Bilateral counterparty risk valuation with stochastic dynamical models and application to Credit Default Swaps

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Joint work with Damiano Brigo
Agenda
Q What is counterparty risk in general?
A The risk taken on by an entity entering an OTC contract with a counterparty having a relevant default probability. As such, the counterparty might not respect its payment obligations.
What is counterparty risk in general?
- The risk taken on by an entity entering an OTC contract with a counterparty having a relevant default probability. As such, the counterparty might not respect its payment obligations.

When is valuation of counterparty risk symmetric?
- When we include the possibility that also the entity computing the counterparty risk adjustment may default, besides the counterparty itself.
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Q: When is valuation of counterparty risk asymmetric?
   A: When the entity computing the counterparty risk adjustment considers itself default-free, and only the counterparty may default.
Some common questions 1

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Q When is valuation of counterparty risk asymmetric?
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Q Which one is computed usually for valuation adjustments?
   A The asymmetric one.
Q What impacts counterparty risk?

A The OTC contract’s underlying volatility, the correlation between the underlying and default of the counterparty, and the counterparty credit spreads volatility.
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   A The amplified risk when the reference underlying and the counterparty are strongly correlated in the wrong direction.
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Existing approaches for the Asymmetric Case

**Capital Adequacy based approach**

- Obtain estimates of expected exposures for the portfolio NPV at different maturities through Monte-Carlo simulations.
- Buy default protection on the counterparty at those maturities through single name or basketed credit derivatives. Notionals follow the expected exposures.

**Problems**

- Ignores correlation structure between counterparty default and portfolio exposure
- In a transaction where wrong-way risk may occur, this approach ignores a significant source of potential loss.
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General Notation

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- “1” will be used to denote the underlying name/risk factor(s) of the contract
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All payoff are seen from the point of view of investor.
The mechanics of Evaluating asymmetric counterparty risk
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- **Counterparty defaults after final maturity**
- **Payoff under counterparty default risk**
- Original payoff of the instrument

\[ \text{Original payoff of the instrument} \]

\[ \quad \]
The mechanics of Evaluating asymmetric counterparty risk

counterparty defaults after final maturity

payoff under counterparty default risk

counterparty defaults before final maturity

original payoff of the instrument

all cash flows before default
⊕ recovery of the residual NPV at default if positive
⊖ Total residual NPV at default if negative
General Formulation under Asymmetry

The fundamental formula for the valuation of counterparty risk when the investor is default free is:

$$\text{NPV}(C) = E_C \{ \Pi(C, T) \} - \text{LGD}_C \cdot E_{t<C \leq T} [\text{NPV}(C)] + \text{First term: Value without counterparty risk.} \text{Second term: Counterparty risk adjustment.} \text{NPV}(C) = E_C \{ \Pi(C, T) \} \text{ is the value of the transaction on the counterparty default date.} \text{LGD}_C = 1 - \text{REC}_C.$$
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What we can observe

- Including counterparty risk in the valuation of an otherwise default-free derivative $\implies$ credit derivative.
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- The inclusion of counterparty risk adds a level of optionality to the payoff.
Often the investor, when computing a counterparty risk adjustment, considers itself to be default-free. This can be either a unrealistic assumption or an approximation for the case when the counterparty has a much higher default probability than the investor.
Including the investor default or not?

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If this assumption is made counterparty risk is asymmetric: if “2” were to consider “0” as counterparty and computed the total value of the position, this would not be the opposite of the one computed by “0”.
Including the investor default or not?

We get back symmetry if we allow for default of the investor in computing counterparty risk. This also results in an adjustment that is cheaper to the counterparty “2”.
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The counterparty “2” may then be willing to ask the investor “0” to include the investor default event into the model, when the counterparty risk adjustment is computed by the investor.
The case of symmetric counterparty risk

Suppose now that we allow for both parties to default. Counterparty risk adjustment allowing for default of “0”? 

0, 2: default times of “0” and “2”. 

T: final maturity
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“0”: the investor;  
“2”: the counterparty;  
(“1”: the underlying name/risk factor of the contract).  
\( \tau_0, \tau_2 \): default times of “0” and “2”.  
\( \tau = \tau_0 \wedge \tau_2 \)  
\( T \): final maturity
The case of symmetric counterparty risk

Formula

\[ \mathbb{E}_t \left\{ \Pi^D(t, T) \right\} = \mathbb{E}_t \left\{ \Pi(t, T) \right\} \]

\[ + \text{LGD}_0 \cdot \mathbb{E}_t \left\{ 1_{\tau=\tau_0 \leq T} \cdot D(t, \tau_0) \cdot [-\text{NPV}(\tau_0)]^+ \right\} \]

\[ - \text{LGD}_2 \cdot \mathbb{E}_t \left\{ 1_{\tau=\tau_2 \leq T} \cdot D(t, \tau_2) \cdot [\text{NPV}(\tau_2)]^+ \right\} \]

\(^a\)Similar formula for interest rate swaps given in Bielecki & Rutkowski (2001)
The case of symmetric counterparty risk

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- 3d term: Counterparty risk adj due to scenarios \( \tau_2 < \tau_0 \).
The case of symmetric counterparty risk

Formula

\[ E_t \left\{ \Pi^D(t, T) \right\} = E_t \left\{ \Pi(t, T) \right\} + LGD_0 \cdot E_t \left\{ 1_{\tau = \tau_0 \leq T} \cdot D(t, \tau_0) \cdot [-NPV(\tau_0)]^+ \right\} - LGD_2 \cdot E_t \left\{ 1_{\tau = \tau_2 \leq T} \cdot D(t, \tau_2) \cdot [NPV(\tau_2)]^+ \right\} \]

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- 2nd term: Counterparty risk adj due to scenarios \( \tau_0 < \tau_2 \).
- 3d term: Counterparty risk adj due to scenarios \( \tau_2 < \tau_0 \).
- If computed from the opposite point of view of “2” having counterparty “0”, the adjustment is the opposite. Symmetry.
The case of symmetric counterparty risk

When allowing for the investor to default: symmetry
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When allowing for the investor to default: symmetry

- One more term with respect to the asymmetric case.

Depending on credit spreads and correlations, the adjustment to be subtracted can now be either positive or negative. In the asymmetric case it can only be positive. Ignoring the symmetry is clearly more expensive for the counterparty and cheaper for the investor. Some counterparties therefore may request the investor to include its own default into the valuation.
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Methodology

1. Assumption: The *investor* enters a transaction with a *counterparty*. 
Assumption: The investor enters a transaction with a counterparty.

We model and calibrate the default time of investor and counterparty using a stochastic intensity default model.
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3. We choose the underlying transaction to be a CDS contract and estimate the deal NPV at default.
Methodology

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4. We allow for correlations between investor, counterparty and underlying CDS reference entity.
Credit default model: CIR stochastic intensity

Model equations

\[
d\lambda_j(t) = k_j(\mu_j - \lambda_j(t))dt + \nu_j \sqrt{\lambda_j(t)}dZ_j(t), \quad j = 0, 1, 2
\]

Cumulative intensities are defined as: \( \Lambda(t) = \int_0^t \lambda(s)ds \).

Default times are \( \tau_j = \Lambda_j^{-1}(\xi_j) \). Exponential triggers \( \xi_0, \xi_1 \) and \( \xi_2 \) connected through a copula with correlation matrix \( R = [r]_{i,j} \).

\(^1\)("0" = investor, "1" = CDS underlying, "2" = counterparty)
We take into account default correlation between \( \tau_0, \tau_1 \) and \( \tau_2 \) and credit spreads volatility \( \nu_j, j = 0, 1, 2 \).

**Important**: volatility can amplify default time uncertainty, while high correlation reduces conditional default time uncertainty. Taking into account \( \rho \) and \( \nu \) \( \Longrightarrow \) better representation of market information and behavior, especially for wrong way risk.
Credit (CDS) Correlation and Volatility Effects

- Spread volatility affects individual times
- Default correlation affects joint times
Credit (CDS) Correlation and Volatility Effects

1. Spread volatility affects individual times
   - Spread volatility
   - Default correlation
   - Joint times

2. Default correlation affects joint times
   - Default correlation
   - Spread volatility
   - Individual times

The diagram illustrates the relationship between spread volatility, default correlation, and their effects on individual and joint times.
Proposition

The bilateral CVA at time $t$ for a receiver CDS contract (protection seller) running from time $T_a$ to time $T_b$ with premium $S$ is given by

$$\begin{align*}
\text{LGD}_2 \cdot \mathbb{E}_t \left\{ \mathbf{1}_{\tau_2 \leq T} \cdot D(t, \tau_2) \cdot \left[ \mathbf{1}_{\tau_1 > \tau_2} \overline{CDS}_{a,b}(\tau_2, S, \text{LGD}_1) \right]^+ \right\} \\
- \text{LGD}_0 \cdot \mathbb{E}_t \left\{ \mathbf{1}_{\tau_0 \leq T} \cdot D(t, \tau_0) \cdot \left[ -\mathbf{1}_{\tau_1 > \tau_0} \overline{CDS}_{a,b}(\tau_0, S, \text{LGD}_1) \right]^+ \right\}
\end{align*}$$

where $\overline{CDS}_{a,b}(T_j, S, \text{LGD}_1)$ is the residual NPV of a receiver CDS between $T_a$ and $T_b$ evaluated at time $T_j$ and given by

$$\begin{align*}
\left\{ S \left[ - \int_{\max\{T_a, T_j\}}^{T_b} D(T_j, t)(t - T_{\gamma(t)-1}) d\mathbb{Q}(\tau_1 > t | G_{T_j}) \\
+ \sum_{i=\max\{a, j\}+1}^{b} \alpha_i D(T_j, T_i) \mathbb{Q}(\tau_1 > T_i | G_{T_j}) \right] + \text{LGD}_1 \left[ \int_{\max\{T_a, T_j\}}^{T_b} D(T_j, t) d\mathbb{Q}(\tau_1 > t | G_{T_j}) \right] \right\}
\end{align*}$$
Let \( \overline{U}_{i,j} = 1 - \exp(-\Lambda_i(\tau_j)) \) and \( F_{\Lambda_i}(x) = \mathbb{P}(\Lambda_i(t) \leq x) \)

**Lemma**

**Survival Probability of reference entity conditional on counterparty default**

\[
1_{\tau = \tau_2 \leq t} 1_{\tau_1 > \tau_2} \mathbb{Q}(\tau_1 > t | G_{\tau_2}) = 1_{t < \tau_2 < \tau_1} + 1_{\tau_2 < t} 1_{\tau_1 \geq \tau_2} \int_{\overline{U}_{1,2}}^{1} F_{\Lambda_1}(t)(-\log(1 - u_1))dC_{1|0,2}(u_1; U_2)
\]

where

\[
C_{1|0,2}(u_1; U_2) := \mathbb{Q}(U_1 < u_1 | G_{\tau_2}, U_1 > \overline{U}_{1,2}, U_0 > \overline{U}_{0,2}) = \\
\left. \frac{\partial C_{1,2}(u_1,U_2)}{\partial u_2} \right|_{u_1 = \overline{U}_{1,2}, U_2} - \left. \frac{\partial C(U_0,2,u_1,U_2)}{\partial u_2} \right|_{U_0 = \overline{U}_{0,2}, U_1 = \overline{U}_{1,2}, U_2} - \left. \frac{\partial C_{1,2}(\overline{U}_{1,2},U_2)}{\partial u_2} \right|_{u_1 = \overline{U}_{1,2}, U_2} + \left. \frac{\partial C(\overline{U}_{0,2},U_1,U_2)}{\partial u_2} \right|_{U_0 = \overline{U}_{0,2}, U_1 = \overline{U}_{1,2}, U_2}
\]
Numerical Computation of Survival Probability

Let \( \overline{U}_{i,j} = 1 - \exp(-\Lambda_i(\tau_j)) \) and \( F_{\Lambda_i}(x) = \mathbb{P}(\Lambda_i(t) \leq x) \)

**Lemma**

*Survival Probability of reference entity conditional on investor default*

\[
1_{\tau=\tau_0 \leq T}1_{\tau_1 > \tau_0} \mathbb{Q}(\tau_1 > t | G_{\tau_0}) = \\
= 1_{t < \tau_0 < \tau_1} + 1_{\tau_0 < t \leq \tau_1} \int_{\overline{U}_{1,0}} F_{\Lambda_1}(t)(-\log(1-u_1)) dC_{1|2,0}(u_1; U_0)
\]

where

\[
C_{1|2,0}(u_1; U_0) := \mathbb{Q}(U_1 < u_1 | G_{\tau_0}, U_1 > \overline{U}_{1,0}, U_2 > \overline{U}_{2,0}) = \\
\frac{\partial c_{0,1}(u_0, u_1)}{\partial u_0} - \frac{\partial c(u_0, u_1, \overline{U}_{2,0})}{\partial u_0} - \frac{\partial c_{0,1}(u_0, \overline{U}_{1,0})}{\partial u_0} + \frac{\partial c(u_0, \overline{U}_{1,0}, \overline{U}_{2,0})}{\partial u_0}
\]

\[
1 - \frac{\partial c_{0,2}(u_0, \overline{U}_{2,0})}{\partial u_0} - \frac{\partial c_{0,1}(u_0, \overline{U}_{1,0})}{\partial u_0} + \frac{\partial c(u_0, \overline{U}_{1,0}, \overline{U}_{2,0})}{\partial u_0}
\]
Case Study 1: Correlation and Volatility Effect

Payer \( x \): BR-CVA when investor is payer and credit spread volatility of “1” is \( x \)
Receiver \( x \): BR-CVA when investor is receiver and credit spread volatility of “1” is \( x 
Investor low risk, Counterparty Medium risk, Reference entity high risk
Credit spreads volatility: \( \nu_2 = 0.01 \) and \( \nu_0 = 0.01 \).
CDS contract on the reference credit has a five-years maturity.

<table>
<thead>
<tr>
<th>Credit Risk Levels</th>
<th>( \lambda(0) )</th>
<th>( \kappa )</th>
<th>( \mu )</th>
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<tbody>
<tr>
<td>Low</td>
<td>0.00001</td>
<td>0.9</td>
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<td>Medium</td>
<td>0.01</td>
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<th>Maturity</th>
<th>Low Risk</th>
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<tbody>
<tr>
<td>1y</td>
<td>0</td>
<td>92</td>
<td>234</td>
</tr>
<tr>
<td>2y</td>
<td>0</td>
<td>104</td>
<td>244</td>
</tr>
<tr>
<td>3y</td>
<td>0</td>
<td>112</td>
<td>248</td>
</tr>
<tr>
<td>4y</td>
<td>1</td>
<td>117</td>
<td>250</td>
</tr>
<tr>
<td>5y</td>
<td>1</td>
<td>120</td>
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<td>124</td>
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<td>1</td>
<td>125</td>
<td>253</td>
</tr>
<tr>
<td>9y</td>
<td>1</td>
<td>126</td>
<td>254</td>
</tr>
<tr>
<td>10y</td>
<td>1</td>
<td>127</td>
<td>254</td>
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Case Study 2: Correlation and Volatility Effect

- **Scenario 1 (Base Scenario).** Investor low risk, reference entity high risk, counterparty middle risk.
- **Scenario 2 (Risky counterparty).** Investor low risk, reference entity middle risk and counterparty high risk.
- **Scenario 3 (Risky investor).** Counterparty low risk, reference entity middle credit risk, investor has high credit risk.
- **Scenario 4 (Risky Ref).** Both investor and counterparty have middle risk, while reference entity has high risk.
- **Scenario 5 (Safe Ref).** Both investor and counterparty have high risk, while reference entity has low risk.

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Concrete Market Scenario Analysis

- Calculate mark-to-market value of 5Y CDS contract involving BA, Lehman and Shell under different correlation scenarios.
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- Contract entered on January 5, 2006 and marked to market by investor on May 1, 2008.
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- Contract entered on January 5, 2006 and marked to market by investor on May 1, 2008.
- CIR processes of the three names calibrated to the CDS quotes are

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<th>Credit Risk Levels (2006/2008)</th>
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<th>( \kappa )</th>
<th>( \mu )</th>
<th>( \nu )</th>
</tr>
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<tbody>
<tr>
<td>Lehman Brothers (name “0”)</td>
<td>0.0001/0.66</td>
<td>0.036/7.879</td>
<td>0.043/0.021</td>
<td>0.055/0.572</td>
</tr>
<tr>
<td>Royal Dutch Shell (name “1”)</td>
<td>0.0001/0.003</td>
<td>0.039/0.183</td>
<td>0.022/0.009</td>
<td>0.019/0.006</td>
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Mark to market procedure: Timeline

- $T_a = January 5, 2006$. Investor compute 5Y risk-adjusted CDS contract ending at $T_b = January 5, 2011$ as

$$CDS_{a,b}(T_a, S_1, LGD_{1,2,3}) = CDS_{a,b}(T_a, S_1, LGD_1) - BR-CVA-CDS_{a,b}(T_a, S_1, LGD_{1,2,3})$$

$S_1 = 5Y$ CDS of name “1” at $T_a$, $CDS_{a,b}(T_a, S_1, LGD_1) = value$ of risk free CDS contract, and $LGD = 0.6$ for each name.

- $T_c = May 1, 2008$. Investor calculates MTM value of CDS contract.
  - Risk-adjusted CDS contract valuation at $T_d$ is

$$CDS_{c,d}(T_c, S_1, LGD_{1,2,3}) = CDS_{c,d}(T_c, S_1, LGD_1) - BR-CVA-CDS_{c,d}(T_c, S_1, LGD_{1,2,3})$$

- Mark-to-market value of the CDS contract is:

$$MTM_{a,c}(S_1, LGD_{1,2,3}) = CDS_{c,d}(T_c, S_1, LGD_{1,2,3}) - \frac{CDS_{a,b}(T_a, S_1, LGD_{1,2,3})}{D(T_a, T_c)}$$
Results from mark-to-market procedure

CDS contract marked-to-market by Lehman on May 1, 2008. The MTM value of the contract without BR-CVA is 84.2(-84.2) bps.

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<th>0.10 15%</th>
<th>0.20 28%</th>
<th>0.30 37%</th>
<th>0.40 42%</th>
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Conclusions

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Analysis including underlying asset, investor and counterparty default correlation requires a credit model.
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- Accurate test scenarios for wrong way risk.
Future Work

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- Handle key aspects of margin agreements such as collateralization thresholds, minimum transfer amount, collateral posting delay and margin call frequency.

\(^2\)CVA computation for counterparty risk assessment in credit portfolios. Assefa et al. (2009)
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Apply the framework to a different class of products.

\(^2\)CVA computation for counterparty risk assessment in credit portfolios. Assefa et al. (2009)
Brigo, D., and Capponi, A.
Bilateral counterparty risk valuation with stochastic dynamical models and application to Credit Default Swaps. Available at ssrn.com or at arxiv.org.
References

Hedging


CVA with Collateral Modeling

CVA and applications to asset classes


