Follow The Money From Boom to Crash

A Behavioral Model of Market Dynamics

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Operational and Credit Risk Analytics
Risk Architecture

April 28, 2010

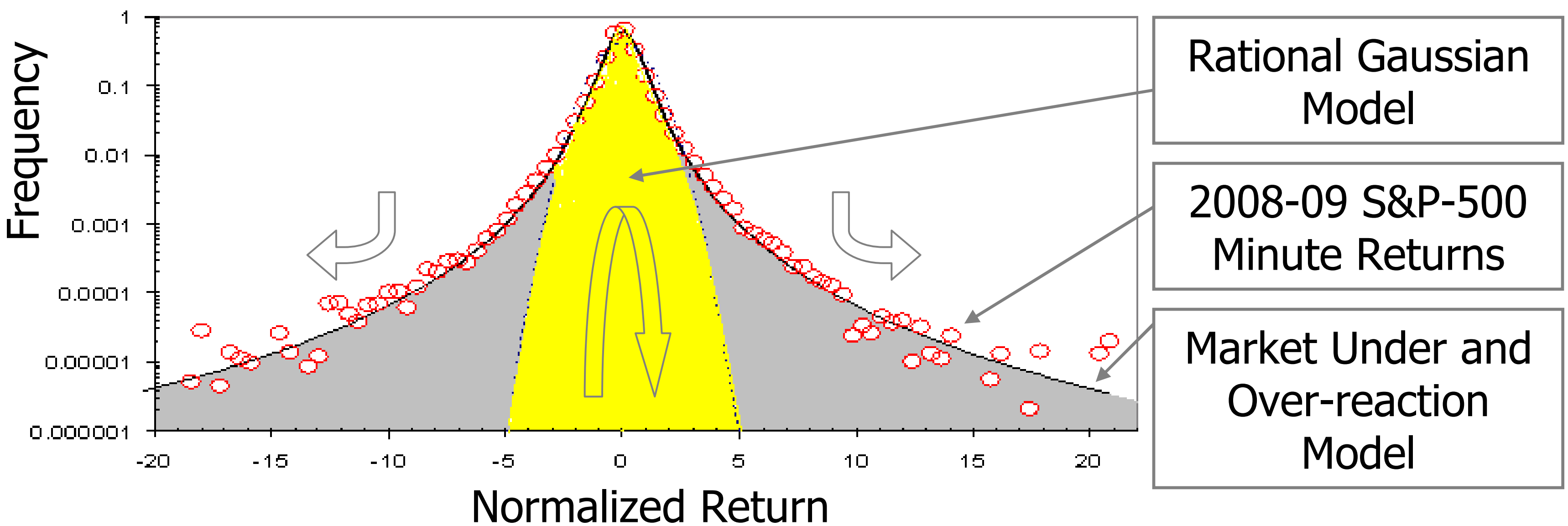
Risk Architecture

Main Points to Make Today

- Market under and over-reaction to price trends,
- Market-specific uncertainty, and
- The recognition of different market dynamics for equity and debt

can lead to better asset pricing and credit models beyond the limitations of idealized models of perfectly informed markets

These issues are more noticeable during periods of high volatility, market illiquidity and market downturns



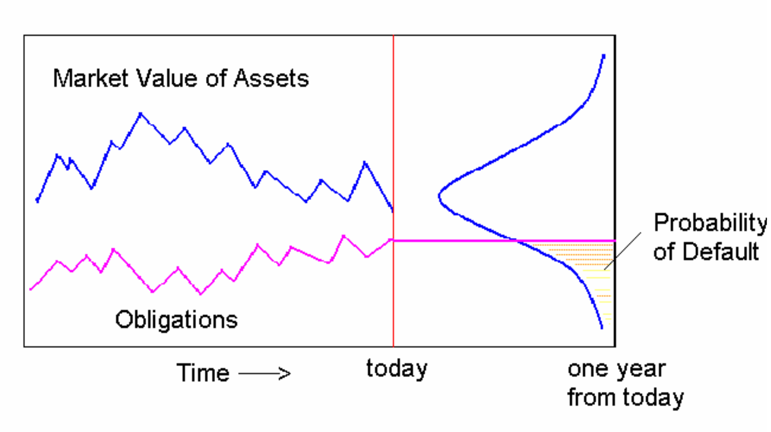
1. Motivation
Market Implied Approaches to Default Risk and Credit Quality Migration

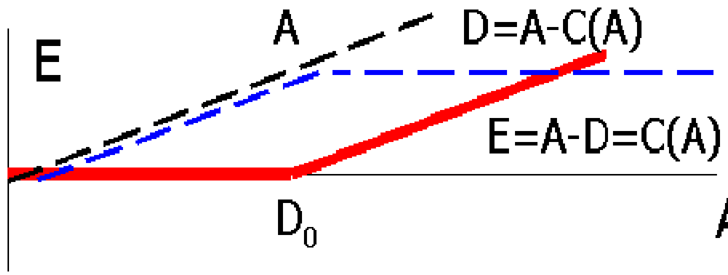
Structural Models of Credit Risk - Equity as an Option

Market Equity = Present Value (Residual Value of the Firm)
Stock Volatility = Leveraged Volatility of Assets

1. Calculate the effective value of the firm's obligations: D_0
2. Use equity information to estimate:
   - Market value of the firm's assets: A (Normal Random Walk)
   - Volatility of assets: V_A
3. Calculate the firm's probability of default at time T-t as

$$PD_M(T-t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-y} e^{-x^2/2}\, dx \qquad y(T-t) = \frac{1}{\sigma\sqrt{T-t}} \log\left(\frac{A e^{(\mu - \sigma^2/2)(T-t)}}{D_0} \right)$$





Equity as an option: ownership of the firm's assets and limited-liability obligation to repay debt.

Limitations of Market-Based Structural Models

The basic assumptions are too restrictive: ideal markets, infinitely divisible securities, costless continuous trading, etc.

Other key issues:

- Measurement issues
  - Implied asset volatility $\Rightarrow$ volatility is not observable
  - Implied market value of assets $\Rightarrow$ market value of assets is not observable
- Default point
  - Uncertainty in future liabilities
  - Uncertainty in business and regulatory environment

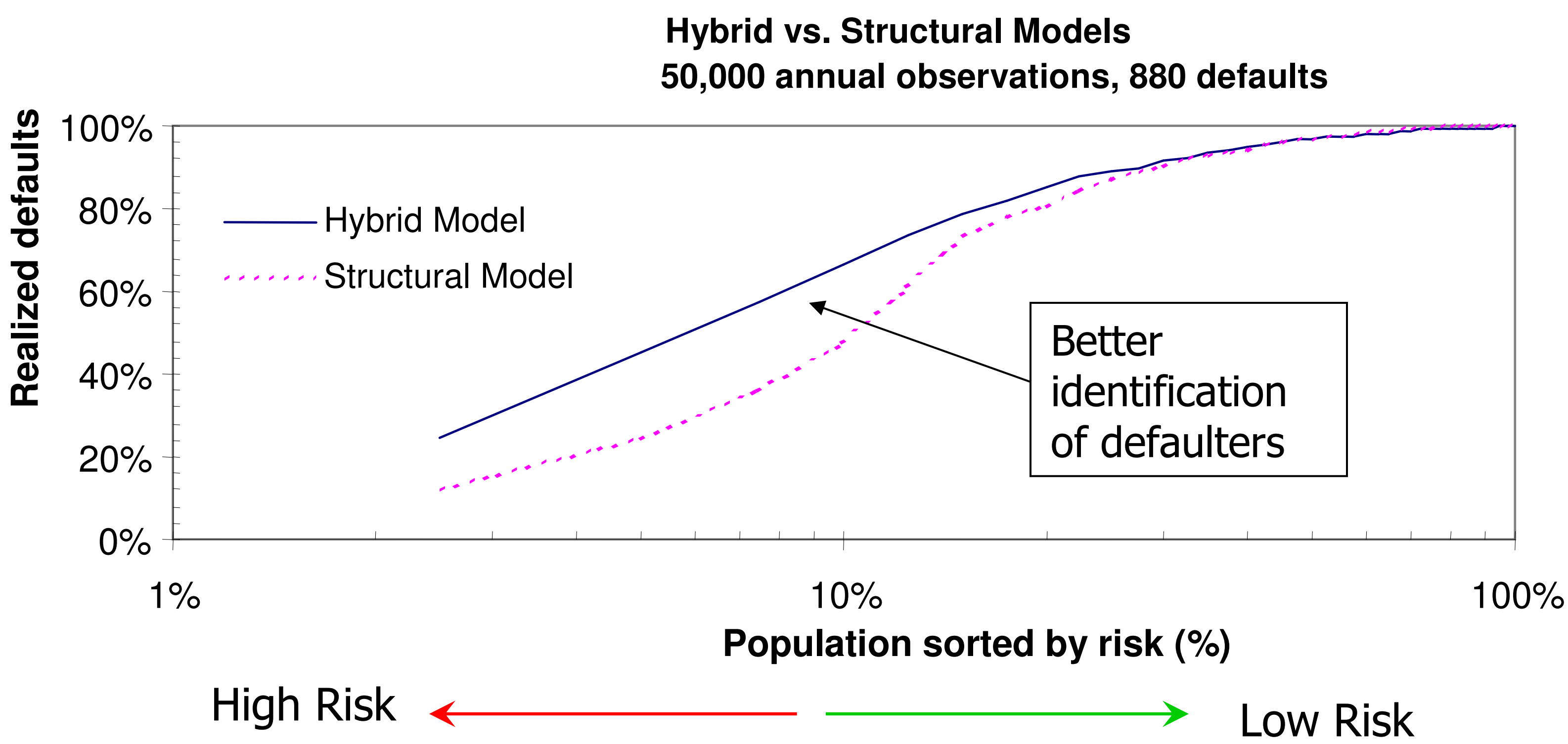
Market dynamics

- Market over- and under-reaction (bubbles, crashes and crises)
- Liquidity limitations, supply-demand effects, marginal share values
- Dynamics of debt and equity markets are very different

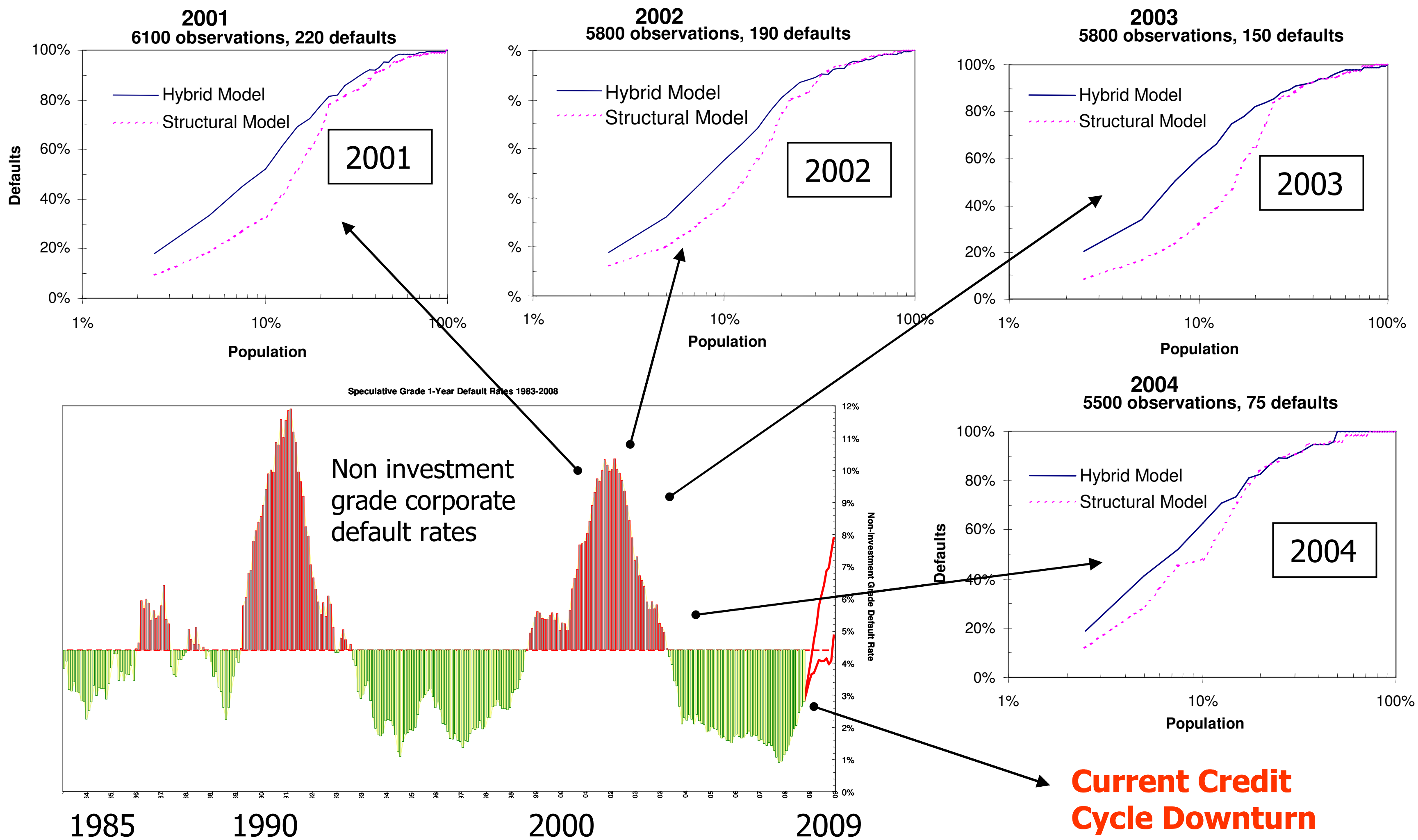
- Debt capacity
  - Inadequate description of the capacity to borrow and/or roll over debt

Default Probability Estimation in Practice

- Model performance tests show that hybrid models with additional financial information and fat-tail effects outperform pure structural models (based on idealized market assumptions)
- The performance gap is greater for low credit quality obligors (exposed to higher levels of uncertainty), and during market downturns

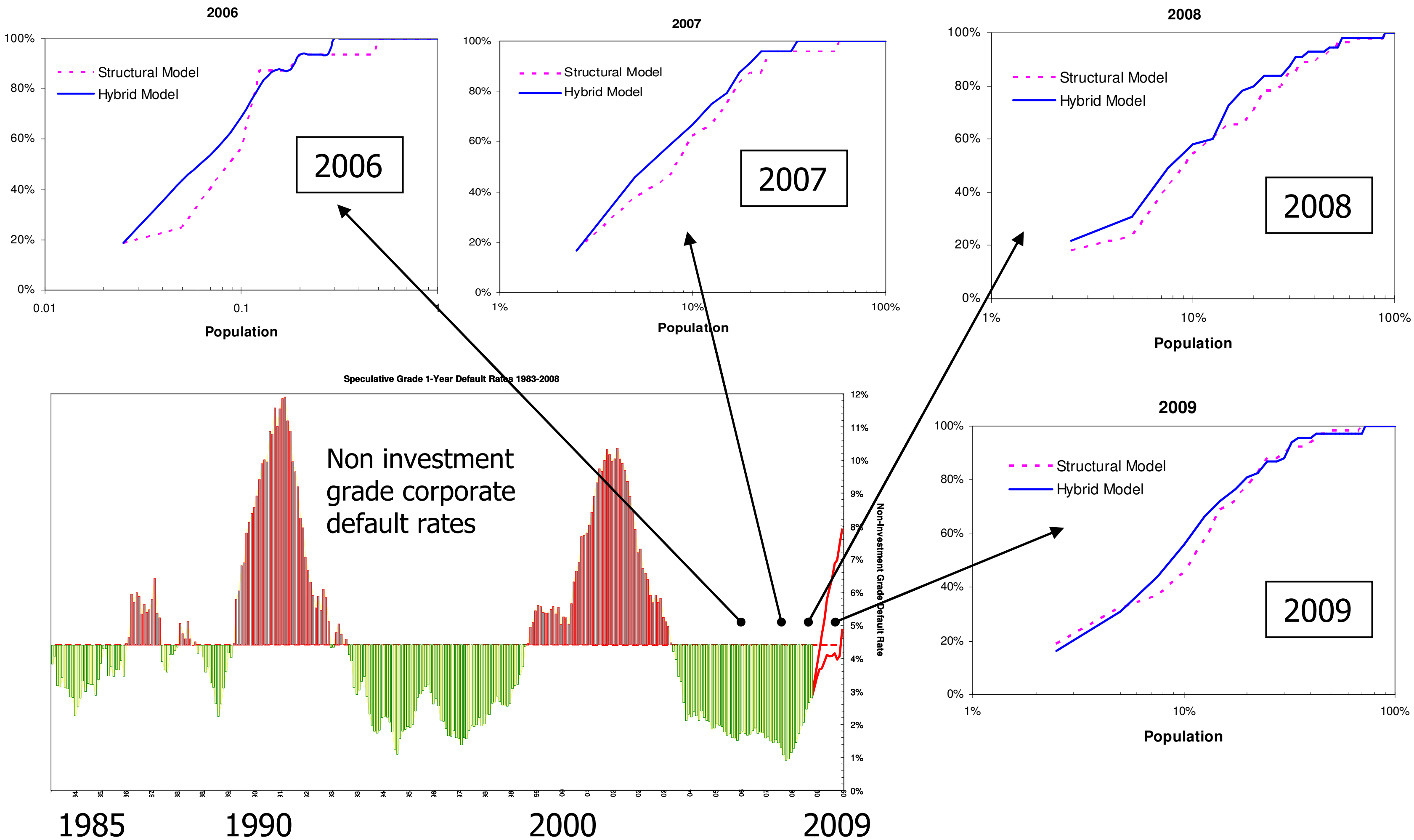


Model Performance in Practice – Downturn Effects (1)



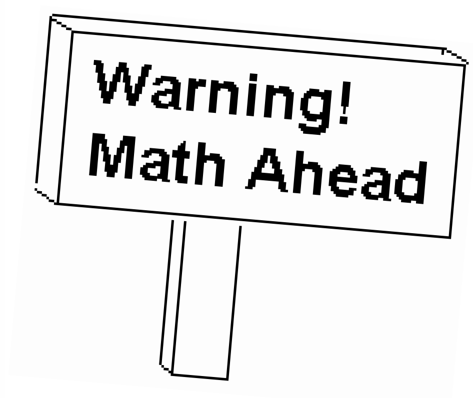
• The model performance gap is greater for low credit quality obligors (higher uncertainty) and during market downturns (higher market volatility)

Model Performance in Practice – Downturn Effects (2)

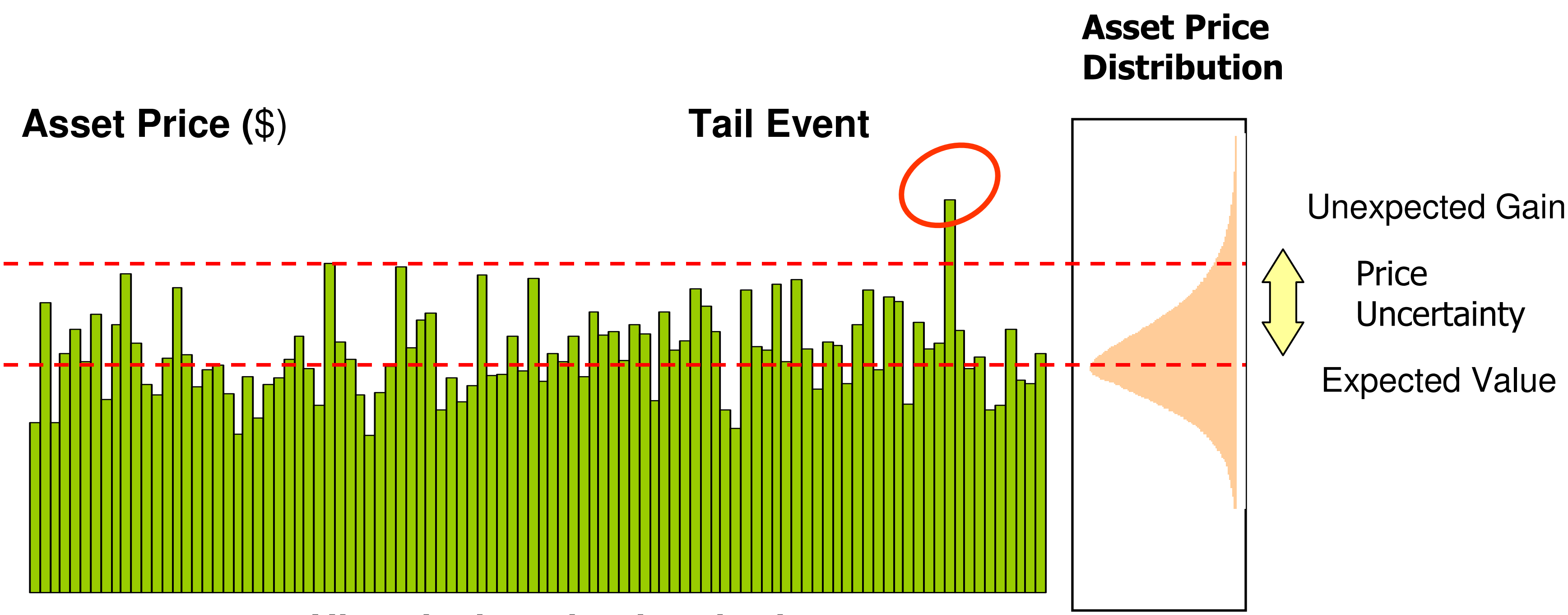


• The model performance gap is greater for low credit quality obligors (higher uncertainty) - Preliminary results based on small samples (2006-2009)

2. Modeling Market Dynamics
Revisiting Key Assumptions
on Investor Behavior



Modeling Asset Prices

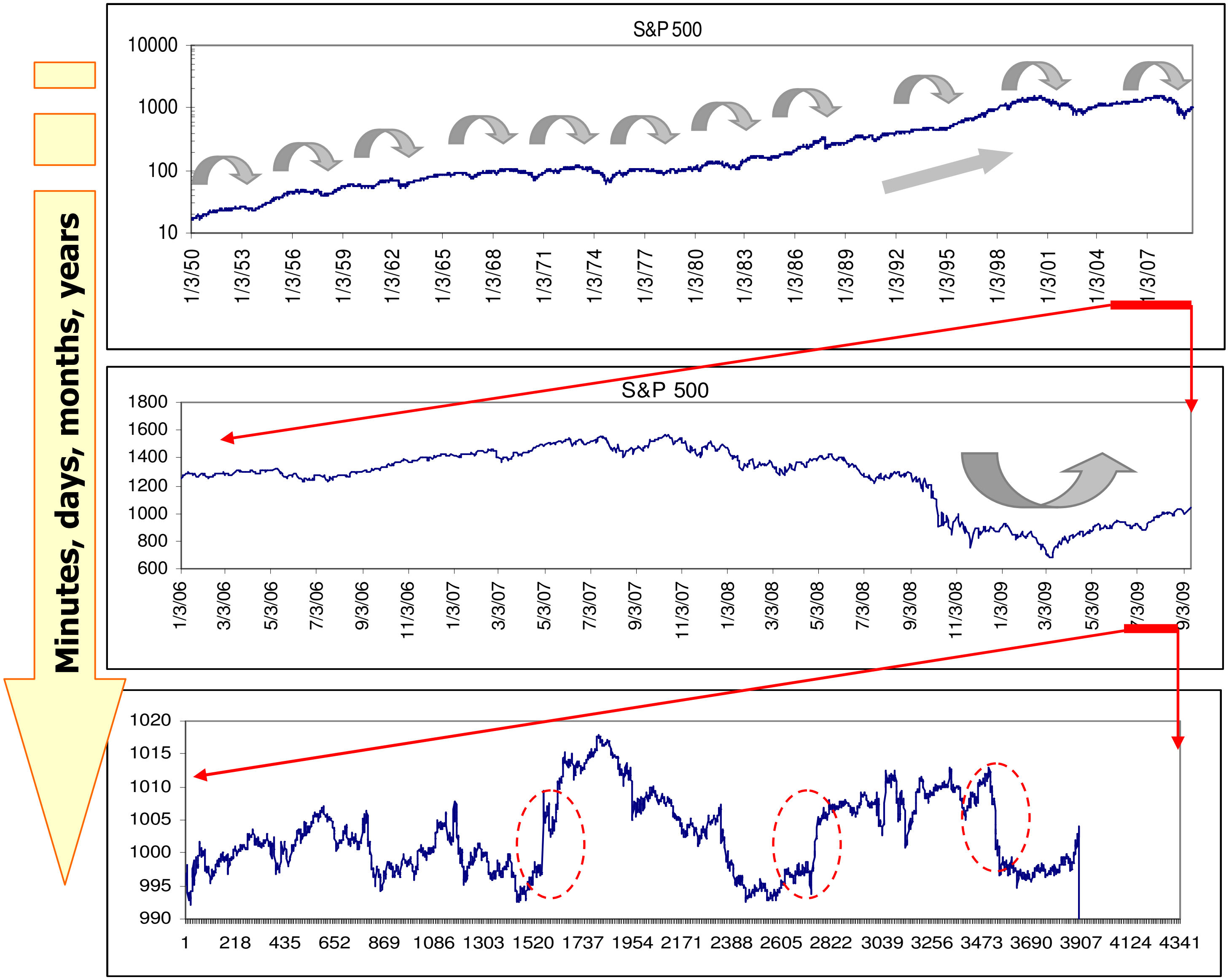


Historical or simulated prices

Modeling asset prices

Question 1. What is the distribution of fundamental values?
Question 2. How do investors react to information?

Modeling Asset Prices – Market Dynamics



Long and medium term trends and cycles, and short term noise

Prices modeled as random walks

Fat tail events

Market Dynamics - Cognitive and Social Patterns

Malkiel, Thaler, Shiller and Shleifer highlighted that prices are sometimes driven by widespread **social** and **cognitive** behavior patterns

Cognitive issues

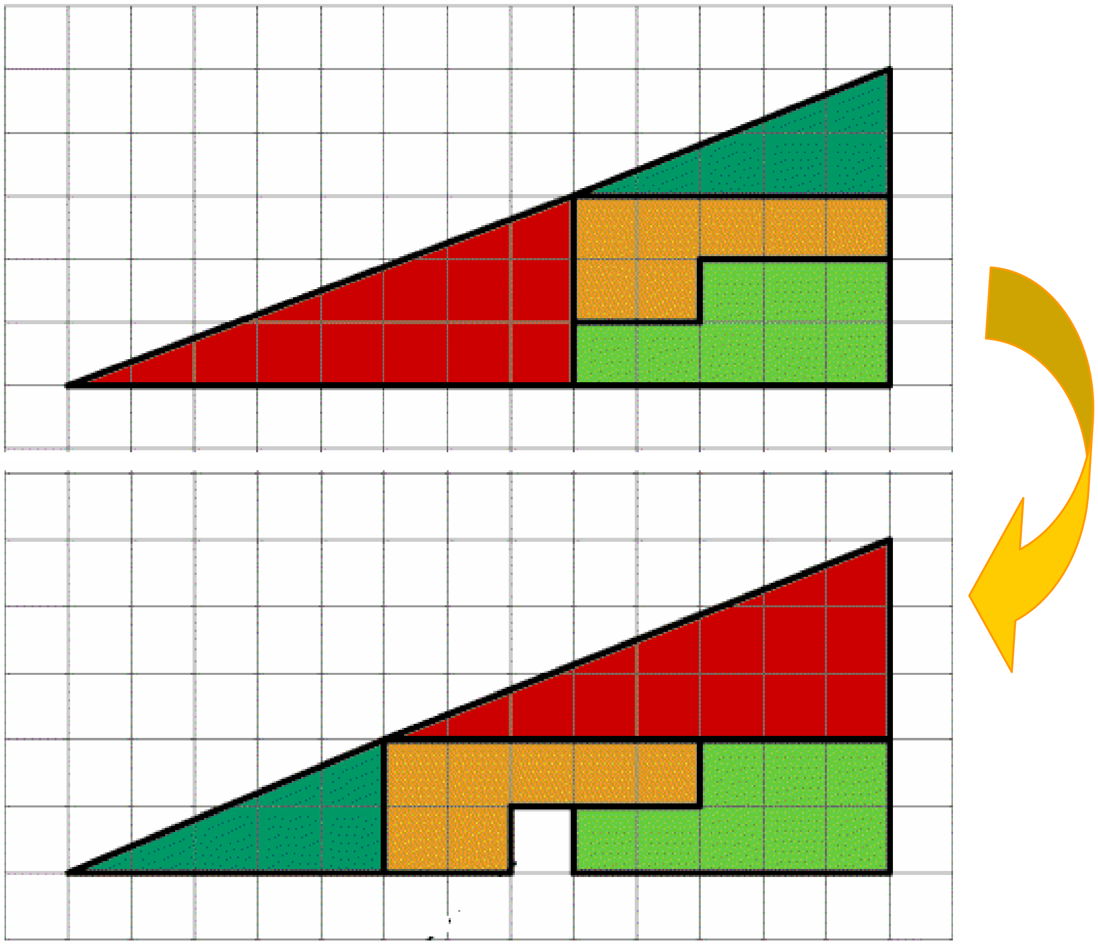
1) **Representativeness heuristic** - failure to account for the correct prior probability of an event
2) **Overconfidence** – overestimation of one's unique abilities and information
3) **Wishful thinking** - assign too high a probability to a desired outcome

Social issues

1) **Herd behavior** - tendency to follow the crowd
2) **Self-reinforced behavior** - positive feedback that can lead to fat tails, bubbles, panics and crashes
   - Following price trends can lead to unstable market feedback
3) Market participants have different <u>motivations</u> and <u>strategies</u>

Here we explore the concepts described above and their impact on prices

Understanding Models And Assumptions



Reordering the pieces
creates a gap
How can this be true?

Review fundamental assumptions

Asset Prices as Random Walks

Let us define the log-asset price as

$$\chi = \log\left(A / A_0\right)$$

Asset prices as a Brownian motion

$$d\chi = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma\, dZ \qquad (1)$$

Here μ is the expected asset growth and σ is the asset volatility
Z follows a Wiener random process

- Markets are rational and perfectly informed. Price changes reflect shocks of new information. They are completely random and have no memory
- The standard random walk assumption for price movements can be traced back to Bachelier (1900)
- Modern literature on prices as random walks usually begins with Samuelson (1965) and the law of iterated expectations
- The articulation of efficient markets and the assumption of price changes as normal random walks were solidified in the 70s (Fama 1970)

Asset Prices as Random Walks - Limitations

- Kendall (1953) highlighted the limitations of the random walk assumption

  "It may be that the motion [of stocks] is genuinely random and that what looks like a purposive movement over a long period is merely a kind of economic Brownian motion. But economists – and I cannot help sympathizing with them – will doubtless resist any such conclusion very strongly"

- However, the relevance of investor behavior in price dynamics was usually dismissed
  - Non-rational investors would tend to lose money, exiting the market
  - Under and overreaction would produce on average no observable net effect

Market Under and Overreaction, and Fat Tails (1)

- Investors behavior may be neither random nor senseless, and can manifest itself as predictable and systematic

- Here we assume that the security's return can deviate temporarily from its fundamental value
  - Random feedback process caused by over and under-reaction to price trends

- We focus primarily on
  - Short-term changes in prices
  - quasi-stationary dynamics
    (no business cycles or crises for this version of the model)

Market Under and Overreaction, and Fat Tails (2)

The economic interpretation of our model is as follows

- During periods of uncertainty investors may tend to react to price trends
  - Bullish investors may attach themselves to an over-optimistic view of the company during upward price trends
  - Bearish investors may attach themselves to an over-pessimistic view during a downward price trend
- In doing so, investors discount the possibility that the bullish (bearish) price trends are the result of aggregated behavioral dynamics rather than a change in the future prospects of the company
- This gives rise to the market overreaction to upward and downward price moves
- This can create positive feedback and potentially unstable dynamics for both stock prices and excess returns

Market Under and Overreaction, and Fat Tails (3)

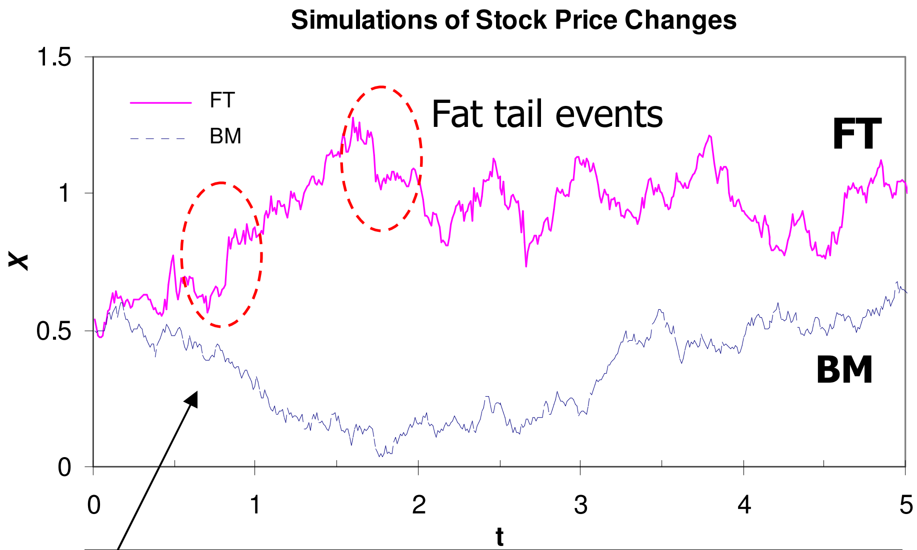
Market under/overreaction

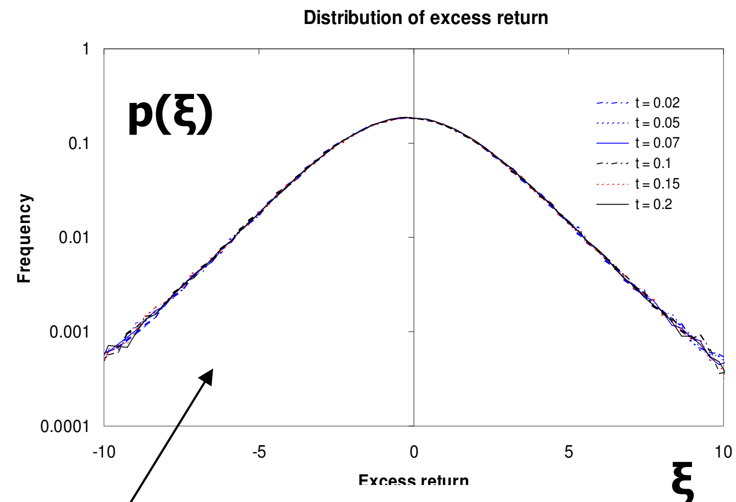
Asset return adjustment $$d\chi = \left(\mu + \xi - \frac{\sigma_\varepsilon^2}{2} \right) dt + \sigma_0 \, dZ_0 \qquad (2)$$

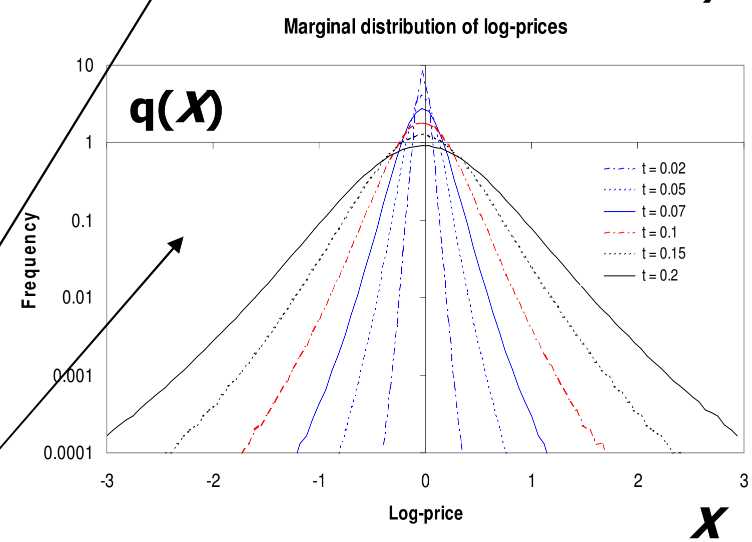
Excess return adjustment $$\boxed{d\xi} = -\xi \left(\theta \, dt + \varepsilon \, dZ_1 \right) + \eta \, dZ_2 \qquad (3)$$

- The model includes a (dissipative) **mean reversion process** for the security's excess return ξ above (below) the mean return μ
- θ is the average sensitivity of market participants to price return discrepancies (mean reversion of returns)
- ε is the strength of the multiplicative price momentum corrections
- η is the magnitude of random additive changes in excess return resulting from investors' under and over-valuation of returns
- Z_0 Z_1 and Z_2 follow Wiener random processes (Brownian motion)

Market Under and Overreaction, and Fat Tails (4)







Numerical realization of equations (2)-(3) (**FT**) for daily price changes, and equation (1) (standard Wiener process) (**BM**)

Diffusion of the distribution p(ξ) of excess returns ξ

Diffusion of the distribution q(X) of log-assets X

Market Under and Overreaction, and Fat Tails (5)

The probability P(X,ξ,t) of asset value X and excess return ξ at time t is given by the following equation

$$\frac{\partial P}{\partial t}+\left(\xi+\mu-\frac{\sigma_\varepsilon^2}{2}\right)\frac{\partial P}{\partial \chi}=\frac{\partial}{\partial \xi}[\theta\xi P]+\boxed{\frac{1}{2}\frac{\partial^2}{\partial \xi^2}\left[\left(\eta^2+\varepsilon^2\xi^2\right)P\right]}+\frac{\sigma_0^2}{2}\frac{\partial^2 P}{\partial \chi^2} \quad (4)$$

The marginal distribution of excess returns

$$p(\xi,t)=\int P(\chi,\xi,t)\,d\chi \quad (5)$$

is determined by the following equation

$$\frac{\partial p}{\partial t}=\frac{\partial}{\partial \xi}\left\{\theta\xi\, p+\frac{1}{2}\frac{\partial}{\partial \xi}\left(\left(\eta^2+\varepsilon^2\xi^2\right)p\right)\right\} \quad (6)$$

Market Under- and Overreaction, and Fat Tails (6)

The distribution of excess returns is given by the following equation

$$p(\xi,t) \approx \frac{\Gamma(\beta+1)}{\Gamma(\beta+\tfrac{1}{2})} \frac{1}{\sqrt{\pi\psi^2}\left(1+(\xi/\psi)^2\right)^{\beta+1}} \qquad (7)$$

$$\psi^2(t) = \delta^2 + (\psi_0^2 - \delta^2)(1 - e^{-2t/\tau}) \qquad \delta = \eta/\varepsilon \qquad \beta = \theta/\varepsilon^2$$

Rational market limit (no under or overreaction to price trends)

$$p_0(\xi) = \frac{\Gamma(\beta+1)}{\Gamma(\beta+\tfrac{1}{2})} \frac{1}{\sqrt{\pi\delta^2}\left(1+(\xi/\delta)^2\right)^{\beta+1}} \underset{\theta>>\varepsilon^2}{\longrightarrow} \boxed{\frac{1}{\sqrt{2\pi\varphi^2}} e^{-\xi^2/2\varphi^2},} \quad \varphi^2 = \delta^2/2\beta \qquad (8)$$

Herding/under and overreaction limit ('irrational exuberance' markets)

$$p_0(\xi) = \frac{\Gamma(\beta+1)}{\Gamma(\beta+\tfrac{1}{2})} \frac{1}{\sqrt{\pi\delta^2}\left(1+(\xi/\delta)^2\right)^{\beta+1}} \underset{\xi>>\delta}{\longrightarrow} \boxed{C\left(\frac{\delta}{|\xi|}\right)^{2(\beta+1)}} \qquad (9)$$

Fat Tailed Bulls and Bears – 2008-2009

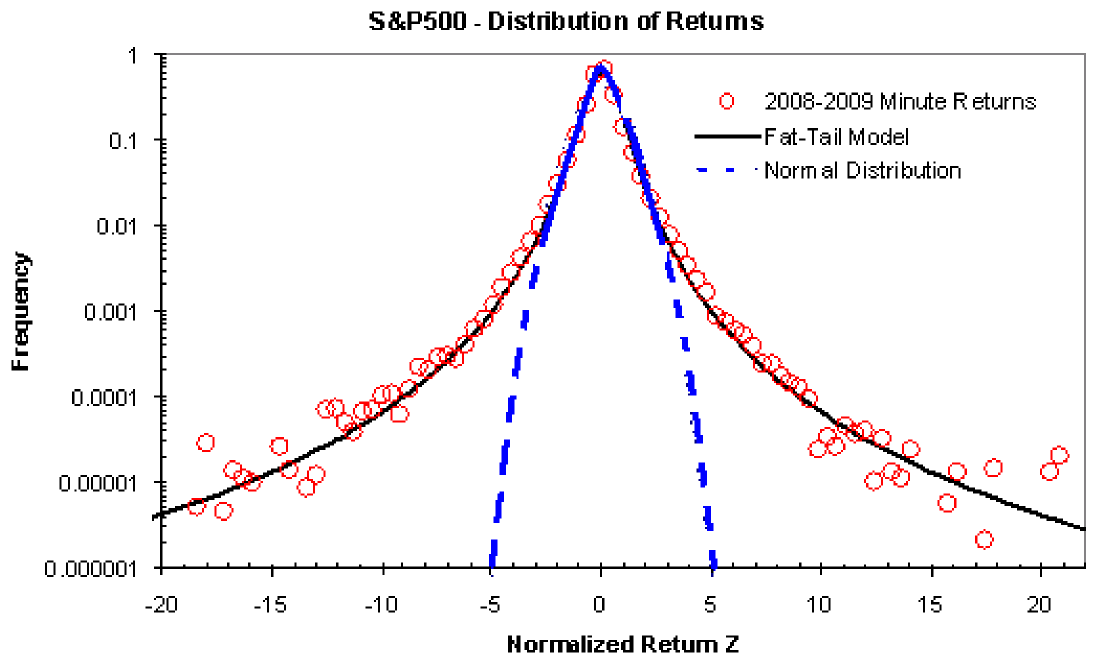
The homogeneous stationary distribution of instantaneous excess returns is a fat tailed distribution

$$p(z) \approx \frac{\Gamma(\beta+1)}{\Gamma(\beta+½)} \frac{1}{\sqrt{\pi(2\beta-1)}\left(1+z^2/(2\beta-1)\right)^{\beta+1}} \qquad (10)$$

Here

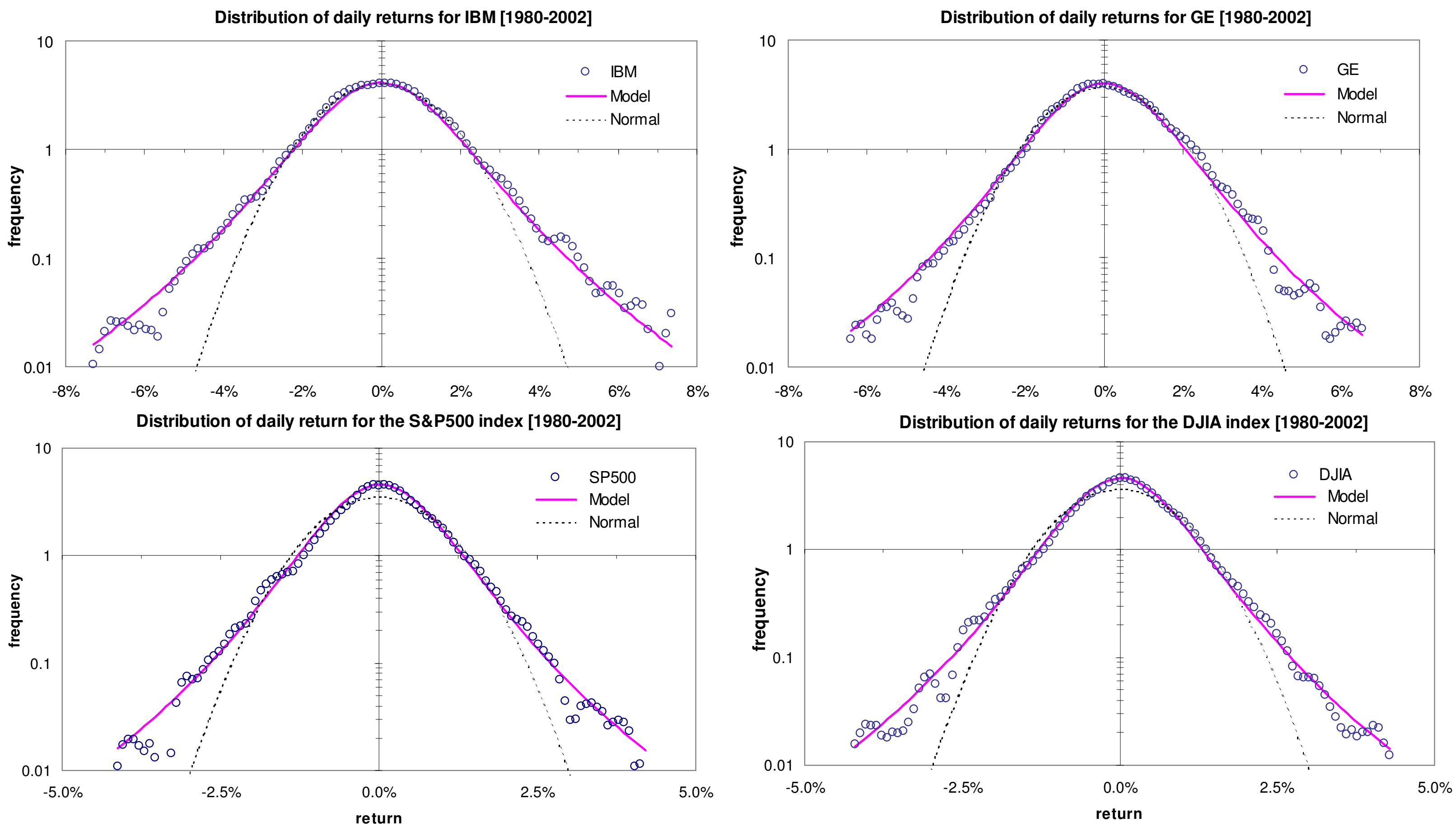
$$z(\xi,t) = \sqrt{2\beta-1}\,\frac{\xi}{\psi(t)}$$

Distribution of **minute equity returns** (symbols), stationary distribution Eq. (10) (solid line) and normal distribution (dashed line)



Fat Tailed Bulls and Bears – Examples 1980-2002

Average distribution of daily equity returns (symbols), stationary distribution Eq. (10) (solid line) and normal distribution (dashed line)



Fat Tailed Distribution of Asset Prices

The actual distribution of asset values is obtained from Equation (4) for an arbitrary log-asset value, excess return and time *t*

$$q(\chi,t)=\int_{-\infty}^{+\infty}P(\chi,\xi,t)\,d\xi$$

$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[G(0,t)+k\frac{\partial G}{\partial k}+\ldots\right]\exp\left\{ik\frac{\varepsilon^3}{\eta}(\chi-\chi_0)-\frac{\varepsilon^2}{2}\left(\delta k^2+2ik\alpha\right)(t-t_0)\right\}\,dk$$

$$\approx\frac{e^{-x^2/2}}{\sqrt{2\pi\sigma^2(t-t_0)}}\sum_{n=0}^{\infty}\frac{a_n(t)H_n(x)}{n!\left(\sigma\sqrt{t-t_0}\right)^n}\xrightarrow[t>>t_0]{}\boxed{\frac{e^{-x^2/2}}{\sqrt{2\pi\sigma^2(t-t_0)}}}+\ldots \qquad (11)$$

Here

$$x(\chi,t)=\frac{\chi-\chi_0-(\mu-\sigma^2/2)(t-t_0)}{\sigma\sqrt{t-t_0}} \qquad \sigma^2\approx\frac{\eta^2}{\theta^2\left(1+\frac{\varepsilon^2}{2\theta}\right)^2}+\sigma_0^2$$

Equation (11) is the fat-tailed distribution with a central peak that resembles a normal distribution and an effective volatility given by the bullish-bearish dynamics of asset prices

Options, Fat Tails and Investor Behavior (1)

Based on the Black-Scholes model, equation (11) and additional technical assumptions, the value of European call options on an asset *A* at time *t* with strike *K* and expiration *T* is

$$C(A,t;K,T,\rho,\sigma) = e^{-\rho(T-t)} E\left(A-K\right)^{+}$$

$$= C_{BS} + \frac{1}{G_0}\sum_{n=1}^{\infty}\frac{i^n}{n!\left(G_0\sigma\sqrt{T-t}\right)^n}\frac{\partial^n G}{\partial k^n}\; B_n \qquad \text{(a)}$$

Here

$$B_n(A,T-t;K,\rho,\sigma) = A\Gamma_n\left(d_1,\sigma\sqrt{T-t}\right) - Ke^{-\rho(T-t)}\,\Gamma_n(d_2,0)$$

$$\Gamma_n(x,y) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} H_n\left(z+y\right)e^{-z^2/2}\,dz \qquad \text{(b)}$$

$$d_1 = \frac{\log(A/K)+(\rho+\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}\,, \qquad d_2 = d_1 - \sigma\sqrt{T-t}$$

Options, Fat Tails and Investor Behavior (2)

Let's calculate an 'implied volatility' from the standard Black-Scholes European call option

$$C_{BS}(A,t;r,\sigma_I) \approx C(A,t;\rho,\sigma) \tag{c}$$

An expansion in terms of the implied volatility and risk premium yields

$$C(A,T-t;r,\sigma) + \frac{\partial C}{\partial r}(\rho - r) \approx C_{BS}(A,T-t;r,\sigma) + \frac{\partial C_{BS}}{\partial \sigma}(\sigma_I - \sigma) \tag{d}$$

Equation (d) has to be solved for the implied volatility

$$\sigma_I \approx \sigma + \sqrt{\frac{2\pi}{T-t}}\frac{K}{A}e^{d_1^2/2}\left[(C - C_{BS}) + \frac{\partial C}{\partial r}(\rho - r)\right] \tag{e}$$

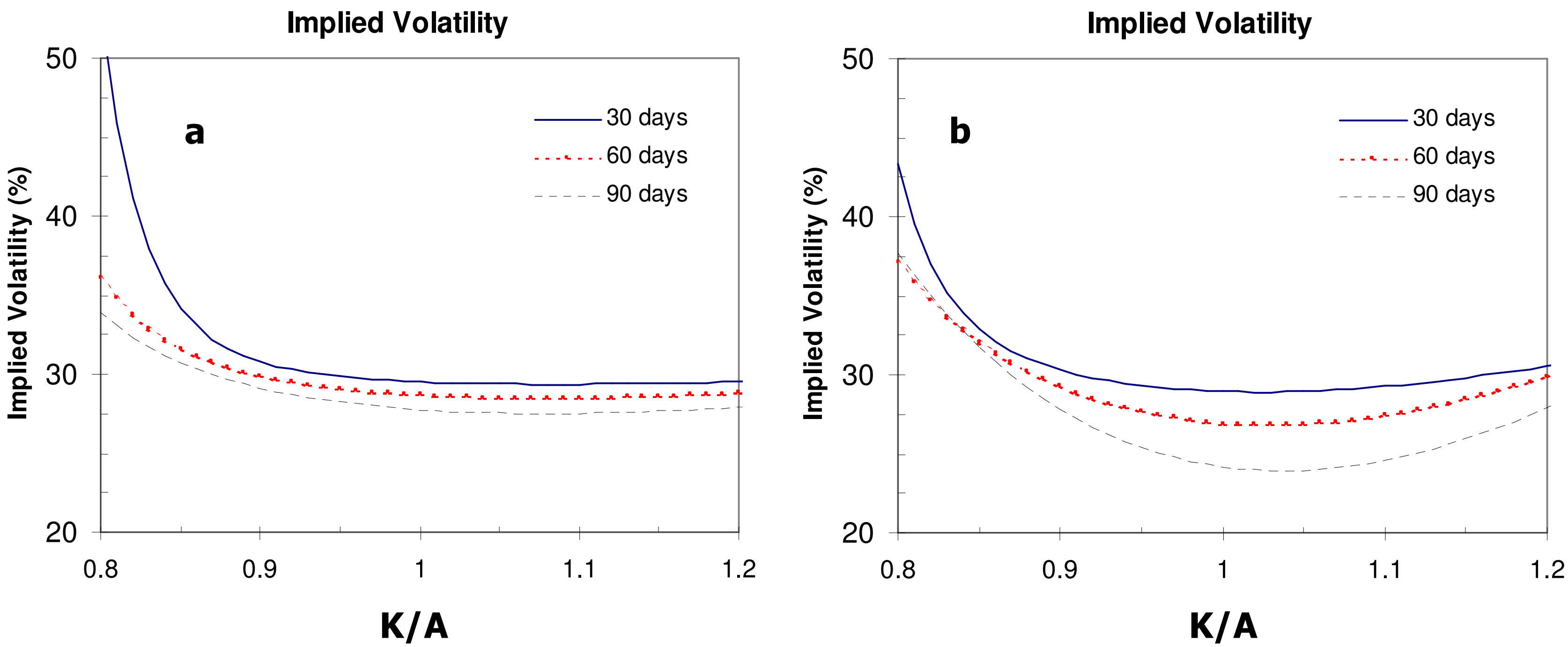
The implied volatility has two contributions:

1) The impact of investors behavior on the distribution of option prices
2) The additional risk premium required for holding an option that cannot be perfectly replicated due to fat-tailed returns

Options, Fat Tails and Investor Behavior (3)

Implied volatility

$$\sigma_I \approx \sigma + \sqrt{\frac{2\pi}{T-t}}\frac{K}{A}e^{d_1^2/2}\left[(C - C_{BS}) + \frac{\partial C}{\partial r}(\rho - r)\right]$$

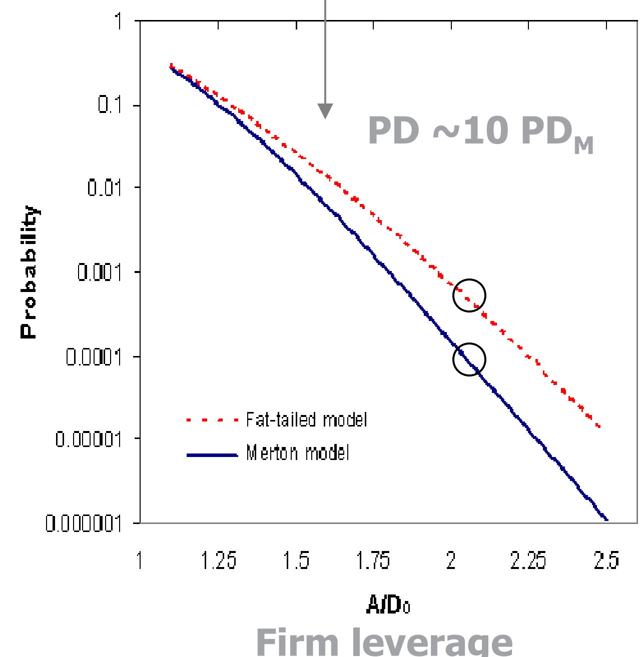


Implied volatility as a function of the option's moneyness for different times to expiration (30, 60 and 90 days). The security's price is A = 100, volatility is 30%, the risk-free rate is 5%, the required risk premium is 0.5%. To illustrate the curves include corrections for different fat tail parameters.

Default Probability, Fat Tails and Investor Behavior

Similarly, based on the Merton model and equation (11), the probability of default *PD* on the firm's obligations *D* is

$$PD = PD_M(d_2) + \frac{1}{G_0}\sum_{n=1}^{\infty}\frac{i^n}{n!\left(G_0\sigma_\varepsilon\sqrt{T-t}\right)^n}\frac{\partial^n G}{\partial k^n}\Gamma_n\left(d_2,0\right) \qquad \text{(f)}$$



Fat-tailed and log-normal distributions

Here

$$\Gamma_n(x,y) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} H_n\left(z+y\right)e^{-z^2/2}\,dz$$

The firm's equity *EQ* is

$$EQ(A,T-t;D,\rho,\sigma) = e^{-\rho(T-t)}E\left(A-D\right)^+$$

$$= EQ_M + \frac{1}{G_0}\sum_{n=1}^{\infty}\frac{i^n}{n!\left(G_0\sigma_\varepsilon\sqrt{T-t}\right)^n}\frac{\partial^n G}{\partial k^n}B_n \qquad \text{(g)}$$

Here

$$B_n(A,T-t;D,\rho,\sigma_\varepsilon) = A\Gamma_n\left(d_1,\sigma_\varepsilon\sqrt{T-t}\right) - De^{-\rho(T-t)}\Gamma_n(d_2,0)$$

Fat Tailed Distribution of Asset Prices in Practice

• The complexity of equation (11) limits its usefulness in practice

• Even simple options require demanding calculations

• Here we introduce a simplified asymptotic expression that can be used for some practical applications (within a <u>meaningful range</u> of asset prices)

• The distribution of normalized log-prices x can be approximated with a t-distribution (Eq. (10)) with slowly-changing degrees of freedom

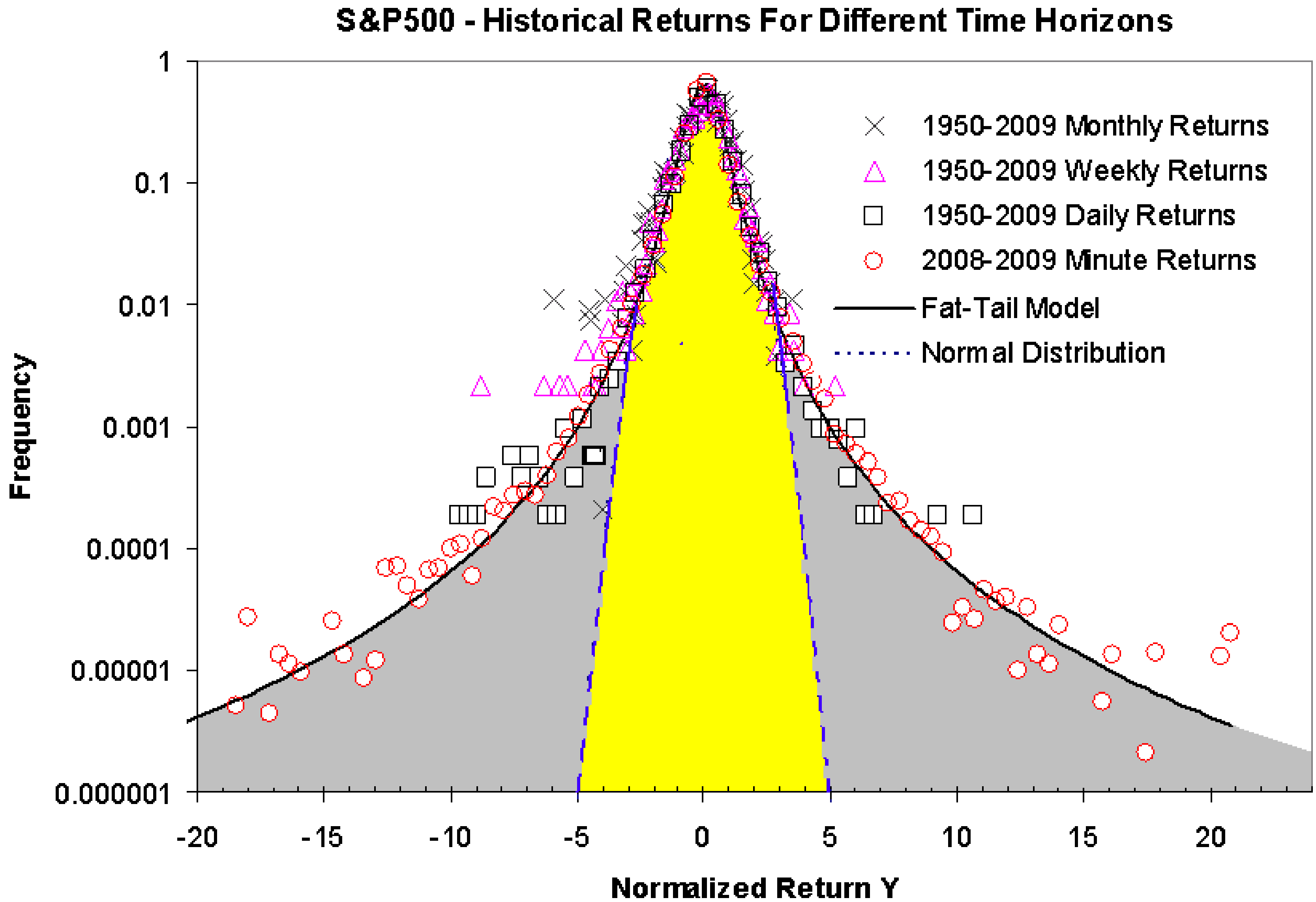
Bell-shape Fat-tailed Distribution

$$\begin{cases} q(y) \approx \dfrac{\Gamma(\gamma+1)}{\Gamma(\gamma+\frac{1}{2})} \dfrac{1}{\sqrt{\pi(2\gamma-1)}\left(1+y^2/(2\gamma-1)\right)^{\gamma+1}} & (12) \\ y(\chi,t) = \dfrac{1}{\rho}\sqrt{2\gamma-1}\left[\dfrac{\chi-\chi_0-(\mu-\sigma^2/2)(t-t_0)}{\sigma\sqrt{t-t_0}}\right] & (13) \end{cases}$$

Here $\rho(t) \sim 1$ and $\gamma(t) \geq \beta$

Market under and overreaction

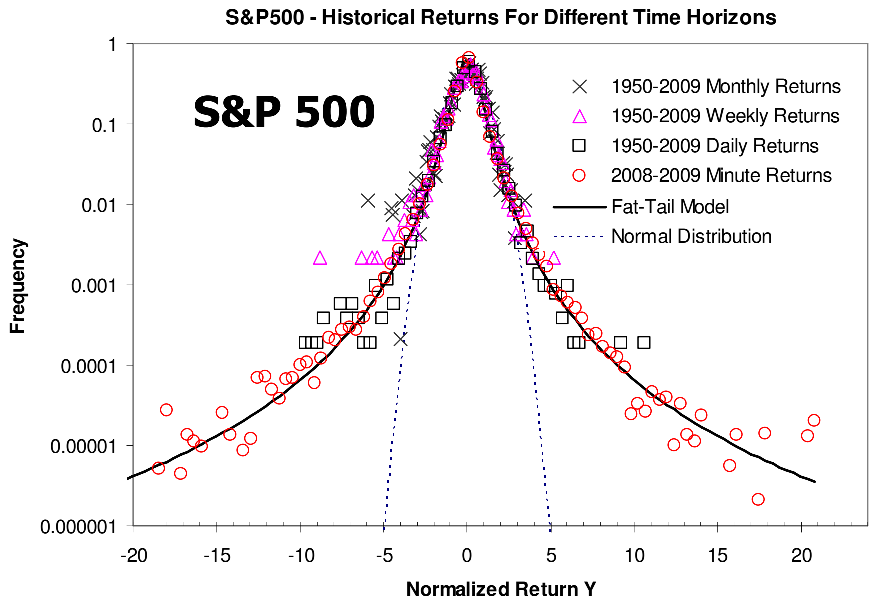
Asymptotic Approximation



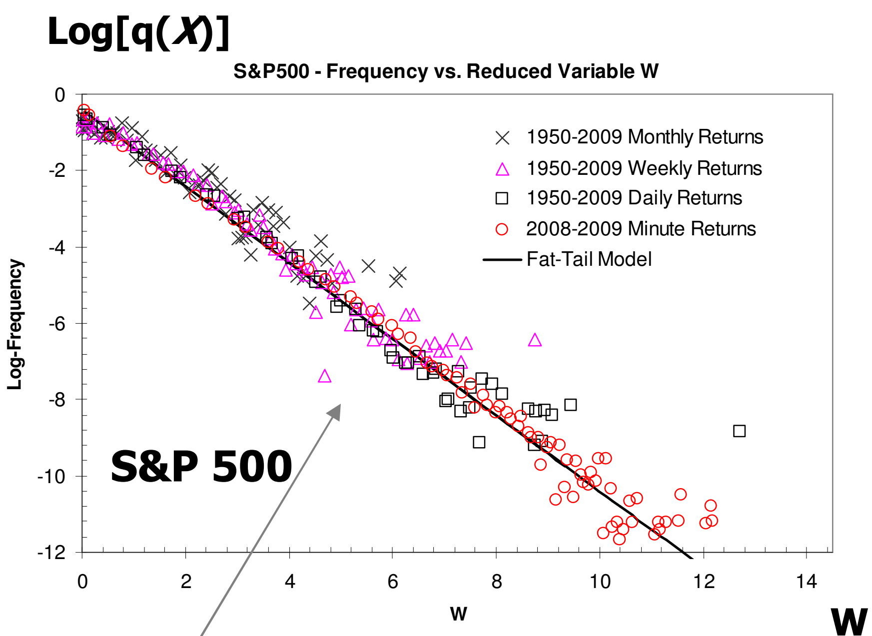
Asymptotic Approximation – Empirical Tests (1)

The distribution of asset prices follows equations (12)-(13) for time period ($t-t_0$)

$$\boxed{q(\chi,t)} = \int_{-\infty}^{+\infty} P(\chi,\xi,t)\,d\xi \approx \frac{c}{\left(1+a.y^2\right)^b} = \boxed{\frac{c}{\left(e^w\right)^b}} \qquad (14)$$



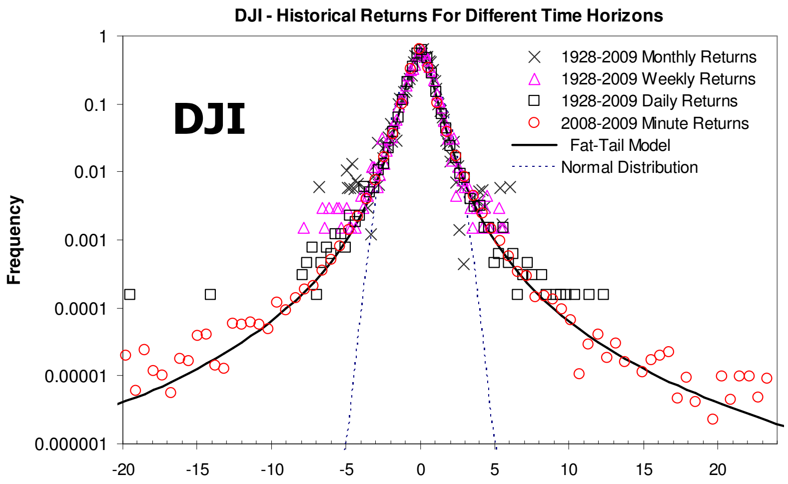
Random feedback $\varepsilon>0$ leads to a narrow distribution with fat tails

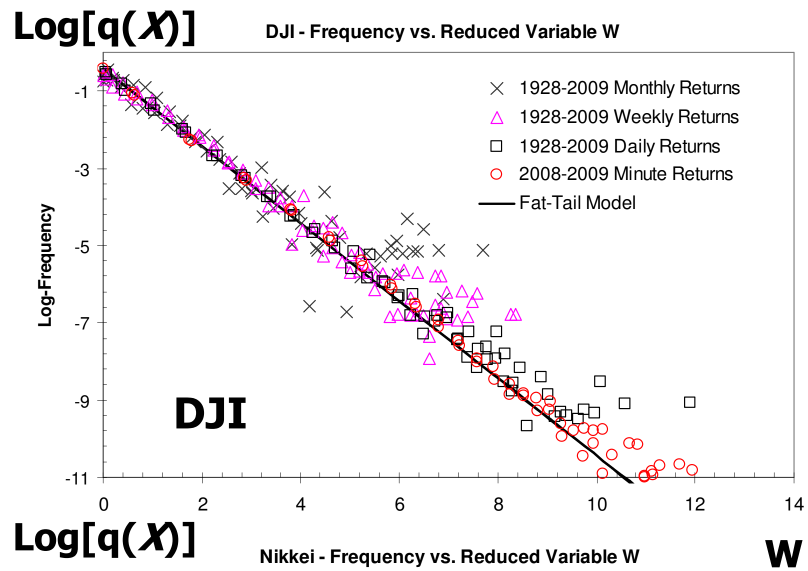


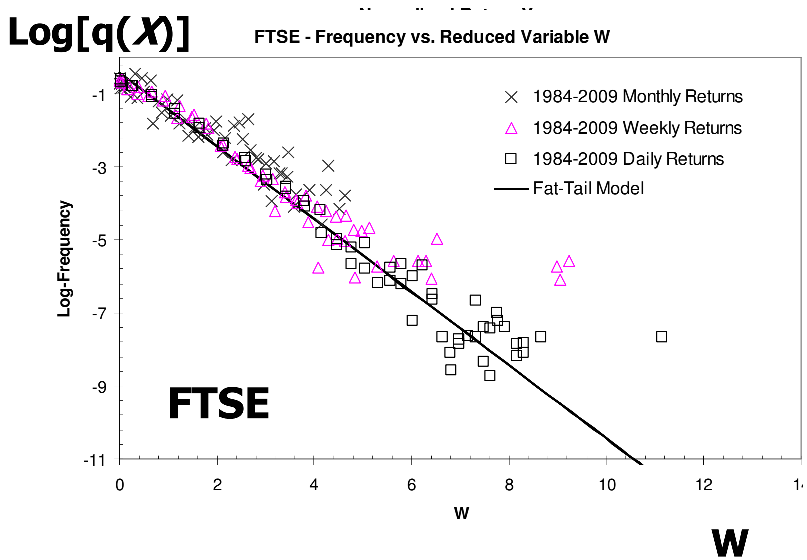
Empirical frequency (symbols) vs. model (solid line) (same parameters 'a' and 'b' for all time horizons)

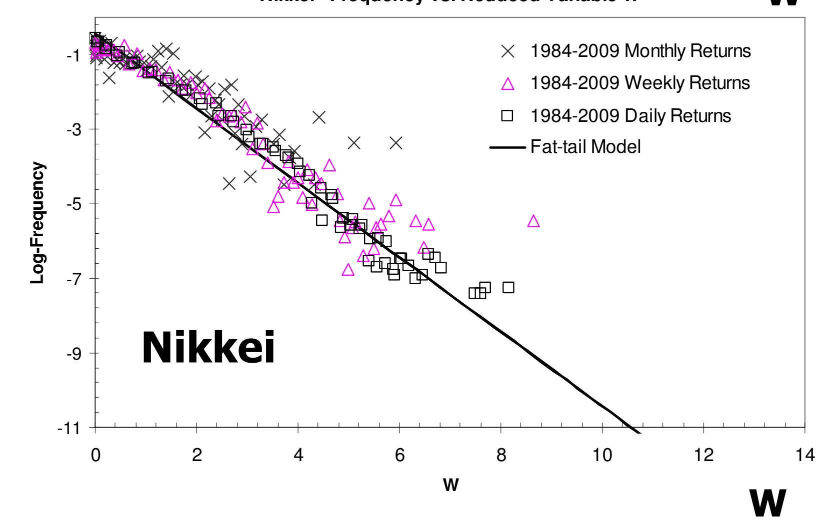
Asymptotic Approximation – Empirical Tests (2)

Empirical results for other market indices (DJI, FTSE, Nikkei)









Summary

- Traditional models of perfectly rational investors may require large parameter changes to capture the observed changes in prices (fat tail events)

- Models based on behavioral patterns driven by <u>price momentum and trends</u> can help us gain a better understanding of the dynamics of asset prices

- These issues seem more noticeable during periods of high volatility and market downturns

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