A model of emissions and the price of carbon

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Context:

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$\longrightarrow$ indirect tax

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- each firm buys a certain number of certificates
- certificate = "right to pollute"
- firms can trade certificates
- end of the year, if global emissions are below κ, then nothing to pay
- else, each firm pays taxes for its emissions not covered by certificates.

As a tradable asset, we assume that the price of certificates satisfies:

$$dY_t = Z_t dB_t$$
$$Y_T = \lambda 1_{[\kappa,+\infty)}(E_T)$$

under a probability $\mathbb{Q}$, where E are the global emissions.

A firm can trade certificates.

$\longrightarrow$ control θ (portfolio).

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$$E = \sum_{i=1}^{n} E^i.$$

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But it is in return of a cost: $c^i : \mathbb{R} \to \mathbb{R}$, C^1, strictly convex and satisfies Inada conditions, for each $t \in [0, T]$:

$$(c^i)'(-\infty) = -\infty \text{ and } (c^i)'(+\infty) = +\infty.$$

The wealth is:

$$X_t^i = X_t^{i,\xi,\theta} = x^i + \int_0^T \theta_t^i dY_t - \int_0^T c^i(\xi_t^i)dt - E_T^i Y_T.$$

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Given a utility function U^i, each firm wants to solve:

$$\sup_{(\xi,\theta)\in\mathcal{A}} \mathbb{E} U^i(X_T^{i,\xi,\theta}).$$

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If Y is given, then there exists an optimal control $(\hat{\xi}^i, \hat{\theta}^i)$ with:

$$\hat{\xi}^i_t = (c')^{-1}(Y_t).$$

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In other words we want to solve:

$$dE_t^i = \{b_t^i - [(c^i)']^{-1}(Y_t)\}dt + \sigma_t^i dB_t, \qquad E_0^i = 0, \text{ for each } i$$
$$dY_t = Z_t dB_t, \qquad Y_T = \lambda 1_{[\kappa,+\infty)}(E_T).$$

For certain b^i's and σ^i's, this leads to the forward-backward SDE:

$$
\begin{aligned}
dE_t &= \{b(t, E_t) - f(Y_t)\}dt + \sigma(t)dB_t, \qquad E_0 = 0 \\
dY_t &= Z_t dB_t, \qquad Y_T = \lambda 1_{[\kappa,+\infty)}(E_T),
\end{aligned}
$$

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We approximate by a sequence of regular FBSDEs and show convergence. Finally: existence and uniqueness of the solution.

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the bigger is the decrease.

Incomplete market: introduction of a Brownian motion by firm. Look for a global equilibrium, as in Karatzas and Shreve. Add the condition:

$$\forall t, \ \sum_{i=1}^{n} \theta_t^i = 0.$$

Short Bibliography

- Fully coupled forward-backward stochastic differential equations and applications to optimal control, Peng and Wu.
- Forward-backward stochastic differential equations and their applications, Ma and Yong, Springer.
- Methods of Mathematical Finance, Karatzas and Shreve, Springer.