Robust Optimization

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Presentation Outline

1. ***Quality in the context of robust optimization.***
    - *Customer Quality*
    - *Engineered Quality*
2. ***Optimization of products/processes***
3. ***Optimization of multivariate information systems.***
4. ***Discussions***

Quality

- **Customer quality**

  - what the customer wants

  (issues related to product planning, species, product function)

- **Engineered quality**

  - what the customer does not want

  (loss to society, failures, defects, pollution, functional variability, etc.)

Customer Quality

- Includes appearance, function & price
- Relates to the customer's personal income & perceived value of the product
- Determines the size of market segment
- Determines specification for the product
- Determined by Management, Marketing & Sales
- Not determined by engineering
- Depends on the company's product strategy

Engineered Quality

- What the customer does not want
- What engineers must improve
- Includes functional failure, operating & maintenance cost, pollution, etc.
- Determines market share within the segment
- Determines the level of global competitiveness

Engineered Quality (Cont.)

- All problems of engineered quality are caused by three types of noise factors:

I. Various usage conditions

- environmental conditions

II. Deterioration and wear

- degradation over time

III. Individual difference

- manufacturing imperfection

Optimization of Products/Processes

Science, Engineering & Technology

- Science is to study natural phenomena
- Laws of nature cannot be changed
- The goal of engineering is to design a product which does not exist in nature, yet performs the customer's desired function

Science, Engineering & Technology (Cont.)

- Numerous engineering studies describe how to design a product that performs certain functions
- However, they *do not* describe:
  - How to minimize the failure of function under various usage conditions
  - How to reduce manufacturing cost by improving robustness
  - How to minimize harmful effects to society (e.g. pollution, etc.)

Science, Engineering & Technology (Cont.)

- These activities economically impact corporate competitiveness
- Taguchi Methods play an important role in these activities
- Taguchi Methods utilize new approaches to improve the efficiency of research and development activities

Taguchi Methods

Focus

- How to measure the product/process functionality against downstream noise conditions during R & D activities
- How to systematically derive the optimum design using orthogonal arrays
- How to economically specify tolerances using the Quality Loss Function

Key Concepts for Robust Design

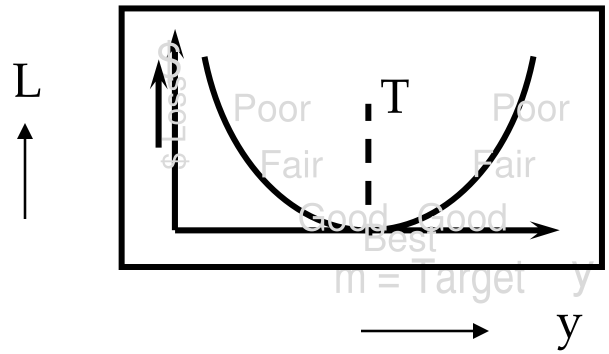
I. Parameter Design (Robust Design)

- What to measure ?
  - Energy Thinking and Signal-to-Noise Ratio
  - Ideal Function & Signal to Noise Ratio
  - P-diagram
- Optimization
  - Inner Array & Outer Array
  - Orthogonal Array for Inner Array to explore the design space
  - Two-step Optimization
  - Prediction & Confirmation

Key Concepts for Robust Design (Continued)

II. Quality Loss Function and Its applications

- § Tolerancing
- § Tolerance Design
- § On-line Quality Engineering



Optimization of Multivariate Information Systems

Also Known as:

Mahalanobis -Taguchi Strategy (MTS)

Introduction to MTS

- MTS is a pattern information processing technique.
- Useful to recognize and evaluate various patterns in multidimensional cases.
    - Medical diagnosis
    - Face/voice recognition
    - Inspection systems etc.,
- Useful to identify driving factors for predicting behavioral patterns.

Objectives of MTS

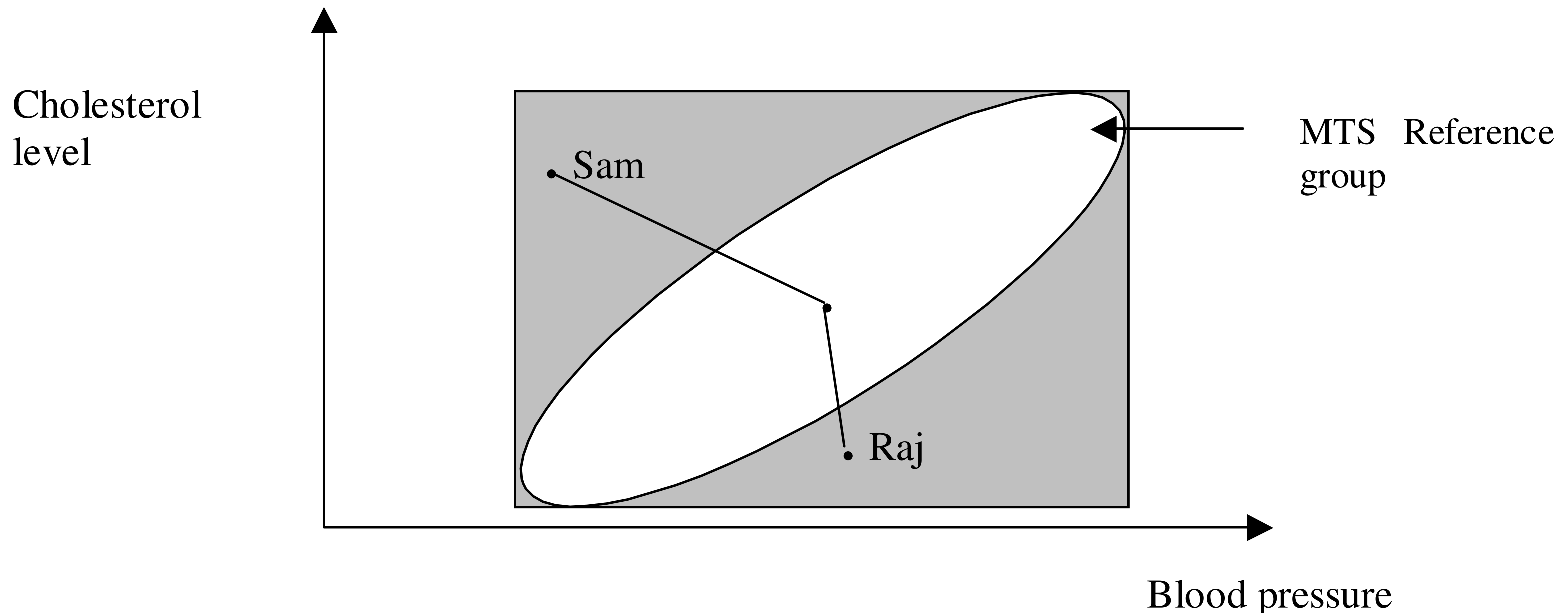
- § Development of measurement scale
  - § Define the variable
  - § Base/reference point
- § Actions based on degree of abnormality
- § Reducing error of predictions (increasing S/N ratios)
- § Data analytic methods.

Mahalanobis Distance (D^2-Statistic)

- Introduced by Prof. P.C. Mahalanobis in 1936.
- Measures distance in Multidimensional spaces.
- Based on multidimensional characteristics.
- Superior to other multidimensional distances.
- Takes correlations into account

In MTS, scaled Mahalanobis distance is used.

Mahalanobis Distance (contd.)



If we look the variables independently, both Sam and Raj are normal or healthy.

When you consider correlations, both are sick and Sam is sicker than Raj.

Mahalanobis distance = $Z_i^{T}\ C^{-1}\ Z_i$

where Z_i= standardized vector of X_i (i=1..k)

C = correlation matrix

Scaled distance

MD = D^2 = (1/k) $Z_i^{T}\ C^{-1}\ Z_i$

where k= no.of characteristics/variables

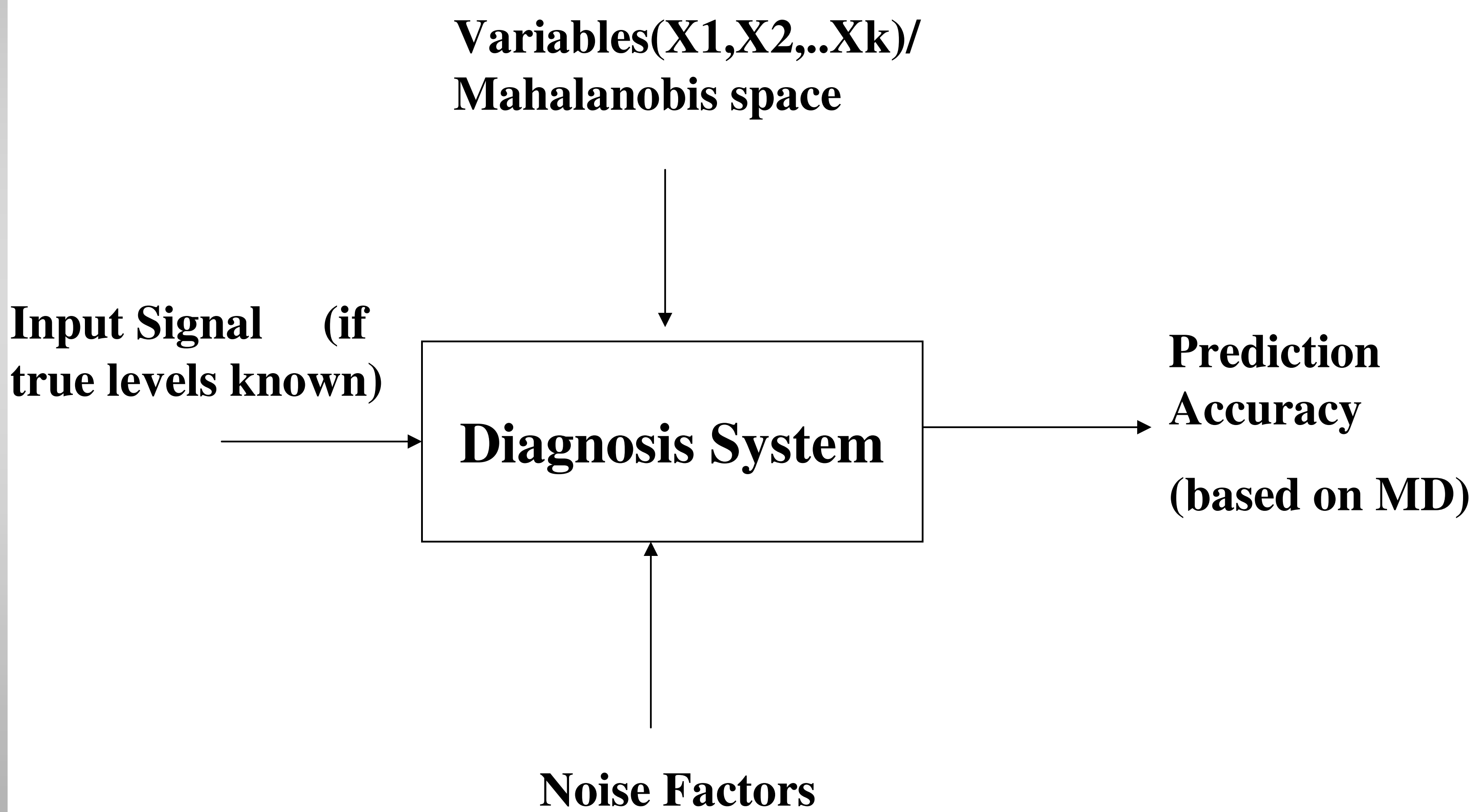
Advantages

- Has a mean of 1.0
- MSD in multidimensional spaces.
- Can be generalized to any number of variables.

Steps in MTS

- Development of Measurement scale
  - Define base or reference point
  - Construction of Mahalanobis space/unit space
- Validation of the scale
  - test known conditions outside MS.
- Developing stage (screening stage)
  - Use of orthogonal arrays and S/N ratios
- Future diagnosis

Multidimensional system



Role of Orthogonal arrays and S/N ratios

L_8 (2^7) Array							
Column	1	2	3	4	5	6	7
Run/variable	X_1	X_2	X_3	X_4	X_5		
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

For each run MD's are computed to calculate S/N ratios.

1:presence of a variable; 2:absence of variable

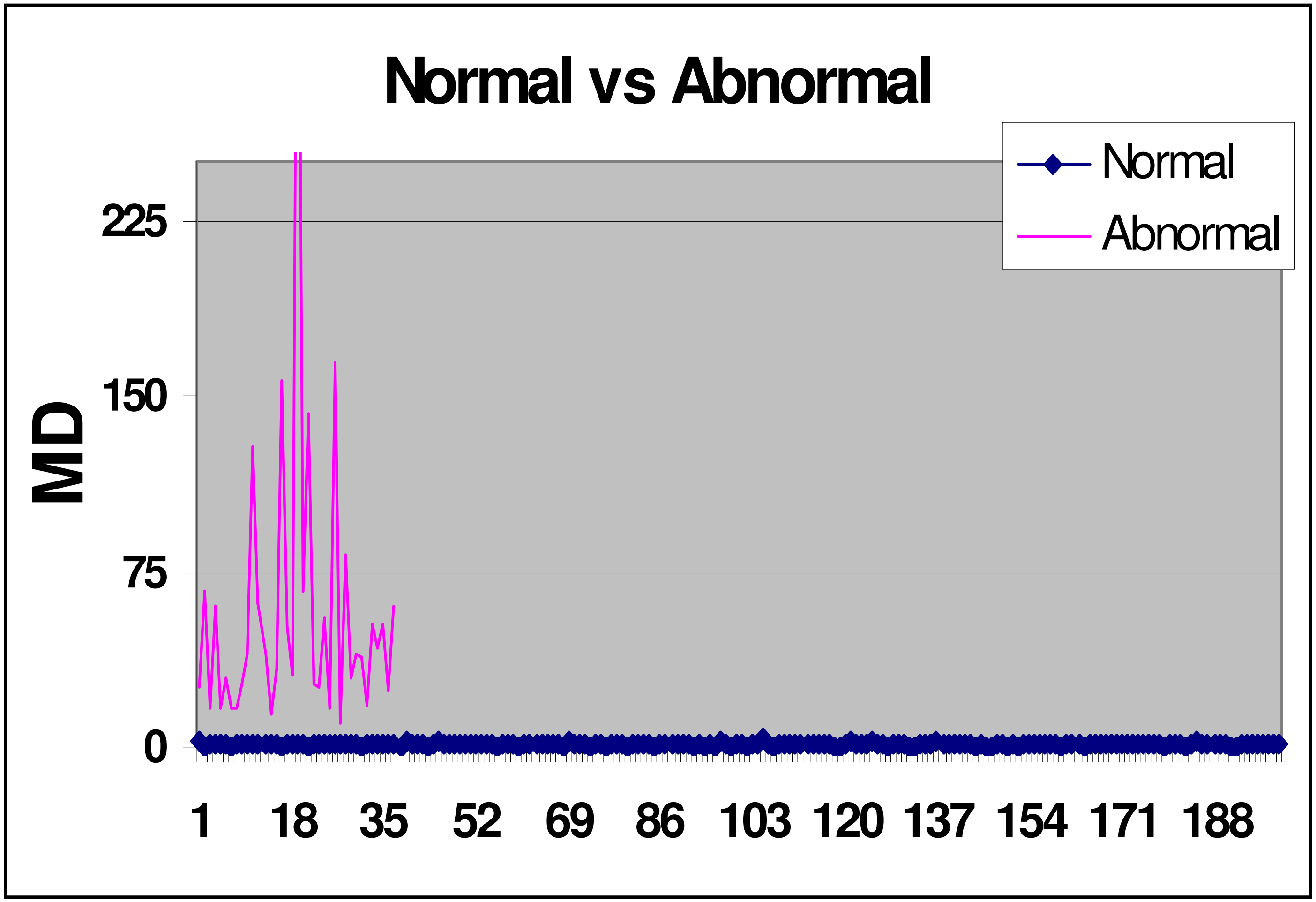
S/N ratio : measure of accuracy of the measurement scale

Medical case study

S.No	Variables	Notation	Notation for Analysis
1	Age		X1
2	Sex		X2
3	Total Protein in Blood	TP	X3
4	Albumin in blood	Alb	X4
5	Cholinesterase	ChE	X5
6	Glutamate O Transaminase	GOT	X6
7	Glutamate P Transaminase	GPT	X7
8	Lactate Dehydrogenase	LHD	X8
9	Alkanline Phosphatase	Alp	X9
10	r-Glutamyl Transpeptidase	r-GPT	X10
11	Leucine Aminopeptidase	LAP	X11
12	Total Cholesterol	TCh	X12
13	Triglyceride	TG	X13
14	Phospholopid	PL	X14
15	Creatinine	Cr	X15
16	Blood Urea Nitrogen	BUN	X16
17	Uric Acid	UA	X17

- **Liver disease test data**
- **200 normals**
- **36 abnormals**

Construction and Validation of the scale



Larger -the-better S/N ratios-MTS method

- **Use of $L_{32}(2^{31})$ array.**
- **S/N ratios for 32 combinations.**
- **Evaluation of gain in S/N ratios**

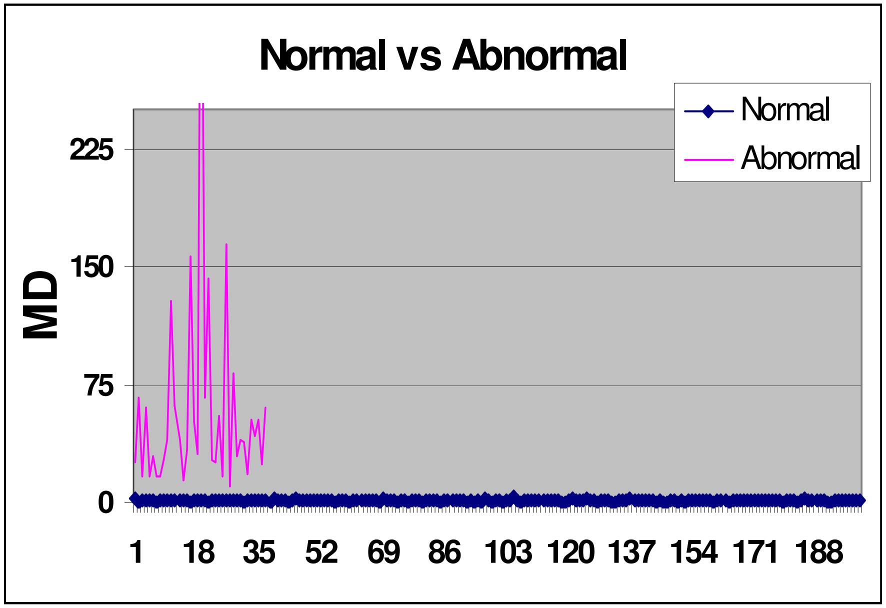
Variable	Level 1	Level 2	Gain
X1	8.73	9.88	-1.15
X2	9.40	9.21	0.19
X3	8.91	9.69	-0.78
...	...	...	...
X16	9.14	9.46	-0.32
X17	9.24	9.37	-0.13

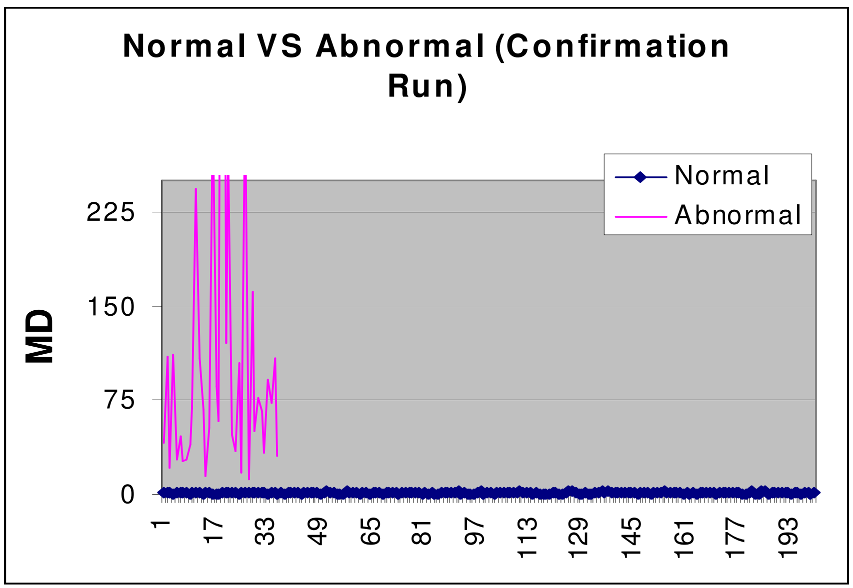
Level 1: presence

Level 2: absence

Useful variables:X_2-X_5-X_7-X_{10}-X_{12}-X_{13}-X_{14}-X_{15}

Confirmation run in both methods gave better results





Avg. abnormal MD

Before: 60.10

After: 121.15

S/N ratio as functionality

S/N ratio-optimal	13.98 dB
S/N ratio-original	11.52 dB
Gain	2.46 dB

This is = 40.2% VRR

Auto Marketing Application

Objectives:

To recognize buying patterns of customers in five segments.

To identify useful variables in each segment for improving the sales.

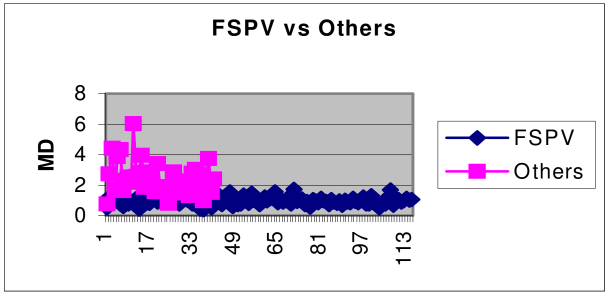
Five Car Segments

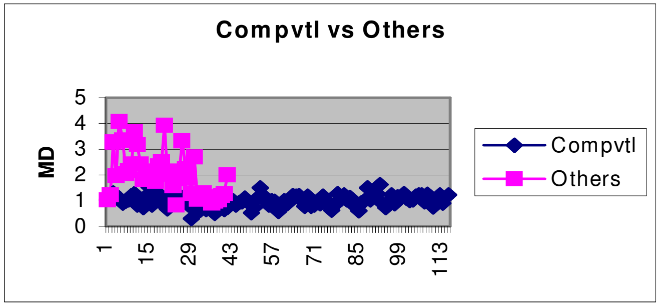
FSPV,Compvtl,Luxury,VPRMD,BSSML

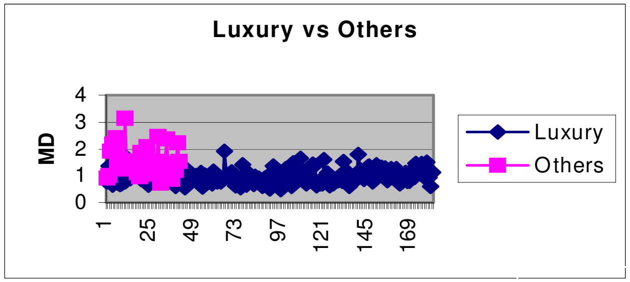
Variables (Total number = 55)

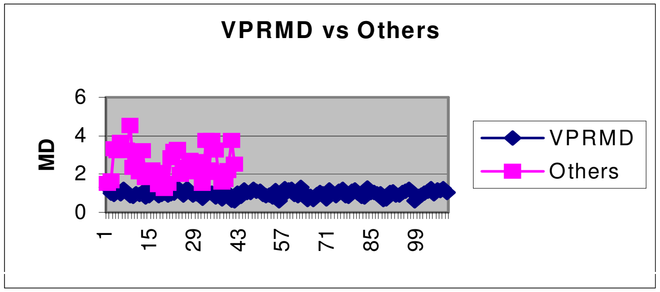
Personal views, Purchase reasons, Demographics

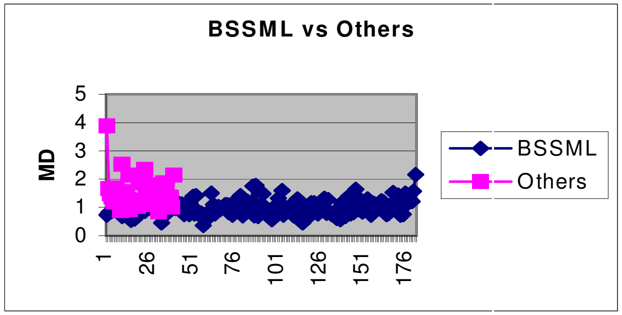
Pattern recognition with useful variables











Case of Fire Alarm System

- Study was conducted by NRLM in Japan.
- There are 17 different fire situations.
- Fire alarm system should recognize them.
- Fire alarm system consists of smoke sensors and temperature sensors.

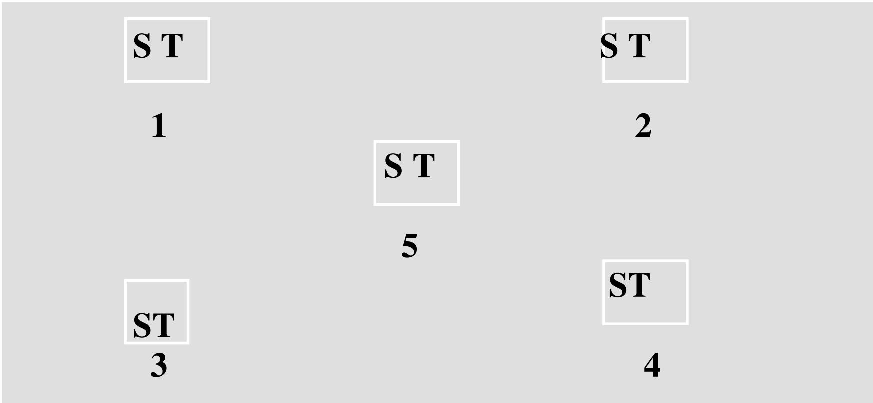


Figure : Positioning of sensors in fire alarm sensor system

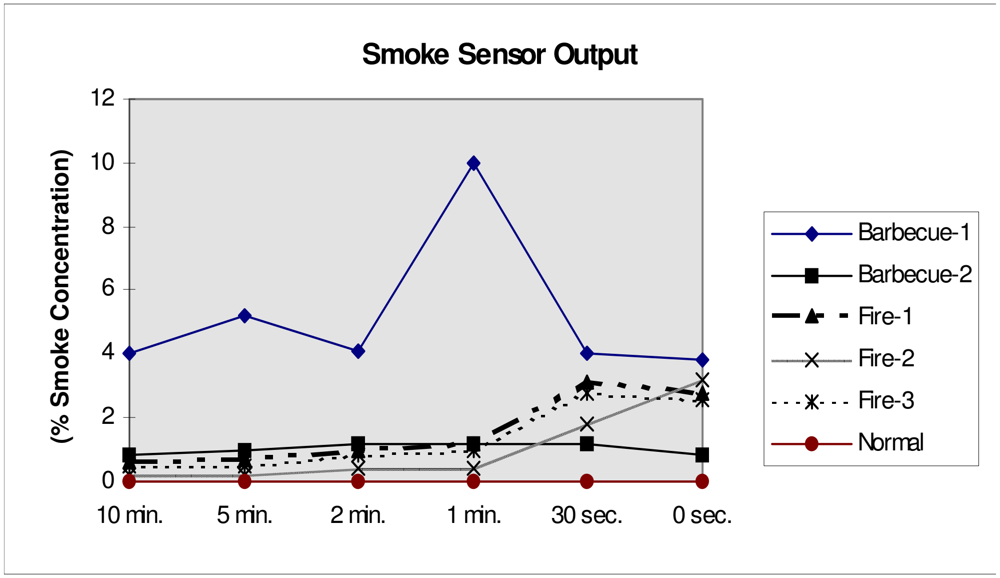


Figure: Output from the smoke sensor

After Application of MTS

Figure : Positioning of sensors in fire alarm sensor system

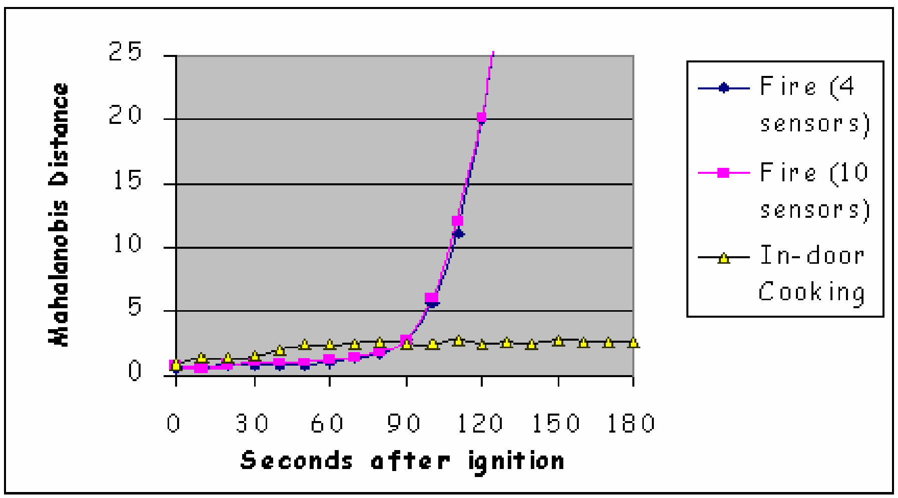


Figure: Output from the sensor system

Direction abnormals

- Some times abnormalities are due to extremely good conditions.
- They also have higher MD values like undesirable (bad) abnormalities.
- It is important to make a distinction between these types of abnormalities.
    - Examples
        - Company diagnosis
        - Student admission system
- Use of Gram Schmidt’s orthogonalization process (MTGS method)

Comparison with classical methods

Problem of multivariate diagnosis in different direction.

- Classification Vs Measurement
- Normals Vs Non-normals
  - *"All Happy Families look alike. Every Unhappy Family is Unhappy After its Own Fashion."*

    -Tolstoy in Ana Karenina
- Probabilistic Vs Data Analytic
- Dimensionality Reduction
  - in terms of original variables
    - helps in cost reduction and time

Comparison with artificial neural networks (ANN)

- Measurement of degree of abnormality.
- Both are data analytic.
- Dimensionality reduction.
- Relation between input and output
- Randomization