Optimal exercise of an executive stock option by an insider

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Outline

Motivation

- Can early exercise of an American ESO be triggered by inside information?
- Empirically, early exercise of ESOs is common.
- Usually attributed to contractual features or to risk aversion.
- Significant abnormal negative returns have been observed after exercise by top managers at small firms (Carpenter and Remmers [10]).
- Hence, inside information may play a role.

ESO literature

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  - Grasselli and Henderson [19]
  - Leung and Sircar [26, 27], Sircar and Xiong [32]
  - Rogers and Scheinkman [31]
- These papers show that market frictions, risk aversion, contractual features, job termination, can lead to early and/or partial exercise.
- Cvitanic, Wiener and Zapatero [13] take early exercise as given.
- We do not include outside trading, or risk aversion or partial exercise opportunities or any other contractual complications.
- Pure optimal stopping problem under the physical measure, maximise expected discouted payoff.
- Goal is to focus exclusively on impact of information.
- Will insider exercise the ESO before maturity, in situations when an agent without inside information would not.

Incorporating inside information

- Pikovsky and Karatzas [30]: initial enlargement through knowldege at time zero of random variable L.
- Amendinger, Imkeller and Schweizer [3], Ankirchner, Dereich and Imkeller [4]: entropic characterisation of additional utility.
- Amendinger, Becherer and Schweizer [2]: monetary value to initial information.
- Imkeller [22, 23]: progressive enlargement to model inside information; Malliavin calculus to characterise the information drift.
- Corcuera, Imkeller, Kohatsu-Higa, Nualart [12]: dynamic flow of inside information.
- Baudoin and Nguyen-Ngoc [5]: information on the law of some random variable.
- Hillairet [20]: comparison of optimal strategies of insiders with different forms of side-information.

Insider literature

- Campi [7]: quadratic hedging problem for an insider.
- Kohatsu-Higa and Sulem [25]: utility maximisation in a market in which insider can influence prices.
- Campi and Çetin [8]: equilibrium with an insider and default.
- Campi, Çetin and Danilova [9]: equilibrium with an insider and evolution of information.
- Øksendal et al [1, 6]: forward integral approach to equilibrium with an insider.
- Danilova, Monoyios and Ng [14]: optimal investment with initial enlargement and drift uncertainty.

Model

- Log-stock price on $(\Omega, \mathcal{F}, P)$, equipped with filtration $\mathbb{F} = (\mathcal{F}_t)_{0 \le t \le T}$ satisfying usual conditions:

$$dX_t = \gamma dt + \sigma dB_t, \quad \gamma := \mu - \frac{1}{2}\sigma^2. \tag{1}$$

- Stock price is $S = e^X$.
- Interest rate $r \ge 0$.
- Agent: an insider (executive) with additional information on future stock price, via knowledge at time zero of an $\mathcal{F}$-measurable random variable L.
- For example

$$L = aX_T + (1-a)\epsilon, \quad 0 < a \le 1. \tag{2}$$

  The insider is barred from trading S and does not trade any outside stocks.
- Regular agent: a non-executive without inside information, with filtration $\mathbb{F}$.

Enlarged filtration

Insider's filtration is $\mathbb{F}^{\sigma(L)} = (\mathcal{F}_t^{\sigma(L)})_{0 \le t \le T}$, defined by

$$\mathcal{F}_t^{\sigma(L)} := \mathcal{F}_t \vee \sigma(L), \quad 0 \le t \le T.$$

For certain choices of L, dynamics of X w.r.t. enlarged filtration are of the form

$$dX_t = a(t, X_t)dt + \sigma dB_t^L, \tag{3}$$

for some $a(t, x)$ which depends on L, and B^L is $(P, \mathbb{F}^{\sigma(L)})$-BM

Example: noisy terminal stock price information

- Suppose $L = aX_T = (1-a)\epsilon$, where $\epsilon \sim \mathrm{N}(0,1)$ independent of B and $a \in (0,1]$.
- Then, with respect to the enlarged filtration $\mathbb{F}^{\sigma(L)}$, the dynamics of the log-stock price are

$$dX_t = \frac{C - X_t}{T_a - t}dt + \sigma dB_t^L, \tag{4}$$

where B^L is an $\mathbb{F}^{\sigma(L)}$-Brownian motion and

$$\begin{aligned} C &= \gamma(T_a - T) + \frac{L}{a}, \\ T_a &= T + \left(\frac{1-a}{a\sigma}\right)^2, \quad 0 < a \leq 1. \end{aligned}$$

Information drift

- Classical enlargement of filtration results:
- With respect to $\mathbb{F}^{\sigma(L)}$ the $\mathbb{F}$-Brownian motion B decomposes as

$$B_t = B_t^L + \int_0^t \frac{g_y(s, L, B_s)}{g(s, L, B_s)} ds, \quad 0 \le t \le T,$$

  where B^L an $\mathbb{F}^{\sigma(L)}$-Brownian motion.
- $g : [0, T] \times \mathbb{R}^2 \to \mathbb{R}^+$ is the conditional density of L given $\mathbb{F}$, which must satisfy

$$\int_0^t |g_y(s, x, y)/g(s, x, y| ds < \infty.$$

- The process

$$\nu_t^L = \frac{g_y(t, L, B_t)}{g(t, L, B_t)}, \quad 0 \le t \le T.$$

  is called the information drift.

Information drift for noisy terminal stock price information

- With L given in (2) and X_0 assumed deterministic, then

$$\mathrm{Law}[L|\mathcal{F}_t] = \mathrm{N}(a(X_0 + \gamma T + \sigma B_t), (a\sigma)^2(T_a - t)).$$

- Hence, the semimartingale decomposition of B with respect to $\mathbb{F}^{\sigma(L)}$ is

$$B_t = B_t^L + \int_0^t \frac{L - a(X_0 + \gamma T + \sigma B_s)}{a(T_a - s)} ds, \tag{5}$$

  where B^L is a Brownian motion with respect to $\mathbb{F}^{\sigma(L)}$.
- Use $X_0 + \sigma B_t = X_t - \gamma t$ to obtain dynamics (4) for X w.r.t. $\mathbb{F}^{\sigma(L)}$.

Sets of stopping times

- Let $\mathcal{T}$ be the set of $\mathbb{F}$-stopping times.
- Let $\mathcal{T}^L$ be the set of $\mathbb{F}^{\sigma(L)}$-stopping times.
- Define the subsets of $\mathcal{T}$ and $\mathcal{T}^L$:

$$
\begin{aligned}
\mathcal{T}_{t,T} &:= \{\tau \in \mathcal{T} | P(\tau \in [t, T]) = 1\}, \quad 0 \leq t \leq T < \infty, \\
\mathcal{T}^L_{t,T} &:= \{\tau \in \mathcal{T}^L | P(\tau \in [t, T]) = 1\}, \quad 0 \leq t \leq T < \infty.
\end{aligned}
$$

- $\mathcal{T}_{0,T} \equiv \mathcal{T}$ and $\mathcal{T}^L_{0,T} \equiv \mathcal{T}^L$.

ESO

- ESO: American call with strike $K > 0$, cash settled.
- The insider's optimal stopping problem is to find a stopping time $\tau^* \in \mathcal{T}^L$ to achieve the maximal expected reward

$$V_0 := \sup_{\tau \in \mathcal{T}^L} E[Y_\tau] = E[Y_{\tau^*}],$$

subject to dynamics (3), and where $Y = (Y_t)_{0 \le t \le T}$ is the reward process:

$$Y_t := e^{-rt}(e^{X_t} - K)^+ = e^{-rt}(S_t - K)^+, \quad 0 \le t \le T,$$

satisfying $E\left[\sup_{0 \le t \le T} Y_t\right] < \infty$.
- The non-executive faces a similar problem, but over $\mathbb{F}$-stopping times, so in this case the dynamics of X are given by (1). His maximal expected reward is given by

$$V_0^0 := \sup_{\tau \in \mathcal{T}} E[Y_\tau].$$

Benchmark case: $\mu \geq r$ and no inside information

- For $\mu \geq r$, the reward process Y_t is a $(P, \mathbb{F})$-submartingale, so the regular agent's value for the American ESO coincides with the European value:

$$V_0^0 = E[Y_T],$$

  and the exercise time $\tau = T$ is optimal for the regular agent.
- (Write $\mu = r - \delta$ for some δ, then $\mu \geq r \Rightarrow \delta \leq 0$. Then we have a standard American call problem with non-positive dividend rate.)
- The goal is to show that inside information on the stock alters this conclusion.
- The inside information makes the drift of the stock level dependent, so we might expect early exercise to be induced.
- Investigate this by analysis of an optimal stopping problem governed by a diffusion with time and state dependent drift.

Starting time $t \in [0, T]$

For a starting time $t \in [0, T]$ the maximal expected discounted payoff is given by the process

$$V_t := \operatorname*{ess\,sup}_{\tau \in \mathcal{T}^L_{t,T}} E[e^{-r(\tau - t)}(S_\tau - K)^+ | \mathcal{F}_t^{\sigma(L)}] = e^{rt} \operatorname*{ess\,sup}_{\tau \in \mathcal{T}^L_{t,T}} E[Y_\tau | \mathcal{F}_t^{\sigma(L)}], \quad 0 \leq t \leq T.$$

So we consider the process U defined by

$$U_t := \operatorname*{ess\,sup}_{\tau \in \mathcal{T}^L_{t,T}} E[Y_\tau | \mathcal{F}_t^{\sigma(L)}], \quad 0 \leq t \leq T, \tag{6}$$

satisfying $U_t = e^{-rt} V_t$ a.s., for $t \in [0, T]$, and

$$U_0 = \sup_{\tau \in \mathcal{T}^L} E[Y_\tau] = V_0. \tag{7}$$

U admits a right-continuous modification, so we shall not distinguish between U and this modification.

Snell envelope

There exists a non-negative càdlàg $(P, \mathbb{F}^{\sigma(L)})$-supermartingale $U = (U_t)_{0 \leq t \leq T}$, the Snell envelope of Y, such that:

- U is the smallest $(P, \mathbb{F}^{\sigma(L)})$-supermartingale that dominates Y:

$$U_t \geq Y_t, \quad 0 \leq t \leq T, \quad \text{and } U_T = Y_T \text{ a.s.}$$

- For every $t \in [0, T]$,

$$\begin{aligned} U_t &= \operatorname*{ess\,sup}_{\tau \in \mathcal{T}^L_{t,T}} E[Y_\tau | \mathcal{F}^{\sigma(L)}_t], \quad \text{a.s.}, \\ E[U_t] = u(t) &:= \sup_{\tau \in \mathcal{T}^L_{t,T}} E[Y_\tau] = E[Y_{\tau^*_t}], \end{aligned}$$

where

$$\tau^*_t := \inf\{\rho \in [t, T] | U_\rho = Y_\rho\} \wedge T, \quad 0 \leq t \leq T,$$

is a stopping time in $\mathcal{T}^L_{t,T}$.

Optimal stopping time

- A stopping time $\tau^* \in \mathcal{T}^L$ is optimal for problem (7) (that is, $E[Y_{\tau^*}] = U_0 = \sup_{\tau \in \mathcal{T}^L} E[Y_\tau]$) if and only if

$$U_{\tau^*} = Y_{\tau^*}, \quad \text{a.s.},$$

  and the stopped supermartingale U^{τ^*} defined by $U_t^{\tau^*} := U_{\tau^* \wedge t}, 0 \leq t \leq T$, is a $(P, \mathbb{F}^{\sigma(L)})$-martingale.

- The supermartingale U admits the Doob-Meyer decomposition

$$U_t = u(0) + M_t - A_t, \quad 0 \leq t \leq T,$$

  where M is a uniformly integrable $(P, \mathbb{F}^{\sigma(L)})$-martingale and A is a non-decreasing, continuous, adapted process with $EA_T < \infty$ and $A_0 = M_0 = 0$.

- The process A is flat off the set

$$\{t \in [0, T] | U_t = Y_t\}.$$

Value functions

- Define the value function $F : [0, T] \times \mathbb{R} \to \mathbb{R}^+$ by

$$F(t, x) := \sup_{\tau \in \mathcal{T}^L_{t,T}} E[f(\tau, X_\tau) | X_t = x]. \quad (8)$$

where $f : [0, T] \times \mathbb{R} \to \mathbb{R}^+$ is continuous, non-negative, convex, given by

$$f(t, x) := e^{-rt}(e^x - K)^+.$$

and X is given by

$$X_t = x + \int_0^t a(u, X_u) du + \sigma \int_0^t dB^L_u, \quad 0 \leq t \leq T.$$

Then F is a continuous function and the process $(F(t, X_t))_{0 \leq t \leq T}$ is the Snell envelope of $Y = (f(t, X_t))_{0 \leq t \leq T}$ (El Karoui [16], El Karoui, Lepeltier and Millet [18]).

ESO value process

- The insider's value process for the ESO is $(V(t, S_t))_{0 \le t \le T}$, where $V : [0, T] \times \mathbb{R}^+ \to \mathbb{R}$ is given by

$$V(t, s) = \sup_{\tau \in \mathcal{T}^L_{t,T}} E[e^{-r(\tau - t)}(S_\tau - K)^+ | S_t = s],$$

so

$$e^{-rt} V(t, s(x)) = F(t, x), \quad \text{with } s(x) := e^x.$$

Continuation and stopping regions

- The (smallest) optimal stopping time for the problem (8) starting at time $t \in [0, T]$ with $X_t = x$ is $\tau^*_{t,x}$ given by

$$\tau^*_{t,x} = \inf\{\rho \in [t, T] | F(\rho, X_\rho) = f(\rho, X_\rho)\}, \quad 0 \leq t \leq T.$$

- In terms of the stock price we have, given $S_t = s$,

$$\tau^*_{t,s} = \inf\{\rho \in [t, T] | V(\rho, S_\rho) = (S_\rho - K)^+\}, \quad 0 \leq t \leq T.$$

- The *continuation region* $\mathcal{C}$ is thus defined by

$$\begin{aligned} \mathcal{C} &:= \{(t, x) \in [0, T] \times \mathbb{R} | F(t, x) > f(t, x)\} \\ &= \{(t, s) \in \mathbb{R}^+ | V(t, s) > (s - K)^+\}. \end{aligned}$$

- Since V is continuous, $\mathcal{C}$ is open.

Critical stock price

- There exists a decreasing function

$$c : [0, T] \to \mathbb{R}, \quad c(t) \geq K, \quad \forall t \in [0, T],$$

  the critical stock price (early exercise boundary), such that the option is exercised the first time the stock price exceeds $c(t)$.

- The continuation region $\mathcal{C}$ can therefore be characterised by

$$\mathcal{C} = \{(t, s) \in [0, T] \times \mathbb{R}^+ | s < c(t)\},$$

  and the stopping region is $\bar{\mathcal{D}}$, the closure of $\mathcal{D}$ defined by

$$\mathcal{D} := \{(t, s) \in [0, T] \times \mathbb{R}^+ | s > c(t)\}.$$

Decreasing exercise boundary

Lemma

The exercise boundary $c(t)$ is decreasing for $t \in [0, T]$.

Proof.

- Assume the map $t \to V(t, s)$ is decreasing on $[0, T]$ for each $s \in \mathbb{R}^+$.
- A non-trivial assumption for price process that is a time-inhomogeneous diffusion.
- See Ekstrom [15] for proof in a stochastic volatility setting (under an EMM) with $r = 0$.
- Choose $(t, s) \in \mathcal{C}$ for some $s \in (0, \infty)$ and consider t' satisfying $0 \le t' < t \le T$. Then $V(t', s) > V(t, s)$, and therefore

$$V(t', s) - (s - K)^+ > V(t, s) - (s - K)^+ > 0, \quad 0 \le t \le T,$$

so $(t', s) \in \mathcal{C}$. That is,

$$s < c(t) \Longrightarrow s < c(t') \Longrightarrow c(t') > c(t), \quad \text{for } t' < t.$$

Properties of $c(t)$

- Concavity of $c(t)$: still to be addressed (see Chen, Chadam, Jian and Zhen [11] for put in BS model).
- Continuity
- $C(T) = K$

Properties of the value function

- Require $x \to F(t, x)$ to be increasing and convex.
- Then familiar properties of standard American options transfer to this problem.
- Convexity does not automatically follow from convexity of the option payoff when returns or volatility are price-dependent (Merton [28], El Karoui, Jeanblanc-Picqué and Shreve [17].
- For one-dimensional diffusion models, under a risk-neutral measure, convexity of American option prices is obtained by:
  - El Karoui, Jeanblanc-Picqué and Shreve [17] using ideas of stochastic flows.
  - Ekstrom [15] using stochastic time changes and a limiting argument based on approximating American option by a Bermudan option.
  - Hobson [21] using coupling methods.
- Conjecture that properties hold if X satisfies a linear SDE, as in the example with $L = aX_T + (1 - a)\epsilon$.

Theorem

- *The map* $x \to F(t, x)$ *is increasing for any* $t \in [0, T]$.
- *Suppose* $a_{xx}(t, x) \geq 0$. *Then the map* $x \to F(t, x)$ *is convex for any* $t \in [0, T]$.

Idea of proof

- Via ideas of stochastic flows.
- For simplicity consider $t = 0$.
- Consider log-stock price with initial condition $X_0 = x$, and write $X \equiv X(x)$:

$$X_t(x) = x + \int_0^t a(u, X_u(x))du + \sigma B_t^L, \quad 0 \leq t \leq T.$$

  We may choose versions of $(X_t(x))_{0 \leq t \leq T}$ which for each $t \in [0, T]$ and each $\omega \in \Omega$ are diffeomorphisms in x from $\mathbb{R} \to \mathbb{R}$. That is, the map $x \to X(x)$ is smooth.

- Define

$$b(t, y) := \frac{\partial}{\partial y} a(t, y),$$

  and

$$D_t(x) := \frac{\partial}{\partial x} X_t(x).$$

Lemma

The map $x \to X(x)$ is increasing, and if $a_{yy}(t,y) \geq 0$, also convex.

Proof.

$$D_t(x) = \exp\left(\int_0^t b(u, X_u(x))du\right) > 0,$$

so $x \to X(x)$ is increasing.
Define

$$c(t,y) := \frac{\partial}{\partial y} b(t,y).$$

Then

$$\frac{\partial}{\partial x} D_t(x) = D_t(x) \int_0^t c(u, X_u(x)) D_u(x) du,$$

which is non-negative if $c(t,y) \geq 0$ for all $(t,y) \in [0,T] \times \mathbb{R}$.
Then $x \to X(x)$ is convex.

$\square$

Properties of value function, continued

- Leads to $x \rightarrow F(t,x)$ being increasing and convex.
- $(t=0)$ Write optimal stopping time given $X_0 = x$ as

$$\tau^*(x) := \inf\{t \in [0,T] | F(t, X_t(s)) = f(t, X_t(x))\}.$$

  Because $t \rightarrow c(t)$ is decreasing, we have:

$$\begin{aligned} x &\rightarrow \tau^*(x) \quad \text{is decreasing,} && (9) \\ x &\rightarrow X_{\tau^*(x)}(x) \quad \text{is increasing.} && (10) \end{aligned}$$

- The value function $F(x) \equiv F(0,x)$ is given by

$$F(x) = E\left[f(\tau^*(x), X_{\tau^*(x)}(x))\right].$$

  Then (9) and (10) imply that $x \rightarrow F(x)$ is increasing.
- Similar ideas for convexity.
- Alternative: prove convexity for European case, then use arguments of El Karoui et al [17] to move to American case.
- Or use coupling methods of Hobson [21].

Free boundary problem

Define the extended generator $\mathcal{L}$ of X by

$$\mathcal{L}g(t,x) := g_t(t,x) + a(t,x)g_x(t,x) + \frac{1}{2}\sigma^2 g_{xx}(t,x).$$

Theorem

F solves the free boundary problem

$$\begin{aligned} \mathcal{L}F &= 0, \quad in \quad \mathcal{C} = \{(t,x) \in [0,T] \times \mathbb{R} | x < \log c(t)\}, \\ F(t, \log c(t)) &= e^{-rt}(c(t) - K), \quad 0 \le t \le T, \\ F(T,x) &= e^{-rT}(e^x - K)^+. \end{aligned}$$

Translate this into a FBP for $V(t,s)$.

Proof.

- If F is $C^{1,2}$

$$F(t,X_t) = F(0,X_0) + \int_0^t \mathcal{L}F(s,X_s)ds + \sigma \int_0^t F_x(s,X_s)dB_s^L.$$

  F convex and increasing so F_x bounded and stochastic integral is martingale.
- Supermartingale condition suggests $\mathcal{L}F \leq 0$. We also have $F \geq f$.
- Need $\mathcal{L}F = 0$ in $\mathcal{C} = \{F > f\}$ so that process A in Doob decomposition of supermartingale $(F_t, X_t)_{0\leq t\leq T}$ only increases on the set $\{F = f\}$.
- For region $\mathcal{R} \subset \mathcal{C}$, consider $C^{1,2}$ solution g to PDE in $\mathcal{R}$, with $g = F$ on $\partial\mathcal{R}$. Classical PDE results guarantee a unique solution.
- Then show that g and F coincide on $\mathcal{R}$. Consider any point in $\mathcal{R}$ and first time process X leaves $\mathcal{R}$.
- Define martingale based on $g(t,X_t)$ with starting point in $\mathcal{R}$, and use fact that stopped Snell envelope process is martingale to show equivalence of this martingale and the one based on g, so $g = V$.

$\square$

Smooth fit condition

Theorem

The function $V : [0, T] \times \mathbb{R}^+ \to \mathbb{R}^+$ *satisfies the smooth fit condition*

$$V_s(t, s^*(t)) = 1, \quad 0 \leq t \leq T.$$

Proof.

- Convexity of F implies convexity of V.
- We have $V(t,s) = (s - K)$ for $s > c(t)$, so $V_s(t, c(t)+) = 1$.
- Convexity of V gives $V_s(t, c(t)-) \leq 1$.
- Then show that $V_s(t, c(t)-) \geq 1$ by considering starting at $(t, s - \epsilon)$, using fact that $c(t)$ is decreasing, and taking limit as $\epsilon \to 0$.

□

Early exercise decomposition

- Free boundary PDE and smooth fit condition lead to decomposition

$$V(t,s) = e^{-r(T-t)}E[(S_T - K)^+ | S_t = s] + p(t,s)$$

  where

$$p(t,s) = \int_t^T e^{-r(u-t)}E[(r - \mathcal{L}^S)(S_u - K)\mathbb{1}_{\{S_u > c(u)\}} | S_t = s]du,$$

  is the early execise premium.
- Probabilistic proof by methods of Jacka [24]?

Integral equation for early exercise boundary

- Early exercise decomposition leads to integral equation for $c(\cdot)$:

$$
\begin{aligned}
c(t) &= K + C(t, c(t)) \\
&+ \int_t^T e^{-r(u-t)} E[\mathbb{1}_{\{S_u > c(u)\}}((r - \mu^L(u, S_u))S_u - rK) | S_t = c(t)] du,
\end{aligned}
$$

where the dynamics of S w.r.t. $\mathbb{F}^{\sigma(L)}$ are

$$dS_t = S_t(\mu(t, S_t)dt + \sigma dB_t^L).$$

- Uniqueness of solution (Peskir [29]).

Numerical result

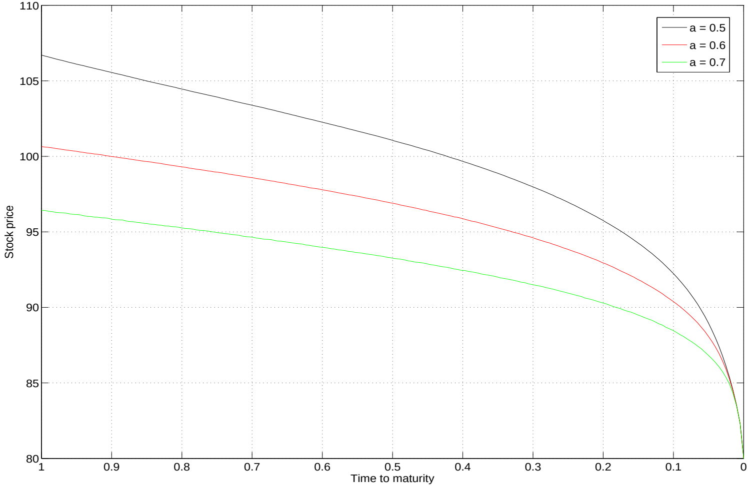

Figure: $r = 0.01$, $\mu = 0.02$, $\sigma = 0.2$, $T = 1$, $K = S_T = 80$.

Conclusions

- Optimal stopping problem for pure exercise of ESO with inside information.
- Leads to Snell envelope of reward process governed by time-inhomogeneous diffusion.
- Establish properties of value function (time decay, convexity, smooth fit) and of exercise boundary (monotone decreasing, concave).
- Numerical results suggest insider will exercise when regular agent will not.
- Extensions: probabilistic proof of value function and exercise boundary properties.
- Different objectives: distorted probabilities, risk aversion, outside trading, ...
- Uncertainty in drift (that is, in model P)
- Other forms of side information, such as when drift changes.

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