A structural risk neutral model for electricity prices

Industrial-Academic Forum
on Commodities, energy markets, and emissions trading

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- Seasonality of spot prices
- Electricity is not storable, thus buy-and-hold strategies on the spot are just not feasible

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- Forward/Futures contracts : spot price is the underlying, trading on a continuous basis, with physical delivery or financially settled.
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- Important part of electricity options trading (Asian, swings...) done OTC

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- In standard stock financial markets : $F_t(T) = P_t e^{r(T-t)}$. This equality relies heavily on costless storability of financial assets, it breaks down when P_t is spot price of electricity
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  - Models for spot and $F_t(T) := \mathbb{E}_Q[P_T | \mathcal{F}_t]$
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Introduction IV : Model idea

- Imagine an fictitious economy where electricity is produced only out of coal with generation with the same efficiency
- Then, electricity spot price $P_t = h_c S_t^c$, where h_c is the coal heat rate.
- Assume no-arbitrage in the market of coal
- Then, preceeding relation hodls also for forwards

$$F_t^e(T) = h_c F_t^c(T)$$

- and no arbitrage relation between spot and forward can be transported to electricity prices:

$$\begin{tikzcd} S_t^c \arrow[r, "g"] \arrow[d, equal] & P_t \arrow[d, equal] \\ F_t^c \arrow[r, "g"] & F_t^0 \end{tikzcd}$$

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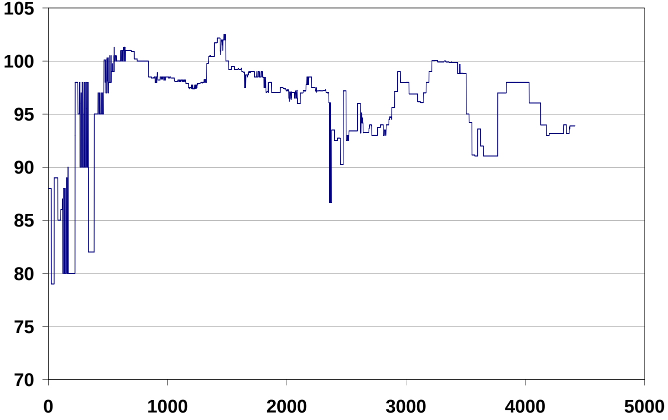

Figure: PolPX spot price - june - december 2000 - www.polpx.pl

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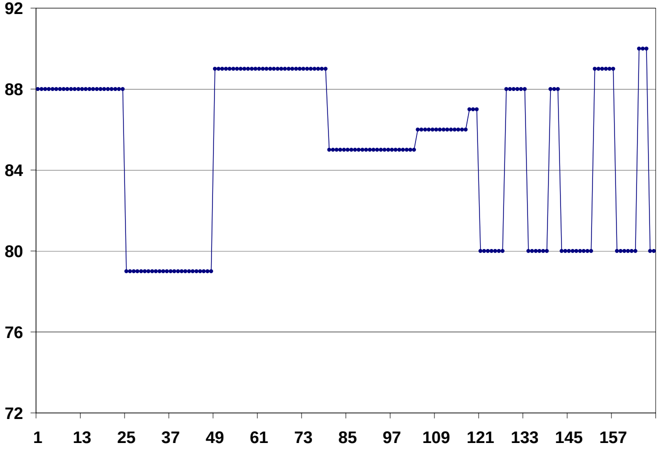

Figure: PolPX spot price - 1st week of july, 2000

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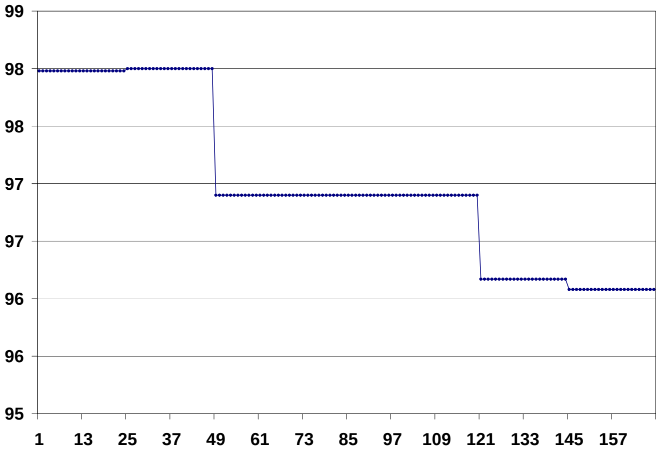

Figure: PolPX spot price, 1st week of november, 2000

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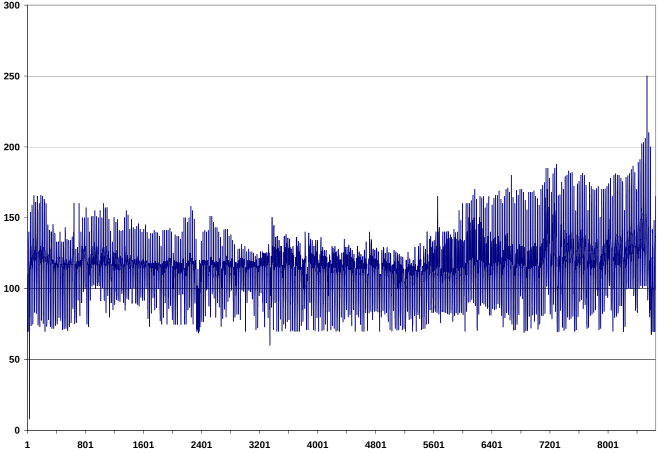

Figure: PolPX spot price 2007

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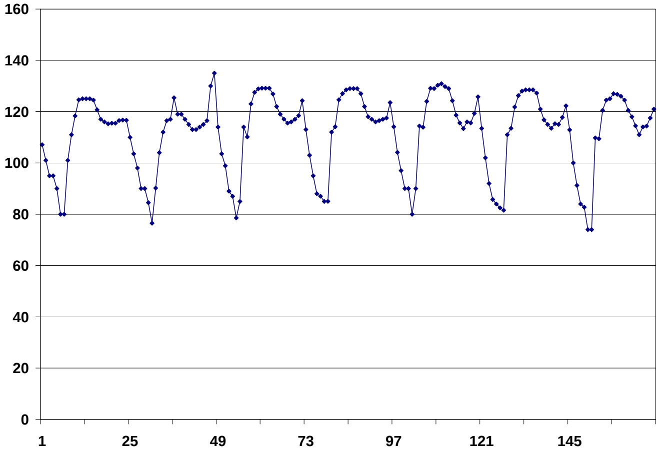

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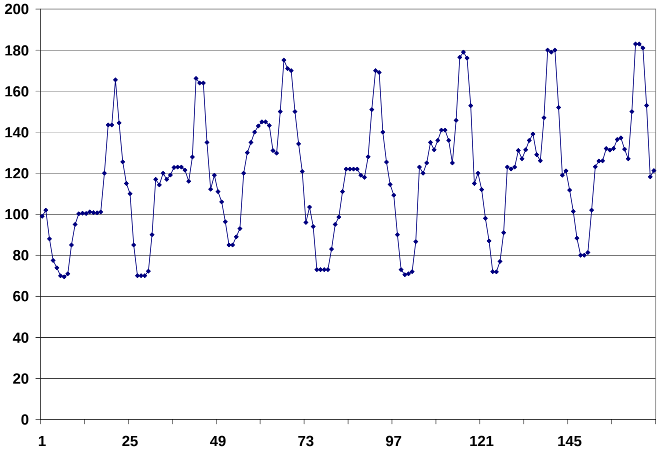

Figure: PolPX spot price 1s week of november, 2007

The Model I

- **Randomness :** $(W^0, W) = (W^0, W^1, \ldots, W^n)$ Wiener process defined on a given $(\Omega, \mathcal{F}, \mathbb{P})$. $\mathcal{F}_t^0$ and $\mathcal{F}_t^W$ are their natural filtrations.
- Riskless asset $S_t^0 = \exp \int_0^t r_u du$, $t \geq 0$, r is $\mathcal{F}_t^0$-adapted and ≥ 0.
- **Commodities market:** $n \geq 1$ commodities (coal, gas, ...) whose prices S^i to produce 1 MWh of electricity follows

$$dS_t^i = S_t^i(\mu_t^i dt + \sum_j \sigma_t^{ij} dW_t^j), \quad t \geq 0.$$

  For simplicity, assume convenience yields and storage costs are zero for all $i = 1, \ldots, n$.
- **Electricity demand:** $D = (D_t)_{t \geq 0}$ $\mathcal{F}_t^0$-adapted, positive process; notice that D is independent of each S^i.

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The Model II

- $\Delta^i > 0$ denotes the maximal capacity generation using the i-th commodity at every instant,
- order commodities prices $S_t^{(1)}(\omega) \leq \ldots \leq S_t^{(n)}(\omega)$ from the cheapest to the most expensive, giving an $\mathcal{F}_t^W$-adapted *random* permutation $\pi_t(\omega)$ of $\{1, \ldots, n\}$
- Price formation mechanism:

$$D_t \in I_k^{\pi_t} := \left[\sum_{i=1}^{k-1} \Delta^{\pi_t(i)}, \sum_{i=1}^{k} \Delta^{\pi_t(i)}\right) \Rightarrow P_t = S_t^{(k)}$$

- ... so that

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No-arbitrage assumptions on commodities market.

Let $T > 0$. There exists $\mathbb{Q} \sim \mathbb{P}$ on $\mathcal{F}_T^W$ such that :

1. each $\tilde{S}^i / S^0$ is a $\mathbb{Q}$-martingale w.r.t. $\mathcal{F}^W$
2. the laws of W^0 and Δ_t^i for all i do not change
3. filtrations $(\mathcal{F}_t^0), (\mathcal{F}_t^W), (\mathcal{F}_t^\Delta)$ are $\mathbb{Q}$-independent

Remarks

1. Property 3 above is satisfied if W^0, W and Δ^i are constructed on the canonical product space and the change of measure affects only the factor where W is defined. Such a $\mathbb{Q}$ is usually called "minimal martingale measure".
2. Being D not tradable, this market is not complete. We choose $\mathbb{Q}$ as the pricing measure.
3. Notation: $\mathcal{F}_t = \mathcal{F}_t^0 \vee \mathcal{F}_t^W \vee \mathcal{F}_t^\Delta$ is the market filtration.

Electricity forward prices: The main formula

Under previous assumptions and if $S_T^i \in L^1(\mathbb{Q}_T)$:

$$F_t(T) = \sum_{\substack{i=1,n \\ \pi \in \Pi_n}} h_{\pi(i)} \cdot F_t^{\pi(i)}(T) \cdot \mathbb{Q}_T[D_T \in I_i^\pi(T) | \mathcal{F}_t^0] \cdot \mathbb{Q}_T^{\pi(i)}[\pi_T = \pi | \mathcal{F}_t^W]$$

where:

- Π_n is the set of all permutations of $\{1, \ldots, n\}$
- $F_t^i(T)$ is T-forward of commodity i
- h_i heat-rate for the i-th commodity
- $d\mathbb{Q}_T^{\pi(i)}/d\mathbb{Q}_T = S_T^{\pi(i)}/\mathbb{E}^{\mathbb{Q}_T}[S_T^{\pi(i)}]$ on $\mathcal{F}_T^W$

Electricity forward prices II : some remarks

This model is able to explain three basic features of electricity market as :

- Observed spikes in electricity spot prices dynamics
- Non-convergence of electricity forward prices towards spot (day-ahead) prices as $t \uparrow T$. Indeed,

$$F_t(T) \to F_{T-}(T) = \sum_{i=1}^{n} S_T^{(i)} \mathbb{Q}_T[D_T \in I_i^{\pi^T}(T) | \mathcal{F}_{T-}].$$

$F_{T-}(T)$

- The paths of electricity forward prices are much smoother than the corresponding spot prices.

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The constant coefficients model: more explicit formulae

- Commodities prices S^i follow n-dim Black-Scholes model with volatilities $\sigma^{ij} > 0$ and interest rate $r > 0$ constant so that, in particular, $\mathbb{Q}_T = \mathbb{Q}$
- Production capacity Δ_t^i are independent compound Poisson processes with two values $(M_i > m_i \geq 0)$
- Demand of electricity : D follows a Ornstein-Uhlenbeck process

$$dD_t = a(b(t) - D_t)dt + \delta dW_t^0, \quad D_0 > 0$$

  with $a, \delta > 0$. $b(t) > 0$ is the seasonality component as in Barlow (2002).
- Under these assumptions probabilities $\mathbb{Q}[D_T \in I_k^\pi(T)|\mathcal{F}_t^0]$ and $\mathbb{Q}_T^{\pi(i)}[\pi_T = \pi|\mathcal{F}_t^W]$ can be *computed explicitly as functions of the parameters.*
- $h_i F_t^i(T) = e^{r(T-t)} S_t^i$ for all commodities $1 \leq i \leq n$

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Back to reality I: What do we know?

Model tested on French market (Powernext). Main features:

1. 80% nuclear production, 12% hydraulique, 7% fossil, 1% renewables
2. Nuclear production is marginal only 10% of the year (during the night)
3. Price fixed by fossil fuels: gaz or fuel

Hence, possibility to simplify the inputs:

- consider only those two technologies
- consider midday hourly prices on peakload to be sure these technologies fixe the price: Gaz plants (S_t^1 = gaz plus CO2), Fuel combustion turbines (S_t^2 = fuel plus CO2)
- heat rates h_i, capacity thresholds $m_i < M_i$ are known for $i = 1, 2$
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Back to reality I: What do we know?

Model tested on French market (Powernext). Main features:

1. 80% nuclear production, 12% hydraulique, 7% fossil, 1% renewables
2. Nuclear production is marginal only 10% of the year (during the night)
3. Price fixed by fossil fuels: gaz or fuel

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Back to reality II: How do we proceed?

We can calibrate the model in two ways:

- on spot prices or
- on futures prices

The data we calibrate on are

- technology costs (spot or forward) S^i or $F^i(T)$
- electricity spot and future prices P_t or $F_t(T)$
- estimated production R^i_t for i-th technology given by :

$$R^1_t = \min(D_t, \Delta^1_t), \quad R^2_t = \min(\Delta^2_t, (D_t - \Delta^1_t)^+)$$

where D_t is the residual demands (without nuclear & hydro).

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Spot price historical estimation

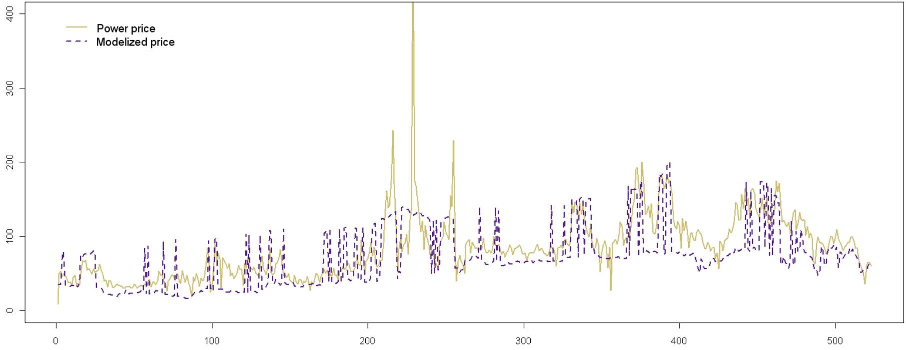

Figure: Historical & modelisd spot price at 12:00 am. Daily data from January 2007 to December 2008.

Futures pricing

Futures have a delivery period (1 month, 1 quarter,..) $\Rightarrow$ the expression must be modified

$$F_t(T_1, T_2) = \frac{1}{T_2 - T_1} \sum_{i=1}^{n} \sum_{\pi \in \Pi_n} \int_{T_1}^{T_2} F_t^{\pi(i)}(T) \times \mathbb{Q}_T^{\pi(i)}[\pi_T = \pi | \mathcal{F}_t^W] \mathbb{Q}_T[D_T \in I_i^\pi(T) | \mathcal{F}_t^{0,\Delta}] dT$$

- integral is computed numerically
- $F_t^i(T)$ are approximated by prices on delivery period $F_t^i(T_1, T_2)$

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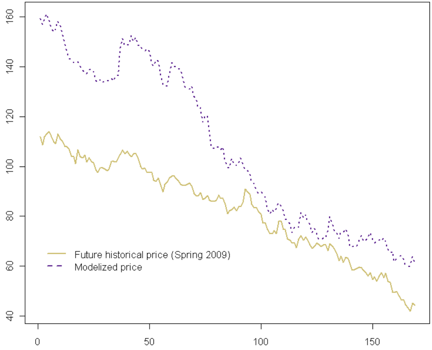
Futures prices & returns



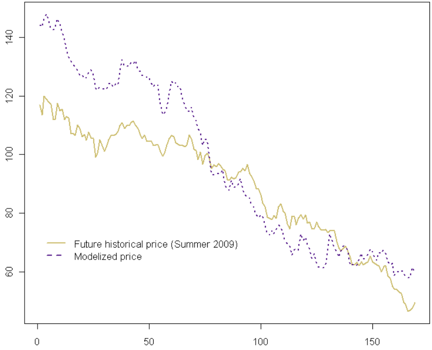
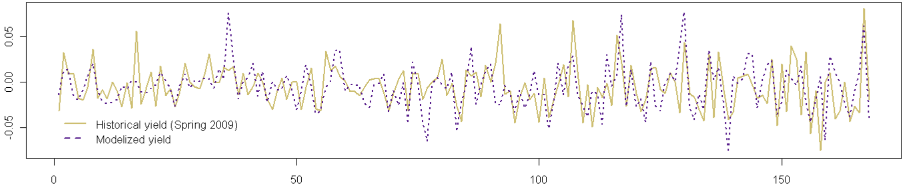
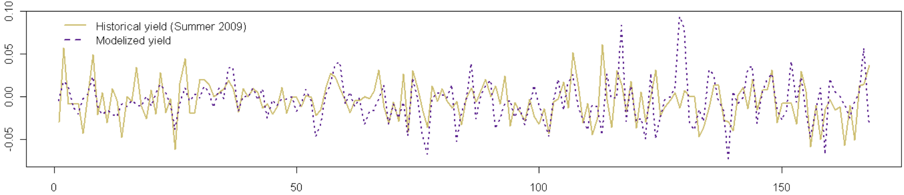


Figure: Historical futures prices (2QAH and 3QAH) and simulated futures prices.

Futures prices & returns





Some papers related to this idea:

- Barlow (2002) : Electricity spot price is defined as an equilibrium between demand and production
- Fleten and Lemming (2003) : Forward curve reconstruction method, relies on external structural model
- Routledge, Seppi and Spatt (2000) : Equilibrium model for term structure of forward prices of storable commodities
- Cartea and Villaplana (2008) : $P_t = \varphi(\Delta_t, D_t)$, with φ exp of affine function, compute forward/future prices, forward premium.
- Coulon and Howison (2009): same idea with also biding modeling (tested on PJM and NEPOOL markets).

Conclusions

What did we get?

- We considered an enlarged market of energy **and fuels**, to justify the use of risk-neutral approach
- Electricity demand introduces correlation between electricity and fuels through capacities
- Our simple model allows explanation of some electricity prices properties

What's next? In progress ...

- more realistic dynamics for capacities
- pricing and hedging formulae for derivatives on energy futures
- study the sign of the electricity risk premium
- develop a two zones model (for regional spread).

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