G10 Swap and Exchange Rates

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http://www.tc.umn.edu/∼jeremy/research/working_papers/FieldsQuantSeminar.pdf

Economic Motivation

- We develop a no-arbitrage model for swap rates and returns in multiple currencies (applied to G10 countries) that allows us to address many interesting questions
    - Does information in global swap markets improve our ability to predict changes in each country's yield curve?
    - How many priced risk factors are there in global swap markets?
    - What is the risk/reward tradeoff, or maximum Sharpe ratio, available to global fixed income investors?

Outline

- Brief review of single currency affine term structure models
  - Fit to cross-section of yields
- Pricing theory with multiple currencies
  - Extension to multi-currency term structure models
- **Parsimonious specification of risk premia in a multi-currency setting**
- Empirical analysis both in- and out-of-sample

The Fundamental Theorem of Pricing

- No arbitrage implies a unique (strictly positive) minimum variance pricing kernel M that prices all payoffs

$$P_t = \mathbb{E}_t\left[\frac{M_T}{M_t} P_T\right] \quad \text{or} \quad P_t = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_u du} P_T\right]$$

  - P_t is the price at t of any future payoff P_T
  - r_u is the short (instantaneous) risk-free rate

- Term structure models take a reduced form approach
  - What pricing kernel, M, is consistent with (international) bond prices and returns?

Risk-Neutral Pricing Measure

- A risk-neutral pricing measure $\mathbb{Q}$ is often used for computing prices/expectations
  - For notational convenience, define r and Z as follows

$$r_t := -\mathbb{E}_t\left[dM_t\right]/M_t dt \quad \text{and} \quad Z_t := e^{\int_0^t r_u \, du}$$

  - Then $Z_t M_t$ is a local martingale (random walk), since

$$\begin{aligned}
d\left(ZM\right)_t &= Z_t M_t \, r_t \, dt + Z_t \, dM_t \\
&= Z_t M_t \left(\, r_t dt + \underbrace{\mathbb{E}_t\left[dM_t\right]/M_t}_{-r_t \, dt} \, \right) + Z_t \left(dM_t - \mathbb{E}_t\left[dM_t\right]\right) \\
&= Z_t M_t \left(d\log M_t - \mathbb{E}_t\left[d\log M_t\right]\right) \\
&\Downarrow \\
\mathbb{E}_t\left[d\left(ZM\right)_t\right] &= 0
\end{aligned}$$

Risk-Neutral Pricing Measure (cont'd)

- With some technical conditions, Girsanov's theorem can be used to define a risk-neutral pricing measure $\mathbb{Q}$ by

$$\left.\frac{d\mathbb{Q}}{d\mathbb{P}}\right|_{\mathcal{F}_t} = (ZM)_t$$

- $\mathbb{Q}$ has the following properties for any process X

$$\mathbb{E}_t^{\mathbb{Q}}[X_T] := \frac{1}{(ZM)_t}\mathbb{E}_t\left[(ZM)_T X_T\right] \quad \text{and} \quad \mathbb{E}_t^{\mathbb{Q}}[dX_t] = \mathbb{E}_t[dX_t] + d\langle X, \log M\rangle_t$$

- In particular, note that

$$\mathbb{E}_t\left[\frac{M_T}{M_t}P_T\right] = \mathbb{E}_t^{\mathbb{Q}}\left[\frac{Z_t}{Z_T}P_T\right] = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_u\,du}P_T\right]$$

Single Currency Affine Yield Models

- Everything follows a continuous diffusion with affine drift [expected change] and quadratic variation ⟨covariance⟩

$$
\begin{aligned}
\overbrace{-\mathbb{E}_t\left[d\log M_t\right]/dt - \frac{1}{2} d\left\langle \log M, \log M\right\rangle_t/dt}^{r_t \;:=\; -\mathbb{E}_t[dM_t]/M_t dt} &= \rho_0 + \rho_1 \cdot X_t \\
d\left\langle X, X^\top\right\rangle_t/dt &= H_0 + \sum H_i X_i(t) \\
\underbrace{\mathbb{E}_t\left[dX_t\right]/dt + d\left\langle X, \log M\right\rangle_t/dt}_{\mathbb{E}_t^{\mathbb{Q}}[dX_t]/dt} &= \theta + \mathcal{K} X_t
\end{aligned}
$$

- This is often written as

$$
\begin{aligned}
r_t &= \rho_0 + \rho_1 \cdot X_t \\
dX_t &= \left[\theta + \mathcal{K} X_t\right] dt + \Sigma_t \, dW_t^{\mathbb{Q}} \\
\Sigma_t \Sigma_t^\top &= H_0 + \sum H_i X_i(t)
\end{aligned}
$$

Affine Yields

- Zero coupon yields are affine since

$$
\begin{aligned}
P_t(\tau) &= \mathbb{E}_t\left[\frac{M_{t+\tau}}{M_t}\cdot 1\right] = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{t+\tau} r_u du}\right] = e^{-A(\tau)-B(\tau)\cdot X_t} =: e^{-Y_t(\tau)\cdot\tau} \\
B(\tau) &= \int_0^{\tau}\left[\rho_1 - \mathcal{K}^{\top}B(u) - \frac{1}{2}\left\{\sum_{i=1}^{m} H_i B_i(u)\right\}B(u)\right]du \\
A(\tau) &= \int_0^{\tau}\left[\rho_0 - \theta\cdot B(u) - \frac{1}{2}B(u)^{\top} H_0 B(u)\right]du
\end{aligned}
$$

  - ODEs for $A(\cdot)$ and $B(\cdot)$ can be solved numerically
  - Known in closed form for special cases such as Gaussian

The Factors X

- There can be many valid pricing kernels, e.g. U', but the minimum variance pricing kernel is the unique projection onto the space of assets being priced

$$M_t = \mathbb{E}\left[U'_t \middle| \sigma\left\{\cup P_t\right\}\right] \quad \Rightarrow \quad M_t = f\left(Y_t\right) \quad \Rightarrow \quad X_t = g\left(Y_t\right)$$

- We choose X_t to be first 2 or 3 principal components of yields

$$X_t = L_0 + L_1 Y_t$$

  - 2 or 3 PCs typically explain most ($\sim$ 97% of) cross-sectional variation in yields

Estimating Pricing Parameters

- Covariance parameters of $d\left\langle X, X^\top \right\rangle_t / dt$ are relatively easy to estimate from time series
- If X is N-dimensional then no-arbitrage imposes $N \times (N+1)$ restrictions on ρ_0, ρ_1, θ, and $\mathcal{K}$

$$\left.\begin{aligned} X_t &= L_0 + L_1 Y_t \\ Y_t &= A_{(\rho_0,\rho_1,\theta,\mathcal{K})} + B_{(\rho_0,\rho_1,\theta,\mathcal{K})} X_t \end{aligned}\right\} \Rightarrow \quad \begin{aligned} L_0 + L_1 A &= 0 \\ L_1 B &= \mathcal{I} \end{aligned}$$

  - Only $N + 1 \approx 4$ remaining free parameters

- **Pricing parameters are easy to estimate**

Cross-Section Empirical Estimates

- Gaussian version of the model (i.e. only $H_0 \neq 0$) using weekly data for G10 countries (USD, GBP, JPY, AUD, EUR, CAD, CHF, NZD, SEK, NOK)
  - DEM used before EUR
- Zero-coupon yields bootstrapped from Libor and swap rates – 3M, 6M, 2Y, 3Y, 5Y, 7Y, 10Y
  - 2 factor NZD - initially missing 6M, 7Y, and 10Y
  - 2 factor NOK - initially missing 7Y and 10Y
- All data is from Bloomberg
  - In-sample estimates 1993-Jan-06 to 2001-Dec-26
  - Out-of-sample testing 2002-Jan-02 to 2009-Mar-28

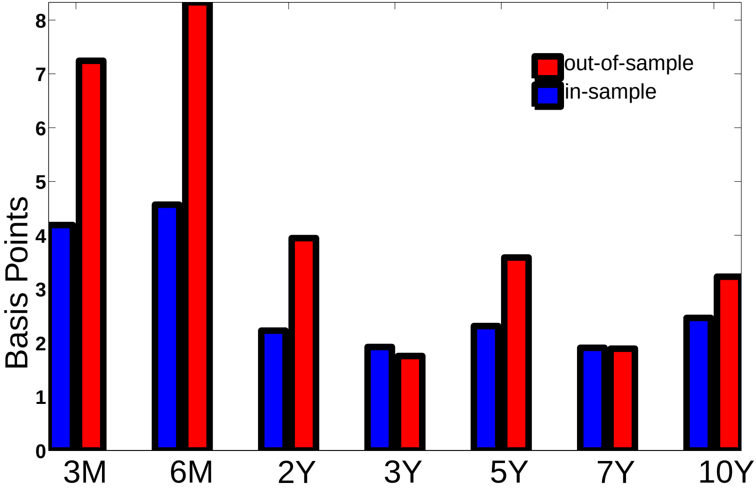
Cross-Sectional Fit to Yields

- Fit to the cross-section of yields is very good
  - Root mean squared errors

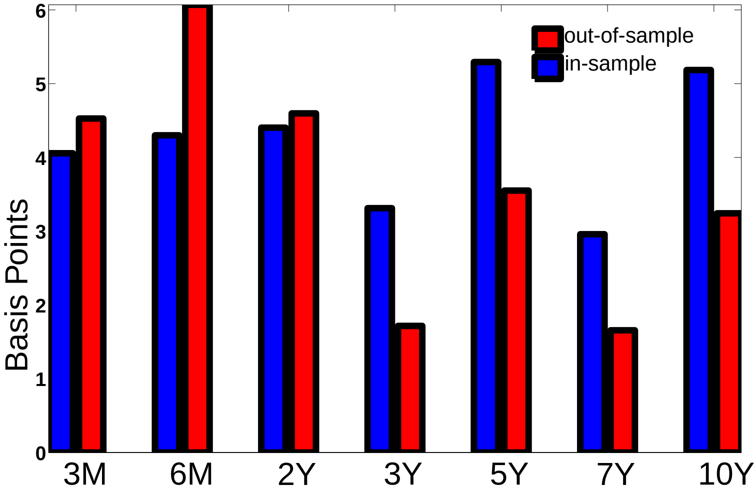
$$\sqrt{\frac{1}{N}\sum_{n=1}^{N}\left[\,Y_{t_n}(\tau)-\underbrace{\left(A(\tau)+B(\tau)X_{t_n}\right)/\tau}_{\text{model yield}}\,\right]^2}\approx 5\text{ bps}$$

- Intuitively, affine models do a good job of describing cross-section of yields given contemporaneous values of “level”, “slope”, and “curvature”

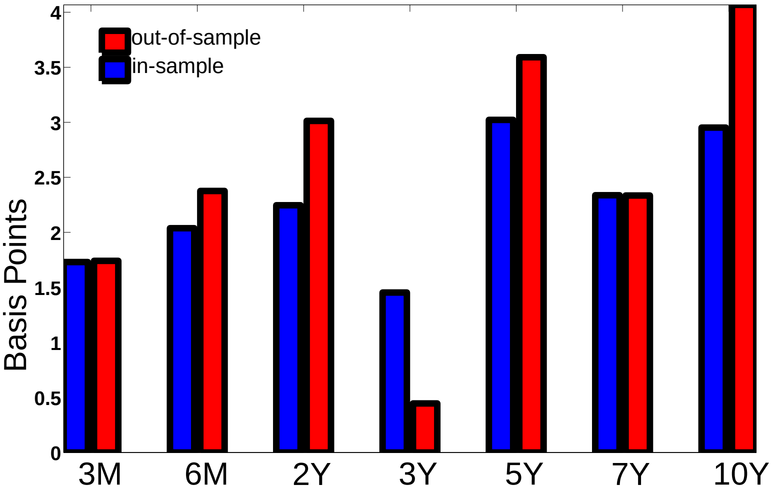
Root Mean Squared Errors - USD



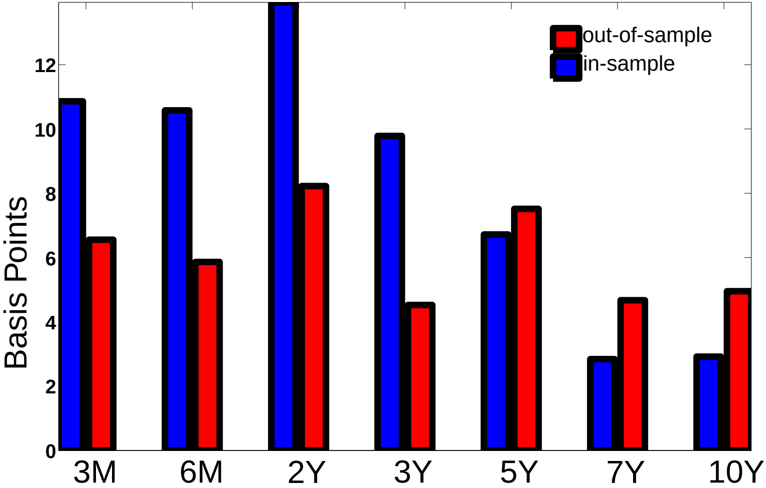
Root Mean Squared Errors - GBP



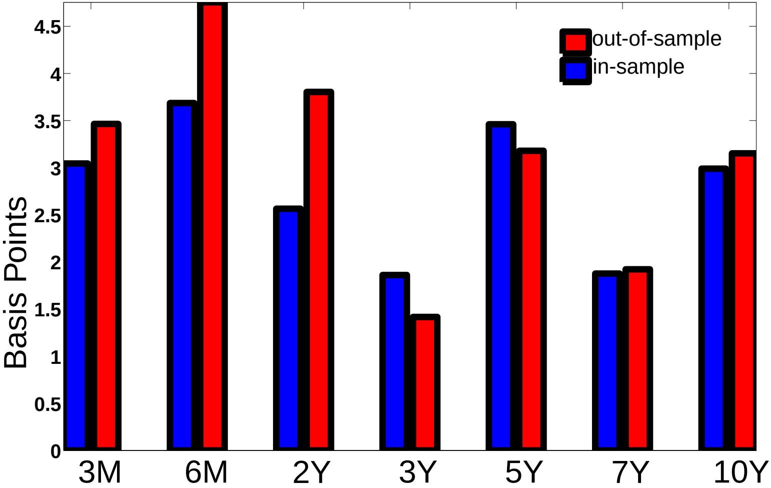
Root Mean Squared Errors - JPY



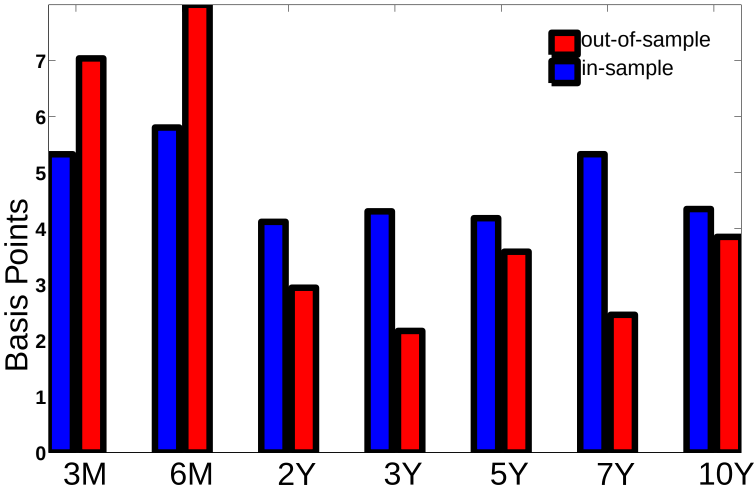
Root Mean Squared Errors - AUD



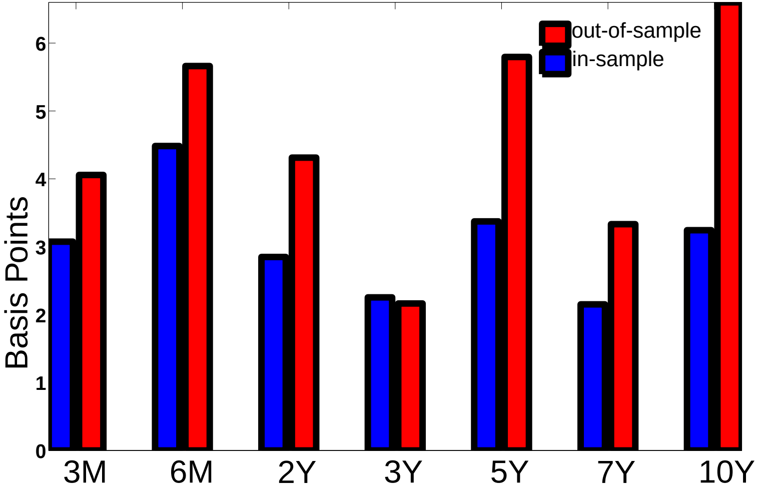
Root Mean Squared Errors - EUR



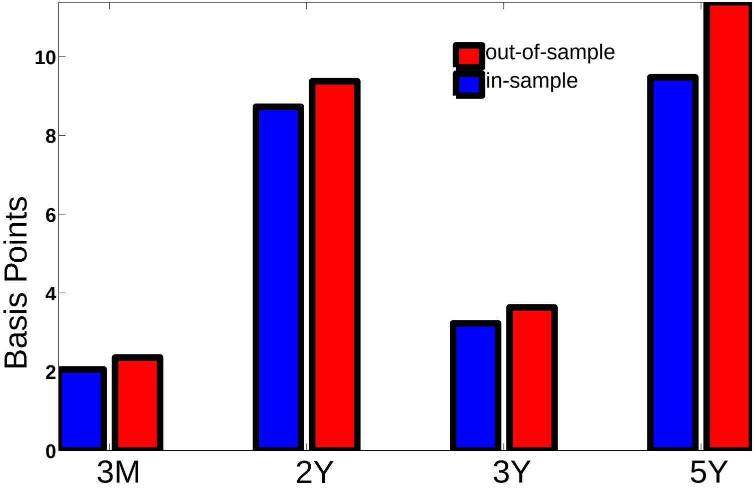
Root Mean Squared Errors - CAD



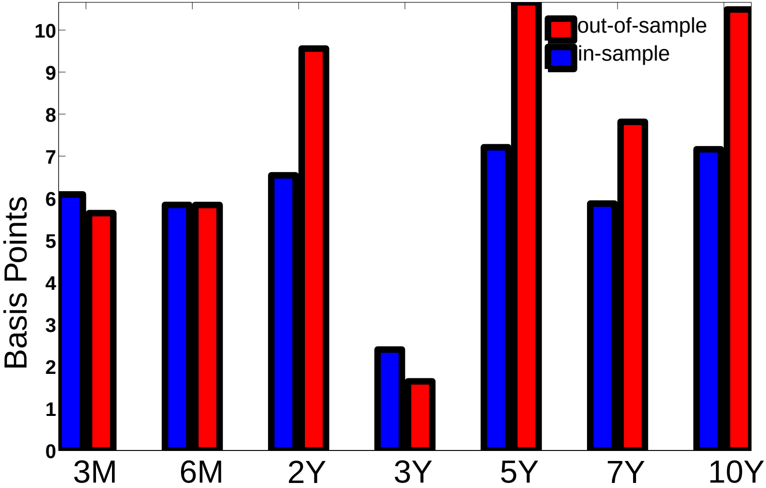
Root Mean Squared Errors - CHF



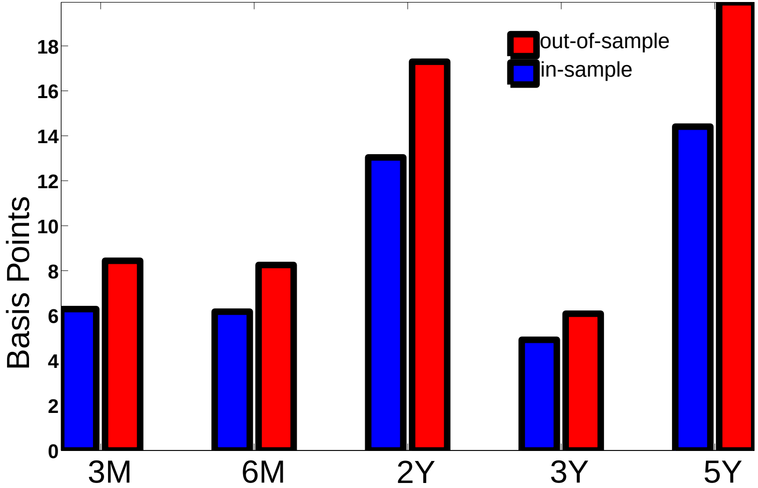
Root Mean Squared Errors - NZD



Root Mean Squared Errors - SEK



Root Mean Squared Errors - NOK



Pricing in an International Setting

- The space of available payoffs can also include assets in foreign currencies ($j = 1, \ldots, J$) when they are exchanged to the domestic currency

$$P_t = \mathbb{E}_t\left[\frac{M_T}{M_t} P_T\right] \quad \textbf{and} \quad S_t^{(j)} P_t^{(j)} = \mathbb{E}_t\left[\frac{M_T}{M_t}\left(S_T^{(j)} P_T^{(j)}\right)\right]$$

  - $P_t^{(j)}$ is the price in currency j of any future payoff $P_T^{(j)}$ in currency j
  - $S^{(j)}$ is the exchange rate for currency j (domestic currency per unit of foreign currency)

Pricing Kernels in Foreign Currencies

- There is a minimum variance pricing kernel $M^{(j)}$ in foreign currency j that also prices any attainable payoff $P_T^{(j)}$

$$\mathbb{E}_t\left[\frac{M_T^{(j)}}{M_t^{(i)}}P_T^{(j)}\right] = P_t^{(j)} = \mathbb{E}_t\left[\frac{M_T S_T^{(j)}}{M_t S_t^{(j)}}P_T^{(j)}\right]$$

- If we choose $P_T^{(j)} = \frac{M_T^{(j)}}{M_t^{(j)}} - \frac{M_T S_T^{(j)}}{M_t S_t^{(j)}}$ then

$$0 = \mathbb{E}_t\left[\left(\frac{M_T^{(j)}}{M_t^{(j)}} - \frac{M_T S_T^{(j)}}{M_t S_t^{(j)}}\right)^2\right]$$

$$\Downarrow$$

$$\frac{M_T^{(j)}}{M_t^{(j)}} = \frac{M_T S_T^{(j)}}{M_t S_t^{(j)}} \quad \text{a.s.}$$

Understanding the Relationship

- Only assumption is that assets in all countries can be traded freely and there are no arbitrage opportunities
  - Swap markets do not need to span foreign exchange markets
  - Empirically, low correlation between exchange rates and interest rates
- If M and $M^{(j)}$ price all of the same assets, but in different currencies, then $M^{(j)} = MS^{(j)}$
- Note that the relationship does not have to hold if the pricing kernels only price assets in the local currency,

$$P_t = \mathbb{E}_t\left[\frac{\tilde{M}_T}{\tilde{M}_t}P_T\right] \quad \textbf{and} \quad P_t^{(j)} = \mathbb{E}_t\left[\frac{\tilde{M}_T^{(j)}}{M_t^{(j)}}P_T^{(j)}\right] \quad \not\Rightarrow \quad \frac{\tilde{M}_T^{(j)}}{\tilde{M}_t^{(j)}} = \frac{\tilde{M}_T S_T^{(j)}}{\tilde{M}_t S_t^{(j)}}$$

Multi-Currency Pricing Specification

- Model the joint dynamics of M and either $M^{(j)}$ or $S^{(j)}$
- Format of model doesn't need to change, simply expand state vector X to include log exchange rates, $\log S$

$$\begin{aligned} r_t &= \rho_0 + \rho_1 \cdot X_t \\ d\left\langle X, X^\top \right\rangle_t / dt &= H_0 + \sum H_i X_i(t) \\ \mathbb{E}_t^{\mathbb{Q}}\left[dX_t\right]/dt &= \theta + \mathcal{K} X_t \end{aligned}$$

Yields in All Currencies are Still Affine

- Yields in all currencies are affine since dynamics are affine under any curreny's risk-neutral measure

$$
\begin{aligned}
r_t^{(j)} &:= -\mathbb{E}_t\left[d\left(MS^{(j)}\right)_t\right] / \left(MS^{(j)}\right)_t dt \\
&= r_t - \mathbb{E}_t^{\mathbb{Q}}\left[d\log S_t^{(j)}\right]/dt - \frac{1}{2} d\left\langle \log S^{(j)}, \log S^{(j)} \right\rangle_t /dt \\
\mathbb{E}_t^{\mathbb{Q}_j}\left[dX_t\right]/dt &:= \mathbb{E}_t\left[dX_t\right]/dt + d\left\langle X, \log\left(MS^{(j)}\right)\right\rangle_t /dt \\
&= \mathbb{E}_t^{\mathbb{Q}}\left[dX_t\right]/dt + d\left\langle X, \log S^{(j)}\right\rangle_t /dt \\
&\Downarrow \\
P_t^{(j)}(\tau) &:= \mathbb{E}_t\left[\frac{M_T S_T^{(j)}}{M_t S_t^{(j)}} \cdot 1\right] \\
&= \mathbb{E}_t^{\mathbb{Q}_j}\left[e^{-\int_t^{t+\tau} r_u^{(j)} du}\right] = e^{-A^{(j)}(\tau) - B^{(j)}(\tau)\cdot X_t}
\end{aligned}
$$

The Devil's in the Factors

- Pricing kernel M has to price yields in all currencies
  - Factors X must span most of the cross-sectional variation of yields in all currencies
- Two modeling choices
  1. Global factors that affect yields in all currencies and local factors that only affect yields in one currency
     - Local factors are independent (by construction)
  2. All factors only affect yields in one currency
     - Local factors for different currencies can be correlated so the two approaches are basically equivalent
- We choose the second approach so the state vector X contains the state vectors of all the single currency models plus the exchange rates

Devilish Factors (cont'd)

- Pricing (i.e. the cross-section of yields) is the **same** in a multi-currency model as in the single currency models
  - Very convenient and excellent fit
  - Intuitively, "level", "slope", and "curvature" in the relevant currency provide sufficient info about yields in that currency
- Only need to estimate quadratic variation $\langle \text{covariance} \rangle$ terms between local factors in different currencies and with the log exchange rates
  - Single currency estimates contains quadratic variation terms for PCs in each currency
  - Need to express all parameters under one common pricing measure

$$\mathbb{E}_t^{\mathbb{Q}}\left[dX_t\right] = \mathbb{E}_t^{\mathbb{Q}_j}\left[dX_t\right] - d\left\langle X, \log S^{(j)} \right\rangle_t$$

Risk Premia Specification

- Still need risk premia $-d\left\langle X, \log M\right\rangle_t$ to complete the time-series specification of the model

$$\mathbb{E}_t\left[dX_t\right] = \underbrace{\left[\theta + \mathcal{K}X_t\right]dt}_{\mathbb{E}_t^{\mathbb{Q}}[dX_t]} - d\left\langle X, \log M\right\rangle_t$$

- PCs capture cross-sectional variation in yields but are not always the best predictors of time-series changes
  - We can let risk premia depend on any combination of yields and log exchange rates (in general, can also depend on other variables as well such as macro factors)

$$-d\left\langle X, \log M\right\rangle_t = \Lambda_0 + \Lambda_1 R_t \quad \text{where} \quad R_t = R_0 + R_1\left[Y_t, \log S_t\right]$$

Risk Premia Specification (cont'd)

- In theory (a perfect world) we shouldn't need different linear combination for risk premia since

$$R_t = R_0 + R_1 \underbrace{[A + BX_t]}_{[Y_t,\ \log S_t]} = (R_0 + R_1 A) + R_1 B X_t$$

- In practice, it's a model so we can't match all yields exactly

$$Y_t = A + BX_t + \varepsilon_t$$

- Intuitively, we can improve the time series fit if we don't ignore the cross-sectional pricing errors
- Λ_0 and Λ_1 are the only new parameters since we assume

$$dR_t = R_1 B\, dX_t$$

Risk Premia Specification

- No arbitrage does not impose any parameter restrictions on risk premia
  - With up to 7 yields each from 10 currencies plus 9 log exchange rates, R_t can contain 74 elements!
  - If X is dimension $N + J = 28 + 9 = 37$ then Λ_0 and Λ_1 can have $37 \times (79 + 1) = \mathbf{2775}!$ free parameters
  - Even if we set $R_t = X_t$ then we still have $37 \times (37 + 1) = \mathbf{1406}!$
- Combination of two approaches to maintain flexible risk premia specification with fewer parameters
  1. Restrict the rank of $[\Lambda_0, \Lambda_1]$
  2. Reduce the dimension of R_t

Rank Restrictions (cont'd)

- We use 10 single currency models to estimate the parameters for $\Lambda_t^{(j)} = \Lambda_0^{(j)} + \Lambda_1^{(j)} Y_t^{(j)}$ for $j = 0, 1, \ldots, 9$
  - $Y_t^{(j)}$ are yields in currency j ($j = 0$ is domestic)
  - We impose

$$\operatorname{rank}\left(\left[\Lambda_0^{(j)}, \Lambda_1^{(j)}\right]\right) = 1 \quad \Rightarrow \quad \Lambda_t^{(j)} = \underbrace{\Lambda^{(j)}}_{N_j \times 1} \times \underbrace{\left[R_0^{(j)} + R_1^{(j)} Y_t^{(j)}\right]}_{1 \times 1}$$

  - 93 total parameters across all $\Lambda_0^{(j)}$, $\Lambda_1^{(j)}$, $R_0^{(j)}$, and $R_1^{(j)}$
- Then estimate full multi-currency model with $R_t = \left[R^{(0)}, R^{(1)}, \ldots, R^{(9)}\right]_t$
  - $\operatorname{rank}([\Lambda_0, \Lambda_1]) = 1 \;\Rightarrow\; 11 + 36 = 47$ free parameters
  - $\operatorname{rank}([\Lambda_0, \Lambda_1]) = 2 \;\Rightarrow\; 2 \times (11 + 35) = 92$
  - $\operatorname{rank}([\Lambda_0, \Lambda_1]) = 3 \;\Rightarrow\; 3 \times (11 + 34) = 135$

Predicting Changes in Yields

- Our measure of time-series fit is

$$R^2 = 1 - \frac{\sum \left(\Delta Y_t^{(j)}(\tau) - \mathbb{E}_t \left[\Delta Y_t^{(j)}(\tau) \right] \right)^2}{\sum \left(\Delta Y_t^{(j)}(\tau) - \mathbb{E}_t^{\mathbb{Q}_j} \left[\Delta Y_t^{(j)}(\tau) \right] \right)^2}$$

  - i.e. $R^2 > 0$ means the model out-performs the "expectations hypothesis"

- $\Delta t = 4$ week overlapping predictions
  - In-sample estimates 1993-Jan-06 to 2001-Dec-26
  - Out-of-sample testing 2002-Jan-02 to 2009-Mar-28

Time-Series Fit - USD R^2 in %

In-Sample

rank	3M	6M	2Y	3Y	5Y	7Y	10Y
single	25.0	20.0	5.7	3.5	1.7	0.9	0.3
1	11.7	7.9	1.2	0.7	0.3	0.2	0.2
2	15.1	9.0	1.7	1.7	2.3	2.7	3.0
3	25.0	18.0	5.1	3.9	3.3	3.2	3.2

Out-of-Sample

rank	3M	6M	2Y	3Y	5Y	7Y	10Y
single	-4.0	4.5	14.5	12.2	7.7	4.6	1.8
1	-2.3	2.2	5.4	4.1	2.7	1.9	1.3
2	-4.7	1.3	4.4	2.2	-0.3	-1.5	-2.4
3	-1.5	4.8	4.9	2.0	-1.0	-2.2	-2.9

Time-Series Fit - GBP R^2 in %

In-Sample

rank	3M	6M	2Y	3Y	5Y	7Y	10Y
single	16.4	7.9	0.2	0.7	0.8	0.5	0.2
1	13.7	8.6	0.7	0.4	0.5	0.8	1.2
2	14.8	8.8	1.9	2.4	3.3	3.7	3.9
3	15.6	9.2	1.8	2.3	3.3	3.7	3.9

Out-of-Sample

rank	3M	6M	2Y	3Y	5Y	7Y	10Y
single	-13.0	-7.0	-0.4	-1.7	-2.8	-2.5	-1.8
1	10.5	10.7	5.0	3.8	3.6	3.7	3.7
2	11.4	11.2	2.4	0.6	0.2	0.8	1.6
3	10.8	10.9	2.6	0.8	0.4	0.9	1.6

Time-Series Fit - JPY R^2 in %

In-Sample

rank	3M	6M	2Y	3Y	5Y	7Y	10Y
single	18.9	21.0	16.1	14.1	11.2	8.9	6.7
1	10.7	13.0	11.7	10.5	8.4	6.6	4.9
2	14.4	17.1	15.1	13.6	11.2	9.0	6.8
3	19.0	22.4	18.6	16.2	12.4	9.5	7.0

Out-of-Sample

rank	3M	6M	2Y	3Y	5Y	7Y	10Y
single	-25.7	-58.1	-76.3	-52.3	-25.8	-15.1	-9.0
1	-17.2	-49.3	-73.6	-49.4	-22.6	-12.1	-6.3
2	1.2	-19.1	-47.4	-33.0	-15.6	-8.6	-4.8
3	-11.0	-45.2	-73.5	-47.7	-20.2	-10.2	-5.2

Time-Series Fit - AUD R^2 in %

In-Sample

rank	3M	6M	2Y	3Y	5Y	7Y	10Y
single	28.4	22.3	7.5	6.1	5.6	5.6	5.4
1	26.3	18.4	3.2	1.9	1.3	1.1	1.1
2	26.9	19.7	6.5	6.5	7.9	8.8	9.3
3	27.6	20.4	6.5	6.3	7.4	8.2	8.7

Out-of-Sample

rank	3M	6M	2Y	3Y	5Y	7Y	10Y
single	1.9	0.7	-1.3	-1.7	-2.3	-2.7	-3.0
1	3.9	5.0	3.4	2.7	2.4	2.2	2.1
2	6.9	7.0	2.6	0.6	-2.0	-3.7	-4.9
3	4.4	2.9	-4.5	-6.8	-9.3	-10.3	-10.8

Time-Series Fit - EUR R^2 in %

In-Sample

rank	**3M**	**6M**	**2Y**	**3Y**	**5Y**	**7Y**	**10Y**
single	10.7	11.7	8.0	6.8	5.4	4.4	3.3
1	2.8	3.9	4.1	3.5	2.6	1.8	1.1
2	4.7	4.1	5.9	6.2	5.8	4.9	3.7
3	5.2	4.4	5.9	6.2	5.8	4.9	3.7

Out-of-Sample

rank	**3M**	**6M**	**2Y**	**3Y**	**5Y**	**7Y**	**10Y**
single	13.3	13.8	7.5	6.2	5.4	4.9	4.2
1	16.6	16.7	7.5	6.1	5.4	4.9	4.2
2	21.5	19.3	5.6	3.7	2.4	1.7	0.9
3	26.6	22.7	5.7	3.6	2.0	0.9	-0.4

Time-Series Fit - CAD R^2 in %

In-Sample

rank	3M	6M	2Y	3Y	5Y	7Y	10Y
single	17.6	15.3	7.5	6.6	6.6	6.8	6.8
1	16.8	14.0	5.3	3.9	3.0	2.6	2.1
2	17.3	14.9	7.2	6.2	5.8	5.6	5.3
3	20.5	17.7	8.3	6.9	6.2	5.9	5.4

Out-of-Sample

rank	3M	6M	2Y	3Y	5Y	7Y	10Y
single	-29.1	-18.7	-2.5	-1.2	0.1	0.9	1.5
1	-92.0	-45.1	5.4	7.7	9.2	9.4	8.7
2	-77.9	-34.7	5.7	6.5	6.4	5.8	4.7
3	-66.7	-31.2	2.3	3.0	3.2	3.1	2.6

Time-Series Fit - CHF R^2 in %

In-Sample

rank	**3M**	**6M**	**2Y**	**3Y**	**5Y**	**7Y**	**10Y**
single	2.5	4.0	5.4	4.8	3.4	2.2	1.3
1	4.1	3.9	2.6	2.4	2.3	2.2	2.0
2	5.0	4.0	4.7	5.5	6.3	6.4	6.0
3	4.5	3.6	4.7	5.5	6.3	6.4	6.0

Out-of-Sample

rank	**3M**	**6M**	**2Y**	**3Y**	**5Y**	**7Y**	**10Y**
single	30.3	26.2	-1.0	-1.6	1.9	4.1	4.9
1	28.6	25.7	10.9	8.6	6.9	5.9	5.1
2	33.7	27.3	5.5	2.8	1.3	0.9	0.8
3	34.8	28.2	5.7	2.9	1.1	0.5	0.2

Time-Series Fit - NZD R^2 in %

In-Sample

rank	**3M**	**2Y**	**3Y**	**5Y**
single	-0.1	5.3	7.9	10.1
1	0.4	-0.0	-0.1	-0.0
2	0.6	2.5	3.0	3.3
3	8.3	6.4	5.4	4.5

Out-of-Sample

rank	**3M**	**2Y**	**3Y**	**5Y**
single	6.0	-32.0	-34.4	-34.7
1	4.7	1.5	0.2	-1.0
2	4.8	-2.3	-4.9	-7.0
3	-39.1	-31.5	-28.3	-25.6

Time-Series Fit - SEK R^2 in %

In-Sample

rank	**3M**	**6M**	**2Y**	**3Y**	**5Y**	**7Y**	**10Y**
single	17.5	12.3	0.9	0.1	-0.0	0.1	0.2
1	22.3	18.3	4.6	2.7	1.8	1.6	1.7
2	22.0	18.1	9.0	8.3	7.6	7.0	6.1
3	23.0	18.6	8.8	8.1	7.6	7.0	6.2

Out-of-Sample

rank	**3M**	**6M**	**2Y**	**3Y**	**5Y**	**7Y**	**10Y**
single	45.8	43.7	12.8	6.4	5.4	7.4	9.4
1	35.5	34.6	14.8	9.7	7.1	6.9	7.1
2	36.0	34.8	9.5	3.1	0.0	0.5	2.0
3	40.5	37.9	4.9	-2.4	-4.7	-3.0	0.0

Time-Series Fit - NOK R^2 in %

In-Sample

rank	**3M**	**6M**	**2Y**	**3Y**	**5Y**
single	14.1	15.1	14.8	12.4	9.3
1	3.2	3.3	2.6	1.9	1.3
2	12.3	13.9	17.8	16.9	14.7
3	12.3	14.0	18.4	17.7	15.7

Out-of-Sample

rank	**3M**	**6M**	**2Y**	**3Y**	**5Y**
single	-74.4	-87.6	-103.5	-91.3	-74.9
1	12.0	13.0	10.8	8.2	5.5
2	8.3	8.0	1.8	-0.7	-2.7
3	6.5	4.7	-6.9	-9.6	-11.0

Time-Series Fit - log FX rate R^2 in %

	Risk Premia Rank					
	In-Sample			Out-of-Sample		
	1	2	3	1	2	3
GBP	0.4	0.4	0.5	-0.7	-0.7	-1.0
JPY	0.1	1.4	1.3	-1.1	-3.7	-3.9
AUD	0.1	0.2	0.4	0.4	0.3	-0.0
EUR	0.4	0.1	1.3	-1.7	-2.6	-2.3
CAD	0.2	0.2	0.3	-0.7	-0.8	-0.8
CHF	0.2	0.1	1.0	-1.3	-1.9	-2.9
NZD	1.0	3.4	3.3	0.7	0.0	0.2
SEK	0.0	0.0	2.6	-0.1	-0.0	-4.8
NOK	0.0	-0.1	0.0	0.2	0.1	-0.1

Zero Coupon Bond Returns

- All zero coupon bond prices are exponential affine in the state vector X
  - Domestic (USD) zero coupon prices are

$$P_t = e^{-A^{(0)} - B^{(0)} X_t^{(0)}}$$

  - Foreign zero coupon prices in USD are

$$S_t^{(j)} P_t^{(j)} = e^{\log S_t^{(j)} \; - A^{(j)} - B^{(j)} X_t^{(j)}}$$

- The vector of prices of zero coupon bonds in all currencies when they are expressed in USD is of the form

$$Z_t = e^{A + B X_t}$$

Zero Coupon Bond Returns (cont'd)

- The pricing kernel prices any payoff Z_T in USD

$$Z_t = \mathbb{E}_t\left[\frac{M_T}{M_t} Z_T\right] \quad \Rightarrow \quad \mathbb{E}_t\left[dZ_t\right] = Z_t\left[r_t\, dt - d\left\langle \log Z, \log M\right\rangle_t\right]$$

- Therefore, for zero coupon bonds in all currencies

$$Z_t = e^{A+BX_t} \quad \Rightarrow \quad \mathbb{E}_t\left[dZ_t\right] = Z_t\left[\, r_t\, dt \underbrace{-B\, d\left\langle X, \log M\right\rangle_t}_{\mu_t}\,\right]$$

and

$$d\left\langle Z, Z^\top\right\rangle_t = Z_t\, d\left\langle Z, Z^\top\right\rangle_t Z_t^\top = Z_t \underbrace{B\, d\left\langle X, X^\top\right\rangle_t B^\top}_{\Omega} Z_t^\top$$

Maximum Sharpe Ratio

- For zero coupon bonds in all currencies, our model provides an estimate of
  - Expected excess return vector μ_t (in USD), and
  - Covariance matrix Ω of returns in USD

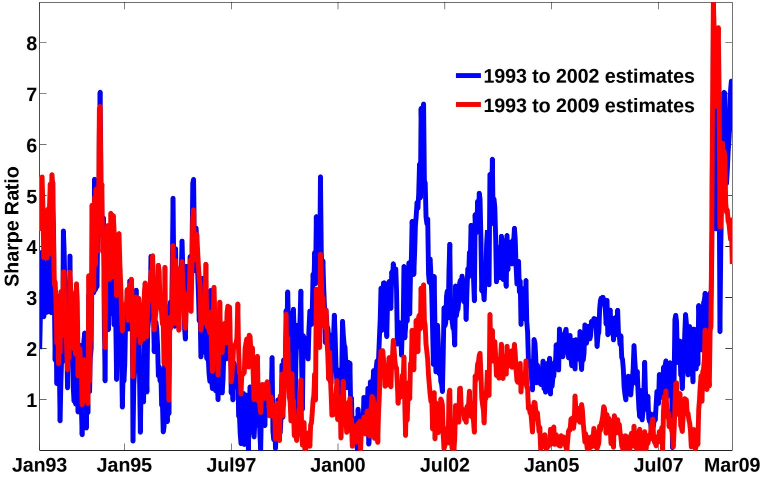
- We can easily compute the global portfolio of all zero coupon bonds, θ_t^*, with the maximum Sharpe ratio

$$\max_{\theta_t} \frac{\theta_t^\top \mu_t}{\sqrt{\theta_t^\top \Omega_t \, \theta_t}} \quad \Rightarrow \quad \theta_t^* \propto \Omega_t^{-1} \mu_t$$

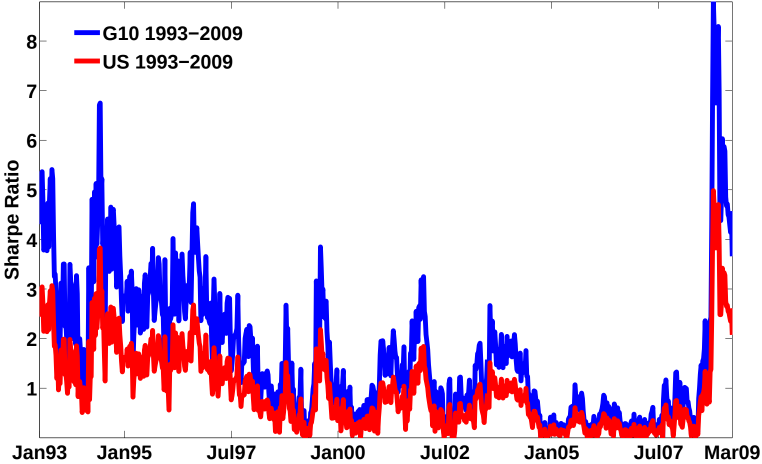
$$\Downarrow$$

$$\frac{\theta_t^{*\top} \mu_t}{\sqrt{\theta_t^{*\top} \Omega_t \, \theta_t^*}} = \sqrt{\mu_t^\top \Omega_t^{-1} \mu_t}$$

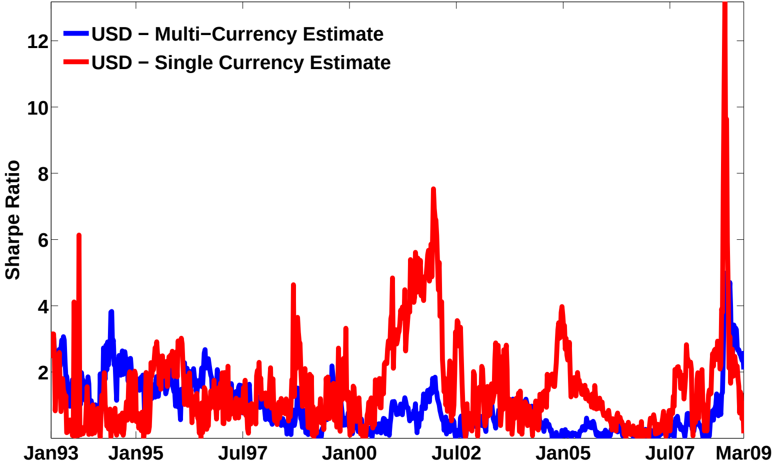
Estimate of Maximum Sharpe Ratio



Global vs. U.S. Sharpe Ratio



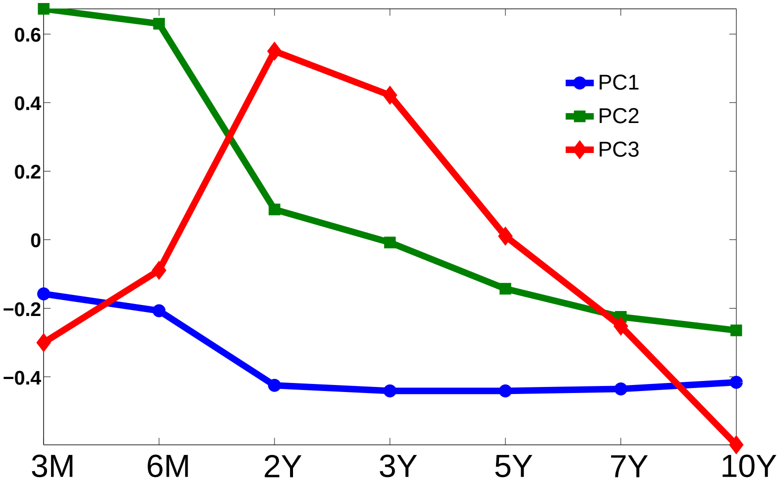
Multi vs. Single U.S. Sharpe Ratio



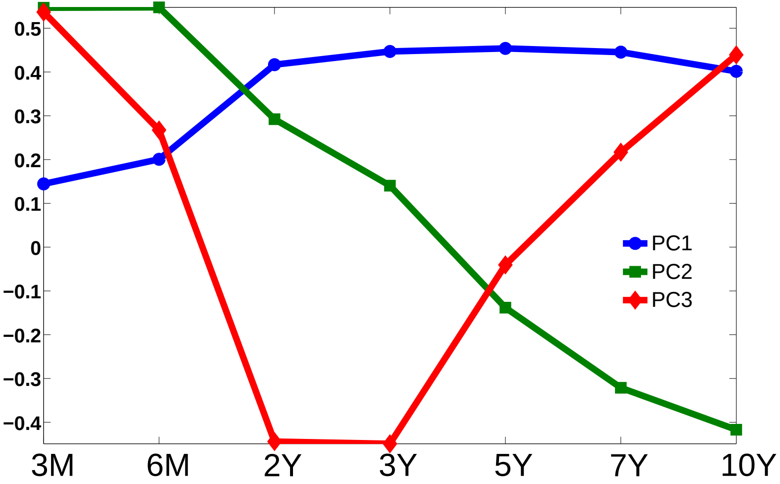
Conclusions

- This paper develops and estimates a no-arbitrage model for swap rates and returns in multiple currencies (applied to G10 countries)
  - Rank restrictions control the number of priced risk factors
- For most countries, except perhaps the US, the predictions of a restricted multi-country model out perform those of a single country
  - Exchange rates and Japanese yields are hard to predict
- Out-of-sample evidence favours one priced global risk factor
- Estimated maximum Sharpe ratio for a global fixed income investor seems to rise as expected during periods of market turmoil
  - However, high mean (2.42 or 1.67) and very volatile
  - Perhaps lower and less volatile with stochastic volatility?

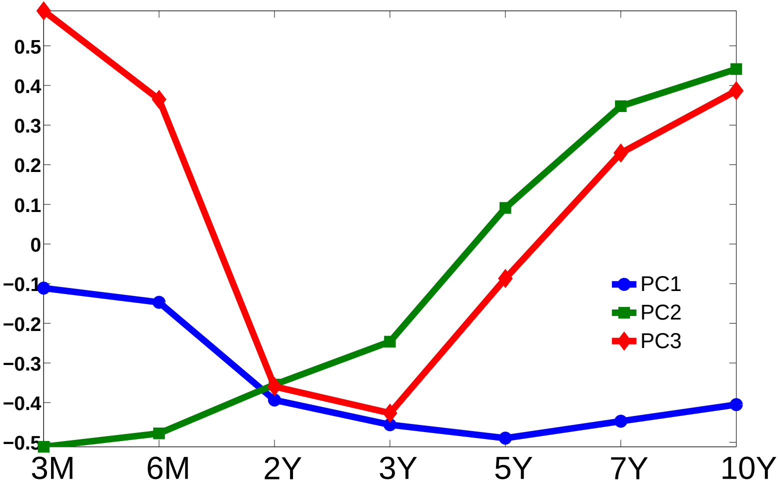
Principal Component Weights - USD



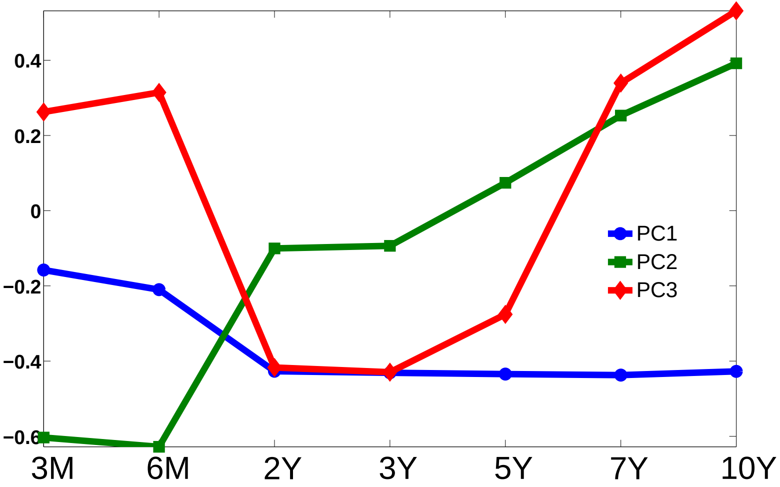
Principal Component Weights - GBP



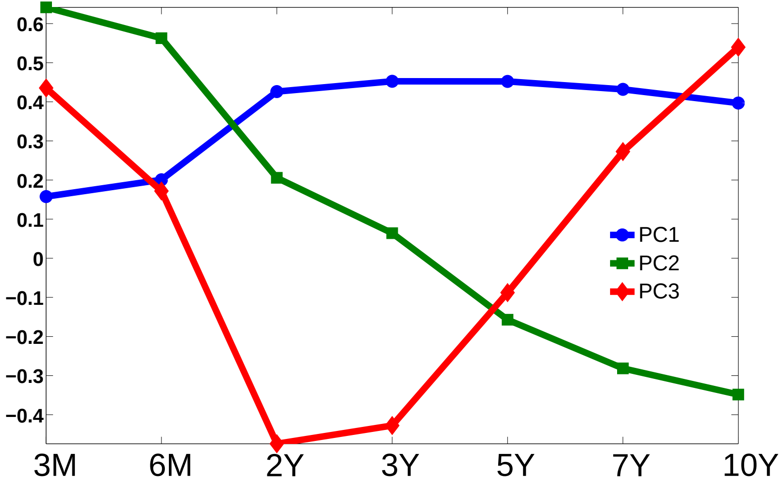
Principal Component Weights - JPY



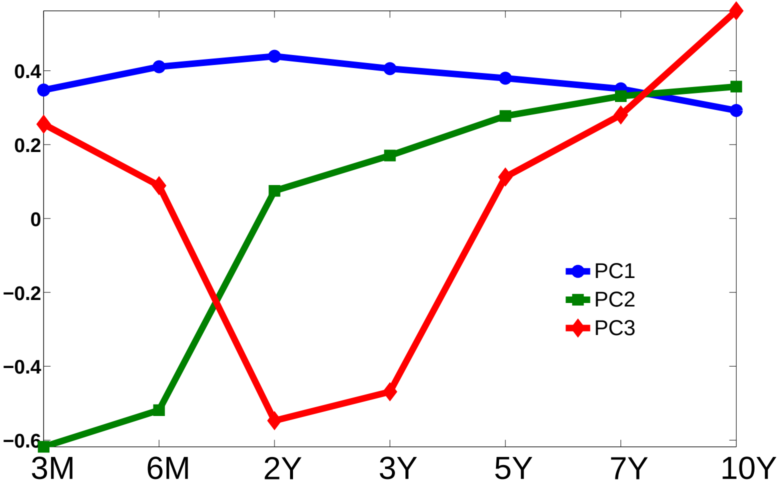
Principal Component Weights - AUD



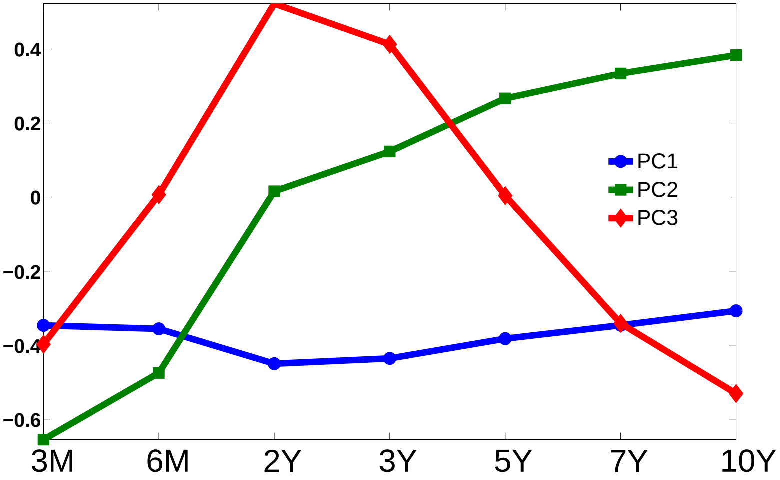
Principal Component Weights - EUR



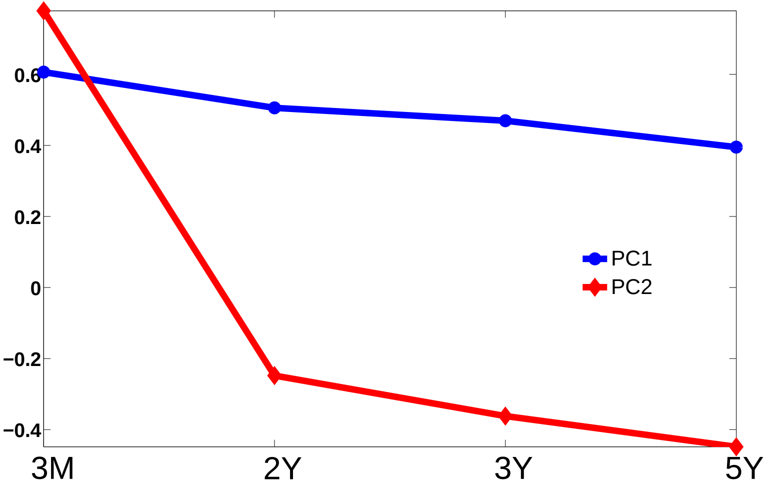
Principal Component Weights - CAD



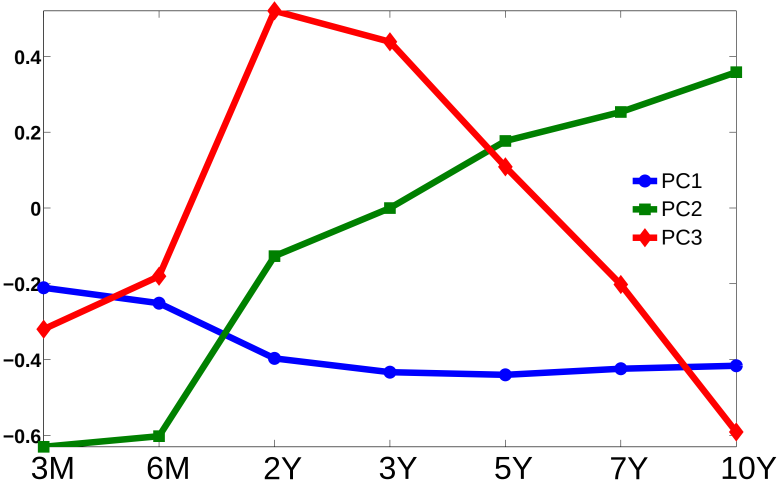
Principal Component Weights - CHF



Principal Component Weights - NZD



Principal Component Weights - SEK



Principal Component Weights - NOK