

# Information Percolation in Segmented Markets

Darrell Duffie  
Stanford GSB

Semyon Malamud  
EPF Lausanne  
Swiss Finance Institute

Gustavo Manso  
MIT Sloan

Foundations of Mathematical Finance  
Fields Institute, Toronto, January 12, 2009

# Information Transmission in Markets

Informational Role of Prices: Hayek (1945), Grossman (1976), Grossman and Stiglitz (1981).

► Centralized Exchanges:

- Wilson (1977), Townsend (1978), Milgrom (1981), Vives (1993), Pesendorfer and Swinkels (1997), and Reny and Perry (2006).

► Over-the-Counter Markets:

- Wolinsky (1990), Blouin and Serrano (2002), Golosov, Lorenzoni, and Tsyvinski (2009).
- Duffie and Manso (2007), Duffie, Giroux, and Manso (2008), Duffie, Malamud, and Manso (2009).

# Contributions of Today's Paper

- ① tractable model of information diffusion in over-the-counter markets with investor segmentation by preferences, initial information, and connectivity.
- ② double auction with common values.
- ③ effects of information and connectivity on profits:
  - more informed/connected investors attain higher expected profits than less informed/connected investors if they can disguise trades.
  - more informed/connected investors may not attain higher expected profits than less informed/ connected investors if characteristics are commonly observed.

# Outline of the Talk

- ① Information Percolation
- ② Segmented Markets
- ③ Double Auction
- ④ Connectedness and Information

# Model Primitives

Duffie and Manso (2007) and Duffie, Giroux, and Manso (2010):

- ▶ Continuum of agents
- ▶ Two possible states of nature  $Y \in \{0, 1\}$ .
- ▶ Each agent is initially endowed with signals  $S = \{s_1, \dots, s_n\}$  s.t.  
 $P(s_i = 1 \mid Y = 1) \geq P(s_i = 1 \mid Y = 0)$
- ▶ For every pair agents, their initial signals are  $Y$ -conditionally independent
- ▶ Random matching, intensity  $\lambda$ .

## Initial Information Endowment

After observing signals  $S = \{s_1, \dots, s_n\}$ , the logarithm of the likelihood ratio between states  $Y = 0$  and  $Y = 1$  is by Bayes' rule:

$$\log \frac{P(Y = 0 \mid s_1, \dots, s_n)}{P(Y = 1 \mid s_1, \dots, s_n)} = \log \frac{P(Y = 0)}{P(Y = 1)} + \sum_{i=1}^n \log \frac{P(s_i \mid Y = 0)}{P(s_i \mid Y = 1)}.$$

We say that the “type”  $\theta$  associated with this set of signals is

$$\theta = \sum_{i=1}^n \log \frac{P(s_i \mid Y = 0)}{P(s_i \mid Y = 1)}.$$

# What Happens in a Meeting?

- ▶ Upon meeting, agents participate in a double auction.
- ▶ If bids are strictly increasing in the type associated with the signals agents have collected, then bids reveal type.

# Information is Additive in Type Space

**Proposition:** Let  $S = \{s_1, \dots, s_n\}$  and  $R = \{r_1, \dots, r_m\}$  be independent sets of signals, with associated types  $\theta$  and  $\phi$ . If two agents with types  $\theta$  and  $\phi$  reveal their types to each other, then both agents achieve the posterior type  $\theta + \phi$ .

This follows from Bayes' rule, by which

$$\begin{aligned} \log \frac{P(Y = 0 \mid S, R, \theta + \phi)}{P(Y = 1 \mid S, R, \theta + \phi)} &= \log \frac{P(Y = 0)}{P(Y = 1)} + \theta + \phi, \\ &= \log \frac{P(Y = 0 \mid \theta + \phi)}{P(Y = 1 \mid \theta + \phi)} \end{aligned}$$



# Information is Additive in Type Space

**Proposition:** Let  $S = \{s_1, \dots, s_n\}$  and  $R = \{r_1, \dots, r_m\}$  be independent sets of signals, with associated types  $\theta$  and  $\phi$ . If two agents with types  $\theta$  and  $\phi$  reveal their types to each other, then both agents achieve the posterior type  $\theta + \phi$ .

This follows from Bayes' rule, by which

$$\begin{aligned} \log \frac{P(Y = 0 \mid S, R, \theta + \phi)}{P(Y = 1 \mid S, R, \theta + \phi)} &= \log \frac{P(Y = 0)}{P(Y = 1)} + \theta + \phi, \\ &= \log \frac{P(Y = 0 \mid \theta + \phi)}{P(Y = 1 \mid \theta + \phi)} \end{aligned}$$

By induction, this property holds for all subsequent meetings.

# Solution for Cross-Sectional Distribution of Information

The Boltzmann equation for the cross-sectional distribution  $\mu_t$  of types is

$$\frac{d}{dt}\mu_t = -\lambda\mu_t + \lambda\mu_t * \mu_t.$$

with a given initial distribution of types  $\mu_0$ .

# Solution for Cross-Sectional Distribution of Information

The Boltzmann equation for the cross-sectional distribution  $\mu_t$  of types is

$$\frac{d}{dt}\mu_t = -\lambda\mu_t + \lambda\mu_t * \mu_t.$$

with a given initial distribution of types  $\mu_0$ .

**Proposition:** The unique solution of (9) is the Wild sum

$$\mu_t = \sum_{n \geq 1} e^{-\lambda t} (1 - e^{-\lambda t})^{n-1} \mu_0^{*n}.$$

## Proof of Wild Summation

Taking the Fourier transform  $\hat{\mu}_t$  of  $\mu_t$  of the Boltzmann equation

$$\frac{d}{dt}\mu_t = -\lambda\mu_t + \lambda\mu_t * \mu_t,$$

we obtain the following ODE

$$\frac{d}{dt}\hat{\mu}_t = -\lambda\hat{\mu}_t + \lambda\hat{\mu}_t^2,$$

whose solution is

$$\hat{\mu}_t = \frac{\hat{\mu}_0}{e^{\lambda t}(1 - \hat{\mu}_0) + \hat{\mu}_0}.$$

This solution can be expanded as

$$\hat{\mu}_t = \sum_{n \geq 1} e^{-\lambda t} (1 - e^{-\lambda t})^{n-1} \hat{\mu}_0^n,$$

which is the Fourier transform of the Wild sum (9).

# Multi-Agent Meetings

The Boltzmann equation for the cross-sectional distribution  $\mu_t$  of types is

$$\frac{d}{dt}\mu_t = -\lambda\mu_t + \lambda\mu_t^{*m}.$$

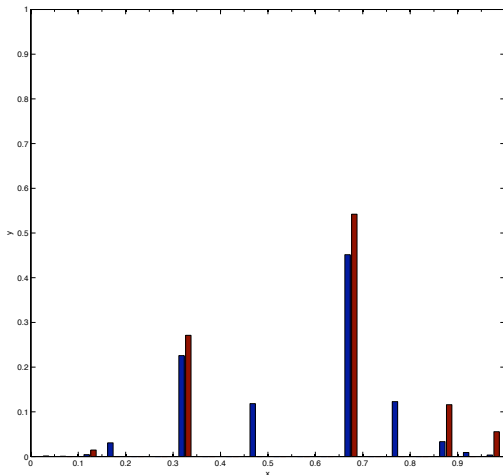
Taking the Fourier transform, we obtain the ODE,

$$\frac{d}{dt}\hat{\mu}_t = -\lambda\hat{\mu}_t + \lambda\hat{\mu}_t^m,$$

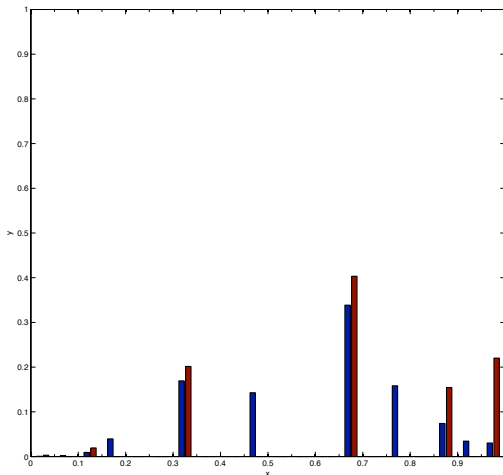
whose solution satisfies

$$\hat{\mu}_t^{m-1} = \frac{\hat{\mu}_0^{m-1}}{e^{(m-1)\lambda t}(1 - \hat{\mu}_0^{m-1}) + \hat{\mu}_0^{m-1}}. \quad (1)$$

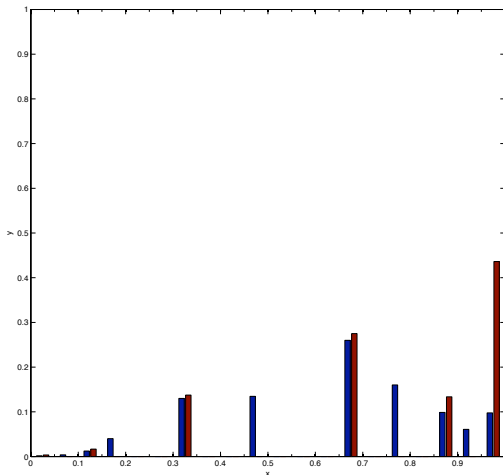
# Groups of 2 (blue) versus Groups of 3 (red)



# Groups of 2 (blue) versus Groups of 3 (red)

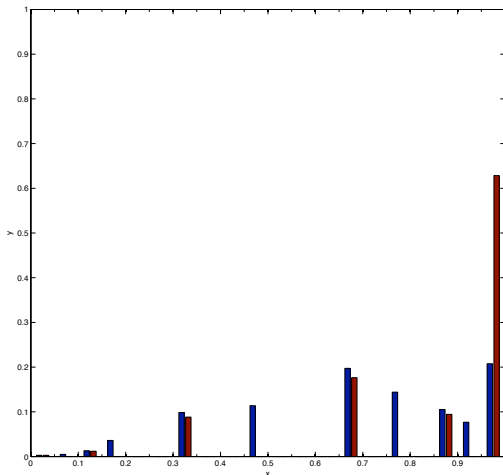


# Groups of 2 (blue) versus Groups of 3 (red)

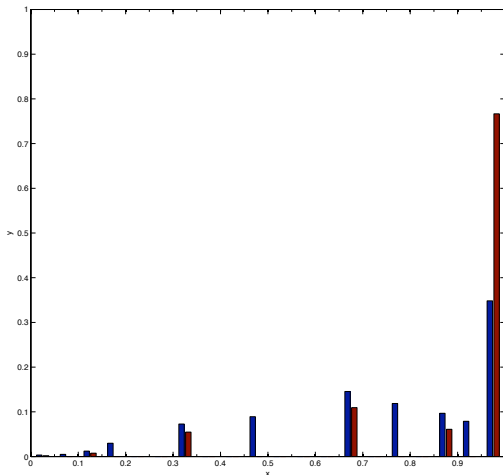




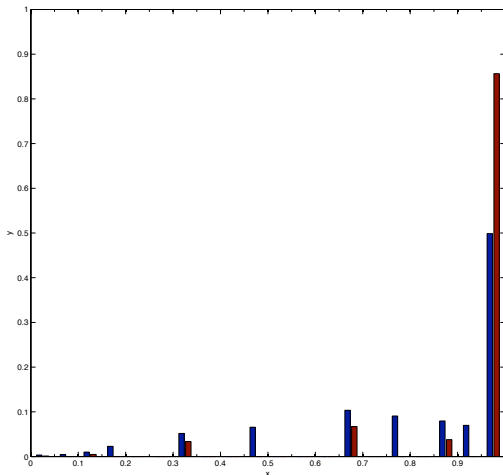
# Groups of 2 (blue) versus Groups of 3 (red)



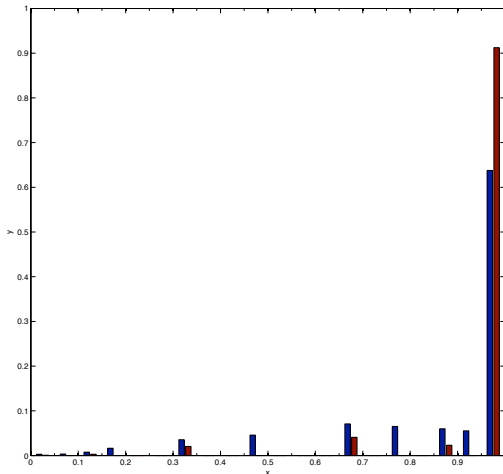
# Groups of 2 (blue) versus Groups of 3 (red)



# Groups of 2 (blue) versus Groups of 3 (red)



# Groups of 2 (blue) versus Groups of 3 (red)



## New Private Information

Suppose that, independently across agents as above, each agent receives, at Poisson mean arrival rate  $\rho$ , a new private set of signals whose type outcome  $y$  is distributed according to a probability measure  $\nu$ . Then the evolution equation is extended to

$$\frac{d}{dt}\mu_t = -(\lambda + \rho)\mu_t + \lambda\mu_t * \mu_t + \rho\mu_t * \nu.$$

Taking Fourier transforms, we obtain the following ODE

$$\frac{d}{dt}\hat{\mu}_t = -(\lambda + \rho)\hat{\mu}_t + \lambda\hat{\mu}_t^2 + \rho\hat{\mu}_t\hat{\nu}.$$

whose solution satisfies

$$\hat{\mu}_t = \frac{\hat{\mu}_0}{e^{(\lambda+\rho(1-\hat{\nu}))t}(1-\hat{\mu}_0) + \hat{\mu}_0}$$

# Other Extensions

- ▶ Public information releases
  - Duffie, Malamud, and Manso (2010).
- ▶ Endogenous search intensity
  - Duffie, Malamud, and Manso (2009).

# Duffie, Malamud and Manso (2010): Public Releases of Information

- ① At public information release random times  $\{T_1, T_2, \dots\}$  (Poisson arrival process with intensity  $\eta$ )  $n$  randomly selected agents have their posterior probabilities revealed to all agents.
- ② We allow for random number of agents in each meeting and in each public information release:
  - Meeting group size  $m$ :  $q_l = P(m = l)$ .
  - Public information release group size  $n$ :  $p_k = P(n = k)$ .

## Evolution of type distribution

**Theorem.** Given the variable  $X$  of common concern, the probability distribution of each agent's type at time  $t$  is  $\nu_t = \alpha_t * \beta_t$ , where  $\alpha_t = h(\mu_0, t)$  is the type distribution in a model with no public releases of information, satisfying the differential equation

$$\frac{d\alpha_t}{dt} = \lambda \left( \sum_{l=2}^{\infty} q_l \alpha_t^{*l} - \alpha_t \right), \quad \alpha_0 = \mu_0, \quad (2)$$

and where  $\beta_t$  is the probability distribution over types that solves the differential equation

$$\frac{d\beta_t}{dt} = -\eta\beta_t + \eta\beta_t * \sum_{k=1}^{\infty} p_k \alpha_t^{*k}, \quad (3)$$

with initial condition given by the Dirac measure  $\delta_0$  at zero.



# The rate of Convergence

Let

$$s \mapsto M(s) = \int e^{sx} d\mu_0(x)$$

and

$$R = \sup_{y \in \mathbb{R}} (-\log M(y)). \quad (4)$$

and

$$\Phi(z) = \sum_{n=1}^{\infty} p_n z^n,$$

**Theorem** Convergence is exponential at the rate  $\lambda + \eta$ , as long as  $\lambda > 0$ . Otherwise, the rate

$$\rho = \eta (1 - \Phi(e^{-R})). \quad (5)$$

is strictly less than  $\eta$ .

# Model Primitives

Same as the previous model except that:

- ▶  $N$  classes of investors.
- ▶ Agent of class  $i$  has matching intensity  $\lambda_i$ .
- ▶ Upon meeting, the probability that a class- $j$  agent is selected as a counterparty is  $\kappa_{ij}$ .

# Evolution of Type Distribution

The evolution equation is given by:

$$\frac{d}{dt}\psi_{it} = -\lambda_i \psi_{it} + \lambda_i \psi_{it} * \sum_{j=1}^N \kappa_{ij} \psi_{jt}, \quad i \in \{1, \dots, N\}.$$

Taking Fourier transforms we obtain:

$$\frac{d}{dt}\hat{\psi}_{it} = -\lambda_i \hat{\psi}_{it} + \lambda_i \hat{\psi}_{it} \sum_{j=1}^N \kappa_{ij} \hat{\psi}_{jt}, \quad i \in \{1, \dots, N\},$$

# Evolution of Type Distribution

The evolution equation is given by:

$$\frac{d}{dt}\psi_{it} = -\lambda_i \psi_{it} + \lambda_i \psi_{it} * \sum_{j=1}^N \kappa_{ij} \psi_{jt}, \quad i \in \{1, \dots, N\}.$$

Taking Fourier transforms we obtain:

$$\frac{d}{dt}\hat{\psi}_{it} = -\lambda_i \hat{\psi}_{it} + \lambda_i \hat{\psi}_{it} \sum_{j=1}^N \kappa_{ij} \hat{\psi}_{jt}, \quad i \in \{1, \dots, N\},$$

## Special Case: $N = 2$ and $\lambda_1 = \lambda_2$

**Proposition:** Suppose  $N = 2$  and  $\lambda_1 = \lambda_2 = \lambda$ . Then

$$\hat{\psi}_1 = \frac{e^{-\lambda t} (\hat{\psi}_{20} - \hat{\psi}_{10})}{\hat{\psi}_{20} e^{-\hat{\psi}_{20}(1-e^{-\lambda t})} - \hat{\psi}_{10} e^{-\hat{\psi}_{10}(1-e^{-\lambda t})}} \hat{\psi}_{10} e^{-\hat{\psi}_{10}(1-e^{-\lambda t})}$$

$$\hat{\psi}_2 = \frac{e^{-\lambda t} (\hat{\psi}_{20} - \hat{\psi}_{10})}{\hat{\psi}_{20} e^{-\hat{\psi}_{20}(1-e^{-\lambda t})} - \hat{\psi}_{10} e^{-\hat{\psi}_{10}(1-e^{-\lambda t})}} \hat{\psi}_{20} e^{-\hat{\psi}_{20}(1-e^{-\lambda t})}.$$

# General Case: Wild Sum Representation

**Theorem:** There is a unique solution of the evolution equation, given by

$$\psi_{it} = \sum_{k \in \mathbb{Z}_+^N} a_{it}(k) \psi_{10}^{*k_1} * \cdots * \psi_{N0}^{*k_N},$$

where  $\psi_{i0}^{*n}$  denotes  $n$ -fold convolution,

$$a'_{it} = -\lambda_i a_{it} + \lambda_i a_{it} * \sum_{j=1}^N \kappa_{ij} a_{jt}, \quad a_{i0} = \delta_{e_i},$$

$$(a_{it} * a_{jt})(k_1, \dots, k_N) = \sum_{l=(l_1, \dots, l_N) \in \mathbb{Z}_+^N, l < k} a_{it}(l) a_{jt}(k - l),$$

and

$$a_{it}(e_i) = e^{-\lambda_i t} a_{i0}(e_i).$$

# Double Auction

- ▶ At some time  $T$ , the economy ends and the utility realized by an agent of class  $i$  for each additional unit of the asset is

$$U_i = v_i Y + v^H(1 - Y),$$

measured in units of consumption, for strictly positive constants  $v^H$  and  $v_i < v^H$ , where  $Y$  is a non-degenerate 0-or-1 random variable whose outcome will be revealed at time  $T$ .

- ▶ If  $v_i = v_j$ , no trade (Milgrom and Stokey (1982)), so that  $\kappa_{ij} = 0$ .
- ▶ Meeting between two agents  $v_i > v_j$ , then  $i$  is buyer and  $j$  is seller.
- ▶ Upon meeting, participate in a double auction. If the buyer's bid  $\beta$  is higher than the seller's ask  $\sigma$ , trade occurs at the price  $\sigma$ .

# Equilibrium

The prices  $(\sigma, \beta)$  constitute an equilibrium for a seller of class  $i$  and a buyer of class  $j$  provided that, fixing  $\beta$ , the offer  $\sigma$  maximizes the seller's conditional expected gain,

$$E \left[ (\sigma - E(U_i | \mathcal{F}_S \cup \{\beta\})) 1_{\{\sigma < \beta\}} \mid \mathcal{F}_S \right],$$

and fixing  $\sigma$ , the bid  $\beta$  maximizes the buyer's conditional expected gain

$$E \left[ (E(U_j | \mathcal{F}_B \cup \{\sigma\}) - \sigma) 1_{\{\sigma < \beta\}} \mid \mathcal{F}_B \right].$$

Counterexample: Reny and Perry (2006)



# Equilibrium

The prices  $(\sigma, \beta)$  constitute an equilibrium for a seller of class  $i$  and a buyer of class  $j$  provided that, fixing  $\beta$ , the offer  $\sigma$  maximizes the seller's conditional expected gain,

$$E \left[ (\sigma - E(U_i | \mathcal{F}_S \cup \{\beta\})) 1_{\{\sigma < \beta\}} \mid \mathcal{F}_S \right],$$

and fixing  $\sigma$ , the bid  $\beta$  maximizes the buyer's conditional expected gain

$$E \left[ (E(U_j | \mathcal{F}_B \cup \{\sigma\}) - \sigma) 1_{\{\sigma < \beta\}} \mid \mathcal{F}_B \right].$$

Counterexample: Reny and Perry (2006)

# Restriction on the Initial Information Endowment

**Lemma:** Suppose that each signal  $Z$  satisfies

$$\mathbb{P}(Z = 1 \mid Y = 0) + \mathbb{P}(Z = 1 \mid Y = 1) = 1.$$

Then, for each agent class  $i$  and time  $t$ , the type density  $\psi_{it}$  satisfies

$$\psi_{it}^H(x) = e^x \psi_{it}^H(-x), \quad \psi_{it}^L(x) = \psi_{it}^H(-x) \quad x \in \mathbb{R}.$$

and the *hazard rate condition*

$$h_{it}^H(x) \stackrel{\text{def}}{=} \frac{\psi_{it}^H(x)}{\int_x^{+\infty} \psi_{it}^H(y) dy} \geq \frac{\psi_{it}^L(x)}{\int_x^{+\infty} \psi_{it}^L(y) dy} \stackrel{\text{def}}{=} h_{it}^L(x).$$

# Bidding Strategies

**Lemma:** For any  $V_0 \in \mathbb{R}$ , there exists a unique solution  $V_2(\cdot)$  on  $[v_i, v^H)$  to the ODE

$$V_2'(z) = \frac{1}{v_i - v_j} \left( \frac{z - v_i}{v^H - z} \frac{1}{h_{it}^H(V_2(z))} + \frac{1}{h_{it}^L(V_2(z))} \right), \quad V_2(v_i) = V_0.$$

This solution, also denoted  $V_2(V_0, z)$ , is monotone increasing in both  $z$  and  $V_0$ . Further,  $\lim_{v \rightarrow v^H} V_2(v) = +\infty$ . The limit  $V_2(-\infty, z) = \lim_{V_0 \rightarrow -\infty} V_2(V_0, z)$  exists. Moreover,  $V_2(-\infty, z)$  is continuously differentiable with respect to  $z$ .

## Bidding Strategies

**Proposition:** Suppose that  $(S, B)$  is a continuous equilibrium such that  $S(\theta) \leq v^H$  for all  $\theta \in \mathbb{R}$ . Let  $V_0 = B^{-1}(v_i) \geq -\infty$ . Then,

$$B(\phi) = V_2^{-1}(\phi), \quad \phi > V_0,$$

Further,  $S(-\infty) = \lim_{\theta \rightarrow -\infty} S(\theta) = v_i$  and  $S(+\infty) = \lim_{\theta \rightarrow +\infty} S(\theta) = v^H$ , and for any  $\theta$ , we have  $S(\theta) = V_1^{-1}(\theta)$  where

$$V_1(z) = \log \frac{z - v_i}{v^H - z} - V_2(z), \quad z \in (v_i, v^H).$$

Any buyer of type  $\phi < V_0$  will not trade, and has a bidding policy  $B$  that is not uniquely determined at types below  $V_0$ .

# Tail Condition

**Definition:** We say that a probability density  $g(\cdot)$  on the real line is of exponential type  $\alpha$  at  $+\infty$  if, for some constants  $c > 0$  and  $\gamma > -1$ ,

$$\lim_{x \rightarrow +\infty} \frac{g(x)}{x^\gamma e^{\alpha x}} = c$$

In this case, we write  $g(x) \sim \text{Exp}_{+\infty}(c, \gamma, \alpha)$ .

# Exponential Tails in Percolation Models

Suppose  $N = 1$ , and let  $\lambda = \lambda_1$  and  $\psi_t = \psi_{1t}$ . The Laplace transform  $\hat{\psi}_t$  of  $\psi_t$  is given by

$$\hat{\psi}_t(z) = \frac{e^{-\lambda t} \hat{\psi}_0(z)}{1 - (1 - e^{-\lambda t}) \hat{\psi}_0(z)}$$

and  $\psi_t(x) \sim \text{Exp}_{+\infty}(c_t, 0, -\alpha_t)$  in  $t$ , where  $\alpha_t$  is the unique positive number  $z$  solving

$$\hat{\psi}_0(z) = \frac{1}{1 - e^{-\lambda z}},$$

and where

$$c_t = \frac{e^{-\lambda t}}{(1 - e^{-\lambda t})^2 \frac{d}{dz} \hat{\psi}_0(\alpha_t)}.$$

Furthermore,  $\alpha_t$  is monotone decreasing in  $t$ , with  $\lim_{t \rightarrow \infty} \alpha_t = 0$ .

# Strictly Monotone Equilibrium

**Proposition:** Suppose that, for all  $t$  in  $[0, T]$ , there are  $\alpha_i(t)$ ,  $c_i(t)$ , and  $\gamma_i(t)$  such that

$$\psi_{it}^H(x) \sim \text{Exp}_{+\infty}(c_i(t), \gamma_i(t), -\alpha_i(t)).$$

If  $\alpha_i(T) < 1$ , then there is no equilibrium associated with  $V_0 = -\infty$ . Moreover, if  $v_i - v_j$  is sufficiently large and if  $\alpha_i(T) > \alpha^*$ , where  $\alpha^*$  is the unique positive solution to  $\alpha^* = 1 + 1/(\alpha^{*2\alpha^*})$  (which is approximately 1.31), then there exists a unique strictly monotone equilibrium associated with  $V_0 = -\infty$ . This equilibrium is in undominated strategies, and maximizes total welfare among all continuous equilibria.

## Class- $i$ Agent Utility

The expected future profit at time  $t$  of a class- $i$  agent is

$$\mathcal{U}_i(t, \Theta_t) = E \left[ \sum_{\tau_k > t} \sum_j \kappa_{ij} \pi_{ij}(\tau_k, \Theta_{\tau_k}) \mid \Theta_t \right],$$

where  $\tau_k$  is this agent's  $k$ -th auction time and  $\pi_{ij}(t, \theta)$  is the expected profit of a class- $i$  agent of type  $\theta$  entering an auction at time  $t$  with a class- $j$  agent.

Agents may be able to disguise the characteristics determining their information at a particular auction. In this case, we denote the expected future profit at time  $t$  of a class- $i$  agent as  $\hat{\mathcal{U}}_i(t, \Theta_t)$ .



# The Value of Initial Information and Connectivity When Trades Can be Disguised

**Theorem:** Suppose that  $v_1 = v_2$ . If  $\lambda_2 \geq \lambda_1$  and if the initial type densities  $\psi_{10}$  and  $\psi_{20}$  are distinguished by the fact that the density  $p_2$  of the number of signals received by class-2 agents has first-order stochastic dominance over the density  $p_1$  of the number of signals by class-1 agents, then

$$\frac{E[\hat{\mathcal{U}}_2(t, \Theta_{2t})]}{\lambda_2} \geq \frac{E[\hat{\mathcal{U}}_1(t, \Theta_{1t})]}{\lambda_1}, \quad t \in [0, T].$$

The above inequality holds strictly if, in addition,  $\lambda_2 > \lambda_1$  or if  $p_2$  has strict dominance over  $p_1$ .

# What if Characteristics are Commonly Observed?

- ▶ trade-off between adverse selection and gains from trade.
- ▶ more informed/connected investor may achieve lower profits than less informed/connected investor.
- ▶ If  $v_1 = v_2 = 0.9$ ,  $v_3 = 0$ ,  $v^H = 1.9$ ,

$$\psi_{10}(x) = 12 \frac{e^{3x}}{(1 + e^x)^5},$$

and  $\psi_{20}(x) = \psi_{10} * \psi_{10}$ .

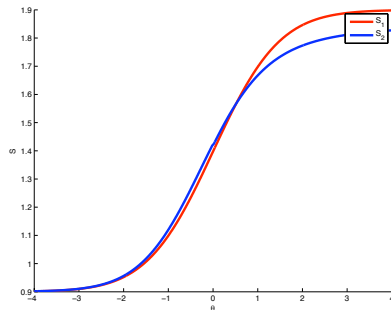
Then,

$$E[\mathcal{U}_2(t, \Theta_{1t})] < E[\mathcal{U}_1(t, \Theta_{2t})]$$

and

$$E[\hat{\mathcal{U}}_1(t, \Theta_{1t})] < E[\mathcal{U}_1(t, \Theta_{2t})].$$

# What if Characteristics are Commonly Observed?



## Even If Characteristics are Commonly Observed Connectivity May be Valuable

**Proposition:** Suppose that  $\kappa_1 = \kappa_2$ ,  $v_1 = v_2$  and  $\lambda_1 < \lambda_2$ , and suppose that class-1 and class-2 investors have the same initial information quality, that is,  $\psi_{10} = \psi_{20}$ , and assume the exponential tail condition  $\psi_{it}^H \sim \text{Exp}_{+\infty}(c_{it}, \gamma_{it}, -\alpha_{it})$  for all  $i$  and  $t$ , with  $\alpha_{10} > 3$ ,  $\alpha_{30} < 3$  and

$$\alpha_{30} > \frac{\alpha_{10} - 1}{3 - \alpha_{10}},$$

and

$$\frac{\alpha_{1t} + 1}{\alpha_{1t} - 1} > \alpha_{3t}, \quad t \in [0, T].$$

If  $\frac{v_1 - v_3}{v^H - v_1}$  is sufficiently large, then for any time  $t$  we have

$$\frac{E[\mathcal{U}_2(t, \Theta_{2t})]}{\lambda_2} > \frac{E[\hat{\mathcal{U}}_2(t, \Theta_{2t})]}{\lambda_2} > \frac{E[\hat{\mathcal{U}}_1(t, \Theta_{1t})]}{\lambda_1} > \frac{E[\mathcal{U}_1(t, \Theta_{1t})]}{\lambda_1}.$$

## Subsidizing Order Flow

- ▶ Investors  $i$  and  $j$  with  $v_i = v_j$  meet at time  $t$ .
- ▶ Enter a swap agreement by which the amount

$$k \left[ (p_j(t) - Y)^2 - (p_i(t) - Y)^2 \right],$$

will be paid by investor  $i$  to investor  $j$  at time  $T$ .

- ▶ Increase connectivity of class  $i$  investors.
- ▶ When would investors want to subsidize order flow?

# Concluding Remarks

- ▶ tractable model of information diffusion in over-the-counter markets.
- ▶ initial information and connectivity may or may not increase profits:
  - more informed/connected investors attain higher profits than less informed connected investors when investors can disguise trades.
  - more informed/connected investors may attain lower profits than less informed connected investors when investors' characteristics are commonly observed.

# Other Applications

- ▶ centralized exchanges, decentralized information transmission
- ▶ bank runs
- ▶ knowledge spillovers
- ▶ social learning
- ▶ technology diffusion

# Thank You !

and

# Bon Apétit !