Bilateral counterparty risk valuation with stochastic dynamical models and application to Credit Default Swaps

Agostino Capponi

California Institute of Technology
Division of Engineering and Applied Sciences
acapponi@caltech.edu

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Agenda

Some common questions 1

Q What is counterparty risk in general?

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Q Which one is computed usually for valuation adjustments?

A *The asymmetric one.*

Some common questions 2

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Existing approaches for the Asymmetric Case

Capital Adequacy based approach

- Obtain estimates of expected exposures for the portfolio NPV at different maturities through Monte-Carlo simulations.
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Problems

- Ignores correlation structure between counterparty default and portfolio exposure
- In a transaction where wrong-way risk may occur, this approach ignores a significant source of potential loss.

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All payoff are seen from the point of view of investor.

The mechanics of Evaluating asymmetric counterparty risk

payoff under counterparty default risk

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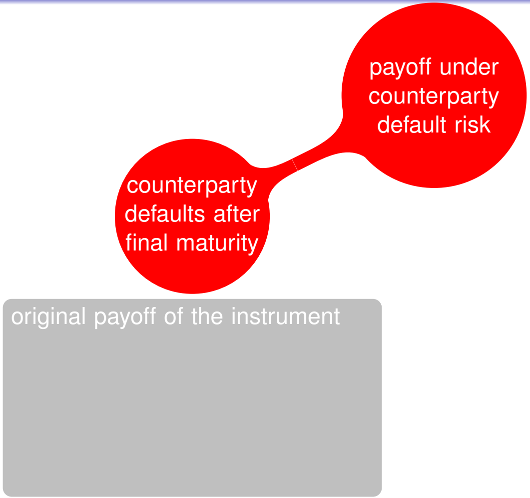

The mechanics of Evaluating asymmetric counterparty risk

payoff under counterparty default risk

counterparty defaults after final maturity

original payoff of the instrument

counterparty defaults before final maturity

all cash flows before default
$\oplus$ recovery of the residual NPV at default if positive
$\ominus$ Total residual NPV at default if negative

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- First term : Value without counterparty risk.
- Second term : Counterparty risk adjustment.
- $\mathrm{NPV}(\tau_C) = \mathbb{E}_{\tau_C}\left\{\Pi(\tau_C, T)\right\}$ is the value of the transaction on the counterparty default date. $\mathrm{LGD}_C = 1 - \mathrm{REC}_C$.

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- The inclusion of counterparty risk adds a level of optionality to the payoff.

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If this assumption is made counterparty risk is asymmetric: if “2” were to consider “0” as counterparty and computed the total value of the position, this would not be the opposite of the one computed by “0”.

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The counterparty “2” may then be willing to ask the investor “0” to include the investor default event into the model, when the counterparty risk adjustment is computed by the investor

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Suppose now that we allow for both parties to default.
Counterparty risk adjustment allowing for default of “0”?

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"0" : the investor;
"2": the counterparty;
("1": the underlying name/risk factor of the contract).
τ_0, τ_2: default times of "0" and "2".
$\tau = \tau_0 \wedge \tau_2$
T: final maturity

The case of symmetric counterparty risk

Formula[a]

$$
\begin{aligned}
\mathbb{E}_t\left\{\Pi^D(t,T)\right\} = \mathbb{E}_t\left\{\Pi(t,T)\right\} & \\
& + \mathrm{LGD}_0 \cdot \mathbb{E}_t\left\{\mathbf{1}_{\tau=\tau_0\le T} \cdot \mathrm{D}(t,\tau_0) \cdot \left[-\mathrm{NPV}(\tau_0)\right]^+\right\} \\
& - \mathrm{LGD}_2 \cdot \mathbb{E}_t\left\{\mathbf{1}_{\tau=\tau_2\le T} \cdot \mathrm{D}(t,\tau_2) \cdot \left[\mathrm{NPV}(\tau_2)\right]^+\right\}
\end{aligned}
$$

[a] Similar formula for interest rate swaps given in Bielecki & Rutkowski (2001)

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- 2nd term : Counterparty risk adj due to scenarios $\tau_0 < \tau_2$.

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\mathbb{E}_t\left\{\Pi^D(t,T)\right\} = \mathbb{E}_t\left\{\Pi(t,T)\right\} \\
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&- \mathrm{LGD}_2 \cdot \mathbb{E}_t\left\{\mathbf{1}_{\tau=\tau_2\le T} \cdot \mathrm{D}(t,\tau_2) \cdot [\mathrm{NPV}(\tau_2)]^+\right\}
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- 2nd term : Counterparty risk adj due to scenarios $\tau_0 < \tau_2$.
- 3d term : Counterparty risk adj due to scenarios $\tau_2 < \tau_0$.
- If computed from the opposite point of view of "2" having counterparty "0", the adjustment is the opposite. Symmetry.

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- Ignoring the symmetry is clearly more expensive for the counterparty and cheaper for the investor.
- Some counterparties therefore may request the investor to include its own default into the valuation

Methodology

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2. We model and calibrate the default time of *investor* and *counterparty* using a stochastic intensity default model.
3. We choose the underlying transaction to be a CDS contract and estimate the deal NPV at default.
4. We allow for correlations between investor, counterparty and underlying CDS reference entity.

Credit default model: CIR stochastic intensity

- Model equations[1]

$$d\lambda_j(t) = k_j(\mu_j - \lambda_j(t))dt + \nu_j\sqrt{\lambda_j(t)}dZ_j(t), \quad j = 0, 1, 2$$

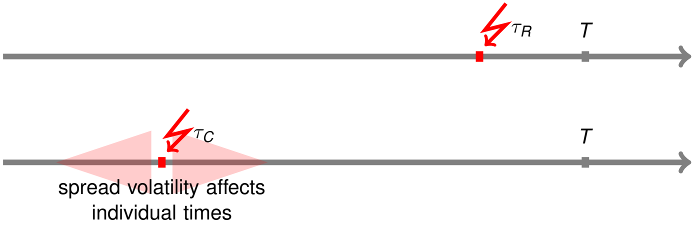
- Cumulative intensities are defined as : $\Lambda(t) = \int_0^t \lambda(s)ds$.
- Default times are $\tau_j = \Lambda_j^{-1}(\xi_j)$. Exponential triggers ξ_0, ξ_1 and ξ_2 connected through a copula with correlation matrix $R = [r]_{i,j}$.

[1] ("0" = investor, "1" = CDS underlying, "2" = counterparty)

Credit default model: CIR stochastic intensity

- We take into account default correlation between τ_0, τ_1 and τ_2 and credit spreads volatility $\nu_j, j = 0, 1, 2$.
- **Important**: volatility can amplify default time uncertainty, while high correlation reduces conditional default time uncertainty.
  Taking into account ρ and $\nu \Longrightarrow$ better representation of market information and behavior, especially for wrong way risk.

Credit (CDS) Correlation and Volatility Effects



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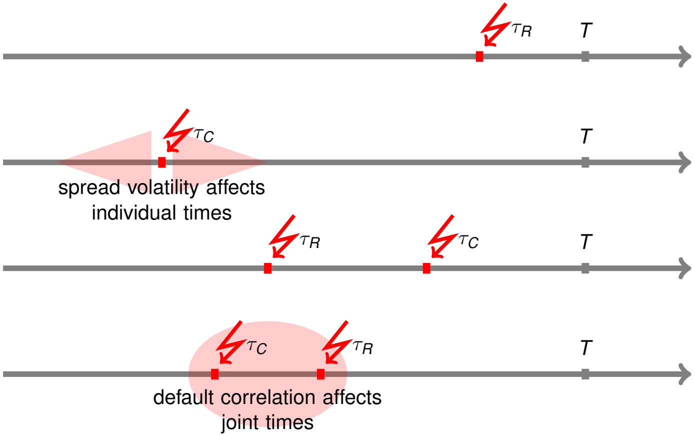

Bilateral Counterparty Risk Valuation Formula

Proposition

The bilateral CVA at time t for a receiver CDS contract (protection seller) running from time T_a to time T_b with premium S is given by

$$\mathrm{LGD}_2 \cdot \mathbb{E}_t \left\{ \mathbf{1}_{\tau=\tau_2 \le T} \cdot \mathrm{D}(t, \tau_2) \cdot \left[\mathbf{1}_{\tau_1 > \tau_2} \overline{CDS}_{a,b}(\tau_2, S, \mathrm{LGD}_1) \right]^+ \right\}$$
$$- \mathrm{LGD}_0 \cdot \mathbb{E}_t \left\{ \mathbf{1}_{\tau=\tau_0 \le T} \cdot \mathrm{D}(t, \tau_0) \cdot \left[-\mathbf{1}_{\tau_1 > \tau_0} \overline{CDS}_{a,b}(\tau_0, S, \mathrm{LGD}_1) \right]^+ \right\}$$

where $\overline{CDS}_{a,b}(T_j, S, \mathrm{LGD}_1)$ is the residual NPV of a receiver CDS between T_a and T_b evaluated at time T_j and given by

$$\left\{ S \left[-\int_{\max\{T_a, T_j\}}^{T_b} \mathrm{D}(T_j, t)(t - T_{\gamma(t)-1}) d\mathbb{Q}(\tau_1 > t | \mathcal{G}_{T_j}) \right.\right.$$
$$\left.\left. + \sum_{i=\max\{a,j\}+1}^{b} \alpha_i \mathrm{D}(T_j, T_i) \mathbb{Q}(\tau_1 > T_i | \mathcal{G}_{T_j}) \right] + \mathrm{LGD}_1 \left[\int_{\max\{T_a, T_j\}}^{T_b} \mathrm{D}(T_j, t) d\mathbb{Q}(\tau_1 > t | \mathcal{G}_{T_j}) \right] \right\}$$

Numerical Computation of Survival Probability

Let $\overline{U}_{i,j} = 1 - \exp(-\Lambda_i(\tau_j))$ and $F_{\Lambda_i(t)}(x) = \mathbb{P}(\Lambda_i(t) \le x)$

Lemma

Survival Probability of reference entity conditional on counterparty default

$$\begin{aligned}\mathbf{1}_{\tau=\tau_2\le T}\mathbf{1}_{\tau_1>\tau_2}\mathbb{Q}(\tau_1 > t|\mathcal{G}_{\tau_2}) &= \\ = \quad \mathbf{1}_{t<\tau_2<\tau_1} + \mathbf{1}_{\tau_2<t}\mathbf{1}_{\tau_1\ge\tau_2}\int_{\overline{U}_{1,2}}^{1} F_{\Lambda_1(t)}(-\log(1-u_1))dC_{1|0,2}(u_1;U_2)\end{aligned}$$

where

$$\begin{aligned}C_{1|0,2}(u_1;U_2) := \mathbb{Q}(U_1 < u_1|\mathcal{G}_{\tau_2}, U_1 > \overline{U}_{1,2}, U_0 > \overline{U}_{0,2}) = \\ = \frac{\frac{\partial C_{1,2}(u_1,U_2)}{\partial u_2} - \frac{\partial C(\overline{U}_{0,2},u_1,U_2)}{\partial u_2} - \frac{\partial C_{1,2}(\overline{U}_{1,2},U_2)}{\partial u_2} + \frac{\partial C(\overline{U}_{0,2},\overline{U}_{1,2},U_2)}{\partial u_2}}{1 - \frac{\partial C_{0,2}(\overline{U}_{0,2},U_2)}{\partial u_2} - \frac{\partial C_{1,2}(\overline{U}_{1,2},U_2)}{\partial u_2} + \frac{\partial C(\overline{U}_{0,2},\overline{U}_{1,2},U_2)}{\partial u_2}}\end{aligned}$$

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Lemma

Survival Probability of reference entity conditional on investor default

$$\begin{aligned}\mathbf{1}_{\tau=\tau_0\le T}\mathbf{1}_{\tau_1>\tau_0}\mathbb{Q}(\tau_1 > t|\mathcal{G}_{\tau_0}) &= \\ = \mathbf{1}_{t<\tau_0<\tau_1} + \mathbf{1}_{\tau_0<t}\mathbf{1}_{\tau_1\ge\tau_0}\int_{\overline{U}_{1,0}}^{1} F_{\Lambda_1(t)}(-\log(1-u_1))dC_{1|2,0}(u_1;U_0)\end{aligned}$$

where

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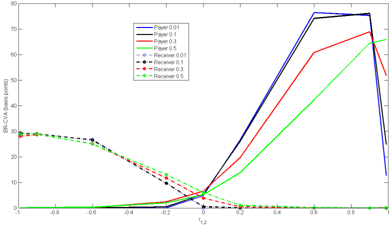
Case Study 1: Correlation and Volatility Effect

Payer x: BR-CVA when investor is payer and credit spread volatility of “1” is x
Receiver x: BR-CVA when investor is receiver and credit spread volatility of “1” is x
Investor low risk, Counterparty Medium risk, Reference entity high risk
Credit spreads volatility: $\nu_2 = 0.01$ and $\nu_0 = 0.01$.
CDS contract on the reference credit has a five-years maturity.

Credit Risk Levels	$\lambda(0)$	κ	μ
Low	0.00001	0.9	0.0001
Medium	0.01	0.80	0.02
High	0.03	0.50	0.05

Maturity	Low Risk	Middle Risk	High risk
1y	0	92	234
2y	0	104	244
3y	0	112	248
4y	1	117	250
5y	1	120	251
6y	1	122	252
7y	1	124	253
8y	1	125	253
9y	1	126	254
10y	1	127	254



Case Study 2: Correlation and Volatility Effect

- Scenario 1 (*Base Scenario*). Investor low risk, reference entity high risk, counterparty middle risk
- Scenario 2 (*Risky counterparty*). Investor low risk, reference entity middle risk and counterparty high risk.
- Scenario 3 (*Risky investor*). Counterparty low risk, reference entity middle credit risk, investor has high credit risk.
- Scenario 4 (*Risky Ref*). Both investor and counteparty have middle risk, while reference entity has high risk.
- Scenario 5 (*Safe Ref*). Both investor and counterparty have high risk, while reference entity has low risk.

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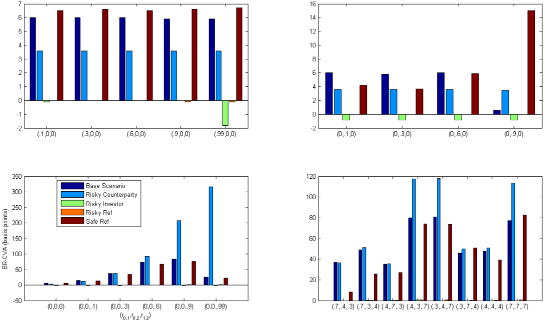

Concrete Market Scenario Analysis

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- Contract entered on January 5, 2006 and marked to market by investor on May 1, 2008.
- CIR processes of the three names calibrated to the CDS quotes are

Credit Risk Levels (2006/2008)	$y(0)$	κ	μ	ν
Lehman Brothers (name "0")	0.0001/0.66	0.036/7.879	0.043/0.021	0.055/0.572
Royal Dutch Shell (name "1")	0.0001/0.003	0.039/0.183	0.022/0.009	0.019/0.006
British Airways (name "2")	0.00002/0.00001	0.026/0.677	0.258/0.078	0.0003/0.224

Maturity	Royal Dutch Shell	Lehman Brothers	British Airways
1y	4/24	6.8/203	10/151
2y	5.8/24.6	10.2/188.5	23.2/230
3y	7.8/26.4	14.4/166.75	50.6/275
4y	10.1/28.5	18.7/152.25	80.2/305
5y	11.7/30	23.2/145	110/335
6y	15.8/32.1	27.3.3/136.3	129.5/342
7y	19.4/33.6	30.5/130	142.8/347
8y	20.5/35.1	33.7/125.8	153.6/350.6
9y	21/36.3	36.5/122.6	162.1/353.3
10y	21.4/37.2	38.6/120	168.8/355.5

Mark to market procedure: Timeline

- T_a = January 5, 2006. Investor compute $5Y$ risk-adjusted CDS contract ending at T_b = January 5, 2011 as

$$CDS^D_{a,b}(T_a, S_1, \mathrm{LGD}_{1,2,3}) = CDS_{a,b}(T_a, S_1, \mathrm{LGD}_1) - \text{BR-CVA-CDS}_{a,b}(T_a, S_1, \mathrm{LGD}_{1,2,3})$$

S_1 = $5Y$ CDS of name “1” at T_a, $\mathrm{CDS}_{a,b}(T_a, S_1, \mathrm{LGD}_1)$ = value of risk free CDS contract, and LGD = 0.6 for each name.

- T_c = May 1, 2008. Investor calculates MTM value of CDS contract.
  - Risk-adjusted CDS contract valuation at T_d is

$$\mathrm{CDS}^D_{c,d}(T_c, S_1, \mathrm{LGD}_{1,2,3}) = \mathrm{CDS}_{c,d}(T_c, S_1, \mathrm{LGD}_1) - \text{BR-CVA-CDS}_{c,d}(T_c, S_1, \mathrm{LGD}_{1,2,3})$$

  - Mark-to-market value of the CDS contract is:

$$MTM_{a,c}(S_1, \mathrm{LGD}_{1,2,3}) = CDS^D_{c,d}(T_c, S_1, \mathrm{LGD}_{1,2,3}) - \frac{CDS^D_{a,b}(T_a, S_1, \mathrm{LGD}_{1,2,3})}{D(T_a, T_c)}$$

Results from mark-to-market procedure

CDS contract marked-to-market by Lehman on May 1, 2008. The MTM value of the contract without BR-CVA is 84.2(-84.2) bps.

(r_{01}, r_{02}, r_{12})	**Vol. parameter ν_1** **CDS Impled vol**	**0.01** **1.5%**	**0.10** **15%**	**0.20** **28%**	**0.30** **37%**	**0.40** **42%**	**0.50** **42%**
(-0.3, -0.3, 0.6)	(LEH Pay, BAB Rec)	39.1(2.1)	44.7(2.0)	51.1(1.9)	58.4(1.4)	60.3(1.7)	63.8(1.1)
	(BAB Pay, LEH Rec)	-84.2(0.0)	-83.8(0.1)	-83.5(0.1)	-83.8(0.1)	-83.8(0.2)	-83.8(0.2)
(-0.3, -0.3, 0.8)	(LEH Pay, BAB Rec)	13.6(3.6)	22.6(3.2)	35.2(2.6)	43.7(2.0)	45.3(2.4)	52.0(1.4)
	(BAB Pay, LEH Rec)	-84.2(0.0)	-83.9(0.1)	-83.6(0.1)	-83.9(0.1)	-83.9(0.2)	-83.8(0.2)
(0.6, -0.3, -0.2)	(LEH Pay, BAB Rec)	83.1(0.0)	81.9(0.2)	81.6(0.3)	82.4(0.3)	82.6(0.3)	82.8(0.4)
	(BAB Pay, LEH Rec)	-55.6(1.8)	-58.7(1.7)	-66.1(1.4)	-71.3(1.1)	-73.2(1.0)	-74.1(0.9)
(0.8, -0.3, -0.3)	(LEH Pay, BAB Rec)	83.9(0.0)	82.9(0.1)	82.3(0.3)	82.9(0.2)	82.9(0.3)	83.0(0.3)
	(BAB Pay, LEH Rec)	-36.4(3.3)	-41.9(3.0)	-55.9(2.2)	-63.4(1.6)	-65.8(1.5)	-66.4(1.5)
(0, 0, 0.5)	(LEH Pay, BAB Rec)	50.6(1.5)	54.3(1.5)	59.2(1.5)	64.4(1.1)	65.5(1.3)	68.8(0.8)
	(BAB Pay, LEH Rec)	-80.9(0.2)	-80.5(0.3)	-80.9(0.4)	-82.3(0.3)	-82.6(0.3)	-82.8(0.3)
(0, 0, 0.8)	(LEH Pay, BAB Rec)	12.3(3.5)	21.0(3.0)	34.9(2.5)	41.3(2.1)	44.6(1.9)	50.6(1.4)
	(BAB Pay, LEH Rec)	-80.9(0.2)	-81.5(0.2)	-81.9(0.3)	-81.9(0.4)	-82.1(0.4)	-82.7(0.3)
(0, 0, 0)	(LEH Pay, BAB Rec)	78.1(0.2)	77.9(0.3)	79.5(0.5)	79.5(0.5)	80.1(0.6)	82.1(0.4)
	(BAB Pay, LEH Rec)	-81.6(0.2)	-81.9(0.2)	-82.3(0.3)	-82.2(0.4)	-82.7(0.3)	-83.2(0.3)
(0, 0.7, 0)	(LEH Pay, BAB Rec)	77.3(0.3)	77.3(0.4)	78.5(0.5)	79.2(0.5)	79.7(0.6)	81.5(0.4)
	(BAB Pay, LEH Rec)	-81.2(0.2)	-81.8(0.2)	-81.9(0.3)	-80.8(1.3)	-82.4(0.3)	-82.6(0.3)
(0.3, 0.2, 0.6)	(LEH Pay, BAB Rec)	54.1(1.4)	56.7(1.3)	62.5(1.1)	63.6(1.1)	66.4(0.9)	69.7(0.6)
	(BAB Pay, LEH Rec)	-81.3(0.2)	-81.7(0.2)	-81.4(0.4)	-81.3(0.5)	-81.6(0.4)	-82.1(0.4)
(0.3, 0.3, 0.8)	(LEH Pay, BAB Rec)	22.8(4.2)	28.8(3.5)	38.6(2.9)	42.6(2.9)	45.9(2.5)	52.0(2.2)
	(BAB Pay, LEH Rec)	-83.0(0.2)	-83.2(0.2)	-82.8(0.3)	-82.4(0.4)	-82.5(0.4)	-82.9(0.4)
(0.5, 0.5, 0.5)	(LEH Pay, BAB Rec)	62.8(0.8)	64.5(0.8)	67.7(0.8)	68.5(0.9)	71.3(0.7)	73.2(0.6)
	(BAB Pay, LEH Rec)	-67.4(1.1)	-70.4(0.9)	-72.9(0.9)	-74.4(0.9)	-75.8(0.8)	-76.7(0.7)
(0.7, 0, 0)	(LEH Pay, BAB Rec)	77.4(0.2)	77.3(0.3)	78.9(0.5)	79.1(0.5)	79.9(0.5)	81.4(0.4)
	(BAB Pay, LEH Rec)	-47.3(2.2)	-55.0(1.9)	-61.6(1.6)	-65.0(1.5)	-67.5(1.3)	-69.6(1.1)

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Future Work

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- Apply the framework to a different class of products.

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