

The StressVaR

How to measure risk with nonlinear models in a scarce data environment

Raphael Douady

Problem

- $X_1 \dots X_n$ are market factors (indices, rates, etc.)
 - Long term history
 - Daily data (even better)
 - Accurate, liquid
 - 1000's of data points
- Y is a hedge fund return for next month
 - Short history: at best a few years
 - Monthly data (sometimes weekly)
 - Illiquid, inaccurate
 - Only a few 10's of data points

Risk Measure Question

Find the distribution of $Y(t + 1)$ including possible extreme events, *looking forward*

Difficulty

The forward distribution may strongly differ from that of past returns, due to hidden risks

Factor Analysis

- Write $Y = \varphi(X_1, \dots, X_n) + Z$
- Estimate φ and the distribution of Z
- Estimate the joint distribution P of (X_1, \dots, X_n) and “confidence sets” E_α with prob. $1 - \alpha$
- Push forward $\varphi_* P$ and merge with the distribution of Z to get that of Y
- Get confidence intervals $\varphi(E_\alpha)$ and compute the VaR

Stress Test

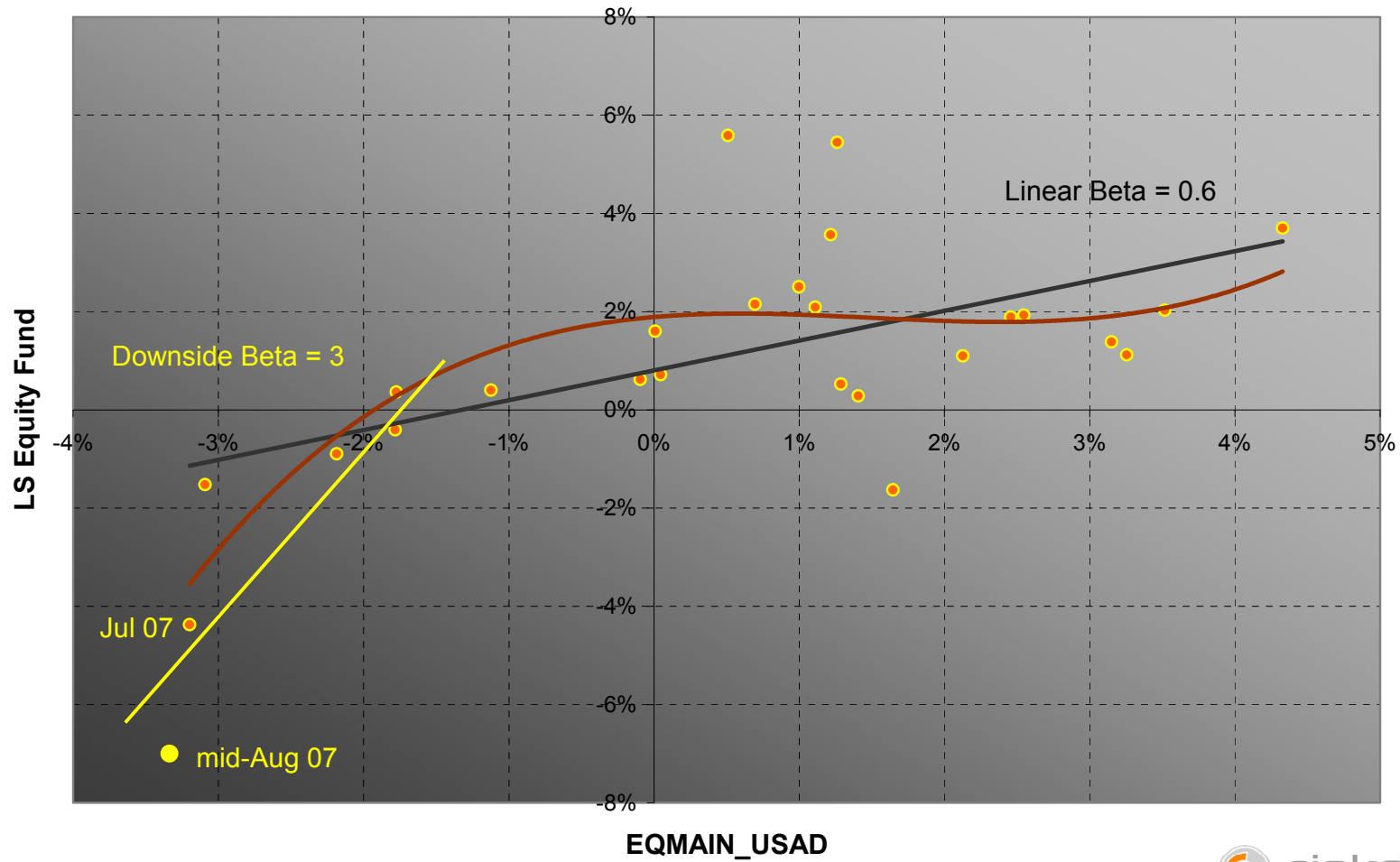
Find the expected impact of a given market move on the fund, i.e. the “Risk Surface”

$$\varphi(x_1, \dots, x_n) = E[Y \mid X_1 = x_1, \dots, X_n = x_n]$$

Difficulties

- φ is strongly nonlinear
- Not enough data points to calibrate a multi-dimensional model

Long-Short Equity Aug 05 - Aug 07



Single Factor Scanning

- Calibrate single factor models, i.e. “Risk Curves” $\varphi_i(x_i) = E[Y | X_i = x_i]$
 - Assume they all have the same expectation, say 0
- Estimate the joint distribution of factors $P(x_1, \dots, x_n)$
- Merge curves to obtain the “Risk Surface” $\varphi(x_1, \dots, x_n)$
- Find $\varphi : \mathbf{R}^n \rightarrow \mathbf{R}$ such that, for any i :
$$E[\varphi(X_1, \dots, X_n) | X_i] = \varphi_i(X_i)$$

Edgeworth Decomposition

- $Y = \varphi_i(X_i) + Z_i$ $\varphi_i(X_i) = \sum_k \beta_{ik} H_k(X_i)$ $H_k = \text{Hermitte polynomials}$
- $Y = \varphi(X_1, \dots, X_n) + Z$ $\varphi(X_1, \dots, X_n) = \sum_{i,k} \lambda_{ik} H_k(X_i)$
- $\Gamma = (\gamma_{ik,j\ell}) = \text{Cov}(H_k(X_i), H_\ell(X_j))$ $C = (c_{ik}) = (\beta_{ik} \gamma_{ik,ik})$
- Solution is given by: $\Lambda = \Gamma^{-1} C$

The method works in L^2 with ∞ sums under *ellipticity condition*: $\left\| \sum_{i=1}^n \varphi_i(X_i) \right\|_{L^2}^2 \geq c \sum_{i=1}^n \|\varphi_i(X_i)\|_{L^2}^2$

Examples with Ellipticity

- The joint distribution (X_1, \dots, X_n) is a Gaussian copula
- X_1, \dots, X_n are independent

Ellipticity fails when there is *tail concentration* of the joint distribution along non axis directions
Unfortunately very common in practice: diversification disappears when crises occur!

Co-linearity

Due to the large number of possible factors and insufficient data, even in the linear case, Γ is not invertible. It is even worse in the nonlinear case. One needs *model selection*

Algorithms for model selection

- Stepwise Regression
- Matching pursuit

These algorithms apply in the nonlinear setting to Hermitte regressors. However, in practice, they miss two essential issues:

- Model selection should depend on the chosen scenario (x_1, \dots, x_n)
- When the scenario is *extreme*, the distribution of factors is very different from under normal conditions (due to *tail concentration*), some factors that usually are uncorrelated, become very correlated and the selection should be made accordingly.

Stress Test: Stepwise Regression with Information Maximization

- Given a scenario $x = (x_1, \dots, x_n)$ we estimate $\hat{\Gamma}(x) = (\text{cov}_x(X_i, X_j))$ by LOESS regression, using weights $\omega_x(t)$ that depend on the proximity of the sample data to the scenario.
 - LOESS = LOcally wEighted Scatter plot Smoothing
- Y is estimated against each X_i by LOESS regression: $Y = \beta_i(x) (X_i - x_i) + \alpha_i(x) + Z_i$
- Let $C(x) = (\beta_i(x) \gamma_{ii}(x))_{i=1 \dots n}$ and $\Lambda(x) = \Gamma(x)^{-1} C(x)$
- Assuming Gaussian inputs, β_i estimate is Fischer, α_i is Gaussian, Γ is Wishart and Λ is inverse Wishart.
- The *efficient number of points* is $n_t = (\sum \omega_i(x))^2 / \sum \omega_i(x)^2$
- For a given subset of indices $I \subset \{1, \dots, n\}$, we can make a joint estimate, also by LOESS regression: $Y = \sum_{i \in I} \lambda_i^I(x) (X_i - x_i) + \alpha_I + Z_I$
- The estimate is $y_I(x) = \alpha_I$ and the variance is $\text{var } Y_I = C_I'(x) \Gamma_I(x) C_I(x) + \text{var } Z_I$
- The estimation error distribution is $\text{var } Y_I + \text{var} \left[\sum_{i \in I} \lambda_i^I(x) (x_i - E(X_i)) \right]$
- The *information ratio* is: $J_I(x) = \frac{y_I(x)^2}{\text{var } Y_I(x)}$

Stepwise Algorithm

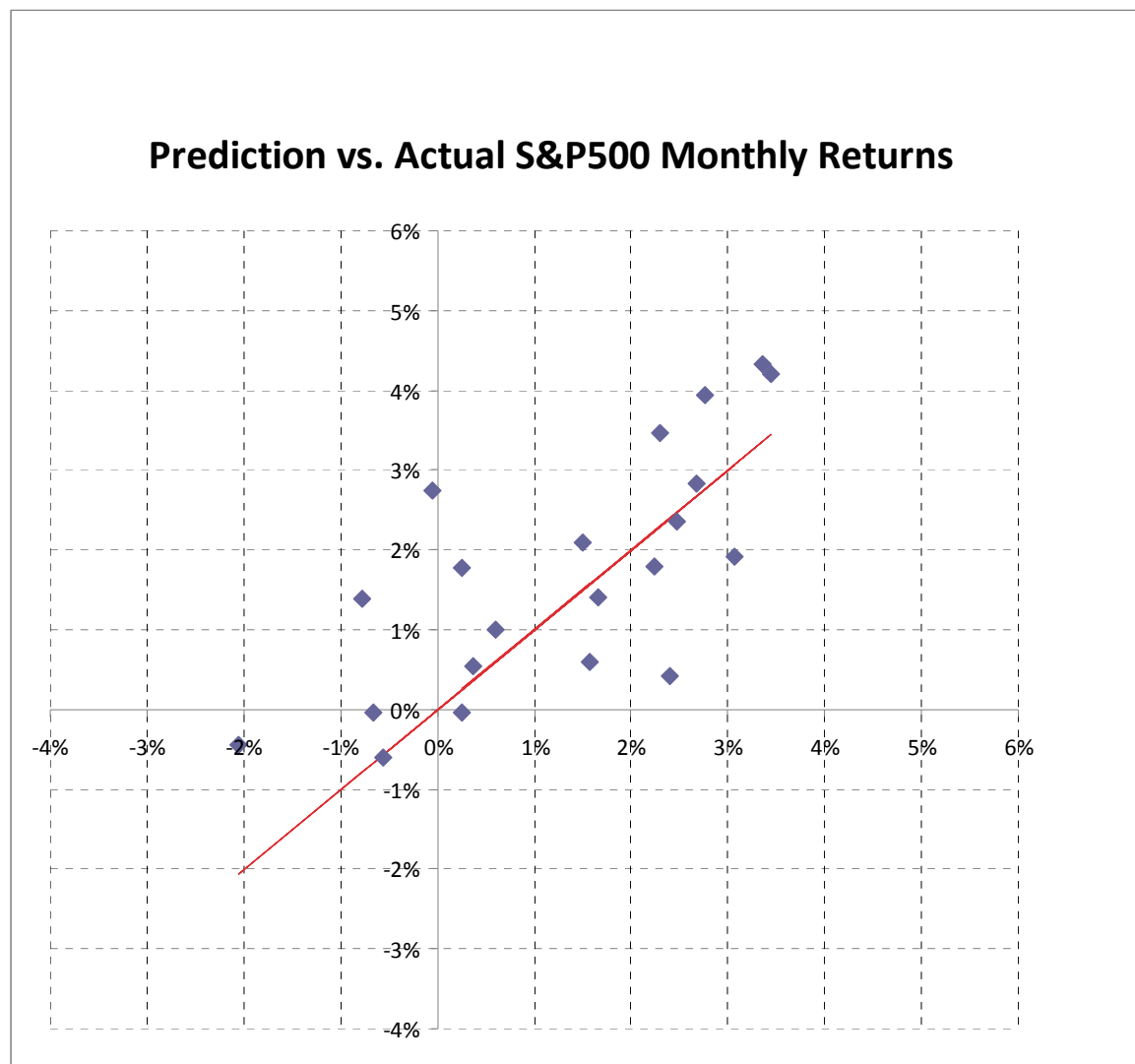
1. Select first a subset of factors that pass some type of individual significance test against the fund (e.g. F-test with LOESS weights)
2. Compute $\Gamma(x)$, $C(x)$ and their estimation error distribution
3. Find the factor X_i that maximizes $J_i(x)$
4. Given i_1 , find the second factor X_{i_2} that maximizes $J_{\{i_1, i_2\}}(x)$
5. Due to the competition between the estimate improvement and the error $\Delta(x)$, after a few iterations, one cannot increase $J_A(x)$. The algorithm stops here.

In practice the algorithm stops very fast: most of the time at the first iteration.
It can be adapted to *matching pursuit*.

Back-test

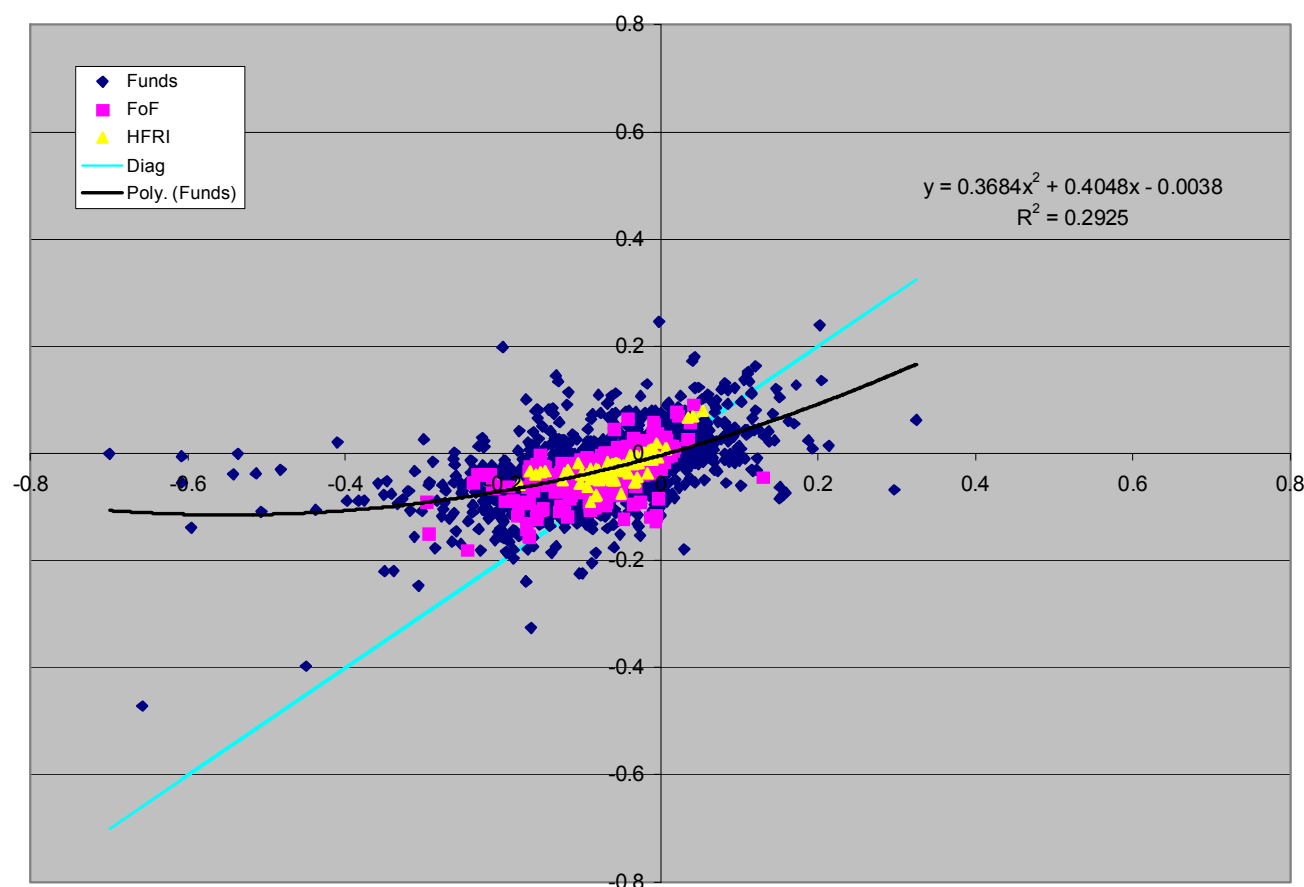
- Selector Factor Universe (X_1, \dots, X_n) and fund Y
- $x_i(t)$ are historical factor returns, $y(t)$ are historical fund returns
- For each time t , compute the “stress test” $\hat{y}(t)$ from factor returns $x_i(t)$, with model calibrated and selected using returns on the period $[0, t-1]$
- Compare $\hat{y}(t)$ to actual return $y(t)$

Example: S&P500 vs. Economic Indicators



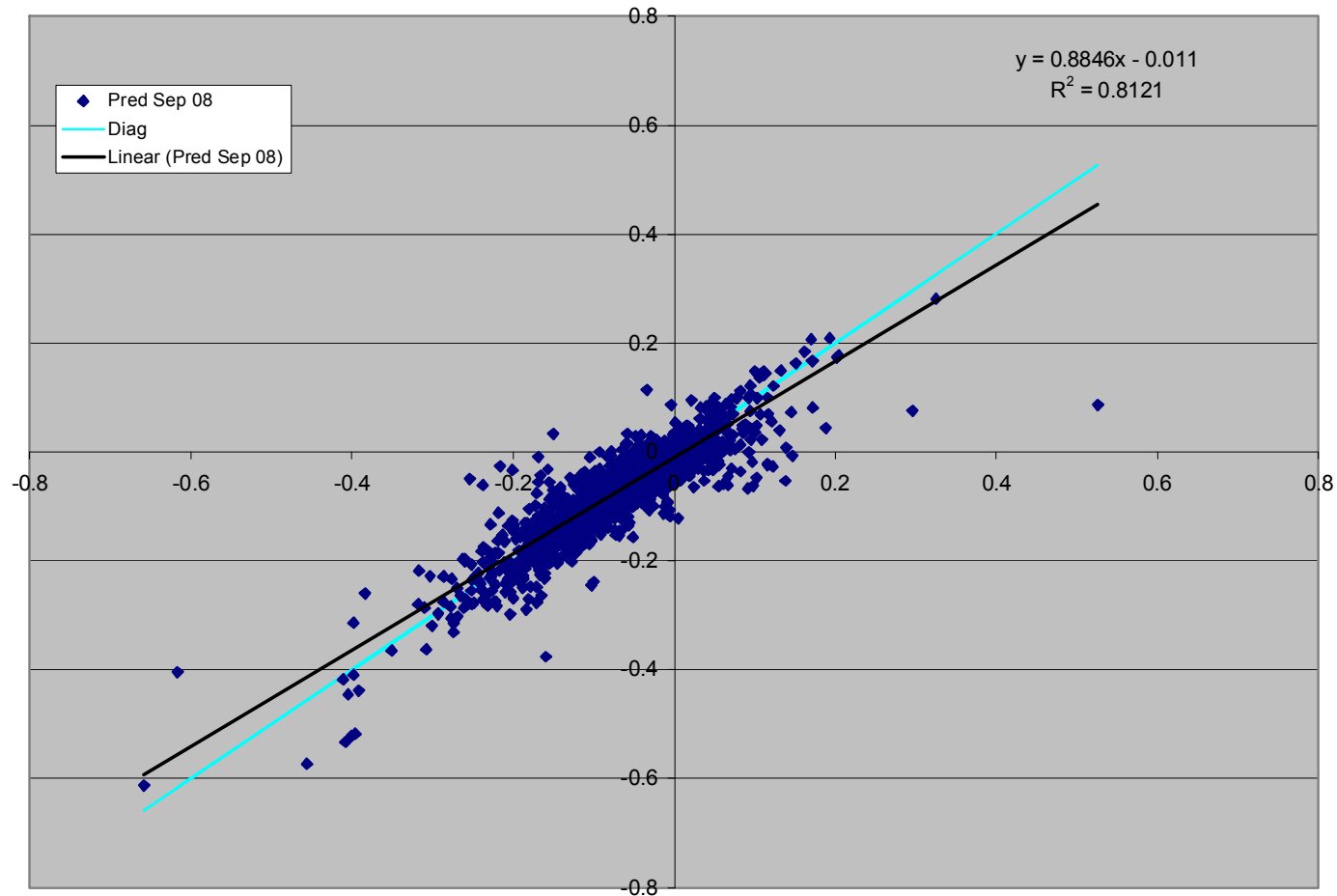
Prediction of 3000 hedge fund returns in Sep 08 with data until Aug 08

Fung-Hsieh 7-factors multi-linear model



Prediction of 3000 hedge fund returns in Sep 08 with data until Aug 08

Nonlinear single factor prediction (173 factors)



Risk Measurement and Portfolio Construction: The Stress VaR

Computing stress tests assume that one *knows* which factor shifts to apply. In a factor-based risk measure, the shifts can be anything within a “confidence set” for a given probability $1-\alpha$. Then the *worst impact* of factors on the fund across this set is a conservative estimate of the VaR of the *explained part* $\hat{Y} = \varphi(X_1, \dots, X_n)$:

$$VaR_{1-\alpha}(\hat{Y}) \leq -\min_{E_\alpha} \varphi(x_1, \dots, x_n)$$

Incorporating Z , one needs to solve an equation involving the cdf of Z and that of \hat{Y} , but we again get a conservative estimate by a simple sum:

$$VaR_{1-\alpha}(Y) \leq -\min_{E_\alpha} \varphi(x_1, \dots, x_n) + VaR_{1-\alpha}(Z)$$

Tail Concentration and Stress VaR

During crises, factors tend to be correlated and funds returns are mostly driven by one single factor. Taking the worst value across all single factor models is sufficient for a good risk estimate. For each factor X_i , define :

$$StressVaR_{1-\alpha}(\hat{Y}, X_i) = - \min_{E_\alpha(X_i)} \varphi_i(x_i)$$

$$StressVaR_{1-\alpha}(Y, X_i) = \sqrt{\min_{E_\alpha(X_i)} \varphi_i(x_i)^2 + VaR_{1-\alpha}(Z_i)^2}$$

Let I be a set of factors that have been identified as having a significant relationship with the fund under extreme conditions. The *StressVaR* is defined by:

$$StressVaR_{1-\alpha}(Y) = \max_{i \in I} StressVaR_{1-\alpha}(Y, X_i)$$

As a risk measure, it is no more coherent than the VaR, but can easily be made coherent if one changes the individual factor StressVaR by a coherent risk measure (see Cherny-Madan). Then the global StressVaR is coherent if one assumes that I is fixed, but can be made non coherent if selecting factors is part of the algorithm.

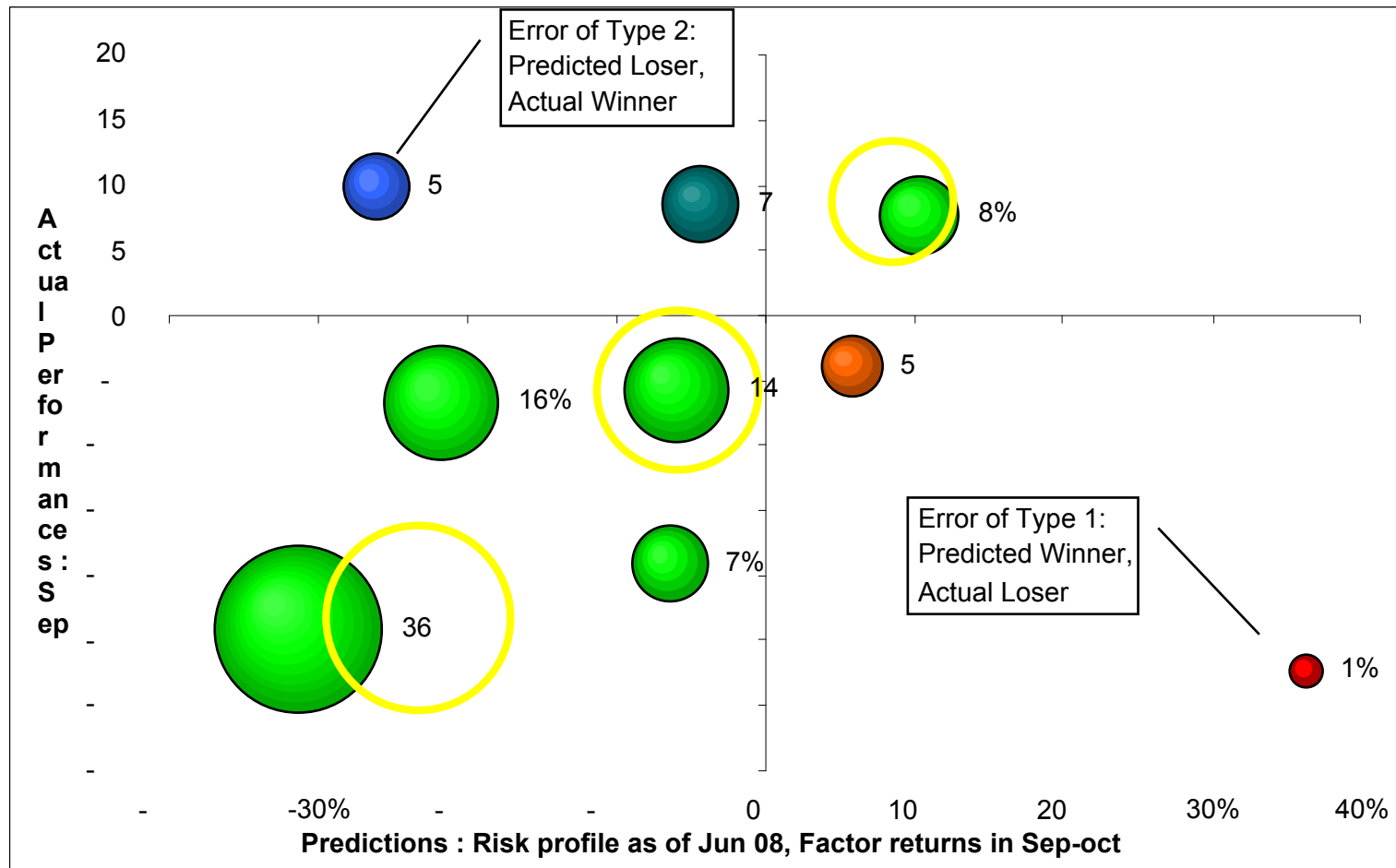
Properties of the StressVaR

This question is, however, a minor issue, regarding the advantages of the StressVaR:

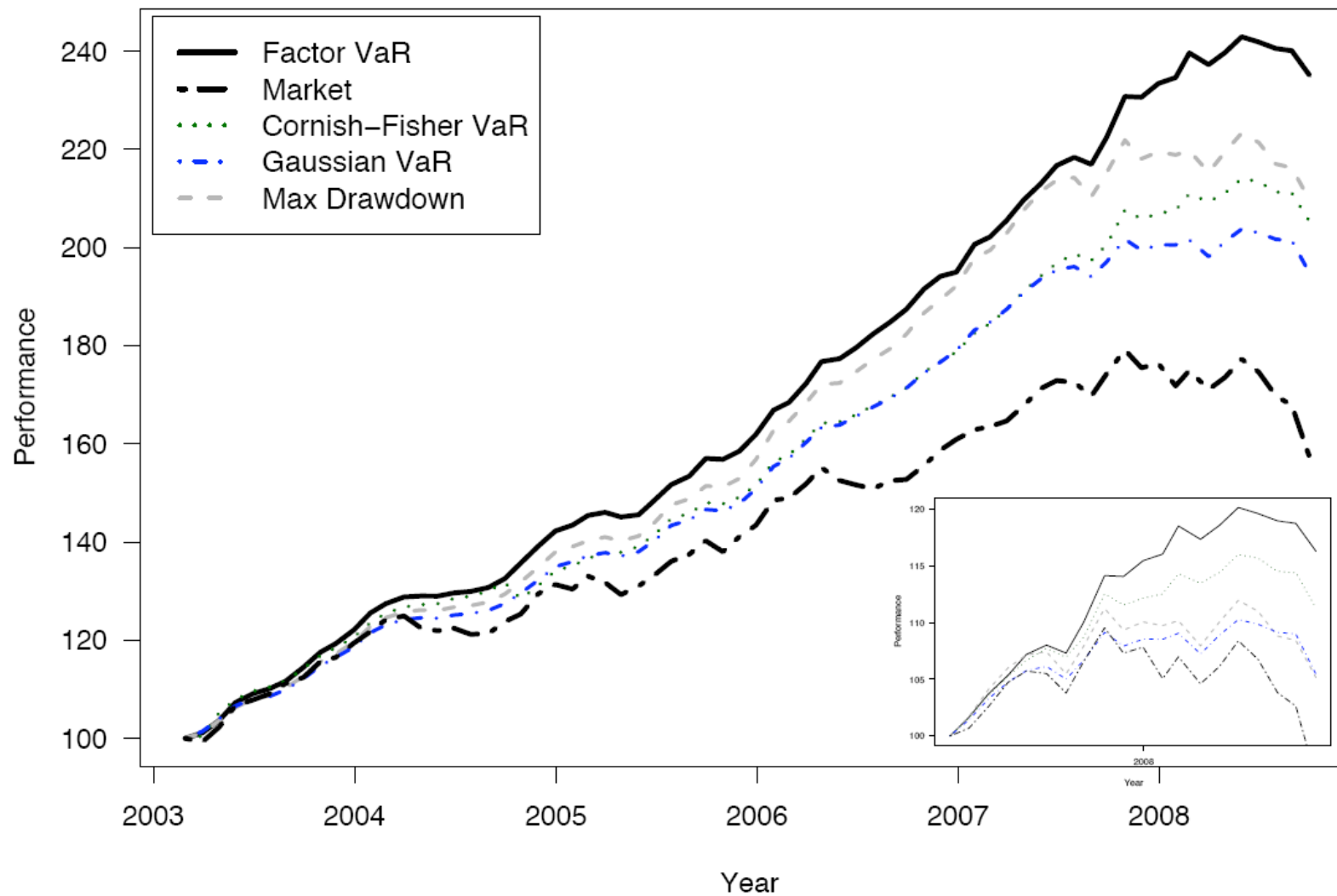
- It is *anticipative* because it relies on a very long term factor history, including past crises when the fund did not exist
- Most *correlation breaks* are the result of the common impact of a single factor on several funds.
- Liquidity crises make the market very correlated: the StressVaR provides a *quantitative* measure of the impact of such crises
- Repeated tests for factor significance are a problem in *normal conditions*, when markets are uncorrelated. But not under *extreme conditions*, when correlations are very high. In other words, the *StressVaR* is only precise when *needed*!
- Hidden Risk Ratio

$$HRR = \frac{StressVaR - MaxDrawDown}{Volatility}$$

Anticipation of Risky Funds during the Crisis, as of June 08



Performance of an Equal-risk distributed Portfolio



Performance of an Optimized Portfolio

