Exploring Statistical Arbitrage Opportunities in the Term Structure of CDS Spreads

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Credit Default Swaps

- The single-name credit default swaps (CDS) are the most liquid and popular credit derivatives with a notional value of $ 42 Trillion by the end of 2008
- Though five-year CDS have been the most liquid contracts, recently a complete credit curve (CDS spreads over different maturities) is available for many companies
- Therefore, the rapid growth of the CDS market makes it possible to speculate on the relative pricing of the credit risk of a company across a wide range of maturities

Research Questions and Importance

- Research Questions: Is the credit risk of a company consistently priced across maturities? Are there "arbitrage" opportunities to be exploited in the term structure of CDS spreads?
- Academic perspective: Can existing credit risk models, either structural or reduced-form, capture the rich term structure behaviors of credit spreads?
- Practical perspective: Can one design trading strategies to exploit potential mispricings along the credit curve?

What We Have Done

- We explore "arbitrage" opportunities in the term structure of CDS spreads (maturities of 1, 2, 3, 5, 7, 10, 15, 20, and 30 years) of 297 N.A. companies between January 4, 2005 and December 31, 2008
- We estimate an affine model of credit risk for each company and identify "mis-valued" CDS contracts relative to the model
- Based on the estimated model parameters, we construct a portfolio of CDS contracts that are both delta- and gamma-neutral to potential changes in credit spread
- Then we would long (short) the portfolio if it is under (over) valued relative to our model and liquidate the portfolio a week later

Main Results

- For in-sample analysis, we estimate model parameters, construct arbitrage portfolios, and calculate trading profits using all the data
- For out-of-sample analysis, we estimate model parameters using the first half of the sample, based on which we construct arbitrage portfolios and calculate trading profits using the second half of the sample
- Our "arbitrage" strategy can be quite profitable both in sample and out of sample
- For most firms, the Sharpe ratio of the weekly returns of this strategy is above one
- For more than half of the firms, the Sharpe ratio can be well above two!

Valuation of CDS Spreads

- Suppose the CDS spread, $S_{CDS}(t,\tau)$, is paid continuously, then the present value of the premium leg of a CDS equals

$$S_{CDS}(t,\tau)\,\mathbb{E}^{Q}\left[\int_t^{t+\tau}\exp\left(-\int_t^u (r_s+h_s)\,\mathbf{d}s\right)\mathbf{d}u \,\middle|\, \mathscr{F}_t\right]$$

- Similarly, the value of the protection leg of a CDS equals

$$\mathbb{E}^{Q}\left[\int_t^{t+\tau} y_u h_u \exp\left(-\int_t^u (r_s+h_s)\,\mathbf{d}s\right)\mathbf{d}u \,\middle|\, \mathscr{F}_t\right]$$

- Premium Leg=Protection Leg $\Leftrightarrow$

$$S_{CDS}(t,\tau) = \frac{\mathbb{E}^{Q}\left[\int_t^{t+\tau} y_u h_u e^{-\int_t^u (r_s+h_s)\mathbf{d}s}\mathbf{d}u \,\middle|\, \mathscr{F}_t\right]}{\mathbb{E}^{Q}\left[\int_t^{t+\tau} e^{-\int_t^u (r_s+h_s)\mathbf{d}s}\mathbf{d}u \,\middle|\, \mathscr{F}_t\right]} \qquad (1)$$

Default Intensity

- We assume the default intensity h_t is driven by the state variable Z_t:

$$h_t = Z_t$$

and

$$\mathbf{d}Z_t = \kappa_Z \left(\theta_Z - Z_t\right) \mathbf{d}t + \sigma_Z \sqrt{Z_t} \mathbf{d}w_Z^Q(t),$$

$$\mathbf{d}Z_t = \left(\kappa_Z \theta_Z - \kappa_Z^P Z_t\right) \mathbf{d}t + \sigma_Z \sqrt{Z_t} \mathbf{d}w_Z^P(t).$$

where $w_Z^Q(t)$ and $w_Z^P(t)$ are standard Brownian motions under the equivalent martingale measure Q and the physical measure P, respectively.

Recovery Rate and Default-Free Interest Rate

- We assume a constant recovery rate and estimate it along with model parameters from CDS spreads
- Specifically, we set the recovery rate as

$$1 - y = \exp(-\beta_0),$$

where we choose $\beta_0 > 0$ to ensure $y \in (0, 1)$.
- We assume independence between h_t and r_t to ensure robustness of model performances by avoiding to estimate a model for the default-free term structure.

The CDS Spread Pricing Formula

- By the independence between h_t and r_t, (1) can be rewritten as

$$S_t^{\tau} = \frac{\left[1 - \exp\left(-\beta_0\right)\right] \int_t^{t+\tau} P\left(t,u\right) \mathbb{E}_2\left(t,u\right) \mathbf{d}u}{\int_t^{t+\tau} P\left(t,u\right) \mathbb{E}_1\left(t,u\right) \mathbf{d}u}, \tag{2}$$

where $P\left(t,T\right)$ is the time-t price of a default-free zero coupon bond that matures at T, and

$$\begin{aligned} \mathbb{E}_1\left(t,u\right) &= \mathbb{E}^Q\left[\exp\left(-\int_t^u Z_s \mathbf{d}s\right) \middle| \mathscr{F}_t\right], \\ \mathbb{E}_2\left(t,u\right) &= \mathbb{E}^Q\left[\exp\left(-\int_t^u Z_s \mathbf{d}s\right) Z_u \middle| \mathscr{F}_t\right]. \end{aligned}$$

Transform Analysis

- Consider the following "transform" and the "extended transform"

$$\Psi(w, Z_t, t, u) = \mathbb{E}^Q\left[\exp\left(-\int_t^u Z_s \mathbf{d}s\right) e^{wZ_u} \middle| \mathscr{F}_t\right], \qquad (3)$$

$$\Phi(v, w, Z_t, t, u) = \mathbb{E}^Q\left[\exp\left(-\int_t^u Z_s \mathbf{d}s\right) vZ_u e^{wZ_u} \middle| \mathscr{F}_t\right]. \qquad (4)$$

- Proposition 1 of Duffie et al. (2000) indicates that (3) has the following form:

$$\Psi(w, Z_t, t, u) = \exp\{A(t, u) + B(t, u) Z_t\},$$

where A and B satisfy a system of ODEs.

Transform Analysis

- Similarly, (4) is given by

$$\begin{aligned}\Phi(v,w,Z_t,t,u) &= \left.\frac{\partial\Psi(\phi v+w,Z_t,t,u)}{\partial\phi}\right|_{\phi=0} \\ &= \Psi(w,Z_t,t,u)\left[C(t,u)+D(t,u)Z_t\right],\end{aligned}$$

  where C and D satisfy a system of ODEs.
- Then we have

$$\begin{aligned}\mathbb{E}_1(t,u) &= \Psi(0,Z_t,t,u), \\ \mathbb{E}_2(t,u) &= \Phi(1,0,Z_t,t,u).\end{aligned}$$

State Space Representation

- The transition equation for Z_t is given as

$$\begin{aligned}
\mathbb{E}_{t-\Delta t}[Z(t)] &= \frac{\kappa_Z \theta_Z}{\kappa_Z^P}\left(1-\exp\left(-\kappa_Z^P \Delta t\right)\right)+\exp\left(-\kappa_Z^P \Delta t\right) Z(t-\Delta t), \\
\mathbb{V}ar_{t-\Delta t}[Z(t)] &= \frac{\kappa_Z \theta_Z \sigma_Z^2}{2\left(\kappa_Z^P\right)^2}\left(1-\exp\left(-\kappa_Z^P \Delta t\right)\right)^2 \\
&+ \frac{\sigma_Z^2\left(\exp\left(-\kappa_Z^P \Delta t\right)-\exp\left(-2\kappa_Z^P \Delta t\right)\right)}{\kappa_Z^P} Z(t-\Delta t).
\end{aligned}$$

- The measurement equation of CDS_t^τ (the observed CDS spread for protection between t and $t+\tau$) is

$$\text{CDS}_t^\tau = S_t^\tau(Z_t)+\varepsilon_\tau,$$

where $\varepsilon_\tau \backsim i.i.d.\ \mathcal{N}\left(0, v^2\right)$ and $\tau = 1, 2, 3, 5, 7, 10, 15, 20$, and 30 years.

Unscented Kalman Filter (UKF)

- We use the UKF in conjunction with QMLE to estimate the credit risk model because CDS spreads are highly nonlinear in the state variable Z_t
- For each measurement occasion t, a set of deterministically selected points, termed *sigma points*, is used to approximate the distribution of the current state estimates at time t using a normal distribution with a mean vector $Z_{t|t-1}$, and a covariance matrix which is a function in the state covariance matrix, $P_{Z,t-1|t-1}$, and conditional covariance $\mathbb{V}ar_{t-1}\left[Z\left(t\right)\right]$

Unscented Kalman Filter (UKF)

- We start the UKF by choosing the initial values of the state variables and their covariance matrix as their steady state values:
$$Z_{0|0} = \frac{\kappa_Z \theta_Z}{\kappa_Z^P}, \; P_{0|0} = \frac{\kappa_Z \theta_Z}{2\left(\kappa_Z^P\right)^2} \sigma_Z^2.$$
- Given $Z_{t-1|t-1}$ and $P_{Z,t-1|t-1}$, the *ex ante* prediction of the state and its covariance matrix are given by

$$\begin{aligned}
Z_{t|t-1} &= \frac{\kappa_Z \theta_Z}{\kappa_Z^P}\left(1 - \exp\left(-\kappa_Z^P \Delta t\right)\right) + \exp\left(-\kappa_Z^P \Delta t\right) Z_{t-1|t-1}, \\
P_{Z,t|t-1} &= e^{-2\kappa_Z^P \Delta t} P_{Z,t-1|t-1} + \frac{\kappa_Z \theta_Z \sigma_Z^2}{2\left(\kappa_Z^P\right)^2}\left(1 - e^{-\kappa_Z^P \Delta t}\right)^2 \\
&+ \frac{\sigma_Z^2\left(e^{-\kappa_Z^P \Delta t} - e^{-2\kappa_Z^P \Delta t}\right)}{\kappa_Z^P} Z_{t-1|t-1}.
\end{aligned}$$

Unscented Kalman Filter (UKF)

- Given an *ex ante* predictions of states $Z_{t|t-1}$, a set of 3 sigma points is selected as

$$\chi_{t|t-1} = \left[\begin{array}{ccc} \chi_{0,t-1} & \chi_{+,t-1} & \chi_{-,t-1} \end{array} \right],$$

where

$$\begin{aligned}
\chi_{0,t-1} &= Z_{t|t-1}, \\
\chi_{+,t-1} &= Z_{t|t-1} + \sqrt{(1+\rho)} \left(\exp\left(-\kappa_Z^P \Delta t\right) \sqrt{P_{Z,t-1|t-1}} + \sqrt{\mathbb{V}ar_{t-1}\left[Z(t)\right]} \right), \\
\chi_{-,t-1} &= Z_{t|t-1} - \sqrt{(1+\rho)} \left(\exp\left(-\kappa_Z^P \Delta t\right) \sqrt{P_{Z,t-1|t-1}} + \sqrt{\mathbb{V}ar_{t-1}\left[Z(t)\right]} \right).
\end{aligned}$$

The term ρ is a scaling constant and given by

$$\rho = \phi^2 (1+\kappa) - 1,$$

where ϕ and κ are user–specified constants. In this paper, we choose $\phi = 0.001$ and $\kappa = 2$.

Unscented Kalman Filter (UKF)

- Nonlinear transformation of the sigma points through measurement function (predictions of measurements) $\chi_{t|t-1}$ is propagated through the nonlinear measurement function $S_t^{\tau}(\cdot)$
$$\mathsf{S}_{t|t-1} = S_t^{\tau}\left(\chi_{t|t-1}\right),$$
where the dimension of $\mathsf{S}_{t|t-1}$ is 9×3. We define the set of weights for covariance estimates as
$$W^{(c)} = diag\left[\ \tfrac{\rho}{1+\rho} + 1 - \phi^2 + 2\ , \frac{1}{2\left(1+\rho\right)}, \frac{1}{2\left(1+\rho\right)}\right];$$
and the weights for mean estimates as
$$W^{(m)} = \left[\ \tfrac{\rho}{1+\rho} \quad \tfrac{1}{2(1+\rho)} \quad \tfrac{1}{2(\omega+\rho)}\ \right]^{\mathsf{T}}.$$

Unscented Kalman Filter (UKF)

- Predicted measurements and the associated covariance matrix are computed as

$$\begin{aligned} S_{t|t-1} &= \mathsf{S}_{t|t-1} W^{(m)}, \\ P_{y_t} &= \left[\mathsf{S}_{t|t-1} - \mathbf{1}_{1\times 3} \otimes S_{t|t-1}\right] W^{(c)} \left[\mathsf{S}_{t|t-1} - \mathbf{1}_{1\times 3} \otimes S_{t|t-1}\right]^{\mathsf{T}} + V, \\ P_{Z_t, y_t} &= \left[\chi_{t|t-1} - \mathbf{1}_{1\times 3} \otimes Z_{t|t-1}\right] W^{(c)} \left[\mathsf{S}_{t|t-1} - \mathbf{1}_{1\times 3} \otimes S_{t|t-1}\right]^{\mathsf{T}}, \end{aligned}$$

  where $V = diag\left[v^2, v^2, \cdots, v^2\right]_{9\times 9}$.
- Finally, the discrepancy between model prediction and actual observations is weighted by a Kalman gain Ξ_t function to yield *ex post* state and covariance estimates as

$$\begin{aligned} Z_{t|t} &= Z_{t|t-1} + \Xi_t \left(S_t - S_{t|t-1}\right), \\ P_{Z,t|t} &= P_{Z,t|t-1} - \Xi_t P_{y_t} \Xi_t^{\mathsf{T}}, \end{aligned}$$

  where $\Xi_t = P_{Z_t, y_t} P_{y_t}^{-1}$.

Unscented Kalman Filter (UKF)

- Assuming the measurement errors are normally distributed, then the transition density of

$$\mathbf{S}(t) = \begin{bmatrix} \text{CDS}_t^1 & \text{CDS}_t^2 & \cdots & \text{CDS}_t^{30} \end{bmatrix}^\intercal$$

given information set $\mathscr{F}_{t-1}$ is a 9-dimensional normal distribution with mean $S_{t|t-1}$ and covariance matrix P_{y_t}, which are outputs from the UKF.

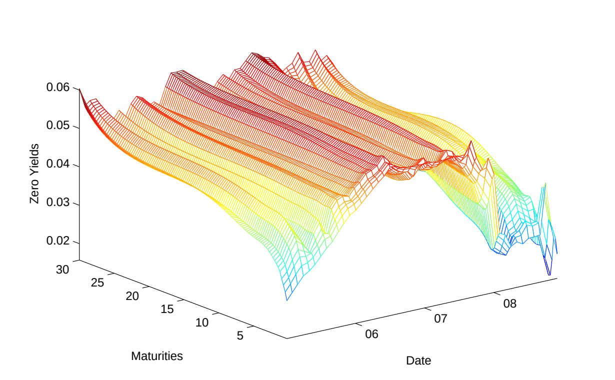
- Then the log-likelihood function is given by

$$\ln \mathscr{L} \propto -\sum_{i=1}^{n} \ln |P_{y_i}| - \sum_{i=1}^{n} \left(\mathbf{S}(i) - S_{i|i-1}\right)^\intercal P_{y_i}^{-1} \left(\mathbf{S}(i) - S_{i|i-1}\right),$$

where n is the sample size.

Default-free Zero Coupon Bond Prices

Figure: Default-free Zero Yields



Zero yields bootstrapped from LIBOR and Swap rates between 2005 and 2008.

CDS Data from Markit

- Focus on CDS quotes that are denominated in US dollars for all US (excluding sovereign) entities
- CDS spreads on senior unsecured issues with modified restructuring clause for 297 firms between 2005 and 2008
- Maturities range: $1, 2, 3, 5, 7, 10, 15, 20$, and 30 years
- Average recovery rates used by data contributors in pricing each CDS contract
- 7 Ratings: AAA, AA, A, BBB, BB, B, and CCC
- 10 Industry Sectors: Basic Material, Consumer Good, Consumer Service, Financials, Industrials, Oil & Gas, Telecommunication, and Utilities

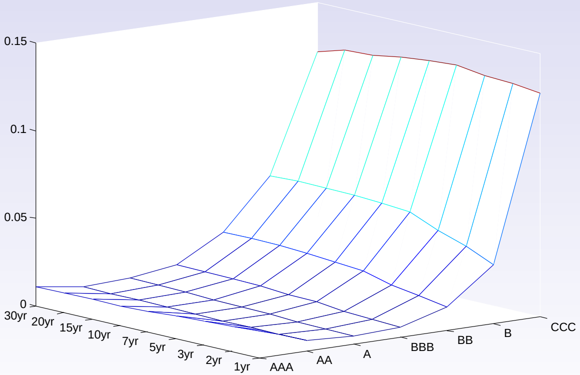
CDS Data by Rating

Table: Summary Statisitics of CDS Spreads by Rating

Rating		1y	2y	3y	5y	7y	10y	15y	20y	30y
AAA	Mean	0.0156	0.0139	0.0136	0.0126	0.0117	0.0112	0.0113	0.0112	0.0113
	Std.	0.0428	0.0349	0.0325	0.0275	0.0240	0.0213	0.0205	0.0199	0.0194
AA	Mean	0.0061	0.0058	0.0059	0.0062	0.0063	0.0066	0.0069	0.0071	0.0072
	Std.	0.0385	0.0306	0.0278	0.0238	0.0213	0.0190	0.0172	0.0178	0.0157
A	Mean	0.0045	0.0048	0.0052	0.0059	0.0064	0.0070	0.0076	0.0079	0.0082
	Std.	0.0191	0.0172	0.0161	0.0141	0.0127	0.0115	0.0111	0.0109	0.0108
BBB	Mean	0.0059	0.0066	0.0074	0.0090	0.0098	0.0108	0.0115	0.0118	0.0120
	Std.	0.0240	0.0203	0.0184	0.0160	0.0142	0.0128	0.0123	0.0121	0.0121
BB	Mean	0.0132	0.0161	0.0187	0.0230	0.0243	0.0257	0.0266	0.0268	0.0268
	Std.	0.0329	0.0326	0.0316	0.0309	0.0285	0.0269	0.0263	0.0256	0.0257
B	Mean	0.0335	0.0406	0.0459	0.0527	0.0540	0.0548	0.0554	0.0555	0.0548
	Std.	0.0723	0.0693	0.0663	0.0608	0.0567	0.0526	0.0516	0.0508	0.0493
CCC	Mean	0.1273	0.1293	0.1303	0.1324	0.1314	0.1295	0.1269	0.1265	0.1216
	Std.	0.2642	0.2325	0.2192	0.2023	0.1943	0.1867	0.1773	0.1772	0.1702

CDS Data by Rating

Figure: Term Structure of Average CDS Spreads by Rating



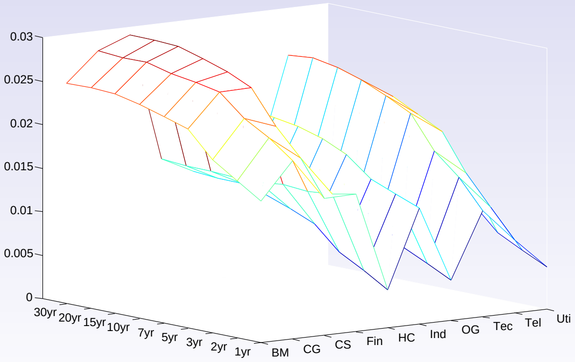
CDS Data by Industry

Table: Summary Statisitics of CDS Spreads by Industry

Sector.		1y	2y	3y	5y	7y	10y	15y	20y	30y
BM	Mean	0.0163	0.0181	0.0199	0.0229	0.0237	0.0246	0.0252	0.0254	0.0253
	Std.	0.0873	0.0762	0.0741	0.0681	0.0630	0.0609	0.0579	0.0588	0.0555
CG	Mean	0.0205	0.0226	0.0242	0.0267	0.0272	0.0277	0.0284	0.0284	0.0286
	Std.	0.0742	0.0667	0.0619	0.0573	0.0532	0.0500	0.0489	0.0475	0.0465
CS	Mean	0.0157	0.0198	0.0229	0.0268	0.0280	0.0290	0.0298	0.0300	0.0300
	Std.	0.0528	0.0552	0.0563	0.0541	0.0524	0.0493	0.0480	0.0471	0.0467
Fin	Mean	0.0158	0.0152	0.0150	0.0152	0.0149	0.0149	0.0151	0.0151	0.0153
	Std.	0.0549	0.0467	0.0421	0.0375	0.0335	0.0307	0.0288	0.0280	0.0273
HC	Mean	0.0044	0.0060	0.0076	0.0103	0.0114	0.0125	0.0132	0.0133	0.0135
	Std.	0.0097	0.0123	0.0143	0.0170	0.0175	0.0178	0.0179	0.0177	0.0177
Ind	Mean	0.0133	0.0144	0.0155	0.0178	0.0190	0.0199	0.0203	0.0208	0.0204
	Std.	0.0632	0.0594	0.0565	0.0564	0.0574	0.0568	0.0546	0.0551	0.0522
OG	Mean	0.0046	0.0060	0.0073	0.0099	0.0110	0.0122	0.0129	0.0131	0.0133
	Std.	0.0090	0.0101	0.0110	0.0127	0.0128	0.0129	0.0132	0.0132	0.0128
Tec	Mean	0.0122	0.0154	0.0179	0.0217	0.0231	0.0244	0.0252	0.0258	0.0255
	Std.	0.0534	0.0513	0.0488	0.0457	0.0444	0.0424	0.0415	0.0411	0.0401
Tel	Mean	0.0084	0.0116	0.0149	0.0191	0.0206	0.0220	0.0229	0.0233	0.0238
	Std.	0.0168	0.0199	0.0232	0.0256	0.0250	0.0243	0.0242	0.0241	0.0240
Uti	Mean	0.0048	0.0062	0.0077	0.0103	0.0114	0.0126	0.0133	0.0136	0.0139
	Std.	0.0093	0.0100	0.0110	0.0123	0.0125	0.0125	0.0128	0.0128	0.0128

CDS Data by Industry

Figure: Term Structure of Average CDS Spreads by Industry



Variations of CDS Spreads Explained by Model

Table: Variance Ratio Summary

	1y	2y	3y	5y	7y	10y	15y	20y	30y
Min	55%	17%	43%	60%	64%	44%	48%	37%	11%
1stQuantile	80%	87%	89%	89%	89%	85%	80%	78%	73%
Median	89%	93%	94%	94%	93%	91%	88%	87%	82%
Mean	81%	89%	91%	92%	91%	88%	85%	83%	79%
3rdQuantile	94%	96%	96%	95%	95%	94%	92%	92%	89%
Max	98%	99%	99%	99%	100%	100%	99%	98%	99%

This table provides distribution of variance ratio, the percentage of variations of CDS spreads explained by the credit risk model, of the 297 firms used in our empirical analysis at 1, 2, 3, 5, 7, 10, 15, 20, and 30 year maturities.

Distribution of Model Parameters

Table: Model Parameters for Default Process

This table reports the distribution of parameter estimates of the 297 firms used in our empirical analysis. Panel (a) is the summary of parameter estimates using full sample period (2005-2008), Panel (b) is the summary of parameter estimates using first sub-sample period (2005-2006)

	(a) Full Sample Summary						(b) First Sub-Sample Summary					
	κ_Z	θ_Z	σ_Z	κ_Z^P	$\exp(-\beta_0)$	ε	κ_Z	θ_Z	σ_Z	κ_Z^P	$\exp(-\beta_0)$	ε
Min	0.0001	0.0080	0.0185	0.0084	0.3679	0.0005	0.0001	0.0260	0.0188	0.0329	0.0000	0.0003
1stQuantile	0.0022	0.4282	0.0455	0.3133	0.4461	0.0009	0.0025	0.4839	0.0620	0.4576	0.1982	0.0006
Median	0.0028	0.5432	0.0693	0.6534	0.5393	0.0017	0.0038	0.9240	0.1046	1.2391	0.5665	0.0010
Mean	0.0110	1.7572	0.1329	1.2472	0.5130	0.0056	0.0077	2.3544	0.1127	1.9424	0.5241	0.0029
3rdQuantile	0.0056	1.1189	0.1353	0.9507	0.5652	0.0037	0.0075	1.9862	0.1493	2.3345	0.8267	0.0023
Max	0.7371	123.65	4.4288	11.342	0.6873	0.0362	0.2165	146.43	0.4411	13.517	0.9535	0.0280

Average Default Rate Under P Measure

Table: Average Estimated $\kappa_Z \theta_Z / \kappa_Z^P$ across Ratings and Sectors

This table reports the average estimated $\frac{\kappa_Z}{\kappa_Z^P}\theta_Z$ which is the mean of default state variable Z_t under P measure. Panel (a) is the result from full sample period (2005-2008), Panel (b) is the result from first sub-sample period (2005-2006)

	(a) Full Sample Result								(b) First Sub-Sample Result							
	AAA	AA	A	BBB	BB	B	CCC	Aver.	AAA	AA	A	BBB	BB	B	CCC	Aver.
BM			0.0016	0.0109	0.0053	0.3933	0.0596	0.0864			0.0030	0.0079	0.0180	0.0045	0.0221	0.0078
CG		0.0006	0.0017	0.0385	0.0240	0.0055	0.0511	0.0218		0.0025	0.0285	0.0024	0.0385	0.0802	0.0076	0.0314
CS		0.0008	0.0020	0.0071	0.0734	0.0414	0.0379	0.0291		0.0026	0.0063	0.0122	0.0243	0.0090	0.1946	0.0196
Fin	0.0007	0.0016	0.0528	0.1319	0.0458		0.3867	0.0779	0.0032	0.0047	0.0081	0.0345	0.0074		0.0033	0.0165
HC		0.0023	0.0017	0.0030	0.0022	0.0034		0.0024		0.0080	0.0066	0.0107	0.0008	0.0051		0.0073
Ind			0.0020	0.0052	0.0053	0.0036	0.0273	0.0044			0.0118	0.0057	0.0058	0.1270	0.0254	0.0152
OG			0.0015	0.0039	0.0124	0.0155		0.0059			0.0040	0.0060	0.0016	0.0042		0.0048
Tec			0.0015	0.0178	0.0095	0.0539		0.0191			0.0050	0.0086	0.0045	0.0738		0.0207
Tel			0.0077	0.0073	0.0071	0.0218		0.0089			0.0017	0.0021	0.0051	0.4307		0.0503
Uti			0.0030	0.0033	0.0581	0.0601		0.0140			0.0130	0.0090	0.0256	0.0015		0.0100
Aver.	0.0007	0.0014	0.0135	0.0265	0.0322	0.0727	0.1001	0.0314	0.0032	0.0044	0.0094	0.0109	0.0201	0.0438	0.0746	0.0177

Average Default Rate Under Q Measure

Table: Average Estimated θ_Z by Rating and Industry

This table reports the average estimated θ_Z which is the mean of default state variable Z_t under Q measure. Panel (a) is the result from full sample period (2005-2008), Panel (b) is the result from first sub-sample period (2005-2006)

(a) Full Sample Result

	AAA	AA	A	BBB	BB	B	CCC	Aver.
BM			0.4459	1.6974	0.7279	3.0345	0.1881	1.4878
CG		0.2787	0.5091	0.9546	2.9236	3.7044	0.2700	1.9467
CS		0.2547	0.5095	1.4168	1.4786	1.4755	2.6547	1.3334
Fin	2.7434	0.2648	0.4625	2.0980	1.0438		2.7703	1.2192
HC		0.6557	0.3059	0.5290	0.3092	4.7409		1.0840
Ind			0.4698	0.7967	2.2839	2.2149	123.65	4.4052
OG			0.4565	0.6899	0.7714	3.3214		0.9804
Tec			0.6577	0.9006	3.6206	1.0110		1.4377
Tel			0.6291	1.8123	1.0393	5.6481		1.8038
Uti			0.5272	0.5996	2.1862	3.1114		1.0003
Aver.	2.7434	0.3056	0.4805	1.1501	1.9640	2.6728	22.0313	1.7572

(b) First Sub-Sample Result

	AAA	AA	A	BBB	BB	B	CCC	Aver.
BM			1.5619	0.8183	1.7551	2.1264	2.7668	1.4487
CG		0.2945	1.1902	0.5824	2.1501	5.6043	0.7684	2.0788
CS		1.6681	1.6955	1.8087	2.3714	5.2998	1.6160	2.7373
Fin	2.0219	0.8488	1.1492	11.0460	1.0663		0.4579	4.5765
HC		1.1106	1.3577	1.4630	0.3853	1.5924		1.3324
Ind			1.9051	1.6657	2.1625	5.6109	0.5541	2.0470
OG			0.7116	1.4033	0.7966	3.4444		1.4385
Tec			2.1909	1.1734	0.4998	5.2130		2.1854
Tel			0.5171	0.7780	1.0062	6.5438		1.4114
Uti			0.8739	0.8446	1.0931	6.4131		1.5108
Aver.	2.0219	0.8460	1.4002	2.4776	1.7712	4.7837	1.2966	2.3544

Average Recovery Rates

Table: Average Estimated Recovery Rate $\exp(-\beta_0)$ by Rating and Sector

This table reports the average estimated recovery rates $\exp(-\beta_0)$ which are assumed to be constant over time.

Panel (a) is the result from full sample period (2005-2008), Panel (b) is the result from first sub-sample period (2005-2006)

(a) Full Sample Result

	AAA	AA	A	BBB	BB	B	CCC	Aver.
BM			0.5622	0.5377	0.4661	0.5392	0.4689	0.5325
CG		0.5398	0.5602	0.5311	0.5119	0.4210	0.3881	0.5052
CS		0.5831	0.5336	0.5017	0.4905	0.4921	0.3859	0.4991
Fin	0.4629	0.5155	0.5509	0.5246	0.5012		0.4197	0.5250
HC		0.5140	0.5232	0.5216	0.4009	0.3679		0.4887
Ind			0.5403	0.5267	0.4033	0.3935	0.3679	0.4989
OG			0.5571	0.5167	0.5172	0.4596		0.5180
Tec			0.5246	0.5615	0.4775	0.4639		0.5120
Tel			0.5684	0.5557	0.5749	0.5016		0.5568
Uti			0.5369	0.5141	0.5031	0.6185		0.5288
Aver.	0.4629	0.5270	0.5453	0.5230	0.4878	0.4764	0.4027	0.5130

(b) First Sub-Sample Result

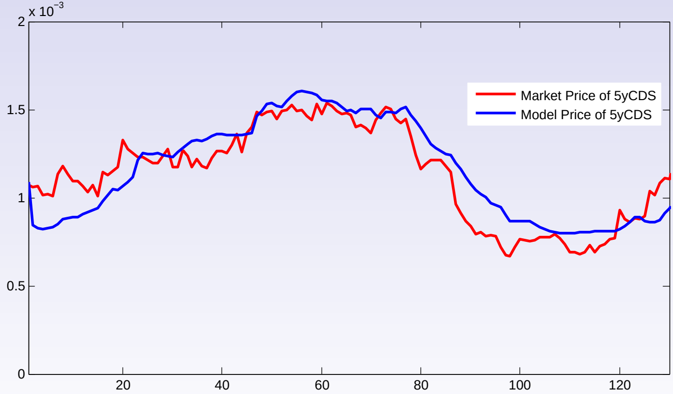
	AAA	AA	A	BBB	BB	B	CCC	Aver.
BM			0.7563	0.5113	0.6399	0.1207	0.7212	0.5158
CG		0.6899	0.7737	0.3798	0.3353	0.3015	0.1077	0.4109
CS		0.9246	0.8096	0.6131	0.3374	0.2793	0.4109	0.5040
Fin	0.8416	0.6096	0.6864	0.5129	0.3601		0.4143	0.5857
HC		0.8482	0.8553	0.7620	0.4325	0.0013		0.6621
Ind			0.7022	0.5777	0.4766	0.4223	0.0535	0.5840
OG			0.6345	0.6160	0.3929	0.2045		0.5405
Tec			0.6816	0.5414	0.2552	0.5958		0.5318
Tel			0.6636	0.5009	0.0322	0.6086		0.4449
Uti			0.7384	0.5841	0.4702	0.0098		0.5328
Aver.	0.8416	0.6810	0.7289	0.5517	0.3682	0.2643	0.3531	0.5241

Strategy Design

- Naive strategy: Fit the model to market CDS spreads as well as possible, then look at the discrepancies between market and model prices. Betting that market prices will converge to model prices, we long (short) under (over) valued CDS contracts. But this does NOT work!
- Market-neutral strategy: Construct market-neutral portfolios of CDS contracts that are immune to both first and second order changes in the default state variable Z_t. Then we would trade the hedged portfolios to take advantage of pricing inefficiency.

An Illustration of the Naive Strategy

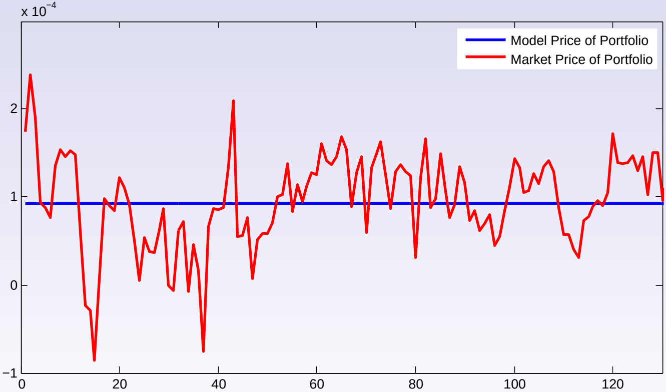
Figure: Model CDS v.s. Market CDS



This figure provides partial time series plots of the Market CDS and Model CDS at 5yr maturity of Colgate Palmolive Co

An Illustration of the Market-Neutral Strategy

Figure: Model Portfolio v.s. Market Portfolio



This figure provides partial time series plots of the Market Portfolio and Model Portfolio at 5yr maturity of Colgate Palmolive Co

Construction of Market-Neutral Portfolio

- Consider a second order expansion of the CDS pricing function at the backed-out state variable $\hat{Z}_t$ with the following first two derivatives $H_1^{\tau}(t) = \frac{\partial \text{CDS}^{\tau}}{\partial Z_t}\Big|_{Z_t=\hat{Z}_t}$ and $H_2^{\tau}(t) = \frac{\partial^2 \text{CDS}^{\tau}}{\partial Z_t^2}\Big|_{Z_t=\hat{Z}_t}$.
- Combine a CDS with maturity τ_0 with two other CDSs with maturities τ_1 and τ_2 to form a hedged portfolio.
- By choosing the appropriate weights of the CDS contracts, we hedge away fluctuations in the value of the portfolio due to changes in Z_t up to the second order.

Market-Neutral Portfolio Weights

- The weights of the other two CDS contracts, $m_1(t)$ and $m_2(t)$, are given as

$$m_1(t) = \frac{H_2^{\tau_0}(t)\,H_1^{\tau_2}(t) - H_1^{\tau_0}(t)\,H_2^{\tau_2}(t)}{H_1^{\tau_1}(t)\,H_2^{\tau_2}(t) - H_2^{\tau_1}(t)\,H_1^{\tau_2}(t)},$$
$$m_2(t) = \frac{H_2^{\tau_0}(t)\,H_1^{\tau_1}(t) - H_1^{\tau_0}(t)\,H_2^{\tau_1}(t)}{H_1^{\tau_2}(t)\,H_2^{\tau_1}(t) - H_2^{\tau_2}(t)\,H_1^{\tau_1}(t)}.$$

- Consequently, the change in the value of the hedged portfolio should be both delta- and gamma-neutral to changes in Z_t.
- To achieve best hedging performance, we choose τ_1 and τ_2 as the two closest maturities to τ_0, e.g., if $\tau_0 = 1$, then $\tau_1 = 2$, and $\tau_2 = 3$; if $\tau_0 = 7$, then $\tau_1 = 5$, and $\tau_2 = 10$; and if $\tau_0 = 30$, then $\tau_1 = 15$, and $\tau_2 = 20$.

Arbitrage Performance

Table: Summary of Strategy Performance

This table reports Minimum, Median, Mean, 1st&3rd Quartiles, and Maximum of 3 important performance measures: Accumulative Profit(Accum.), Sharpe Ratio, and Max Drawdown(MDD). Panel(a) is the in sample result, Panel(b) is the out of sample result

(a) In Sample Result

	Accum.	Sharpe	MDD
Min	0.3429	-4.6183	0.09%
1stQ	1.4885	1.5768	0.84%
Median	2.0763	2.1575	1.16%
Mean	2.5093	2.0890	1.57%
3rdQ	2.8655	2.6424	1.86%
Max	14.7611	4.4609	8.99%

(b) Out of Sample Result

	Accum.	Sharpe	MDD
Min	0.2516	-3.4593	0.10%
1stQ	1.1825	1.2944	0.61%
Median	1.7791	2.0630	0.98%
Mean	2.2467	1.8793	1.45%
3rdQ	2.7860	2.5670	1.55%
Max	14.0332	4.3198	17.49%

Arbitrage Accumulative Profits by Rating and Industry

Table: Average of Accumulative Profits by Rating and Industry

This table reports average of accumulative profits distribution across rating and sectors, Panel(a) is the in sample result, Panel(b) is the out of sample result

(a) In Sample Result

	AAA	AA	A	BBB	BB	B	CCC	Aver.
BM			1.5009	1.7233	4.6531	2.8971	14.7611	2.7777
CG		1.6386	1.9535	2.0978	2.0882	5.0524	7.7142	2.7565
CS		0.9860	1.8132	2.4461	3.3465	4.0862	4.8683	2.9933
Fin	0.8950	2.2107	2.0378	2.4996	2.5342		2.3221	2.2227
HC		0.9449	1.4488	1.9696	2.1362	2.3618		1.7636
Ind			1.9689	2.4235	4.4776	5.7950	2.7210	2.8179
OG			1.8497	1.6589	2.0274	1.9919		1.7863
Tec			1.7230	2.2787	3.2227	5.1235		2.9318
Tel			2.0277	2.3961	3.0945	1.7483		2.3975
Uti			1.2296	1.6944	2.7375	1.8505		1.7212
Aver.	0.8950	1.8472	1.8177	2.1230	3.0098	3.8545	6.2092	2.5093

(b) Out of Sample Result

	AAA	AA	A	BBB	BB	B	CCC	Aver.
BM			1.1039	1.5688	3.5429	5.6203	4.8224	2.6345
CG		1.2007	1.4863	2.1653	2.1549	4.2058	6.0833	2.5163
CS		0.5916	1.1331	1.9001	3.6959	4.3283	2.6993	2.7422
Fin	0.7613	1.7794	1.6173	2.0930	4.0508		2.2903	2.0081
HC		0.8059	0.8500	1.2109	1.8140	2.6416		1.3074
Ind			1.4789	1.9827	3.1345	5.6116	2.5728	2.2176
OG			1.5218	1.2883	2.1015	2.3053		1.5657
Tec			1.3900	2.3063	3.4783	2.9043		2.4238
Tel			1.8209	2.2996	4.2478	1.2452		2.5090
Uti			0.9507	1.3588	2.9215	3.9449		1.7146
Aver.	0.7613	1.4475	1.3653	1.7992	3.0666	4.0838	3.5279	2.2467

Average Sharpe Ratio by Rating and Industry

Table: Average of Sharpe Ratios Across Rating and Sectors

This table reports average of Sharpe ratios distribution across rating and sectors, Panel(a) is the in sample result, Panel(b) is the out of sample result

(a) In Sample Result

	AAA	AA	A	BBB	BB	B	CCC	Aver.
BM			1.6520	1.9278	2.1097	1.7564	2.6436	1.8778
CG		1.6442	1.9896	2.0634	1.8116	2.8300	3.1743	2.1385
CS		1.2891	2.0641	2.5359	2.7529	2.3001	2.1903	2.4175
Fin	0.3712	1.6742	1.5973	2.2787	2.0249		1.7130	1.8312
HC		0.9483	1.7021	1.9645	1.1250	2.3036		1.7730
Ind			1.8744	2.4576	3.0184	2.4483	3.2454	2.3844
OG			2.2582	1.7363	2.2625	1.7297		1.9100
Tec			1.3480	2.6522	2.2215	2.0189		2.0516
Tel			2.1223	2.1753	3.1749	1.6904		2.3318
Uti			1.1005	1.8008	3.2397	1.7120		1.7935
Aver.	0.3712	1.5571	1.7710	2.1561	2.3704	2.2373	2.5261	2.0890

(b) Out of Sample Result

	AAA	AA	A	BBB	BB	B	CCC	Aver.
BM			0.8930	1.7946	1.8387	2.1581	2.4812	1.6837
CG		1.0654	1.7556	2.1086	2.4238	2.5959	3.0154	2.2213
CS		-0.37	1.1108	2.2688	2.6648	2.2363	2.3778	2.1252
Fin	0.1349	1.4143	1.4562	1.7454	2.4791		1.6167	1.5887
HC		0.6878	0.9602	1.2115	0.9592	2.6846		1.2818
Ind			1.4200	2.1564	3.0072	1.8399	2.9237	2.0504
OG			1.9905	1.3173	2.4263	1.9783		1.6788
Tec			0.9119	2.7571	2.2747	2.4254		2.0554
Tel			2.0577	2.0999	3.1950	1.2203		2.2362
Uti			0.0778	1.3819	3.5323	2.2080		1.4420
Aver.	0.1349	1.0931	1.2909	1.8654	2.5587	2.2764	2.4654	1.8793

Average Max Drawdown by Rating and Industry

Table: Average of Max Drawdown Across Rating and Sectors

This table reports average of Max Drawdown distribution across rating and sectors, Panel(a) is the in sample result, Panel(b) is the out of sample result

(a) In Sample Result

	AAA	AA	A	BBB	BB	B	CCC	Aver.
BM			1.08%	1.52%	1.61%	2.21%	1.66%	1.57%
CG		1.10%	1.28%	1.45%	1.14%	2.49%	2.95%	1.56%
CS		0.43%	1.01%	1.34%	1.71%	2.31%	6.13%	1.76%
Fin	3.12%	2.08%	1.69%	1.71%	1.61%		1.14%	1.80%
HC		0.35%	1.15%	1.48%	4.42%	1.24%		1.46%
Ind			1.47%	1.18%	1.43%	0.89%	7.03%	1.45%
OG			0.83%	1.32%	0.96%	1.81%		1.24%
Tec			1.39%	0.87%	1.84%	3.32%		1.75%
Tel			0.87%	2.00%	1.12%	1.23%		1.47%
Uti			1.54%	1.03%	0.85%	2.56%		1.27%
Aver.	3.12%	1.55%	1.32%	1.36%	1.47%	2.24%	4.17%	1.57%

(b) Out of Sample Result

	AAA	AA	A	BBB	BB	B	CCC	Aver.
BM			0.82%	1.29%	1.01%	4.40%	1.11%	1.76%
CG		0.99%	1.15%	1.46%	1.17%	1.80%	2.65%	1.41%
CS		0.32%	0.70%	0.89%	2.68%	3.50%	5.05%	2.00%
Fin	0.80%	1.46%	1.37%	1.25%	2.28%		1.02%	1.39%
HC		0.24%	0.37%	0.67%	3.86%	1.60%		0.91%
Ind			1.22%	0.83%	1.05%	0.63%	7.92%	1.16%
OG			0.70%	0.98%	1.11%	2.55%		1.13%
Tec			0.86%	0.82%	2.21%	1.45%		1.26%
Tel			0.47%	1.81%	1.61%	1.50%		1.43%
Uti			0.91%	0.74%	0.84%	4.39%		1.19%
Aver.	0.80%	1.13%	0.98%	1.05%	1.70%	2.80%	3.80%	1.45%

Arbitrage Performance for 10 Firms

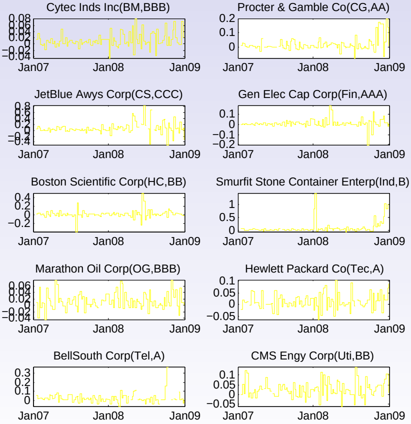
Table: The P&L of the Statistical Arbitrage Strategy for 10 Firms

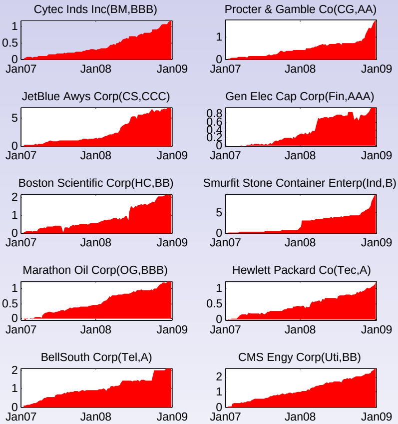
This table provides summary statistics of P&L for the statistical arbitrage strategies for ten individual firms, which include the max, min, mean, standard deviation, accumulated profit, Sharpe ratio, and Max Drawdown. Panel(a) is the in sample result, Panel(b) is the out of sample result

			(a) In Sample Result							(b) Out of Sample Result						
	Sector	Rating	Max	Min	Aver.	Std.	Accum.	Sharpe	MDD	Max	Min	Aver.	Std.	Accum.	Sharpe	MDD
Cytec Inds Inc	BM	BBB	0.08	-0.04	0.01	0.02	1.20	1.68	0.5%	0.07	-0.04	0.01	0.02	1.05	1.43	0.4%
Procter & Gamble Co	CG	AA	0.20	-0.08	0.02	0.05	1.75	1.78	0.8%	0.07	-0.04	0.01	0.02	0.83	0.64	0.4%
JetBlue Awys Corp	CS	CCC	0.85	-0.49	0.07	0.19	6.69	2.35	3.3%	0.26	-0.11	0.02	0.05	2.37	2.40	1.1%
Gen Elec Cap Corp	Fin	AAA	0.19	-0.20	0.01	0.04	0.96	0.50	2.3%	0.17	-0.12	0.01	0.03	0.95	0.63	1.3%
Boston Scientific Corp	HC	BB	0.51	-0.38	0.02	0.09	2.14	1.12	4.4%	0.46	-0.36	0.02	0.08	1.81	0.96	3.9%
Smurfit Stone Cont.	Ind	B	1.45	-0.06	0.10	0.24	9.61	2.78	0.7%	1.46	-0.07	0.11	0.27	10.29	2.67	0.8%
Marathon Oil Corp	OG	BBB	0.08	-0.04	0.01	0.02	1.24	1.70	0.5%	0.07	-0.03	0.01	0.02	1.06	1.37	0.4%
Hewlett Packard Co	Tec	A	0.11	-0.06	0.01	0.03	1.25	1.38	0.7%	0.05	-0.03	0.01	0.02	0.88	1.03	0.3%
BellSouth Corp	Tel	A	0.38	-0.07	0.02	0.05	2.08	2.13	1.0%	0.24	-0.03	0.02	0.03	1.42	1.81	0.3%
CMS Engy Corp	Uti	BB	0.14	-0.06	0.03	0.04	2.64	3.63	0.6%	0.16	-0.06	0.03	0.05	3.04	3.87	0.7%

Figure: In Sample P&L and Accumulative Profits

This figure provides the in sample P&L time series and paths of accumulated profits of the statistical arbitrage strategy for 10 firms between January 17, 2007 and December 31, 2008. Left two panels present P&L time series; right two panels present paths of accumulative profits

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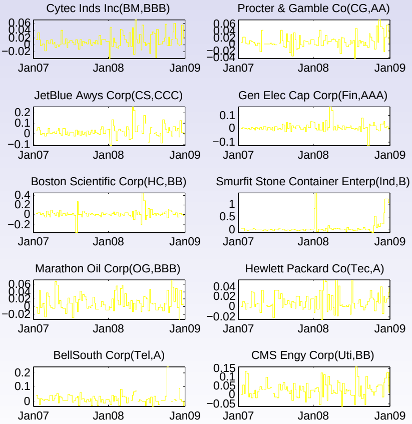
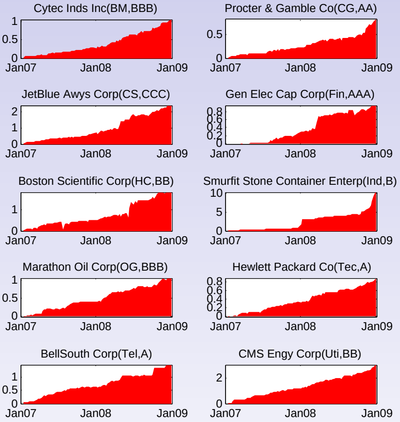
Procter & Gamble Co(CG,AA)
JetBlue Awys Corp(CS,CCC)
Gen Elec Cap Corp(Fin,AAA)
Boston Scientific Corp(HC,BB)
Smurfit Stone Container Enterp(Ind,B)
Marathon Oil Corp(OG,BBB)
Hewlett Packard Co(Tec,A)
BellSouth Corp(Tel,A)
CMS Engy Corp(Uti,BB)
Jan07
Jan08
Jan09

Figure: Out of Sample P&L and Accumulative Profits

This figure provides the out of sample P&L time series and paths of accumulated profits of the statistical arbitrage strategy for 10 firms between January 17, 2007 and December 31, 2008. Left two panels present P&L time series; right two panels present paths of accumulative profits




Cytec Inds Inc(BM,BBB)
Procter & Gamble Co(CG,AA)
JetBlue Awys Corp(CS,CCC)
Gen Elec Cap Corp(Fin,AAA)
Boston Scientific Corp(HC,BB)
Smurfit Stone Container Enterp(Ind,B)
Marathon Oil Corp(OG,BBB)
Hewlett Packard Co(Tec,A)
BellSouth Corp(Tel,A)
CMS Engy Corp(Uti,BB)
Jan07
Jan08
Jan09

Conclusion and Caveats

- We have developed market-neutral strategies to explore potential "arbitrage" opportunities in the term structure of CDS spreads
- Our strategy performs well both in sample and out of sample and achieves high Sharpe ratios
- We have not explicitly accounted for bid-ask spreads, transactions costs, and liquidity concerns, which could eat into our profits.
- Nonetheless, the impressive Sharpe ratios our strategy generates do point out great potentials for “statistical” arbitrage in the term structure of CDS spreads.