Hidden Liquidity and the Optimal Display of Iceberg Orders

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dbqpl quantitative products laboratory

Outline

- Hidden liquidity

- A simple model of placing iceberg orders

    - Hidden liquidity with higher time priority

    - Net order arrival dynamics

- Statistical analysis of hidden liquidity:

    - How much and where

    - Explanatory variables

- Some calibration results

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Limit Order Books

- Almost all electronic exchanges are based on Limit Order Books (LOBs)

  - Market Orders: immediate execution

  - Limit Orders: stored in the LOB

- Orders may be schielded from public view.

- Orders are executed according to a set of Priority Rules:

  - Price Priority

  - Display Priority

  - Time Priority

- Large orders (limit or market) move the market

Hidden Liquidity

- Increasing proportion of liquidity is hidden: reduced market impact

- Dark pools:

    - Typically associated with a primary venue (e.g. LSE)

    - Trade settled only if matching liquidity is/becomes available

- Iceberg orders:

    - Only a fraction of the order is openly displayed in the LOB

    - The hidden part loses its time priority

- Uncertainty of execution

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Iceberg Orders

- Only a fraction of the order is openly displayed in the LOB

- The hidden part loses time priority over the displayed part

- There may nor may not be a minimal display size

    - „Refill" upon execution of the visible part

- Synthetic iceberg order

    - Repeated submission of visible orders of a certain size

    - Inferior to icebergs in terms of time priority

Data Issues

- No (Limited) real time data on hidden liquidity available

- Reconciling trade and quote data, executed hidden liquidity may be detected (difficult!)

- We use NASDAQ Modelview Data

  - One minute snapshots of the entire book (hidden and visible)

  - Stock Universe: S&P 500

  - Period: November/December 2008.

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Sample Snapshot

Ticker	Time	Side	Price	Visible	Hidden
MSFT	11:51	B	27.11	27720	0
MSFT	11:51	B	27.12	34020	0
MSFT	11:51	B	27.13	39100	0
MSFT	11:51	B	27.14	35201	1800
MSFT	11:51	B	27.15	6920	37400
MSFT	11:51	S	27.16	44433	0
MSFT	11:51	S	27.17	41940	100
MSFT	11:51	S	27.18	41395	100
MSFT	11:51	S	27.19	34350	100
MSFT	11:51	S	27.20	41199	5000

The Model

The Model

- What is the optimal display size of an iceberg order?

- Model assumptions:

    - Order placed at a single price level (top of book or in spread)

    - Select display size to maximize excepted execution volume

    - Visible orders have a market impact

- Model parameters:

    - Visible and hidden liquidty with higher priority

    - Order (market and limit) arrival rates
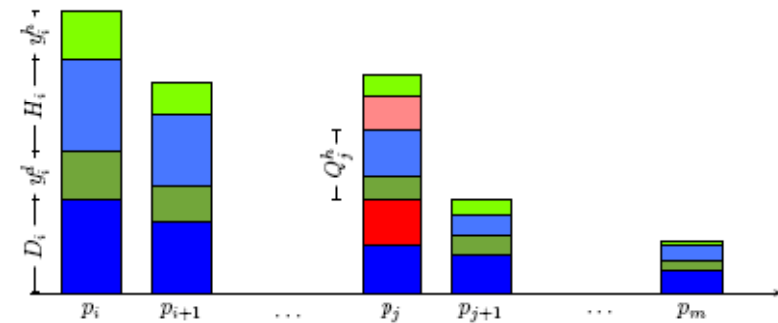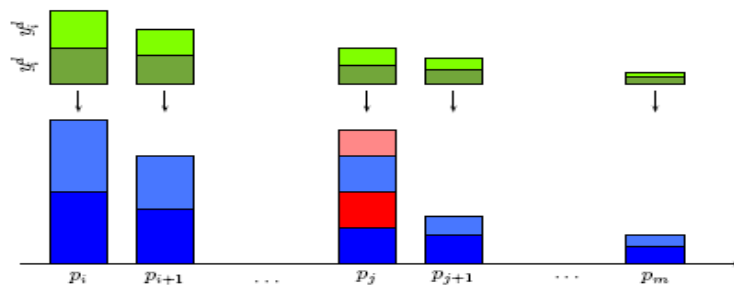
    - Market impact parameter

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The Model
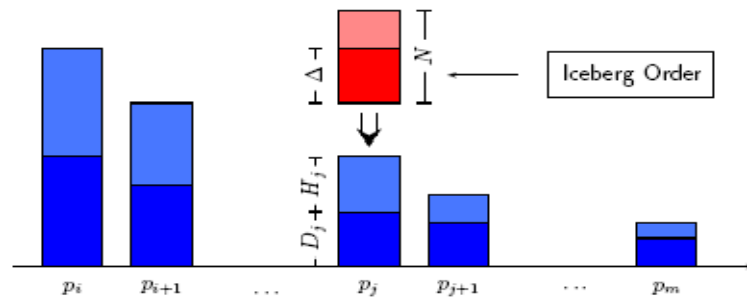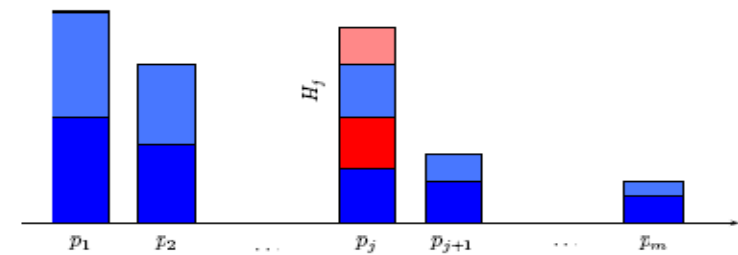
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Order Arrival Rates

- **Aggregate (sell) volume** at more competitive prices follows an exponential distribution with parameter

$$\beta_j(\Delta)$$

where Δ is the display size.

- Volume at the submission price level is exponentially distributed

- Aggregate market order (MO) volume is exponentially distributed

- Both distributions may depend on Δ (market impact) and the market imbalance.

Discrete Execution Profile of Iceberg Orders

- Visible part of the iceberg is executed if the MO exceeds sum of

  - Displayed liquidity with higher priority at t=0

  - Visible liquidity with higher priority arriving at t=1

- Entire iceberg is executed if MO exceeds all of above plus

  - Hidden liquidity at t=0

- The expected execution size as a function of the display size is

  - Linear for low market impact

  - Bell-shaped for medium market impact

  - Strictly decreasing for strong market impact

The Optimal Display Size

- Expected execution volume can be given in closed form

- Optimal display size:

  - Full display if the order size is small enough

  - Display size increases with the expected amount of hidden liquidity

  - Display size decreases with market impact

- To calibrate the model, we need an estimate of

  - Hidden liquidity (as a function of …)

 - Net order arrival rates (as a function of volume imbalance, spread)

Statistical Properties of Hidden Liquidity

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Statistical Properties of Hidden Liquidity

- Our empirical analysis was focussed on the following questions:

  - How much is hidden?

  - How much HL is in the spread/on top of the book?

  - Which macroscopic variables correlate with the HL volume in the spread/on top of the book?

- Can we estimate the HL volume in the spread/on top of the book?

- Hidden liquidity distribution in the spread

How Much is Hidden?

Hidden Liquidity

Mean: 0.1694
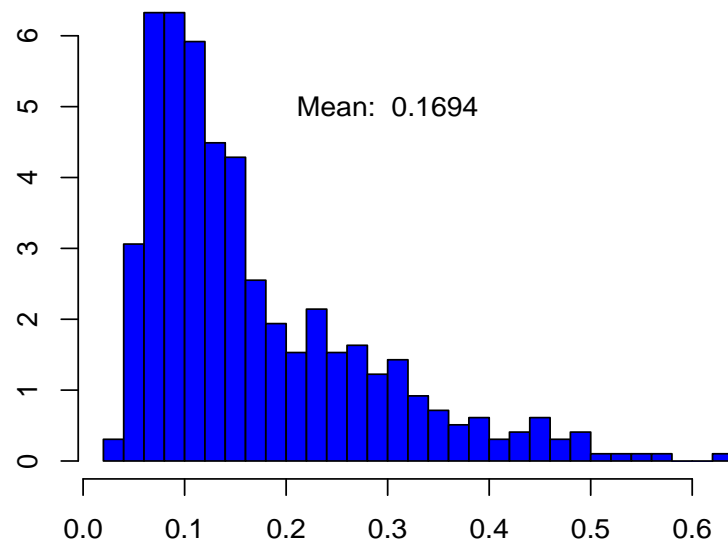
Average Ratio of Posted HL

Hidden Liquidity at Top of the Book

Mean: 0.1599

Average Ratio of Posted HL at Top

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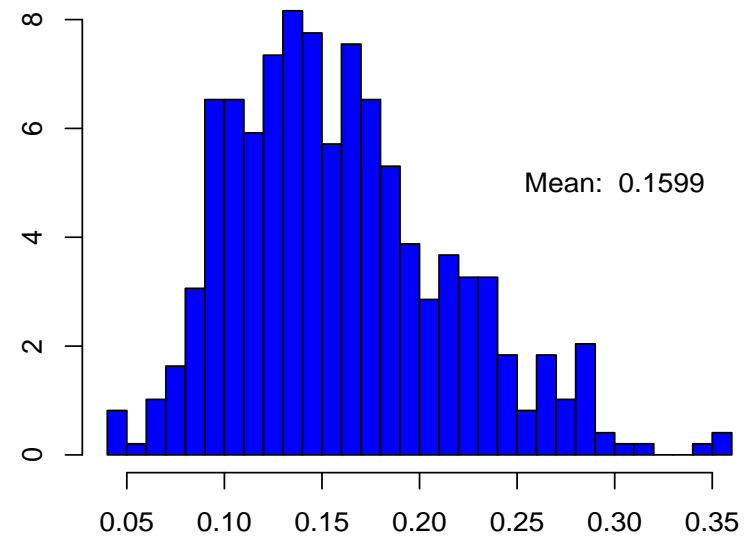
Probability of Finding an Empty Spread

Probability of Finding an Empty Spread

Mean: 0.715
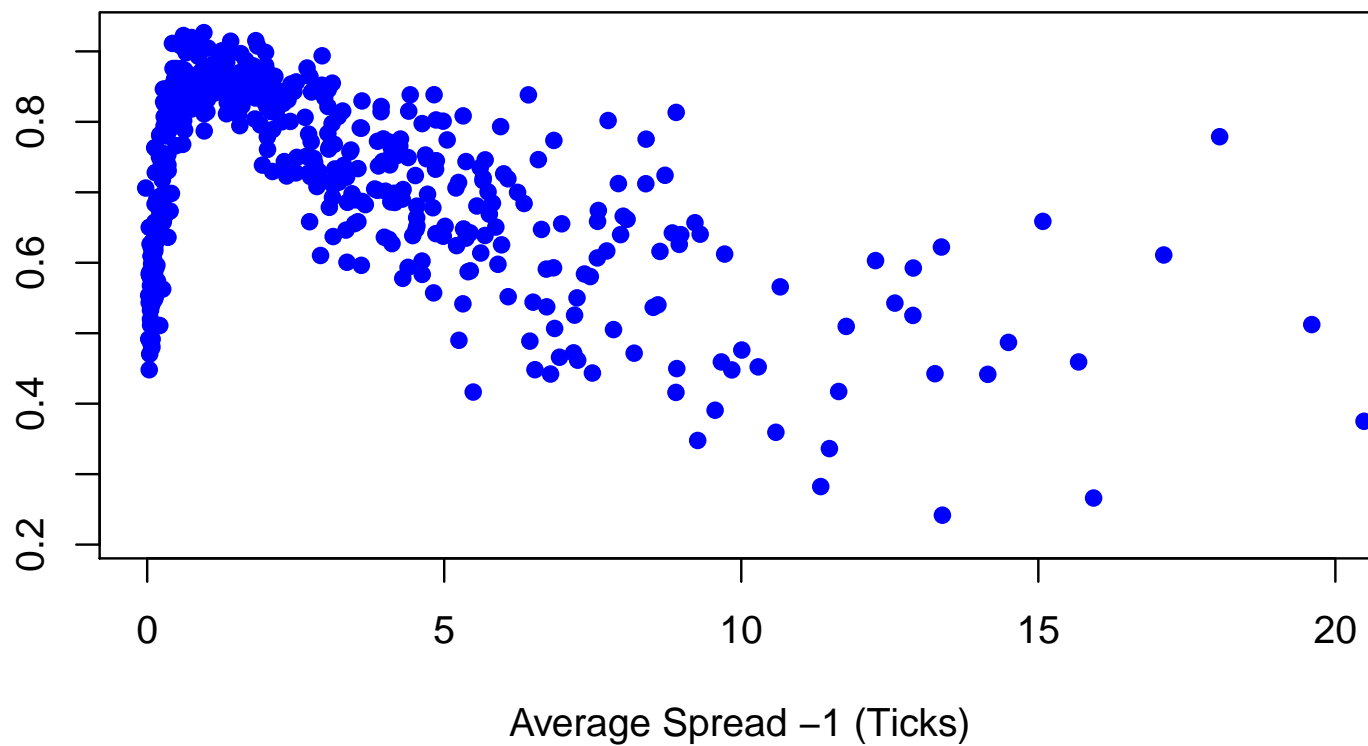
Probability to find empty Spread
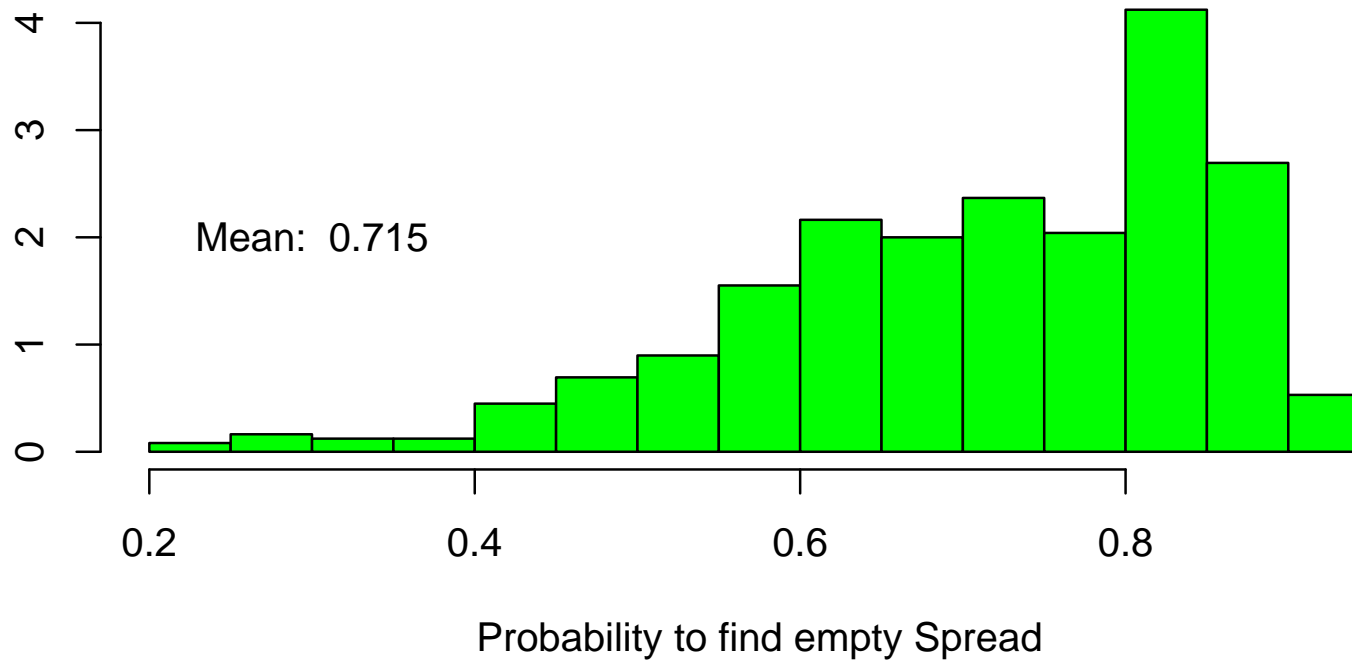
Hidden Liquidity in the Spread

- Averaged over the S&P 500 the spread is empty 70% of time

- Small spreads (<1 tick, MSFT, EBAY, DELL): good chance of HL in spread

- Medium spreads (1- 5 ticks, e.g. AMZN): empty > 70% of time

- Larger spreads (5+ ticks, e.g. GOOG): good chance of HL in spread

- What variables correlate with the HL volume in the spread?

HL Volume\Ratio in the Spread

Hidden Liquidity in the Spread

- Correlation of hidden liquidity volume with:

    - Average Spread: 0.474

    - Average Price: 0.404

    - 1/(Average Order Size):   0.374

    - Average Daily Trading Volume (ADV):   0.223

    - Average Trade Size: 0.084

- HL volume not well explained by either quantity ($R^2 < 0.4$)

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Hidden Liquidity Ratio in the Spread

- Correlation of hidden liquidity ratio with:

    - Average Spread: 0.859

    - Average Price: 0.755

    - 1/(Average Order Size):   NA

    - Average Daily Trading Volume (ADV):   -0.212

    - Average Trade Size: -0.322

- HL ratio well explained by average spread.

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Hidden Liquidity Ratio in the Spread

- HL Ratio in the spread is well explained by the spread (R^2 > 0.7):

$$H_{Ratio} = -0.04 + 0.09\sqrt{Spread}$$

- For stocks with average spreads between 5 and 10:

    - Linear relation between spread and HL ratio

    - Slope is well estimated using ADV

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Hidden Liquidity Ratio in the Spread

Average Spread−1 (Ticks)

Hidden Liquidity Ratio in the Spread

GR

R^2: 0.982

Avg. Spread: 5.05

Relative Spread

CLX

R^2: 0.963

Avg. Spread: 6

Relative Spread

GD

R^2: 0.988

Avg. Spread: 5.2

Relative Spread

AVB

R^2: 0.961

Avg. Spread: 13.26

Relative Spread

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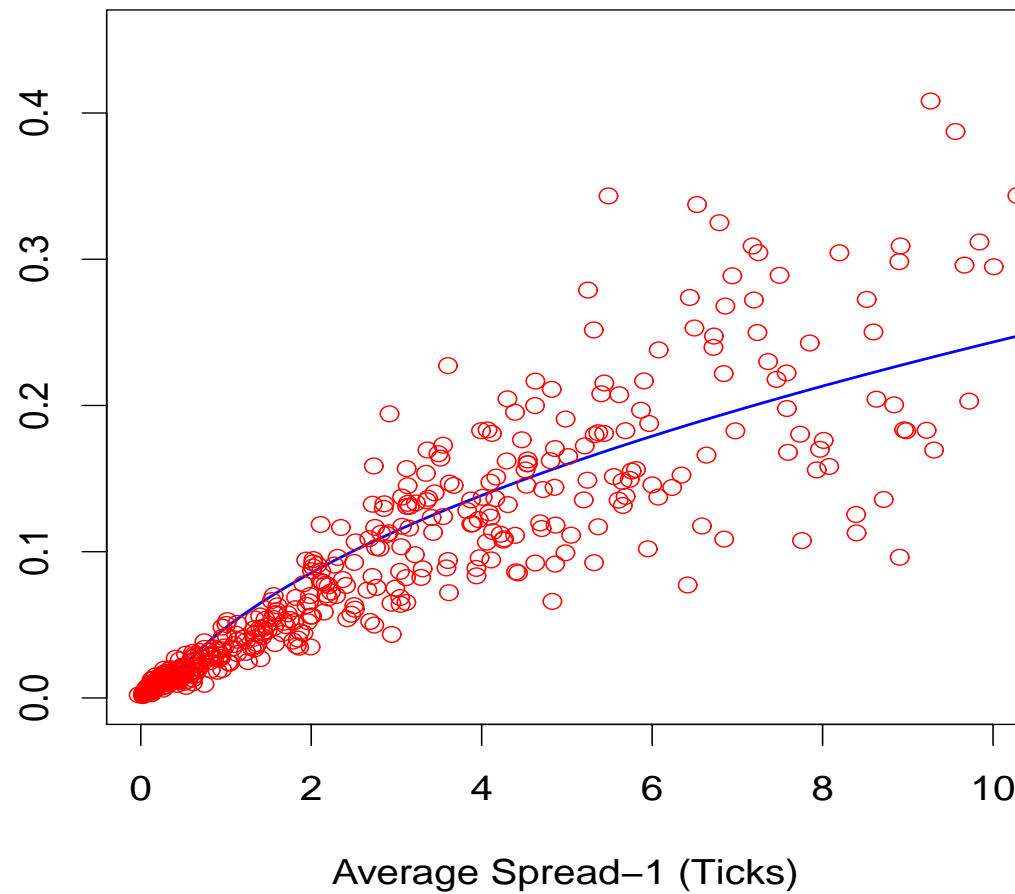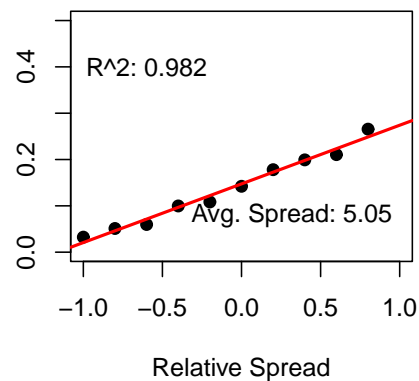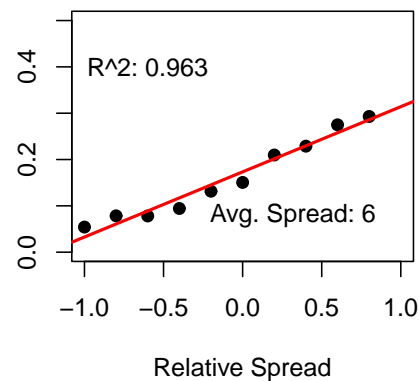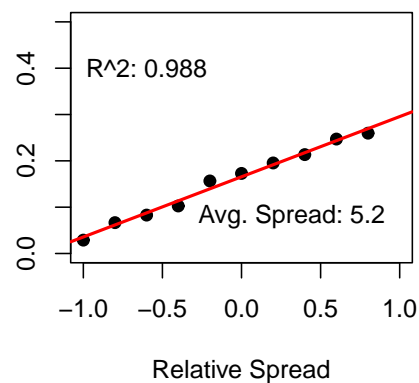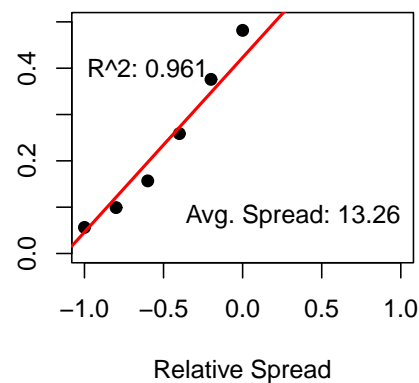
Hidden Liquidity Ratio in the Spread: S&P 500

Relative Spread
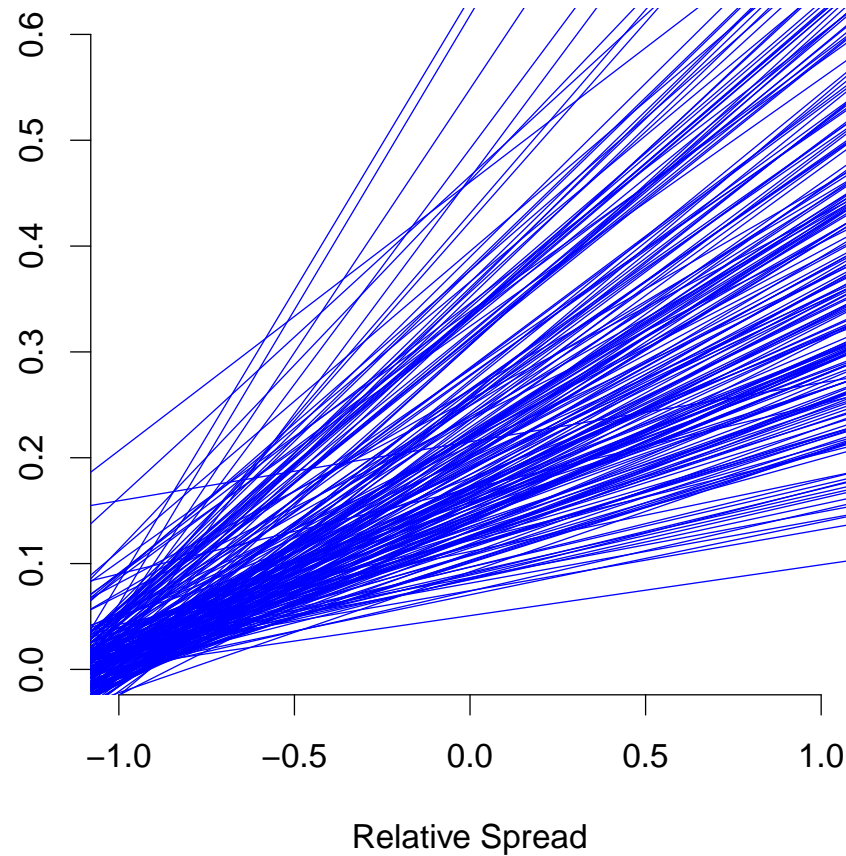
HL Distribution in the Spread
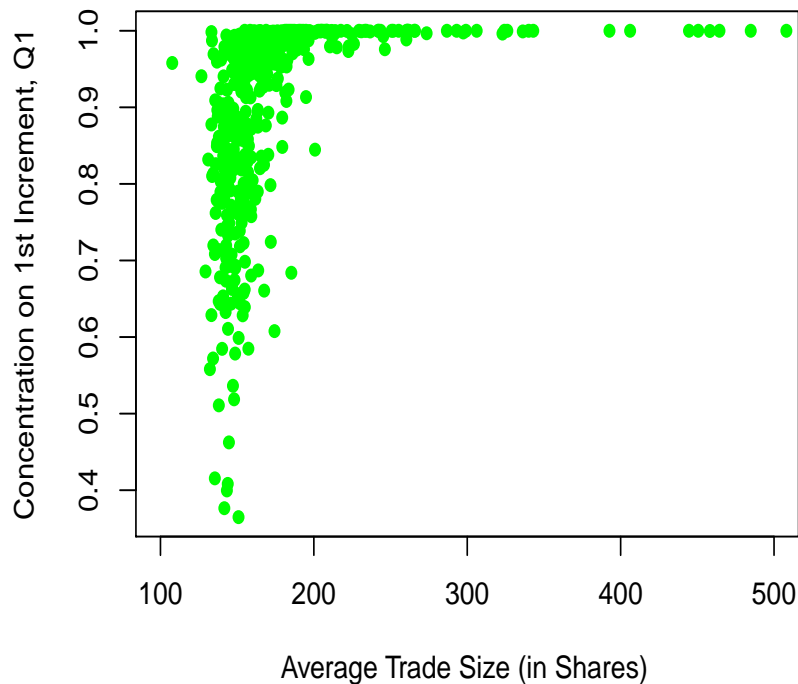
Hidden Liquidity Concentration in the Spread
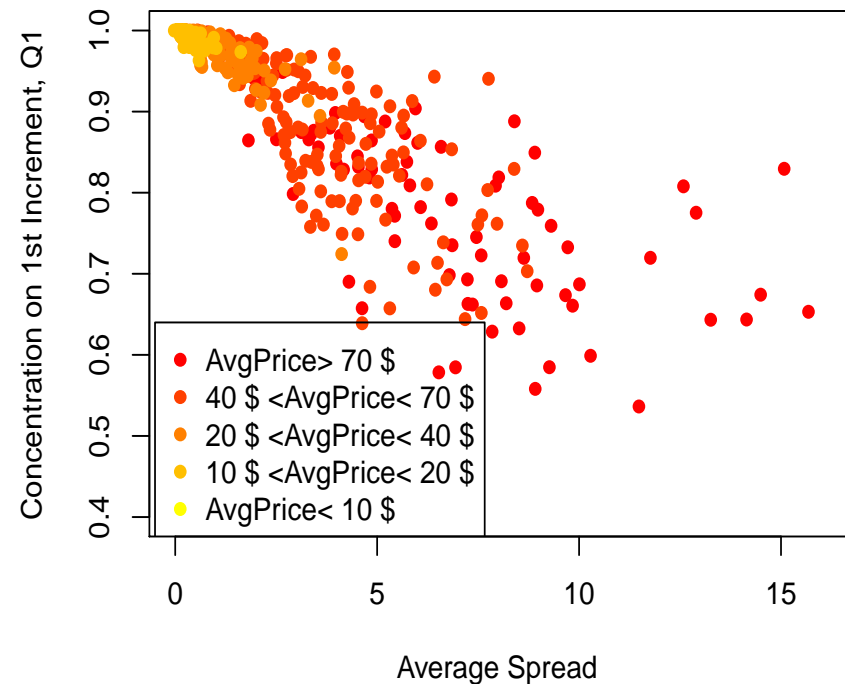
# Hidden Liquidity Concentration in the Spread

- Spread-liquidity mainly concentrated on the first best increment

- Take at „First-Increment-in-Spread" – not at Midpoint

- Exception: Google with 34% of liquidity on the first increment

  - Spread/price ratio exceptionally high

  - Stepping deeper into the spread is cheap

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Hidden Liquidity in the Spread (AMZN)

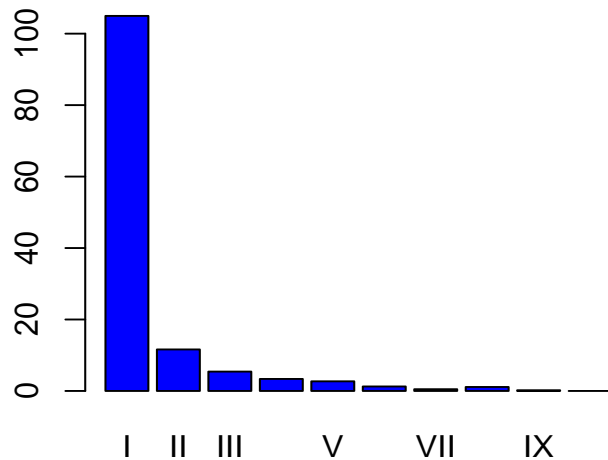**Stock: AMZN ,   Avg. Spread: 2.92 Ticks**   **ADV: 11.74 (M Shares), PR: 44.52 $**
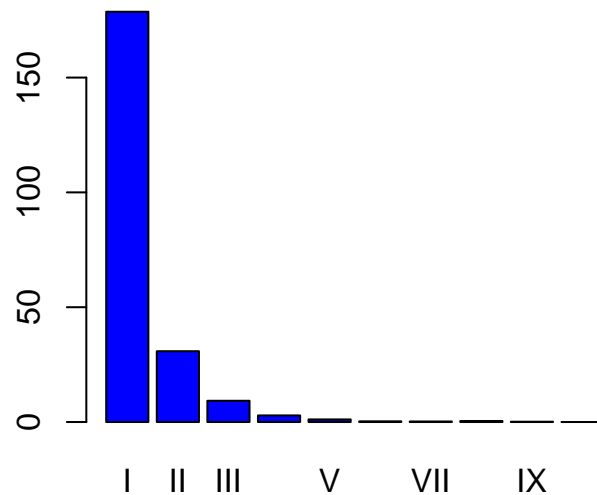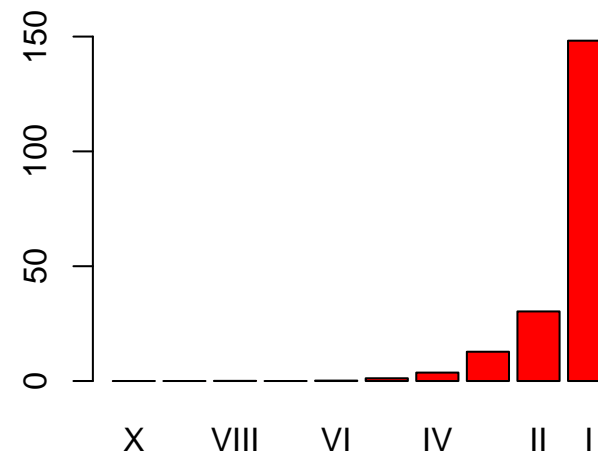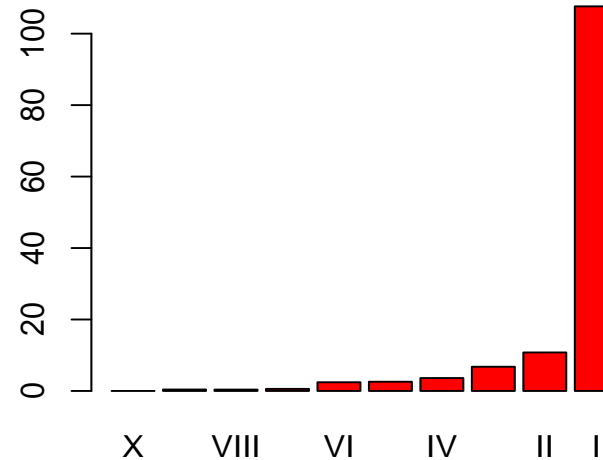
Hidden Liquidity in the Spread (IBM)

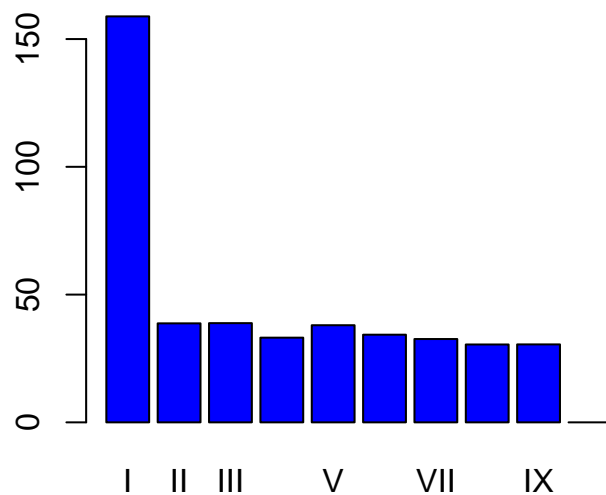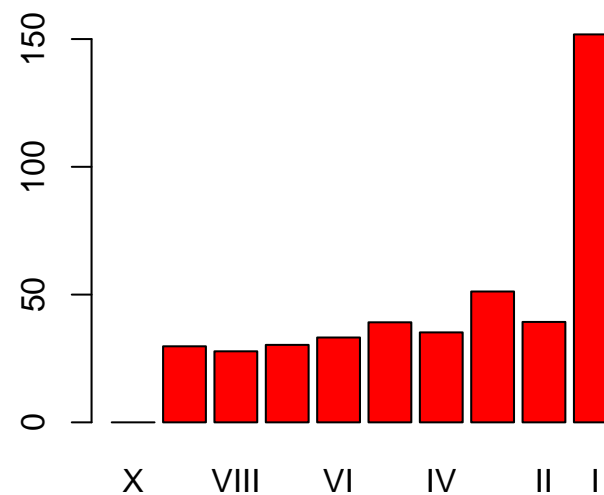**Stock: IBM ,   Avg. Spread: 5.69 Ticks**   **ADV: 10.51 (M Shares), PR: 82.33 $**

Hidden Liquidity in the Spread (GOOG)

Stock: GOOG ,   Avg. Spread: 30.15 Ticks

ADV: 8.19 (M Shares), PR: 305.46 $

Hidden Liquidity in the Spread/on Top-Of-The Book

- Amount of HL in the spread not well explained by macroscopic variables

- HL ratio well estimated by average spread (plus ADV)

- Hidden liquidity at the top of the book correlates strongly with

  - average trade size

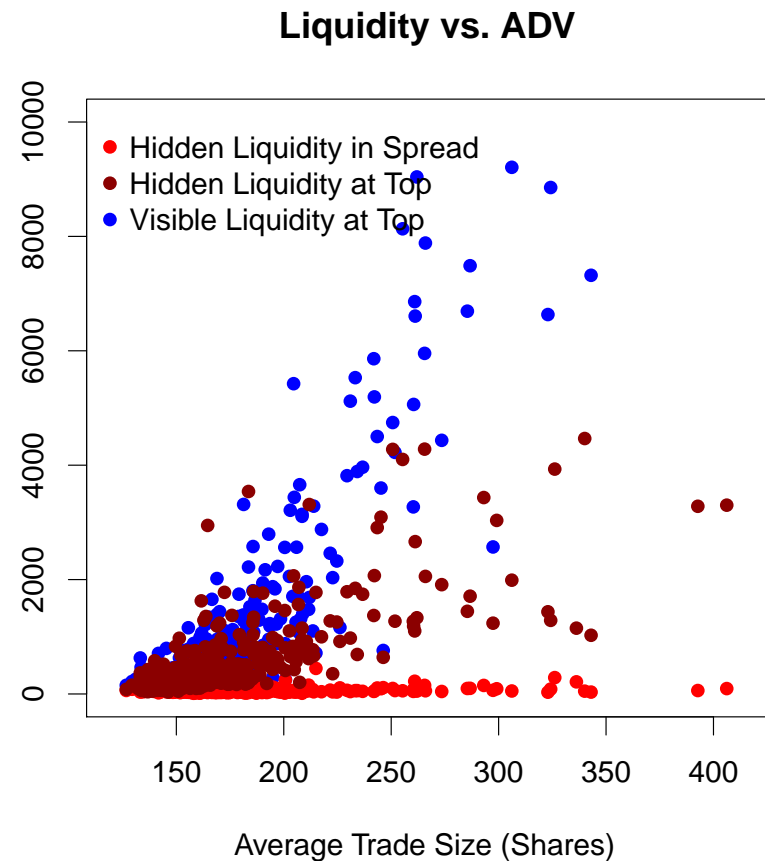 - average top-of-the book volume

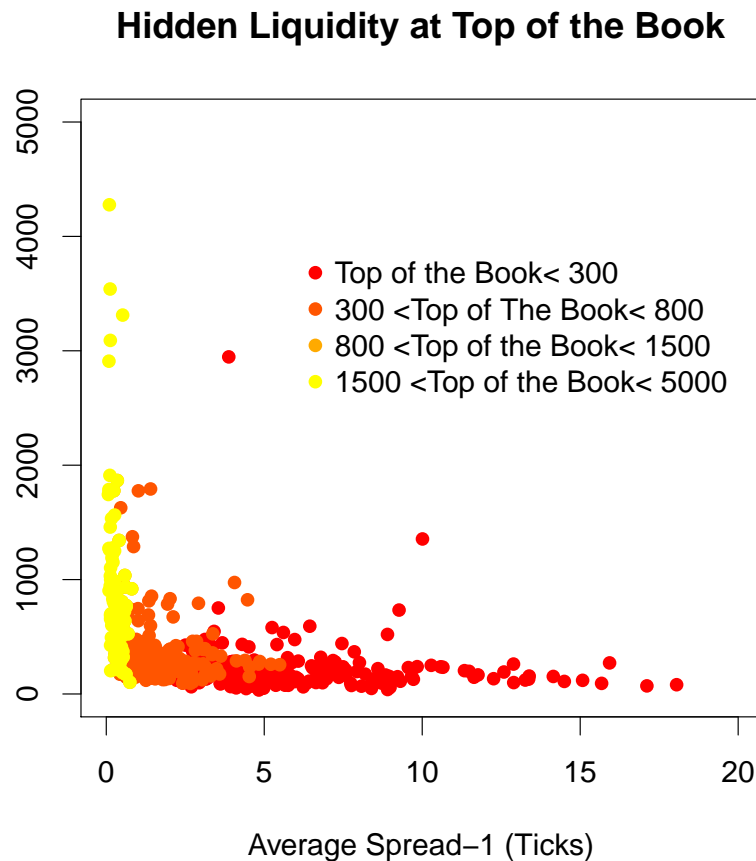 - inverse average price and spread

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HL at Top of the Book

Hidden Liquidity at Top of the Book

- Correlation of hidden liquidity volume at TOB with:

  - Average Trade Size: 0.783

  - Average (Visible) Volume at TOB :   0.691

  - 1/(Average Price):   0.677

  - 1/(Average Spread):   0.669

  - ADV: 0.508

- HL volume at TOB highly correlated with …

- Linear relation for HL ratio as before.

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Hidden Liquidity at Top of the Book

Hidden Liquidity at Top of the Book

Liquidity vs. ADV

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Back to the Model

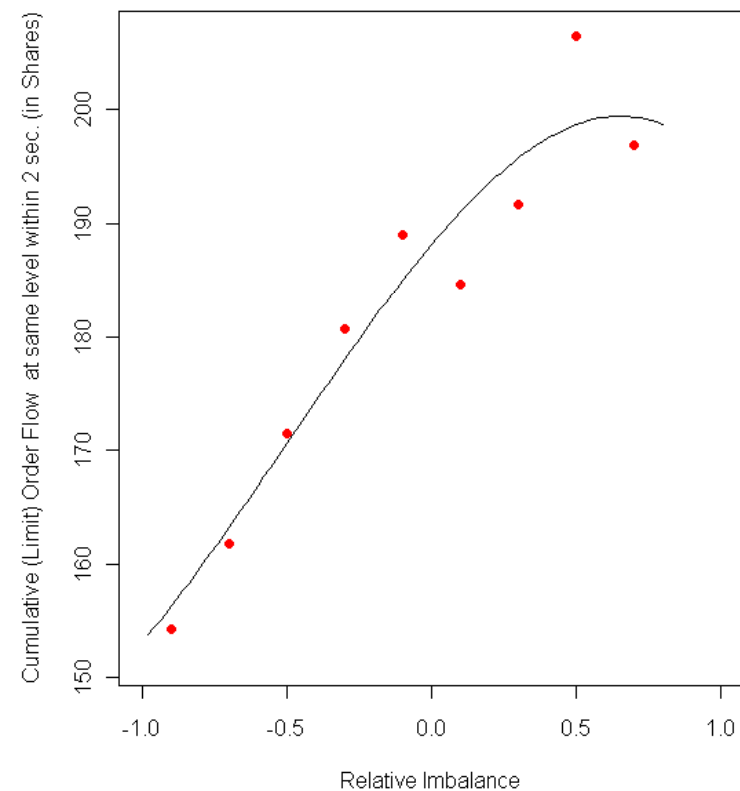# Calibration Results for Apple: Parameter Setting

- Calibration period: Jan 01, 2008 – Jan 15, 2008

- Submitted order size: 2000 Shares

- Submission price level: best bid/ask

- Trading horizon: 2 seconds

- Top of the book size: 300 shares

- We calibrated the order arrival rates as a function of market imbalance.

Calibration Results for Apple: Order Arrivales

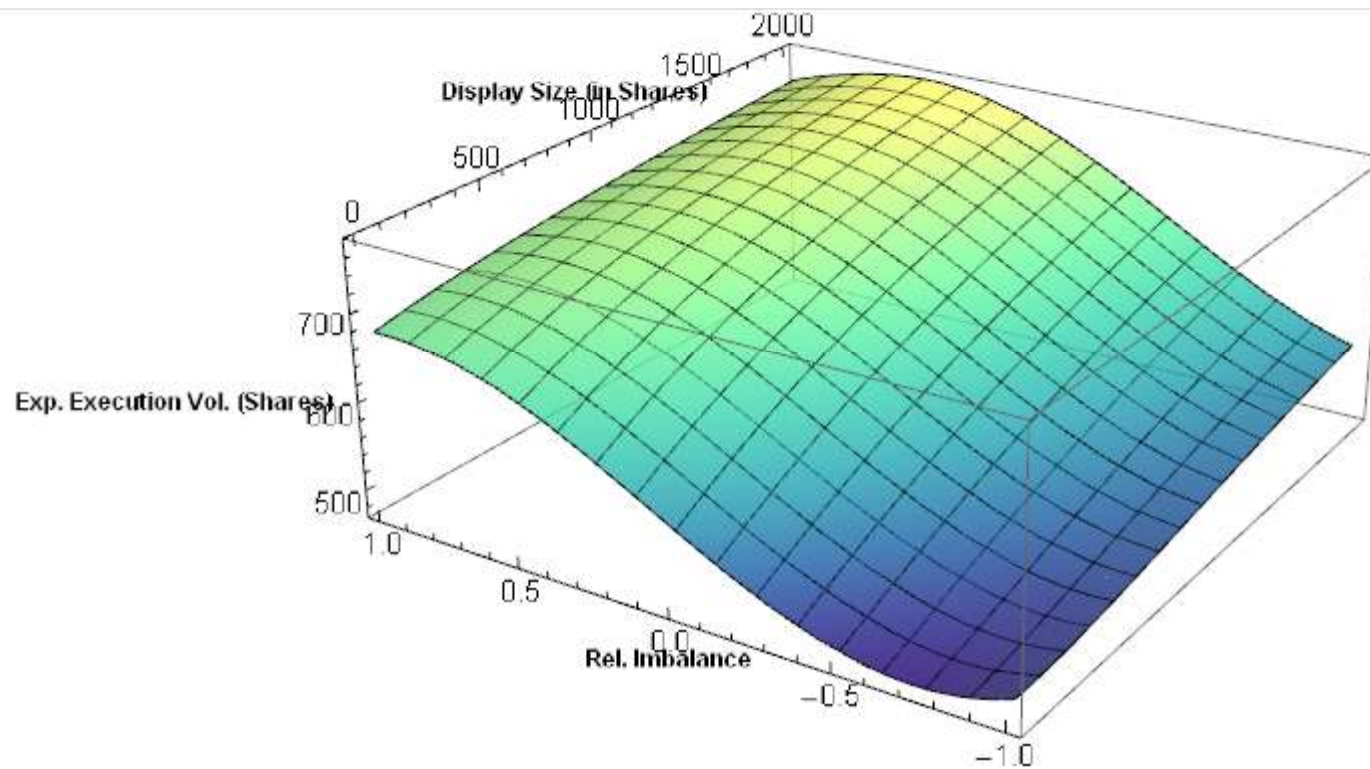
Market Impact I (Market Order Flow)
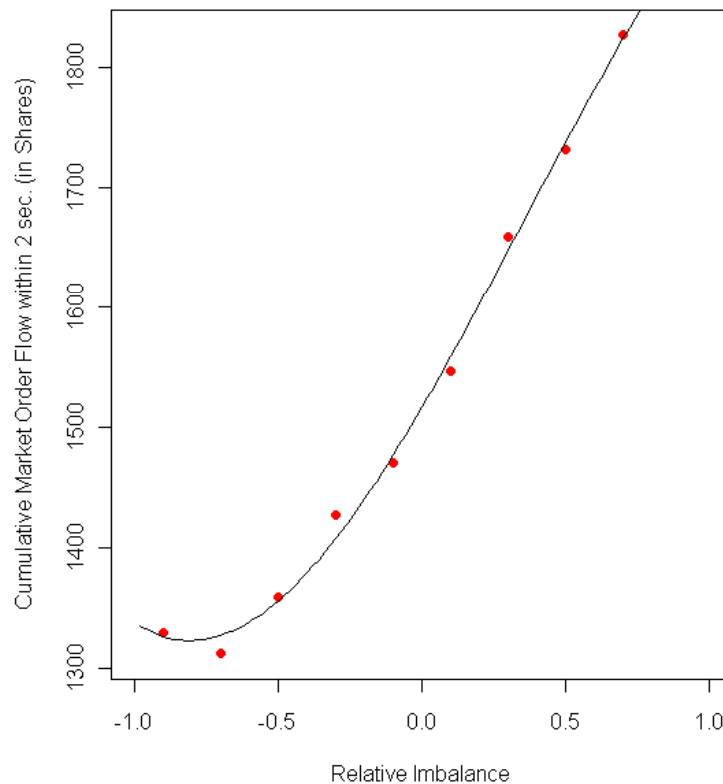
Market Impact II (Limit Order Flow at Price Level)

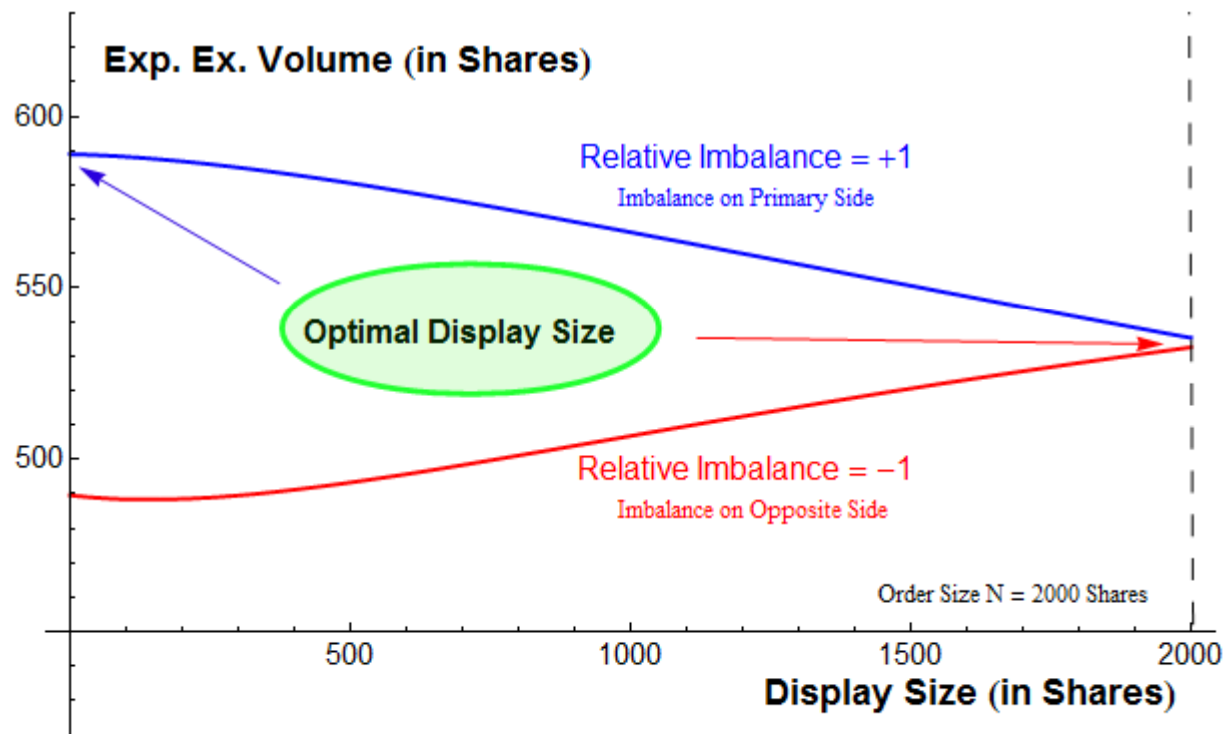# Calibration Results for Apple: What to Expect

- „Benchmark price" for liquidity providers: spread to midpoint

- Small spread: full display

- Medium \Large spreads:

  - More liquidity on the opposite side: display

  - Much more liquidity on my side: hide

- Similar effects, but amplified by spread

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Execution Profile for Narrow Spreads (< 2 Cents)

Display Order

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Execution Profile for Large Spreads (3-12 Cents)

Execution Profile

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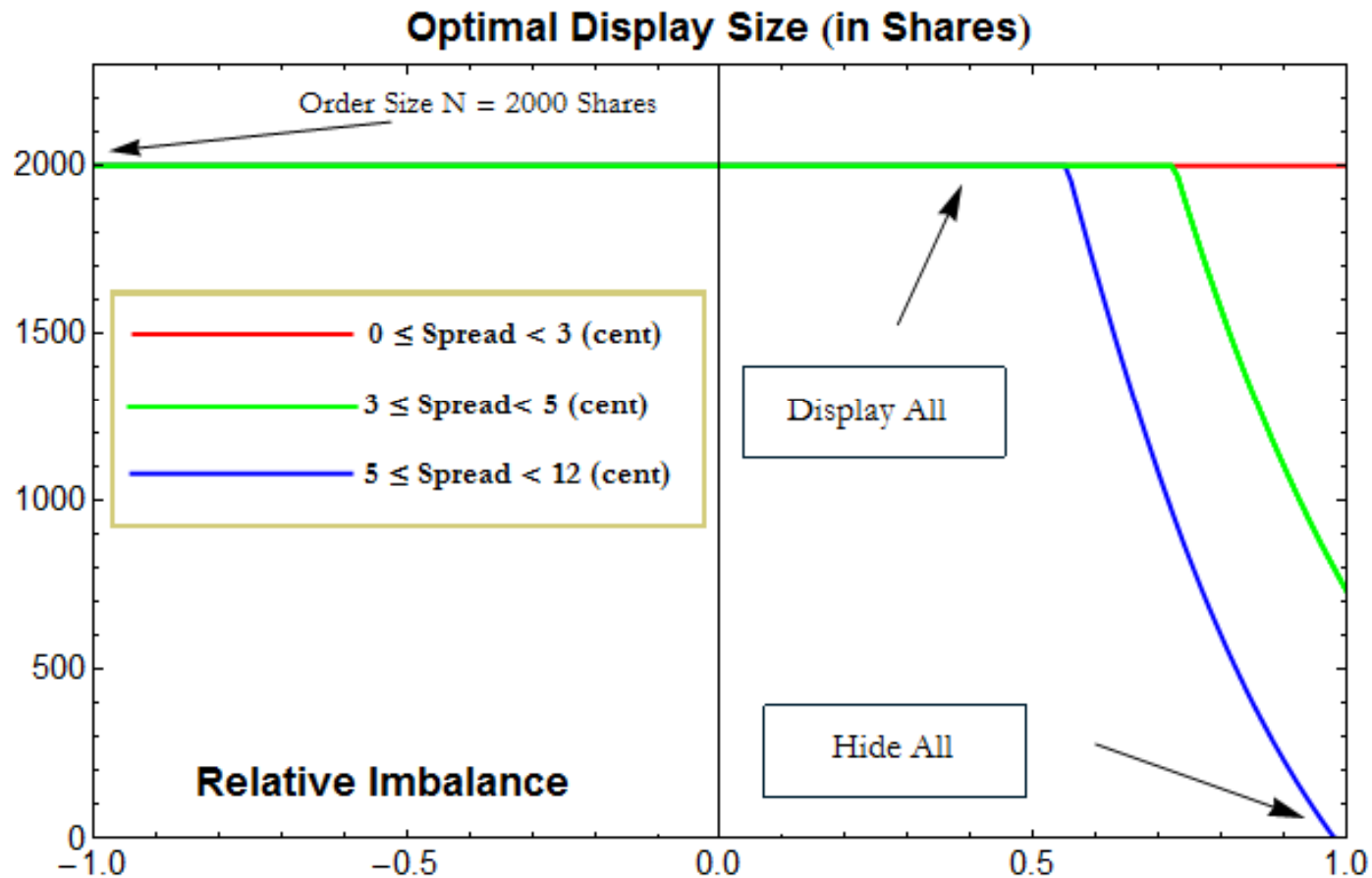
Conclusion

- Simple model for the optimal display of icebergs

- HLRatio well explained the ADV and VL at the top of the book

- Calibration for Apple:

  - Small spread: full display

  - Large Spread: dependence on market imbalance

- Future research topics:

  - Dynamic models

  - High frequency data

  - …

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