The Electricity Bid Stack:

Linking the dynamics of fuel, power and carbon prices

Michael Coulon

April 10th, 2010

mcoulon@princeton.edu

ORFE Department

Princeton University

Structural Models for Power

Hybrid / structural models are alternatives to reduced-form. Key steps:

- Choice of Factors - Demand, Fuel Prices, Outages, etc.
- Choice of function $S_t = B(t, D_t, G_t, \ldots)$ to map to spot power.
- Calibration method for both components above.

Structural Models for Power

Hybrid / structural models are alternatives to reduced-form. Key steps:

- Choice of Factors - Demand, Fuel Prices, Outages, etc.
- Choice of function $S_t = B(t, D_t, G_t, \ldots)$ to map to spot power.
- Calibration method for both components above.

Pros and Cons:

- Exploit strong and intuitive relationships with easily observable underlying price drivers (eg, load).
- Many factors and complex market structures leads to difficulty in creating both realistic and mathematically tractable models.

Structural Models for Power

Hybrid / structural models are alternatives to reduced-form. Key steps:

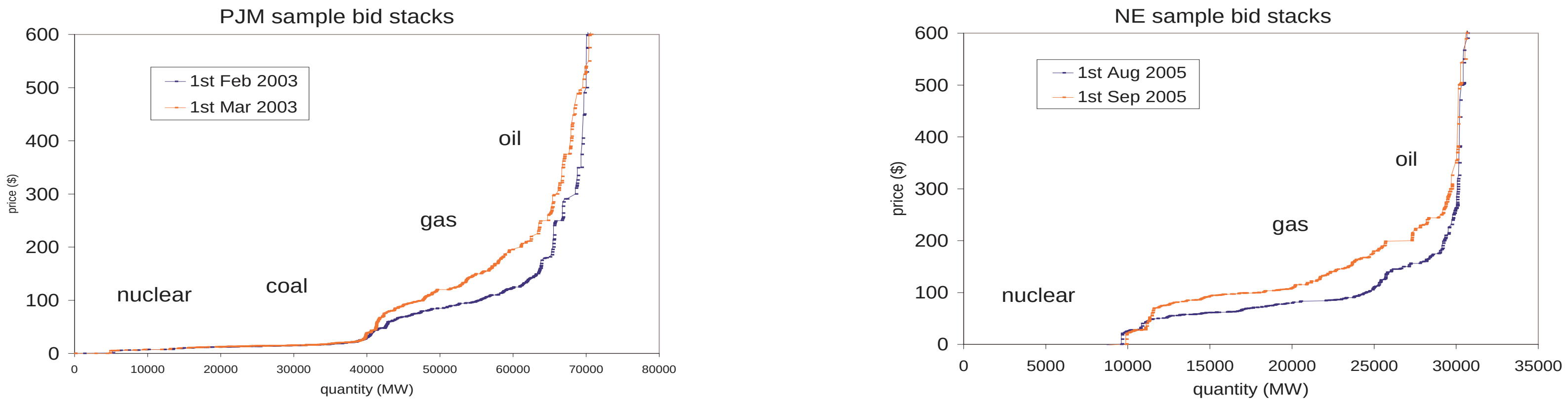
- Choice of Factors - Demand, Fuel Prices, Outages, etc.
- Choice of function $S_t = B(t, D_t, G_t, \ldots)$ to map to spot power.
- Calibration method for both components above.

Pros and Cons:

- Exploit strong and intuitive relationships with easily observable underlying price drivers (eg, load).
- Many factors and complex market structures leads to difficulty in creating both realistic and mathematically tractable models.

Examples include: Eydeland & Wolyniec (2003), Burger *et al* (2004), Cartea, Figueroa and Geman (2009), Davison *et al* (2002), Pirrong & Jermakyan (2008), Aid *et al* (2009)

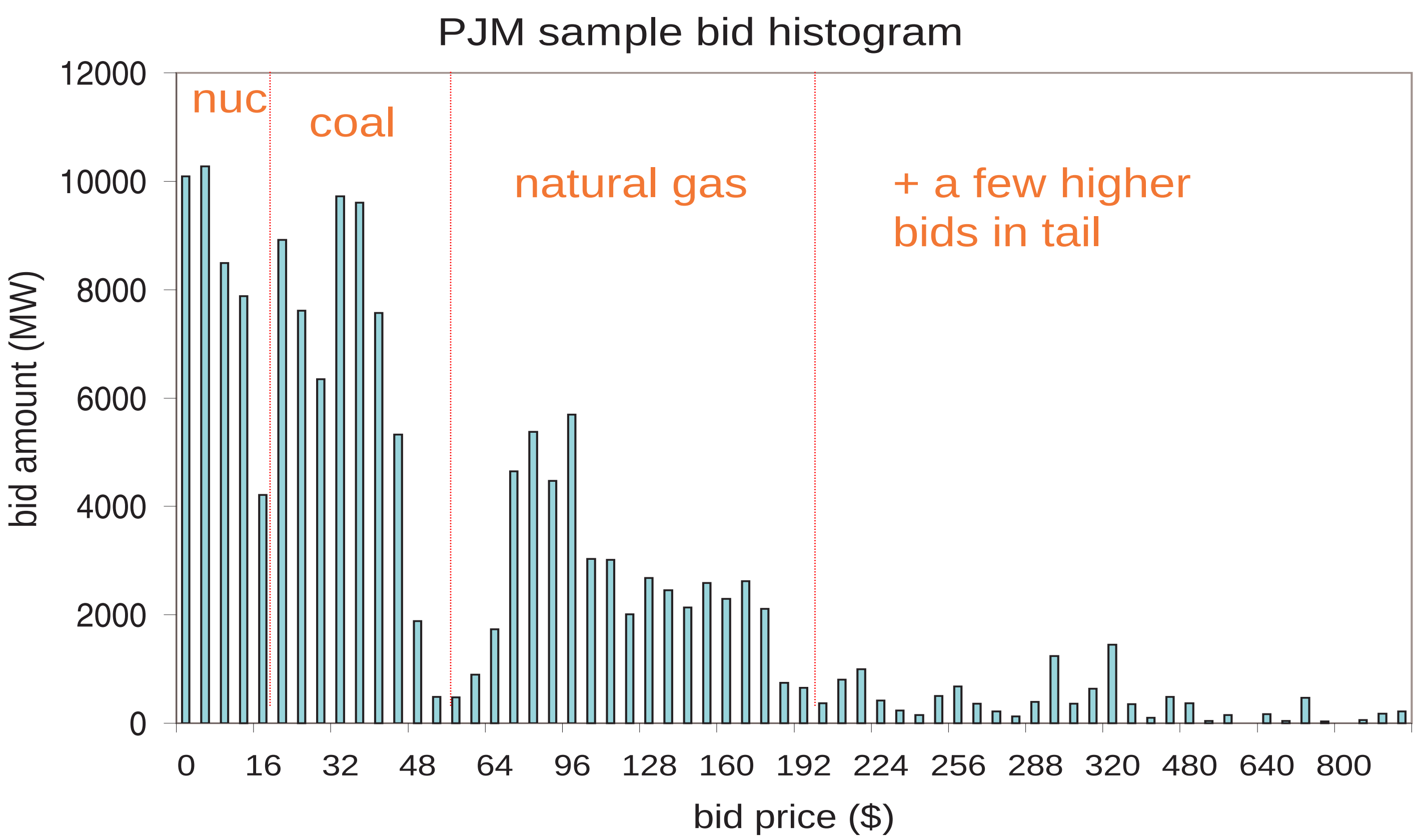
The bid stack function



- Generators make day-ahead bids based on production costs
- Arrange by price (merit order) to form the **bid stack**
- Spot price S_t (market clearing price) is set by finding highest bid needed to match demand D_t (often assumed inelastic).
- Higher cost units are thus only needed for peak demand.
- Actual bid data available in many US markets

An alternative perspective

- Can look at bid stack as a histogram of bids
- Merit order is often visible through clusters of bids



Distributions of bids

Coulon, Howison (2009) - spot price / demand / capacity / fuel price

- Let $F_1(x), \ldots, F_N(x)$ equal the proportion of bids below x dollars for generators of fuel type $i = 1, \ldots, N$, with weights $w_1, \ldots, w_N$.
- We require $0 < \frac{D_t}{C^{max}} < 1$. (demand cannot exceed max capacity)
- Then the spot power price S_t solves:

$$\sum_{i=1}^{N} w_i F_i(S_t) = \frac{D_t}{C^{max}}$$

- Hence the bid stack is the "inverse cdf" of our distribution of bids.
- C^{max} can be replaced by a process C_t for capacity available.
- Assuming $0 < b_L < \frac{D_t}{C_t} < b_U < 1$, we can rescale the region (b_L, b_U) to equal $(0, 1)$. This will improve the fit by ignoring highest and lowest bids (typically set $b_L = 0.2$ and $b_U = 0.9$ or 0.95).

Distribution-based Bid Stack Model

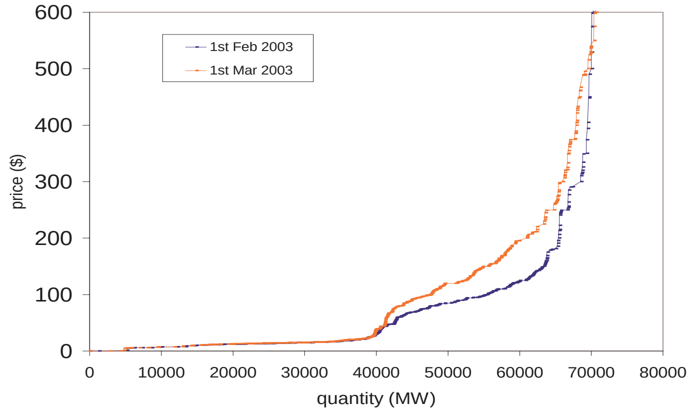
- Now choose two-parameter distributions for bids (location m_i, scale s_i) such as Gaussian, Logistic, Cauchy, Weibull.
- *One Fuel Case: (New England Market):*
  - Gaussian: $S_t = m_1 + s_1 \Phi^{-1}\left(\frac{D_t}{C_t}\right)$
  - Logistic: $S_t = m_1 + s_1 \left(\log(D_t) - \log(C_t - D_t)\right)$
  - Cauchy: $S_t = m_1 + s_1 \tan\left(\pi\left(\frac{D_t}{C_t} - \frac{1}{2}\right)\right)$
  - Weibull: $S_t = -m_1 \left(\ln(C_t - D_t) - \ln(C_t)\right)^{1/s_1}$
- *Two Fuel Case: (PJM Market, with* $w_1 \approx 0.5$*):*
  - e.g. Gaussian: S_t solves

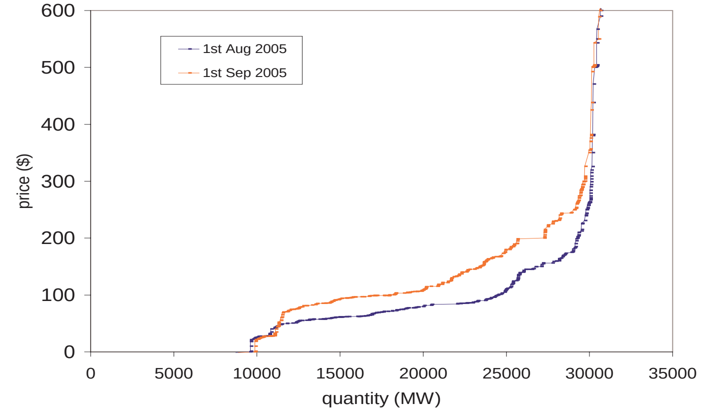
$$w_1 \Phi\left(\frac{S_t - m_1}{s_1}\right) + w_2 \Phi\left(\frac{S_t - m_2}{s_2}\right) = \frac{D_t}{C_t}$$

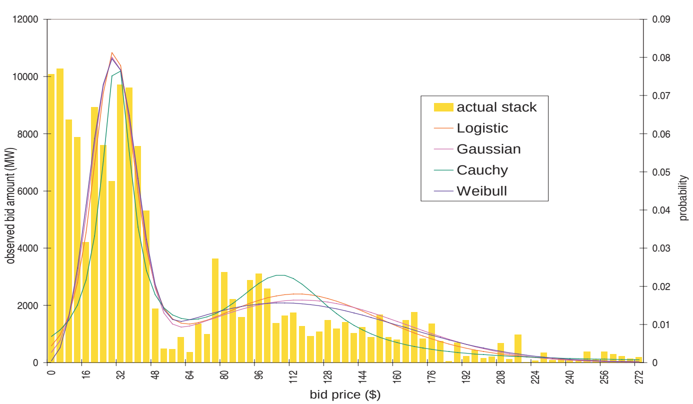
- We estimate m_1, s_1, m_2, s_2 by MLE independently for each day, and then observe the relationship with fuel prices.

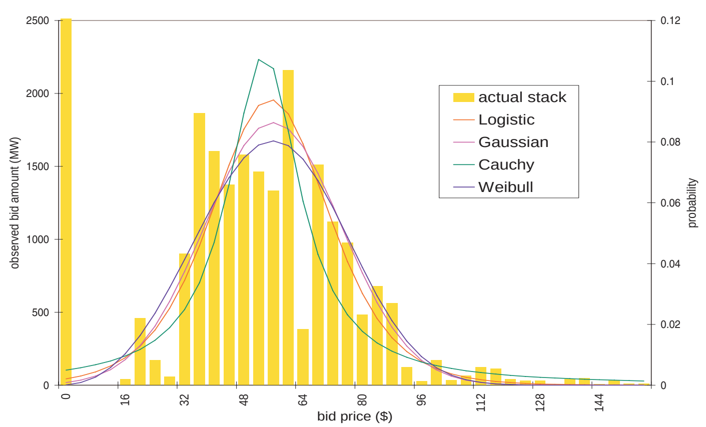
Sample Bid Stack Fitting

- Sample bid stacks for PJM (left) and NEPOOL (right) along with histogram representations and fits below:



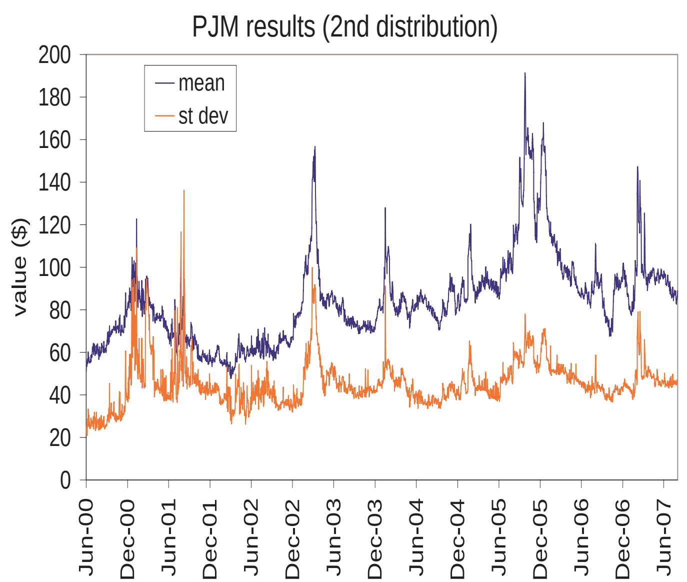


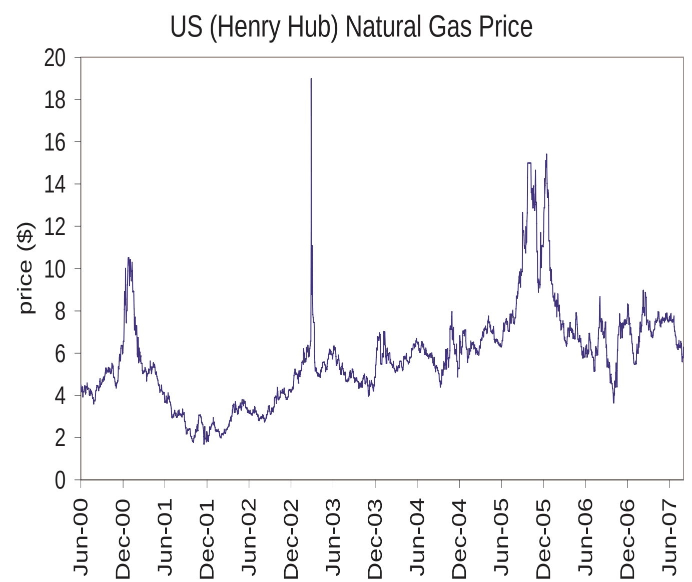




PJM Results (June 00 - Jul 07) for m_2 and s_2

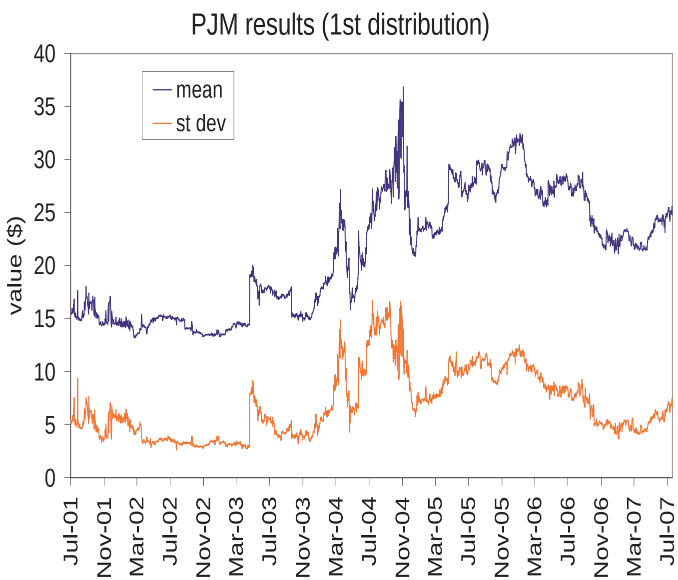
- As expected, the second distribution's parameters show very high correlation with natural gas prices (as high as 96% for m_2 in recent years).

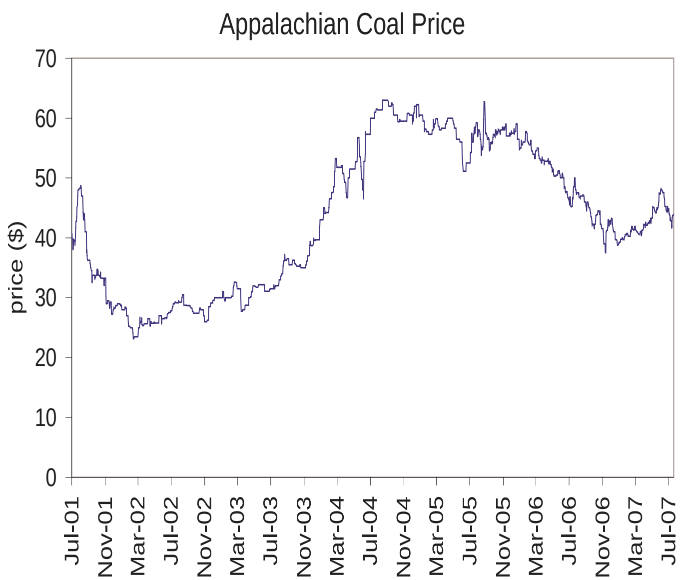




PJM Results (June 00 - Jul 07) for m_1 and s_1

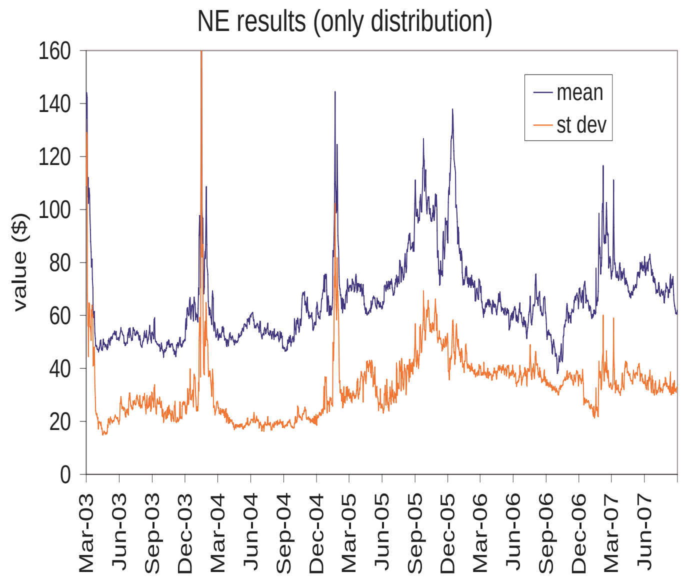
- The first distribution for PJM also shows reasonable correlation with coal prices (86% for m_1 over entire dataset).

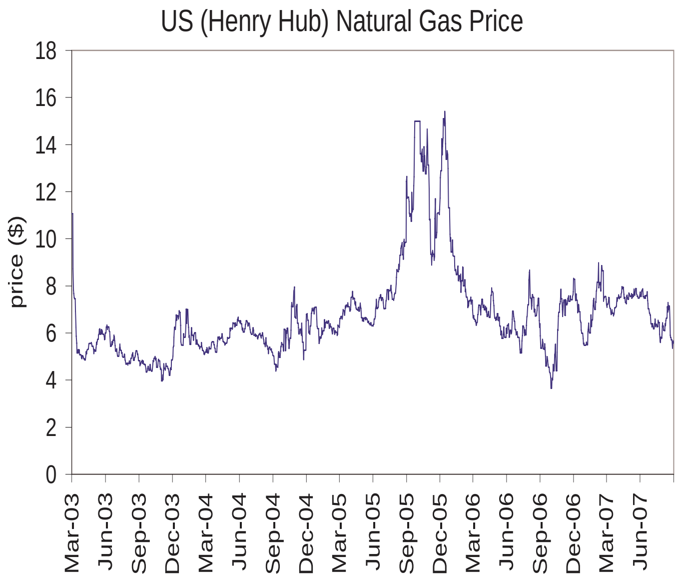




NE Results (Mar 03 - Jul 07) for m_2 and s_2

- Again, very high correlation with gas prices (as high as 95% for m_2 in recent years).





Regression Results

- Thus assume a linear dependence of parameters on fuel prices: (G_t = gas price, P_t = coal price)
- For PJM, $$m_1 = \tilde{\alpha}_0 + \tilde{\alpha}_1 P_t, \quad s_1 = \tilde{\beta}_0 + \tilde{\beta}_1 P_t,$$ $$m_2 = \alpha_0 + \alpha_1 G_t, \quad s_2 = \beta_0 + \beta_1 G_t$$
- For NE, $m_1 = \alpha_0 + \alpha_1 G_t, \quad s_1 = \beta_0 + \beta_1 G_t$
- Regression results are encouraging, particularly for recent years.

		m_1 or m_2			s_1 or s_2		
		inter	slope	R^2	inter	slope	R^2
PJM (Coal)	Entire dataset	3.38	0.408	0.727	-1.57	0.123	0.703
	Last 2yrs	7.02	0.390	0.749	-5.32	0.198	0.869
PJM (Gas)	Entire dataset	35.15	8.51	0.833	17.82	1.25	0.233
	Last 2yrs	31.03	9.20	0.927	15.23	1.53	0.674
NE (Gas)	Entire dataset	17.35	7.67	0.701	7.36	1.29	0.168
	Last 2yrs	27.36	6.58	0.908	8.63	1.11	0.557

Fuel Price Models

- For G_t, use two-factor Schwartz model, common for commodities:

$$
\begin{aligned}
dX_t^1 &= \kappa(\mu_1 - X_t^1)dt + \sigma_1 dW_t \\
dX_t^2 &= \mu_2 dt + \sigma_2 d\tilde{W}_t \\
G_t &= \exp(g(t) + X_t^1 + X_t^2).
\end{aligned}
$$

- For coal price P_t, a simple one-factor model:

$$
\begin{aligned}
dX_t^3 &= \mu_3 dt + \sigma_3 dW_t^3, \\
dW_t^1 dW_t^3 &= \rho_{13} dt \\
dW_t^2 dW_t^3 &= \rho_{23} dt \\
P_t &= \exp(X_t^3).
\end{aligned}
$$

(Parameter estimation using MLE, Kalman Filtering and historical gas forward curves. Can then calibrate to current coal and gas forwards.)

Demand and Capacity Model

- Assume D_t inelastic and bid stack a function of $\frac{D_t}{C_t}$.
- Assume D_t and C_t are independent of fuel prices G_t, P_t

Demand and Capacity Model

- Assume D_t inelastic and bid stack a function of $\frac{D_t}{C_t}$.
- Assume D_t and C_t are independent of fuel prices G_t, P_t
- For C_t, calculate implied capacity available: $S_t = B^{obs}\left(\frac{D_t}{C_t^{imp}}\right)$
- C_t^{imp} describes an aggregation of several factors:
  Seasonal maintenance schedules (planned outages), Unplanned / forced generator outages, Transmission constraints, Operational Constraints, Imports & Exports, Other effects.

Demand and Capacity Model

- Assume D_t inelastic and bid stack a function of $\frac{D_t}{C_t}$.
- Assume D_t and C_t are independent of fuel prices G_t, P_t
- For C_t, calculate implied capacity available: $S_t = B^{obs}\left(\frac{D_t}{C_t^{imp}}\right)$
- C_t^{imp} describes an aggregation of several factors:
  Seasonal maintenance schedules (planned outages), Unplanned / forced generator outages, Transmission constraints, Operational Constraints, Imports & Exports, Other effects.
- Modelling D_t and C_t separately leads to difficulty in satisfying $0 < \frac{D_t}{C_t} < 1$, as required for the bid stack approach.
- Problem avoided by using margin as a factor: $M_t = C_t - D_t$
- Model $\tilde{D}_t = \log D_t$ and $\tilde{M}_t = \log M_t$ to ensure $0 < \frac{D_t}{D_t + M_t} < 1$

Instead: Demand and Margin Model

- Log-demand well modelled as the sum of a seasonal component and an Ornstein-Uhlenbeck process:

$$\begin{aligned} \log D_t &= f(t) + Y_t \\ f(t) &= a_1 + a_2 t + a_3 \cos(2\pi t + a_4) + a_5 \cos(4\pi t + a_6) \\ dY_t &= \kappa_D(\mu_D - Y_t)dt + \sigma_D dB_t \end{aligned}$$

- Log-margin features short-term outages and recoveries. Thus we choose a regime switching process with 'normal' and 'spike' regimes:

$$\log M_t = \begin{cases} Z_t^{OU} & \text{with probability } 1 - p_i \\ Z_t^{SP} & \text{with probability } p_i \end{cases}$$

$$\begin{aligned} \text{where } Z_t^{OU} &= \kappa_Z \left(\mu_Z - Z_t^{OU}\right) dt + \sigma_Z d\tilde{B}_t, \qquad dB_t d\tilde{B}_t = \rho\, dt, \\ \text{and } Z_t^{SP} &= \alpha - J\,, \qquad J \sim \text{Exp}(\lambda_i), \quad \text{for seasons } i = 1, 2, 3, 4. \end{aligned}$$

Parameter Estimates

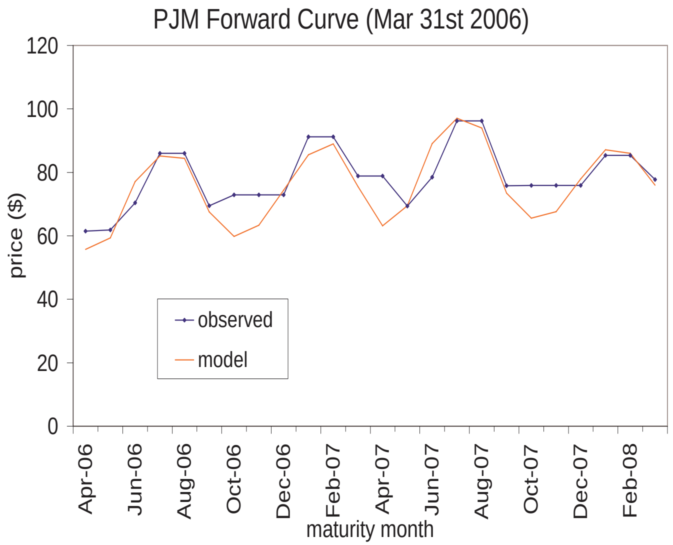
- Parameters of D_t estimated by maximum likelihood methods.
- Parameters of M_t estimated by a combination of maximum likelihood and moment matching techniques.
- Dynamics of underlying factors different significantly in time-scales:

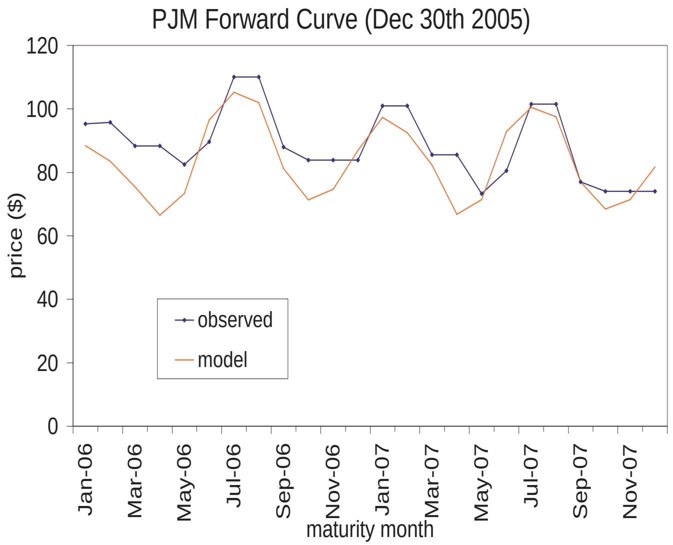
		Mean-Reversion	Volatility
Gas	X_t^2	none	0.14
	X_t^1	1.14	0.70
PJM	Y_t	64.2	1.39
	Z_t^{OU}	133.6	6.12
	Z_t^{SP}	(seas avg: $p = 0.129$,	$\lambda = 1.21$, $\alpha = -1.91$)
NE	Y_t	132.1	2.70
	Z_t^{OU}	76.0	4.79
	Z_t^{SP}	(seas avg: $p = 0.072$,	$\lambda = 1.89$, $\alpha = -2.05$)

Summary of Model Performance

Encouraging results in terms of:

- Simulated forward curves vs market data (eg, for PJM below)
- Behaviour / statistics of simulated price paths vs market data
- Regression coefficients in line with market heat rates
- Implied generation volumes (ie, % of power from gas vs coal)





Additional Challenges

While the basic assumptions here provide a good approximation to PJM and NEPOOL, other markets require additional modifications.
E.g., in the German market (EEX), there are many more challenges to tackle:

- Non-mandatory bidding means that total capacity (and weights w_i) in the bid stack vary significantly.
- The total available capacity depends strongly on highly volatility wind capacity (can provide $\approx 25\%$ of demand) as well as imports / exports.
- Demand elasticity to price is significant $\implies$ combined bid and offer curve behaviour replaces bid stack.
- Big variety of power sources: hydro, wind, nuclear, lignite, coal, gas, oil.
- Carbon market in EU introduces a key additional underlying factor.

Carbon Price Modelling

- The European Union Emissions Trading Scheme (EU ETS) begun in 2005 has added a new dimension to energy markets.
- A North American market will/may/should some day do the same?
- We let A_t represent carbon spot (current vintage year) prices. From a structural perspective, A_t joins fuel prices as a new cost of power generation.
- Thus the bid stack model can also be adapted to understand these relationships:
  - Parameters of bid distributions m_i, s_i depend also on allowance price and emission rates of coal and gas
  - Demand for allowances is driven both by exogenous variables (eg weather, economy) and the merit order of the bid stack
  - Fuel-switching as an abatement tool captured via changes in w_i.

Carbon Price Modelling - Literature

Common simplifying assumptions:

- There is finite maturity T corresponding to the end of the trading period, where a fixed penalty is paid per ton of CO_2 over-polluted.
- Supply is strictly fixed by the annual emissions cap.
- Penalties are not paid before T due to intra-period borrowing.

Carbon Price Modelling - Literature

Common simplifying assumptions:

- There is finite maturity T corresponding to the end of the trading period, where a fixed penalty is paid per ton of CO_2 over-polluted.
- Supply is strictly fixed by the annual emissions cap.
- Penalties are not paid before T due to intra-period borrowing.

Key results:

- Equivalence of representative agent optimisation problem.
- A unique equilibrium allowance price A_t exists.
- Discounted allowance prices must be martingales and satisfy:

$$A_t = (\text{discount}) \times (\text{penalty}) \times (\text{probability of shortfall at T})$$

Papers include: Seifert, Uhrig-Homburg, Wagner (2008), Chesney and Taschini (2008), Fehr and Hinz (2006), Carmona *et al* (2008).

Carbon Price Modelling - Bid Stack Model

How do we adapt the bid stack model to carbon markets?

- A simple assumption is that m_1, m_2 are also a linear in A_t:

$$m_1 = \tilde{\alpha}_0 + \tilde{\alpha}_1 P_t + \tilde{\alpha}_2 A_t, \qquad m_2 = \alpha_0 + \alpha_1 G_t + \alpha_2 A_t$$

- Similarly for s_1 and s_2 (due to heat rate differences)
- We would expect $\tilde{\alpha}_2 = e_C$, and $\alpha_2 = e_G$, the average emission rates of coal and gas generators (per MWh of power).

Carbon Price Modelling - Bid Stack Model

How do we adapt the bid stack model to carbon markets?

- A simple assumption is that m_1, m_2 are also a linear in A_t:

$$m_1 = \tilde{\alpha}_0 + \tilde{\alpha}_1 P_t + \tilde{\alpha}_2 A_t, \qquad m_2 = \alpha_0 + \alpha_1 G_t + \alpha_2 A_t$$

- Similarly for s_1 and s_2 (due to heat rate differences)
- We would expect $\tilde{\alpha}_2 = e_C$, and $\alpha_2 = e_G$, the average emission rates of coal and gas generators (per MWh of power).

How do fuel price movements lead to equilibrium carbon price?

- Note that $e_C >> e_G \implies$ merit order changes for high A_t.
- 'Fuel switching' (changes in w_1, w_2) amplifies the effect of merit order changes.

Carbon Price Modelling - Bid Stack Model

- At expiry T, A_T has is worth either 0 or the penalty price π, like a digital option. Thus for the single-period model,

$$A_t = e^{-r(T-t)}\mathbb{E}_t^{\mathbb{Q}}\left[\pi 1_{\{Y_T > K\}}\right],$$

  where Y_t is the total emissions process, and K is total market cap.
- Write $Y_T = \sum_{u=1}^{T} X_u$, where X_t is CO_2 emissions in $[t-1, t]$.
- Using our framework, X_t is a function of G_t, C_t, D_t, M_t, but also A_t.
- For the Gaussian bid stack model, in simplest case of constant emissions rates e_C and e_G, we have (where $D^{\max}$ is max demand) :

$$X_t = w_1 D^{\max} e_C \Phi\left(\frac{S_t - m_1(C_t, A_t)}{s_1(C_t, A_t)}\right) + w_2 D^{\max} e_G \Phi\left(\frac{S_t - m_2(G_t, A_t)}{s_2(G_t, A_t)}\right),$$

$$\text{where} \quad w_1 \Phi\left(\frac{S_t - m_1(C_t, A_t)}{s_1(C_t, A_t)}\right) + w_2 \Phi\left(\frac{S_t - m_2(G_t, A_t)}{s_2(G_t, A_t)}\right) = \frac{D_t}{D_t + M_t}.$$

- High dimensional problem to solve (numerically) for A_t and S_t

Carbon Price Modelling - Simplest Bid Stack

- Let the bid stack consist of two point masses, one for coal at $b_c = f_c + h_c C_t + e_c A_t$ and one for gas at $b_g = f_g + h_g G_t + e_g A_t$.
- Then A_t depends on coal and gas only through the fuel switching price $F_t = \frac{h_g G_t - h_c C_t + f_g - f_c}{e_c - e_g}$, like in Fehr and Hinz (2006).
- Suppose D_t is power demand over $(t, t+1]$ and that $D_t + M_t = 1$.
- Emissions X_t are a piecewise linear function of demand D_t

$$X_t = \begin{cases} X_t^c = e_c D_t + (e_c - e_g)(D_t - w_1)^+ & \text{if } A_t < F_t \\ X_t^g = e_g D_t + (e_g - e_c)(D_t - (1 - w_1))^+ & \text{if } A_t > F_t \end{cases}$$

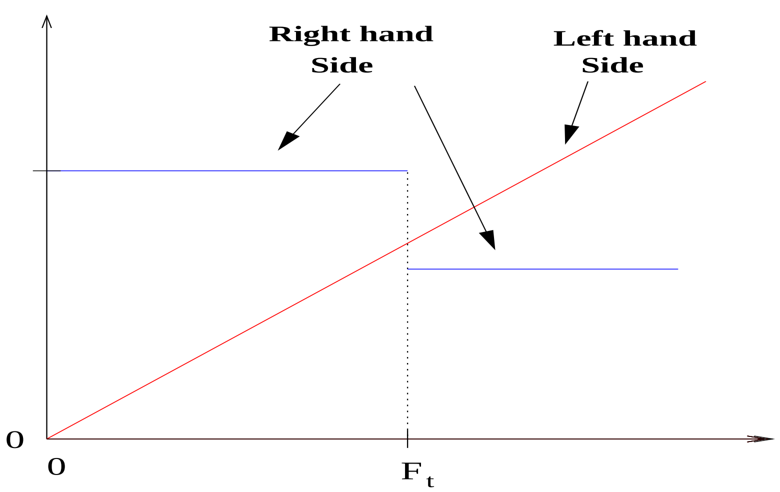
- As before, for a given current emissions level Y_t and fuel switching price F_t, carbon price A_t solves

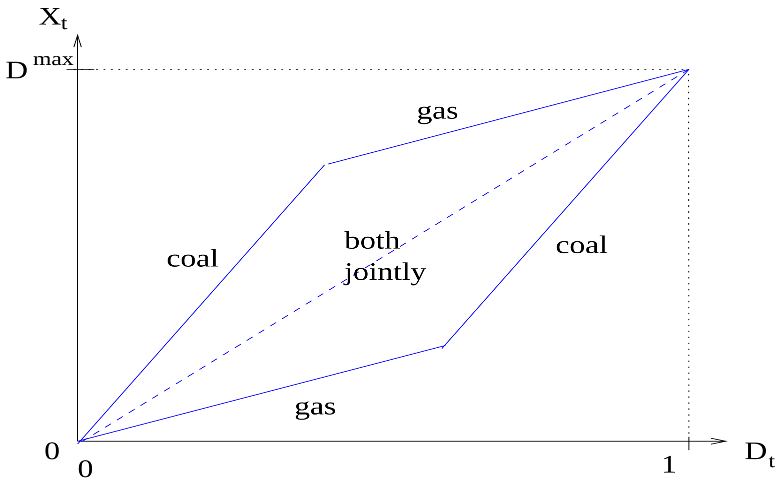
$$A_t = \pi e^{-r(T-t)} P \left\{ Y_t + \sum_{u=t+1}^{T} X_u(D_u, F_u, A_u) > K \right\}.$$

Carbon Price Modelling - Simplest Bid Stack

- Over each time step Δt, we have $A_t = e^{-r\Delta t}\mathbb{E}^{\mathbb{Q}}[A_{t+\Delta t}|A_t]$.
- A_t can be found as RHS above is a step function decreasing in A_t.
- Let $\mathbb{E}_t^{\mathbb{Q}}\left[A_{t'}|X_t^c\right] = e^{-r\Delta t}\mathbb{E}_t^{\mathbb{Q}}\left[A_{t+\Delta t}|Y_t, G_t, X_t = X_t^c\right]$ and same for g.
- Working backwards from maturity,

$$A_t = \begin{cases} \mathbb{E}_t^{\mathbb{Q}}\left[A_{t'}|X_t^c\right] & \text{if} \quad F_t > \mathbb{E}_t^{\mathbb{Q}}\left[A_{t'}|X_t^c\right] \\ F_t & \text{if} \quad \mathbb{E}_t^{\mathbb{Q}}\left[A_{t'}|X_t^g\right] < F_t < \mathbb{E}_t^{\mathbb{Q}}\left[A_{t'}|X_t^c\right] \\ \mathbb{E}_t^{\mathbb{Q}}\left[A_{t'}|X_t^g\right] & \text{if} \quad F_t < \mathbb{E}_t^{\mathbb{Q}}\left[A_{t'}|X_t^g\right]. \end{cases}$$





Carbon Price Modelling - Adding more bids

It is easy to make this simple model more realistic by adding more point masses of bids (eg, categories of cleaner and dirty gas generators):

- Assume n_C different groups of coal generators, with:
  - weights $w_c^1, w_c^2, \dots, w_c^{n_C}$,
  - heat rates $h_c^1, h_c^2, \dots, h_c^{n_C}$ ($h_c^i \le h_c^j$ for $i < j$),
  - emissions rates $e_c^1, e_c^2, \dots, e_c^{n_C}$ ($e_c^i \le e_c^j$ for $i < j$),
  - fixed costs $f_c^1, f_c^2, \dots, f_c^{n_C}$ ($f_c^i \le f_c^j$ for $i < j$).
- Analogous assumptions for n_G groups of gas generators
- Of course require $\sum_{i=1}^{n_C} w_c^i + \sum_{i=1}^{n_G} w_g^i = 1$
- For appropriate parameter choices, $n_C = n_G \approx 6$ already gives a good approximation to carbon dynamics in the full bid stack.

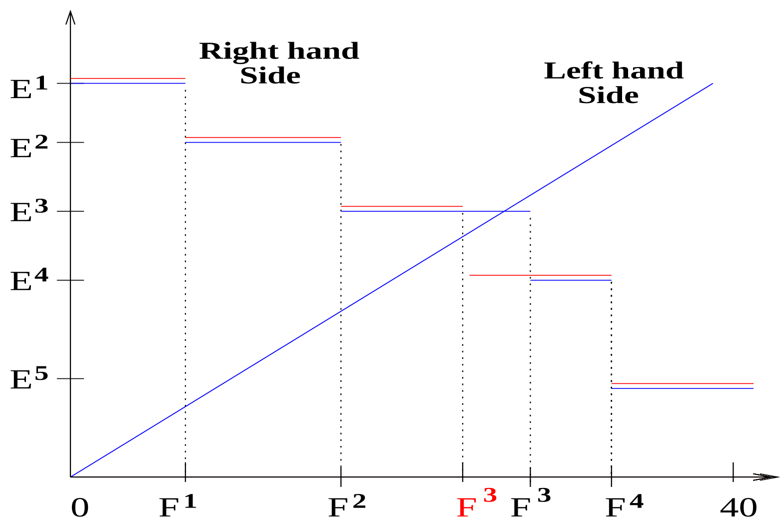
Then at any t, there are up to $n_C + n_G$ possible power prices S_t, and up to $n_C n_G + 1$ possible permutations of the merit order (plus up to $n_C n_G$ cases of matching coal and gas bids), with corresponding cases for $X_t(D_t)$.

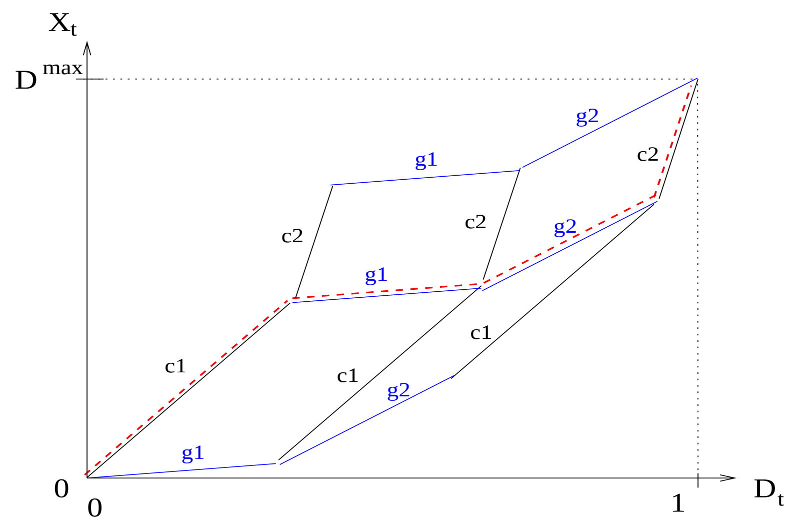
Carbon Price Modelling - Adding more bids

As before, for each node (G_t, Y_t) on our grid, $A_t = \mathbb{E}_t^{\mathbb{Q}}\left[A_{t+1}|Y_t, G_t\right]$ We now have a matrix $\mathbf{F}_t$ of possible 'fuel-switching prices':

$$\mathbf{F}_t = \left(\begin{array}{ccc} \frac{h_g^1 G_t - h_c^1 C_t + f_g^1 - f_c^1}{e_c^1 - e_g^1} & \cdots & \frac{h_g^1 G_t - h_c^{nC} C_t + f_g^1 - f_c^{nC}}{e_c^{nC} - e_g^1} \\ \vdots & \ddots & \vdots \\ \frac{h_g^{nG} G_t - h_c^1 C_t + f_g^{nG} - f_c^1}{e_c^1 - e_g^{nG}} & \cdots & \frac{h_g^{nG} G_t - h_c^{nC} C_t + f_g^{nG} f_c^{nC}}{e_c^{nC} - e_g^1 nG} \end{array} \right),$$

although only elements in the range $[0, \pi]$ are relevant. If $n_C = n_G = 2$:



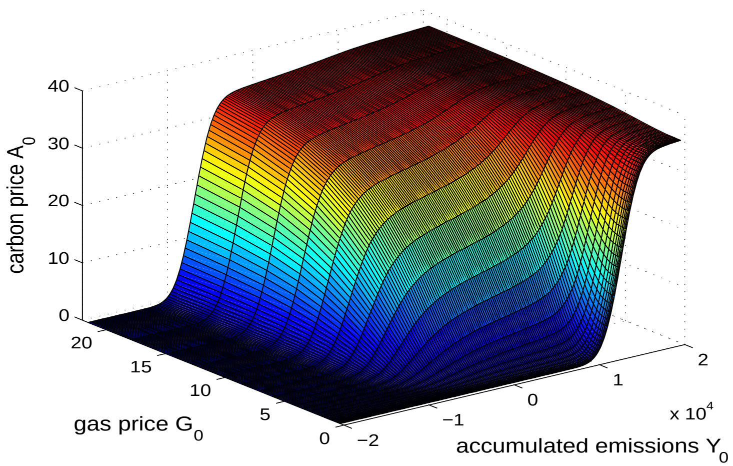


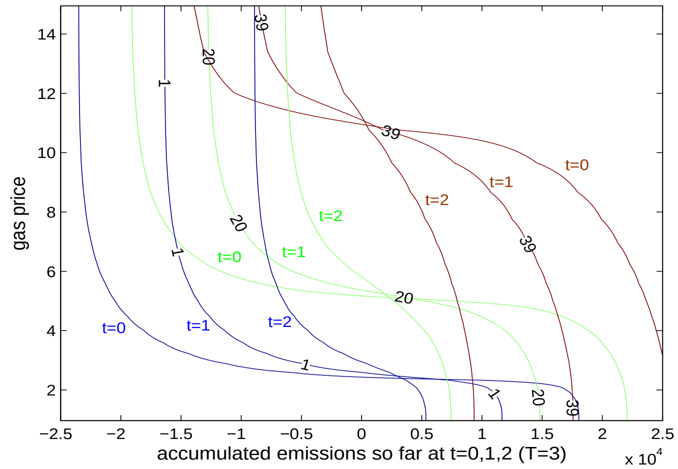
Carbon Price Modelling - One-Period Results

- To reduce dimensionality of $A_t(C_t, G_t, D_t, Y_t)$, we can assume power demand D_t is i.i.d, and C_t fixed (since impact is similar that of G_t).
- Then discretize G_t (trinomial tree with mean reversion) and Y_t (non-recombining grid) and solve backwards from T for $A_t(G_t, Y_t)$.

Carbon Price Modelling - One-Period Results

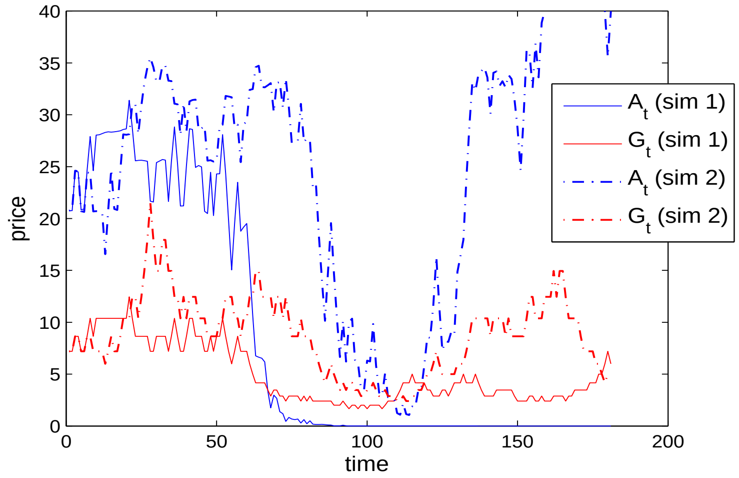
- To reduce dimensionality of $A_t(C_t, G_t, D_t, Y_t)$, we can assume power demand D_t is i.i.d, and C_t fixed (since impact is similar that of G_t).
- Then discretize G_t (trinomial tree with mean reversion) and Y_t (non-recombining grid) and solve backwards from T for $A_t(G_t, Y_t)$.
- Three regions emerge away from maturity (here $\pi = 40, r = 0.05$):
  - In outer regions, high sensitivity to Y_t, as we approach 0 or π.
  - In middle region, high correlation with G_t due to 'merit order abatement mechanism'. Fuel switching widens this region.

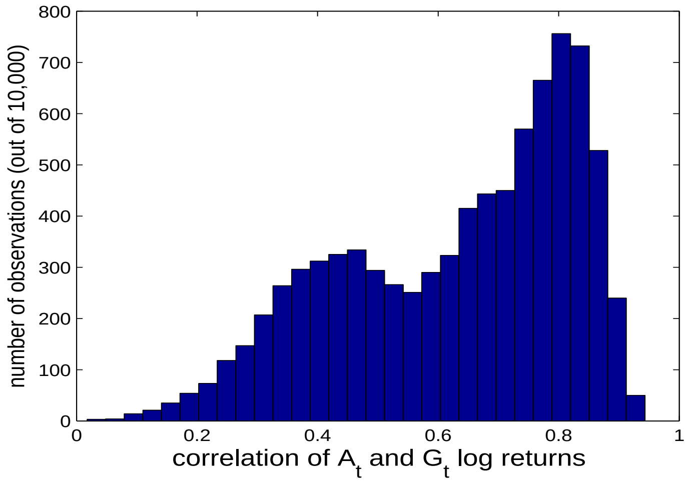




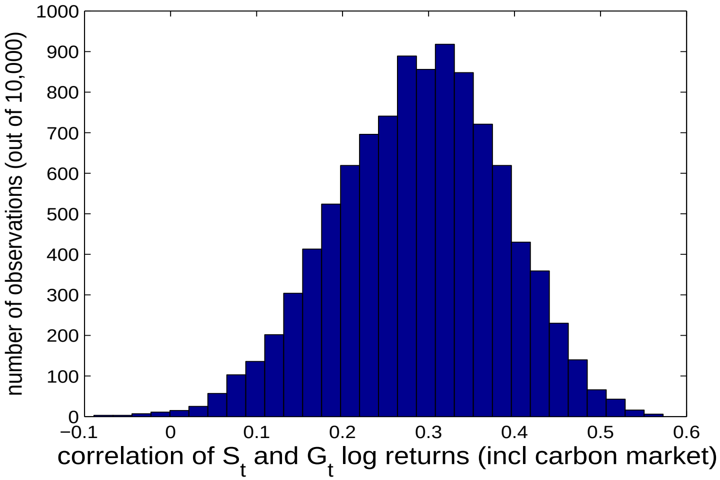
Carbon Price Modelling - One-Period Results

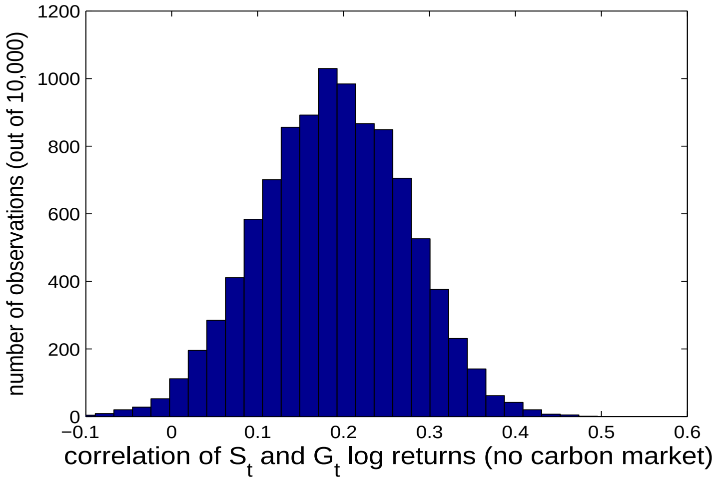
Simulations reveal interesting correlation structure between carbon and gas.





Carbon market also increases gas to power correlation





Carbon Price Modelling - Multi-Period Model

Amended banking rules now imply no final maturity T.

- Trading phases (n-years each) end at T_i for $i = 1, 2 \ldots$
- Shortfall at T_i implies payment of penalty π (since no borrowing) as well as a debt carried into the next phase.
- Assuming fixed cap K per period, let $\tilde{Y}_t = Y_t - \frac{K}{n} t$.
- Hence at time $t = T_i - \Delta t$,

$$A_t = e^{-r\Delta t} \mathbb{E}^{\mathbb{Q}} \left[A_{T_i} + \pi 1_{\{\tilde{Y}_{T_i} > 0\}} | Y_t, G_t \right].$$

- Within n-year trading periods, annual penalty also possible but less likely. eg, at $t = T_i - 1 - \Delta t$,

$$A_t = e^{-r\Delta t} \mathbb{E}^{\mathbb{Q}} \left[A_{T_i - 1} + \pi 1_{\{\tilde{Y}_{T_i - 1} > \frac{K}{n}\}} | Y_t, G_t \right].$$

- Analogy: stock paying annual dividend in event of non-compliance.

Carbon Price Modelling - Multi-Period Model

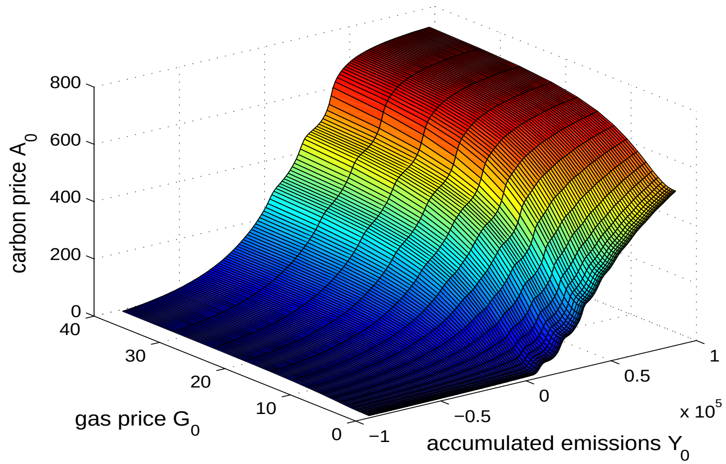
Can solve an infinite horizon problem for $A_t(G_t, Y_t)$ under the following (very unrealistic!) assumptions:

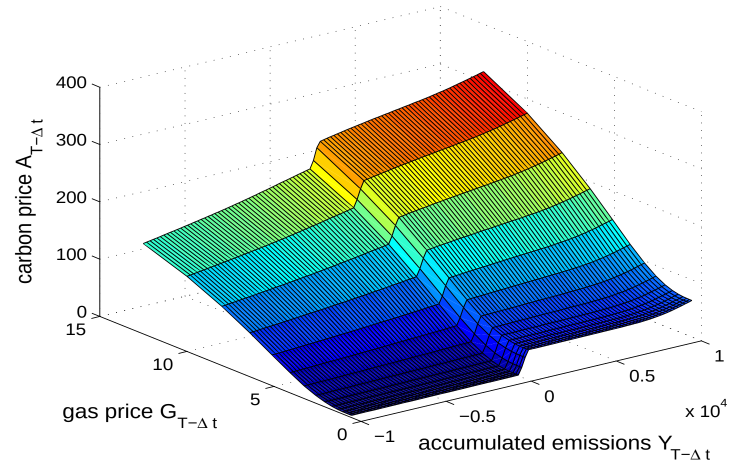
- Fixed carbon market structure (eg, constant K, π)
- Fixed power market structure (eg, constant generation mix and technology including w_i, h_i, e_i).
- No impact from non-power sector, government intervention, and new supply sources (eg, CDM credits), etc.

Now $A_t \leq \frac{\pi}{r}$ (ie, price of a risk-free perpetual bond), allowing the dynamic programming algorithm to converge to a solution $A_t(G_t, Y_t)$

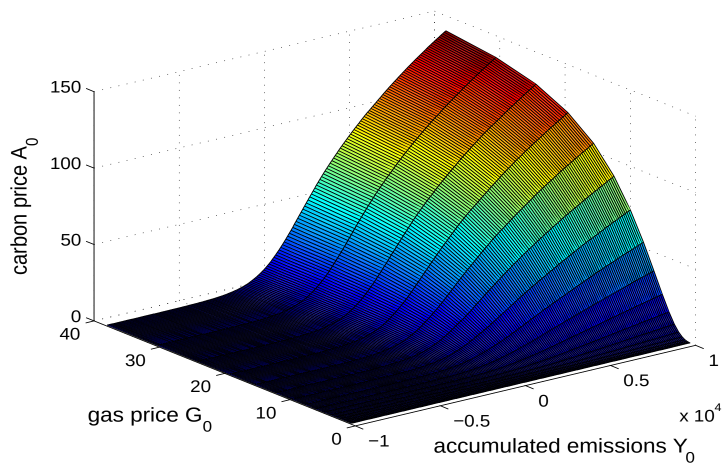
Carbon Price Modelling - Infinite-Horizon

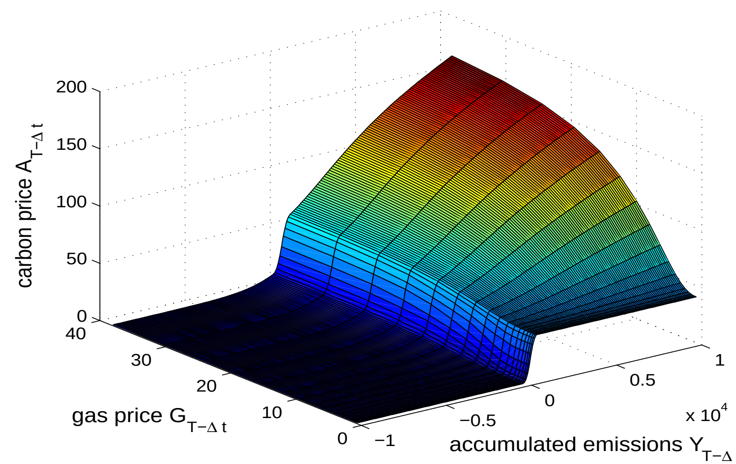
Using a cap similar to the average emissions over all generation scenarios:





Using a more lenient cap:(Left graphs $t = 0$, right graphs $t = T_i - \Delta t$)





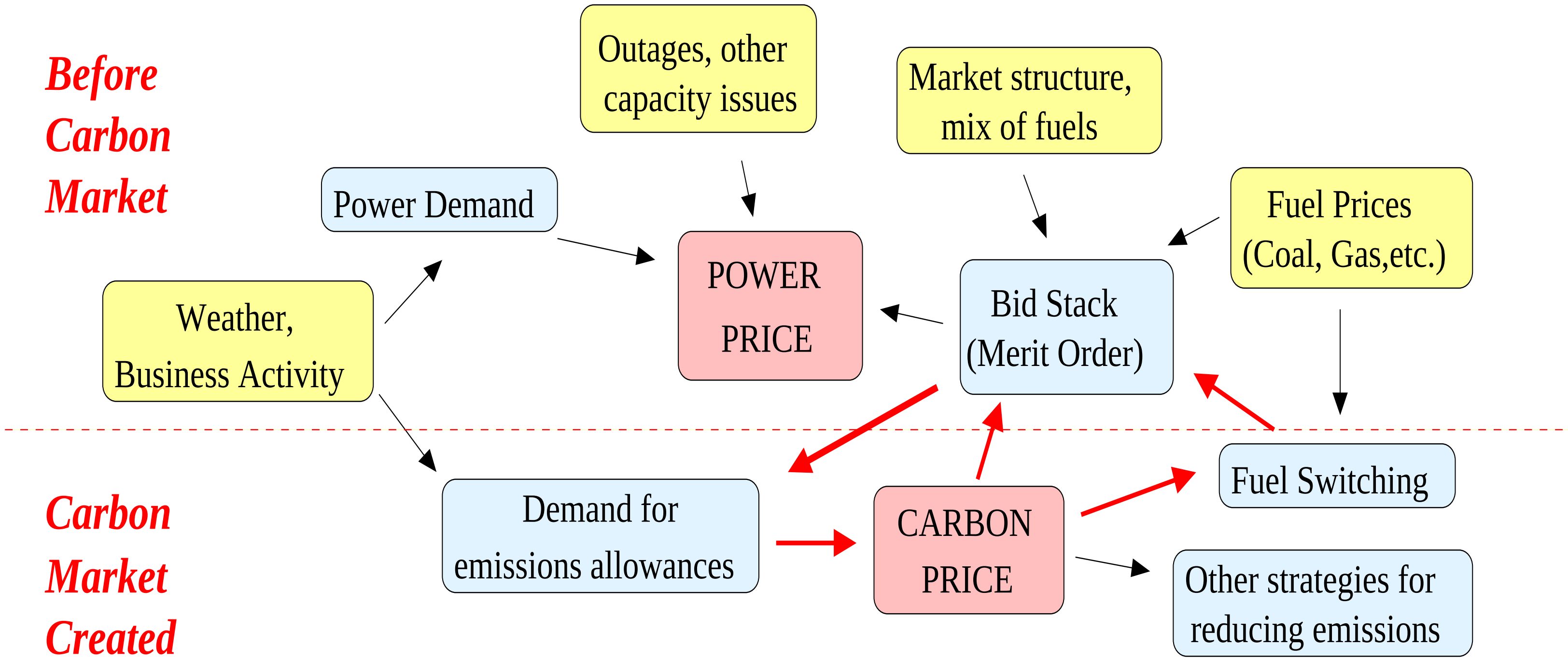
Carbon Markets - Back to Reality

Industry research suggests:

- Majority of remaining power-sector demand before 2012 is for forward hedging (next phase compliance).
- Utilities are hedging 30-50% two-years ahead and 60-80% one-year ahead $\Longrightarrow$ penalty payment currently very unlikely
- Significant supply uncertainty due to NER mechanism, early auctioning, CER credits, industry (non-power sector) selling, and rate of long-term cap level decrease.
- Long-term supply elasticity to price also likely (eg, abatement technologies, offset project creation, government intervention) $\Longrightarrow$ 'production' and 'storage' decisions relevant.

While many modifications are needed, the simple model can give us some intuition about the key price dependencies.

Summary of Dependencies in Energy Markets



- Higher carbon prices automatically lead to emissions reductions in the power market (through merit order changes and/or fuel switching), thus reducing carbon prices again - an equilibrium exists.
- Strength of gas, power and carbon relationships depend on which scenario we consider (eg high or low demand, high or low gas/coal price ratio)

Carbon Model Conclusions

- The bid stack model successfully captures power price behaviour and dependencies on supply and demand drivers.

Carbon Model Conclusions

- The bid stack model successfully captures power price behaviour and dependencies on supply and demand drivers.
- Extension to carbon markets is straightforward under artificially simple assumptions. Power sector abatement can be intuitively understood through bid stack changes.

Carbon Model Conclusions

- The bid stack model successfully captures power price behaviour and dependencies on supply and demand drivers.
- Extension to carbon markets is straightforward under artificially simple assumptions. Power sector abatement can be intuitively understood through bid stack changes.
- Behaviour of non-power sector and offset supply less clear.

Carbon Model Conclusions

- The bid stack model successfully captures power price behaviour and dependencies on supply and demand drivers.
- Extension to carbon markets is straightforward under artificially simple assumptions. Power sector abatement can be intuitively understood through bid stack changes.
- Behaviour of non-power sector and offset supply less clear.
- Multi-phase modelling necessary but very challenging, due especially to non-stationarity of markets and political or regulatory uncertainty.

Carbon Model Conclusions

- The bid stack model successfully captures power price behaviour and dependencies on supply and demand drivers.
- Extension to carbon markets is straightforward under artificially simple assumptions. Power sector abatement can be intuitively understood through bid stack changes.
- Behaviour of non-power sector and offset supply less clear.
- Multi-phase modelling necessary but very challenging, due especially to non-stationarity of markets and political or regulatory uncertainty.
- Model testing and calibration to historical data is limited, but increased option price data could be beneficial.

Carbon Model Conclusions

- The bid stack model successfully captures power price behaviour and dependencies on supply and demand drivers.
- Extension to carbon markets is straightforward under artificially simple assumptions. Power sector abatement can be intuitively understood through bid stack changes.
- Behaviour of non-power sector and offset supply less clear.
- Multi-phase modelling necessary but very challenging, due especially to non-stationarity of markets and political or regulatory uncertainty.
- Model testing and calibration to historical data is limited, but increased option price data could be beneficial.
- Many interesting topics for research!