify the governing equations or the boundary conditions*.

$$U(t,x) = A + sin(x + Ct)$$

- Here applied to two different problems:
  - two sets of governing PDE's
  - two sets of boundary conditions

The Method of Manufactured solution - 3

Example 1 : Burgers equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \alpha\frac{\partial^2 u}{\partial x^2}$$

$$L(u) = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \alpha\frac{\partial^2 u}{\partial x^2} = 0$$

However, $U(x, t)$ is not a solution of above

$$\begin{aligned} L(U(t,x)) &\neq 0 \\ L(U(t,x)) &= Q_1(t,x) \\ Q_1(t,x) &= L(U(t,x)) \end{aligned}$$

The Method of Manufactured solution - 4

$$
\begin{aligned}
Q_1(t,x) = \quad & C \quad cos(x + C\,t) \\
& + \quad [A + sin(x + C\,t)]cos(x + C\,t) \\
& + \quad \alpha sin(x + C\,t)
\end{aligned}
$$

$U(x,t) = A + sin(x + C\ t)$ is then solution of modified PDE

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \alpha\frac{\partial^2 u}{\partial x^2} + Q_1(t,x)$$

with compatible initial and boundary conditions

The Method of Manufactured solution - 6

Note:

- Domain not specified,
- Domain could be $0 \leq x \leq 1$
- Domain could be $-10 \leq x \leq 100$
- Boundary conditions will differ for different domains
- Boundary conditions type not specified
- Same $U(t, x)$ can be solution of many different BC combinations

Example 2 : Burgers-like equation

Idealized 1-D mixing length turbulence model

$$\begin{aligned}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x} &= \alpha\frac{\partial^2 u}{\partial x^2}+\lambda\frac{\partial}{\partial x}\left[\left(x\frac{\partial u}{\partial x}\right)^2\right]\\ &= \alpha\frac{\partial^2 u}{\partial x^2}+2\lambda\left[x\left(\frac{\partial u}{\partial x}\right)^2+x^2\frac{\partial^2 u}{\partial x^2}\right]\end{aligned}$$

$$\begin{aligned}Q_2(t,x)=L(u)=u_t+uu_x &- \alpha u_{xx}\\ &- 2\lambda\left[x(u_x)^2+x^2u_{xx}\right]\end{aligned}$$

The Method of Manufactured solution

$$U(t,x) = A\, sin(x + Ct)$$

is solution of 2 PDE's:

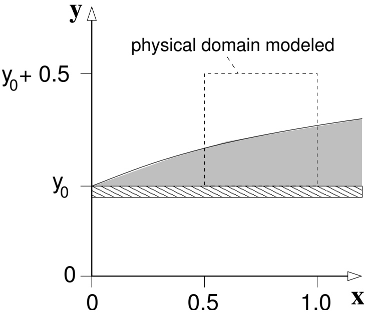
$$
\begin{aligned}
u_t + uu_x &= \alpha u_{xx} + & &+Q_1(t,x) \\
u_t + uu_x &= \alpha u_{xx} + 2\lambda \left[x(u_x)^2 + x^2 u_{xx} \right] & &+Q_2(t,x)
\end{aligned}
$$

Note:
The same solution can be used to verify **two** different 'codes' solving **two** different governing differential equations!
The source term changes to maintain the solution across codes (PDE's).

Use of MMS

- Error: $E = f_h - f_{ex} = Ch^p$
- Grid refinement study : Monitor p

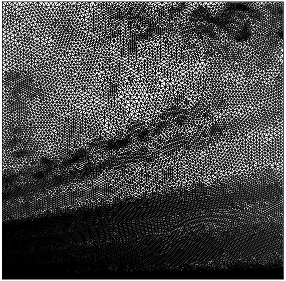
h	$E = c_1 h$	$E = c_2 h^2$
$h_1 = h$	$E_1 = E_0$	$E_1 = E_0$
$h_2 = \frac{h}{2}$	$E_2 \approx \frac{h}{2} = \frac{E_0}{2}$	$E_2 \approx (\frac{h}{2})^2 = \frac{E_0}{4}$
$h_3 = \frac{h}{4}$	$E_3 \approx \frac{h}{4} = \frac{E_0}{4}$	$E_3 \approx (\frac{h}{4})^2 = \frac{E_0}{16}$
$h_4 = \frac{h}{8}$	$E_4 \approx \frac{h}{8} = \frac{E_0}{8}$	$E_4 \approx (\frac{h}{8})^2 = \frac{E_0}{64}$



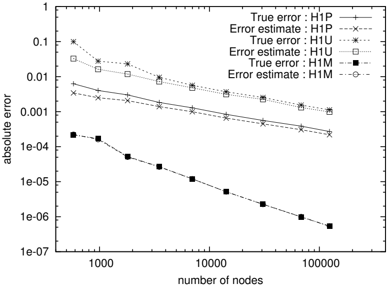
Geometry

$$
\begin{aligned}
u &= \operatorname{erf}(\eta) \\
v &= \frac{1}{\sigma\sqrt{\pi}}\left(1 - e^{-\eta^2}\right) \\
p &= 0.5\ln\left(2x - x^2 + 0.25\right)\ln \\
&\quad * \left(4y^3 - 3y^2 + 1.25\right) \\
k &= k_{max}\eta_\nu^2 e^{1-\eta_\nu^2} + \alpha_k \\
\epsilon &= 0.36\frac{k_{max}^2}{\nu_{max}} e^{-\eta_\nu^2} + \alpha_\epsilon
\end{aligned}
$$

MMS - Turbulent boundary layer



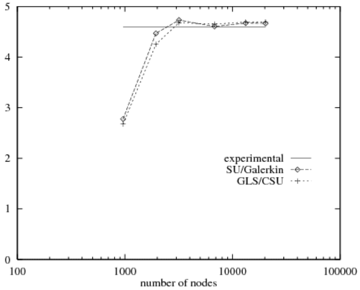
Grid 5

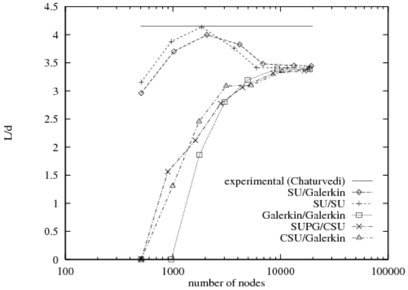


Error Trajectories

Code is verified!

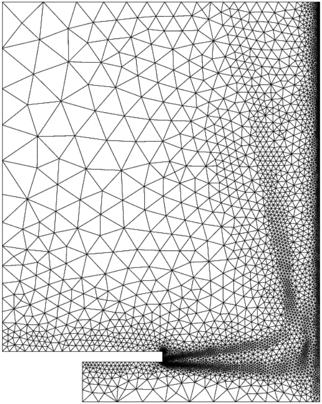
Outline



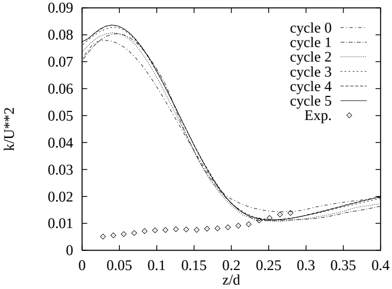


Pipe expansion
Verified
Validated

30° diffuser
Verified
NOT Validated



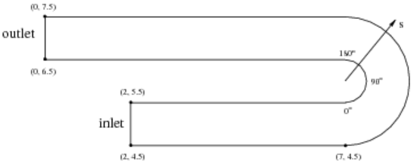
Grid 7



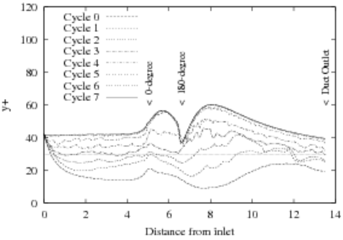
TKE : k/U^2 at $r/d = 0.5$

Verified

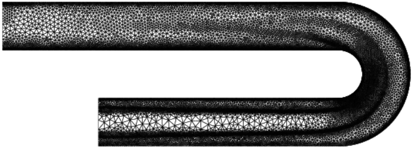
NOT Validated



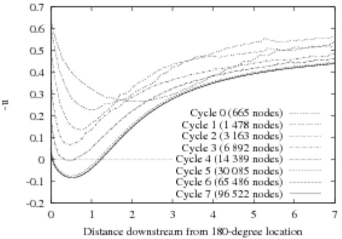
Domain



y^+

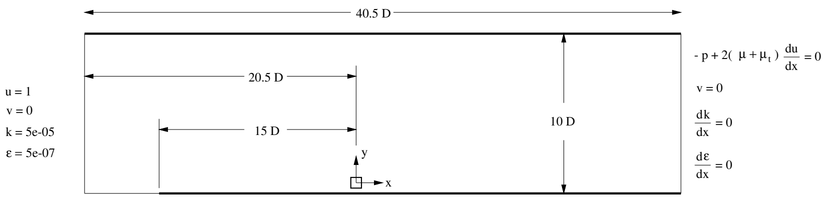


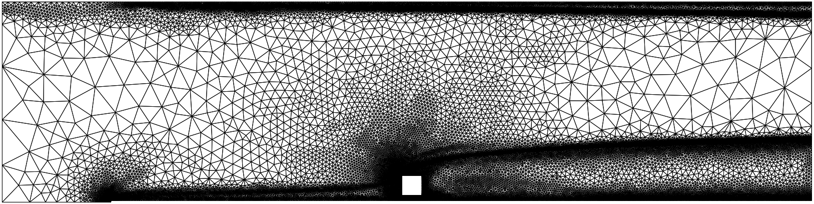
Grid 7 (96 522 nodes)



Slip velocity

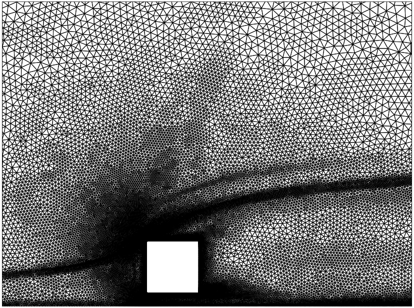
Square Cylinder close to Ground



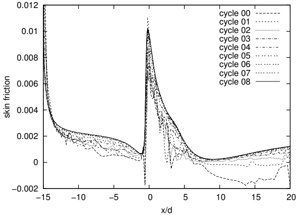


Grid 6 = 220,000 nodes (Grid 8 = 556,567)

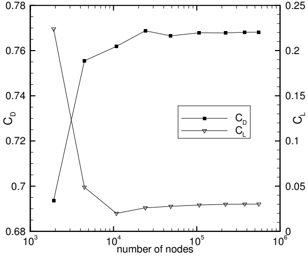
Square Cylinder close to Ground



Grid 6
(Grid 8 = 556,567 nodes)

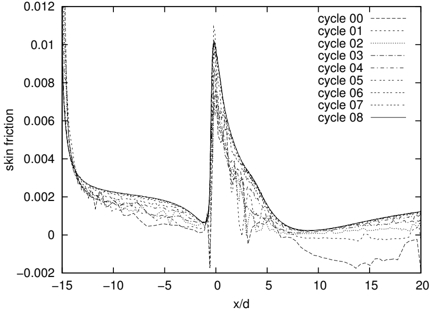


C_f

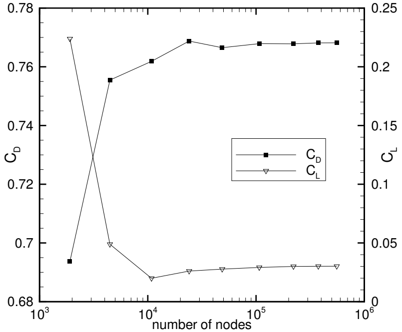


C_L and C_D

Square Cylinder close to Ground

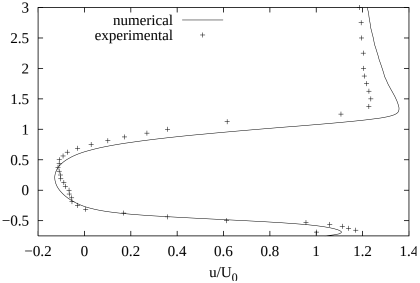


(c) Skin friction c_f



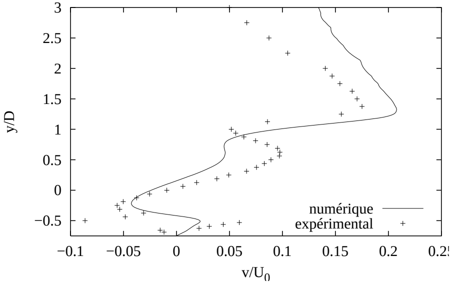
(d) Loads on the square cylinder

Figure: Grid convergence with adaptive cycles



U at x = 1 D

Validated



V at x = 1 D

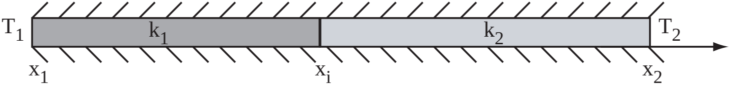
Validated ?

Validation: summary

- Code Verification = code and its use are reliable
- Simulation Verification = solution accuracy estimated
- Simulation Validation = numerical model accurate and reiable
-
- Very accurate, exhaustive data needed to ensure that simulation and experiment are for same problem

Outline

Sensitivities: Definition



$$T = T(x; T_1, T_2, \kappa_1, \kappa_2, x_1, x_i, x_2)$$

Sensitivity with respect to a

$$s_T = \frac{\partial T}{\partial a}$$
$$a \in \{T_1, T_2, \kappa_1, \kappa_2, x_1, x_i, x_2\}$$

Sensitivities: uses

Gradient based optimization

$$\min \; J(\boldsymbol{u}(\alpha), p(\alpha); \alpha)$$

$$\frac{dJ}{d\alpha} = \frac{\partial J}{\partial \boldsymbol{u}} \underbrace{\frac{\partial \boldsymbol{u}}{\partial \alpha}}_{\boldsymbol{s}_u} + \frac{\partial J}{\partial p} \underbrace{\frac{\partial p}{\partial \alpha}}_{s_p} + \frac{\partial J}{\partial \alpha}$$

Fast nearby solution via Taylor series

$$C_p(x; \alpha_0 + \delta\alpha) = C_p(x; \alpha_0) + \underbrace{\frac{\partial C_p}{\partial \alpha}}\bigg|_{\alpha_0} \delta\alpha + \underbrace{\frac{\partial^2 C_p}{\partial \alpha^2}}\bigg|_{\alpha_0} \frac{\delta\alpha^2}{2}$$

$$C_f(x; \alpha_0 + \delta\alpha) = C_f(x; \alpha_0) + \underbrace{\frac{\partial C_f}{\partial \alpha}}\bigg|_{\alpha_0} \delta\alpha + \underbrace{\frac{\partial^2 C_f}{\partial \alpha^2}}\bigg|_{\alpha_0} \frac{\delta\alpha^2}{2}$$

Forward uncertainty propagation

Cascade input data uncertainty into CFD outputs

1st Order:

$$\sigma_F^2 = \sum_{i=1}^{n} (\underbrace{\frac{\partial \boldsymbol{F}}{\partial a_i}}_{s_F^{a_i}} \sigma_{a_i})^2$$

2nd Order:

$$\sigma_F^2 = \sum_{i=1}^{n} (\underbrace{\frac{\partial \boldsymbol{F}}{\partial a_i}}_{s_F^{a_i}} \sigma_{a_i})^2 + \frac{1}{2!} \sum_{i,j=1}^{n} (\underbrace{\frac{\partial^2 \boldsymbol{F}}{\partial a_i \partial a_j}}_{s_F^{a_i a_j}} \sigma_{a_i} \sigma_{a_j})^2$$

Flow and 1st Order Sensitivity Equations

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \nabla \cdot \left[\mu \left(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T \right) \right]$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

SEM: differentiate then discretize

$$\frac{\partial}{\partial a} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] = 0 \qquad \rightarrow \qquad \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial a} \right) + \frac{\partial}{\partial y} \left(\frac{\partial v}{\partial a} \right) = 0$$

$$\boxed{\frac{\partial s_u}{\partial x} + \frac{\partial s_v}{\partial y} = 0}$$

1st Order Momentum Sensitivity

$$\frac{\partial}{\partial a}\left[\rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \nabla \cdot \left[\mu\left(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T\right)\right]\right]$$

$$\Downarrow$$

$$\rho'_a \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \rho \boldsymbol{s}_u^a \cdot \nabla \boldsymbol{u} + \rho \boldsymbol{u} \cdot \nabla \boldsymbol{s}_u^a = -\nabla s_p^a$$
$$+ \nabla \cdot \left[\mu'_a\left(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T\right) + \mu_a\left(\nabla \boldsymbol{s}_u^a + (\nabla \boldsymbol{s}_u^a)^T\right)\right]$$

Newton linearization 1 linear system of PDE / parameter

2nd Order Sensitivity Equations

$$\frac{\partial}{\partial b}\left[\begin{array}{l} \rho'_a \boldsymbol{u}\cdot\nabla\boldsymbol{u} + \rho \boldsymbol{s}_u^a\cdot\nabla\boldsymbol{u} + \rho\boldsymbol{u}\cdot\nabla\boldsymbol{s}_u^a = -\nabla s_p^a \\ +\nabla\cdot\left[\mu'_a\left(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\right) + \mu_a\left(\nabla\boldsymbol{s}_u^a + (\nabla\boldsymbol{s}_u^a)^T\right)\right] \end{array}\right]$$

$$\Downarrow$$

$$\rho\boldsymbol{s}_u^{ab}\cdot\nabla\boldsymbol{u} + \rho\boldsymbol{u}\cdot\nabla\boldsymbol{s}_u^{ab} + \nabla\boldsymbol{s}_p^{ab} - \nabla\cdot\left(\mu(\nabla\boldsymbol{s}_u^{ab} + (\nabla\boldsymbol{s}_u^{ab})^T)\right) =$$

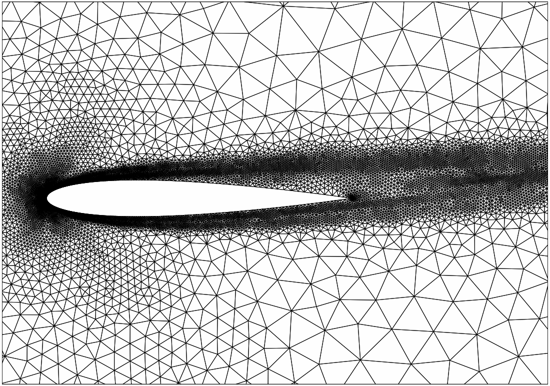
$$-\left[\rho'_{ab}\boldsymbol{u}\cdot\nabla\boldsymbol{u} + \rho'_a(\boldsymbol{s}_u^b\cdot\nabla\boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{s}_u^b) + \rho'_b(\boldsymbol{s}_u^a\cdot\nabla\boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{s}_u^a)\right.$$

$$\left. +\rho(\boldsymbol{s}_u^a\cdot\nabla\boldsymbol{s}_u^b + \boldsymbol{s}_u^b\cdot\nabla\boldsymbol{s}_u^a)\right]$$

$$+\nabla\cdot\left[\mu'_{ab}(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T) + \mu_a(\nabla\boldsymbol{s}_u^b + (\nabla\boldsymbol{s}_u^b)^T) + \mu_b(\nabla\boldsymbol{s}_u^a + (\nabla\boldsymbol{s}_u^a)^T)\right]$$

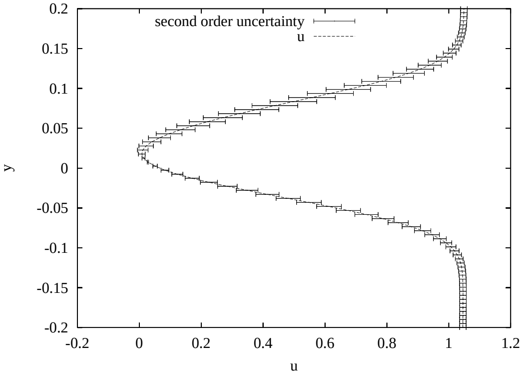
Uncertainty analysis

$$Re = 2000 \text{ - } \alpha = 3^\circ \pm 1\% \text{ and } U_\infty = 1 \pm 1\%$$

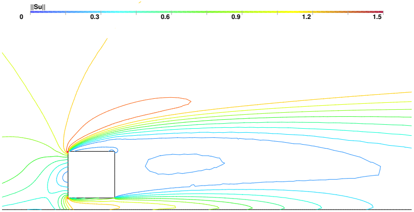


Uncertainty in Near Wake , $x = 1.05$

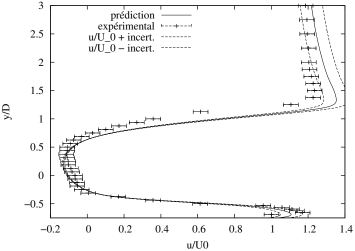
$$Re = 2000 \text{ - } \alpha = 3^\circ \pm 1\% \text{ and } U_\infty = 1 \pm 1\%$$



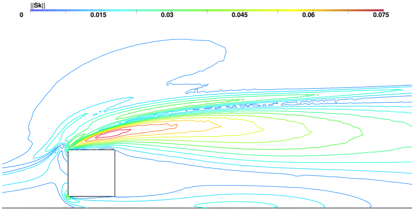
Square obstacle: sensitivity w.r.t U_0 and k_0



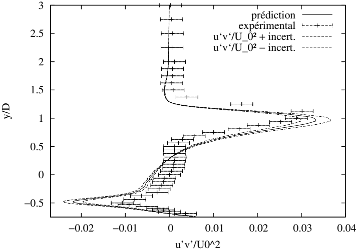
w.r.t. U_0



U/U_0



w.r.t. k_0

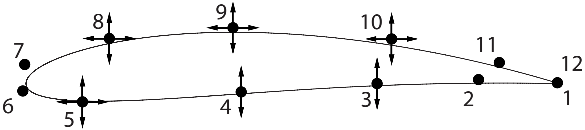


$-\overline{u'v'}/U_0^2$

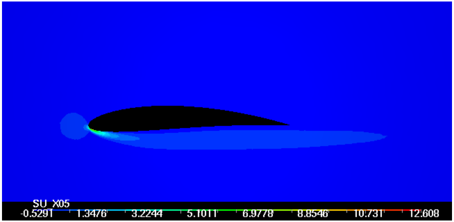
Outline

Maximize Lift to Drag ratio

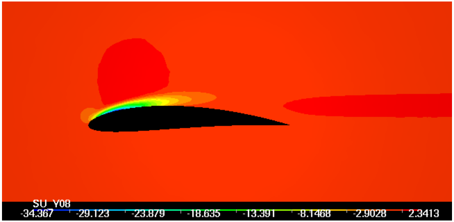
- Baseline: NACA 4512 at 0^o and $Re = 1000$
- NURBS representation:12 control points



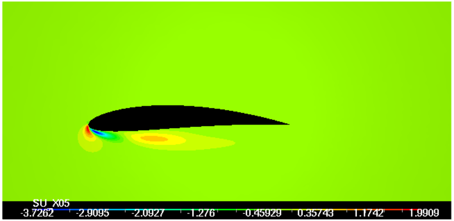
- Selection of the most influent parameters: 10 degrees of freedom
- Maximization of $\mathcal{J} = \dfrac{C_L}{C_D}$
- Initial guess: $C_D^0 = 0.1234$, $C_L^0 = 0.01902$ and $\mathcal{J}^0 = 0.1541$



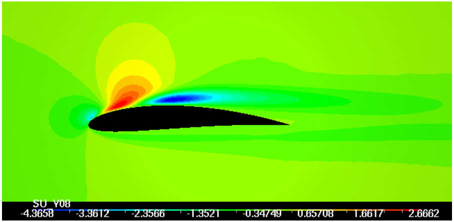
CESEM $s_{U_{X_5}}$



CESEM $s_{U_{Y_8}}$



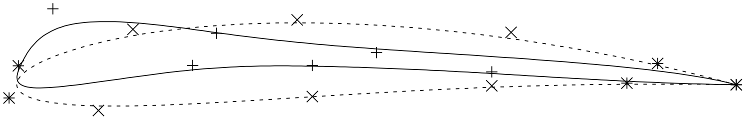
CLSEM $S_{U_{X_5}}$



CLSEM $S_{U_{Y_8}}$

Results: CLSEM - 7 iterations

	Initial	Optimized
C_D	0.1234	0.1190
C_L	0.01902	0.1960
$\mathcal{J}$	0.1541	1.647



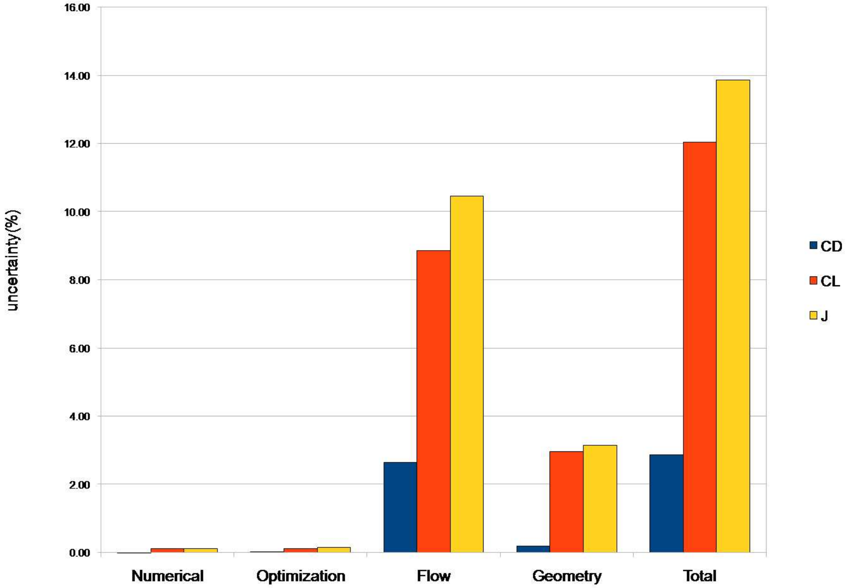
Uncertainty analysis

- C_L, C_D and $\mathcal{J}$ suffer from uncertainty due to
  - Numerical uncertainties:discretization errors
  - Stopping criteria δX^{opt} (active parameters only)
  - Uncertain inlet flow δRe, $\delta\alpha$
  - Geometrical uncertainties δX^{geo} (coordinates of the control points)
- Evaluation of the uncertainties:

$$\Delta C_D = \sum_{\alpha_i} \left| \frac{DC_D}{D\alpha_i} \right| \Delta\alpha_i \quad \text{and} \quad \Delta C_L = \sum_{\alpha_i} \left| \frac{DC_L}{D\alpha_i} \right| \Delta\alpha_i$$

$$\Delta\mathcal{J} = \frac{C_D \Delta C_L + C_L \Delta C_D}{C_D^2}$$

Results of the uncertainty analysis



Outline

Conclusion

- CFD can be a very powerful tool if used properly:
  - not a black box by a lng shot
  - much research is fet to do
- Vrification: tackles Accuracy:
  - Code Verification using MMS : Cude accuracy and reliability
  - Simulation Verification: accuracy of PDE solution
- Simulation Validation: Reliability/Realism of mathematical model
  - Not as trivial as it seems
- Sensitivity equation method:
  - Provides insight into complex flows
  - Provides uncertainty bands on flow response
  - Provides quantitative data on which parameter exerts most influence on the flow and where
- successful application to airfoil optimization. Then again ...

Conclusion

- There is much left to discover and to do.
- Job security does not look so bad for some of us!

CFD is like scientific computing...

Hamming

1973: *The purpose of computing is insight, not numbers.*

anonymous

1980: *The purpose of computing is is not yet in sight*