Jumps and Information Flow in Financial Markets

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- Historical evidence of two types of risks: jump and volatility
- Time varying hedging demand from investors
- Fits better data over time
- Time-varying excess kurtosis and skewness (higher moments)
- Time-varying implied volatility smiles and smirks in option markets
- Time-varying degrees of market incompleteness
- $\Rightarrow$ Need for dynamic models separately for jump and volatility risk

- Asset pricing with mean return predictability: Lo and Wang (1995)

- Asset pricing with stochastic volatility: Heston (1993)

- Asset pricing with stochastic volatility and/or jump:
  Merton (1976), Bakshi, Cao, and Chen (1997), Bates (1996), Duffie, Pan, and Singleton (2000), Ait-Sahalia (2002, 2004), Andersen, Benzoni, and Lund (2002), Chernov, Gallant, Ghysels, and Tauchen (2003), Eraker, Johannes, and Polson (2003), Carr and Wu (2003,2004), Li, Wells, and Yu (2008)

- *Asset pricing with jump event:*
  Piazzesi (2003), Dubinsky and Johannes (2006)

Jump Predictor Test Motivation

- Identifying jump predictors
- Sorting predictors in terms of impact size and precision
- Separating systematic/macro jumps from idiosyncratic jumps
- Extending prediction models: Regression model, ARCH, GARCH, Stochastic Volatility, and then, what's next?
- Market timer for high frequency program trading popular in hedge fund industry

Outline

- A new two-stage semi-parametric jump predictor test (JPT)
  - Stage I: Nonparametric jump tests
  - Stage II: Maximum partial likelihood predictor test
- Inference theory and guideline (likelihood for continuous-time jump process within jump diffusion model)
- Monte Carlo simulation study
- Empirical evidence on short-term jump predictors (macro and firm-specific) in U.S. individual equity markets
- Implications for systematic jump components over time

- Under a probability space $(\Omega, \mathcal{F}, P)$: market information
- $S(t)$ Asset price under P (data-generating measure in continuous-time)
- T Time horizon
- n Number of observation within $[0, T]$
- Observe $S(t)$ at $0 = t_0 \leq t_1 \leq .. \leq t_n = T$
- $\Delta t_i = t_i - t_{i-1} = \Delta t = \frac{T}{n}$

$$d \log S(t) = \mu(t)dt + \sigma(t)dW(t) + Y(t)dJ(t) \qquad (1)$$

Theoretical Framework for Jump Predictor Test
Sub-model for Irregular Jump Arrivals

- $J(t) = \int_0^t dJ(s)$: Doubly-stochastic Poisson process (also used in Duffie, Saita, and Wang (2007))
- $\Lambda_\theta(t) = \gamma(t, X(t); \theta)$: Integrated stochastic jump intensity
- θ Effect parameter in Euclidean space
- $X(t)$ Information: $\mathcal{F}_t$-predictable process observed at discrete times $0 = t_0 \leq t_1 \leq .. \leq t_n = T$
- $Y(t)$ Jump size with any distribution with its mean $\mu_y(t)$ and variance $\sigma_y^2(t)$

Assumption C & D for Drift, Diffusion, Jump Intensity

- Very general assumptions
- Drift μ and Diffusion σ do not "dramatically" change over a short period of time, can be stochastic, depend on itself
- $d\Lambda_\theta(t)$ should be continuous & 3 times differentiable to apply martingale central limit theorem, etc.

Intuition for Stage I
Nonparametric test by Lee & Mykland (2008) and Lee & Hannig (2009)
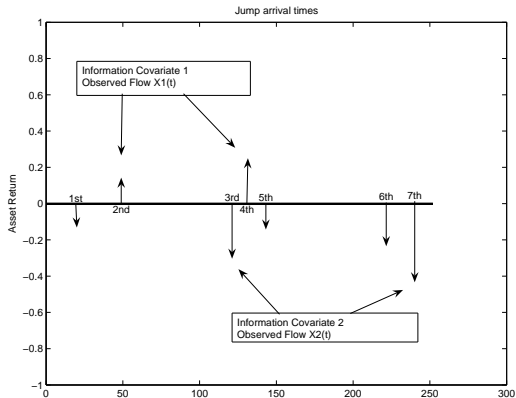
- Assess if the Kth obs in moving widows of size K are jumps
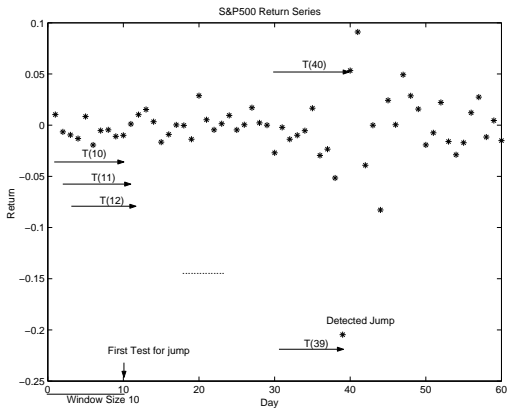
Intuition for Stage II
Maximum partial likelihood test

- Come up with information predictors and functional relationship with jump intensity
- Determine the effect of information predictors on jump arrivals

Jump Detection Tests Admissible for the Stage I

- The statistic $\mathcal{L}(i)$, which tests at time t_i whether there was a jump from t_{i-1} to t_i, is defined as

$$\mathcal{L}(i) \equiv \frac{\log S(t_i)/S(t_{i-1})}{\widehat{\sigma(t_i)}\sqrt{\Delta t}},$$

where $\widehat{\sigma(t_i)}$ can be chosen from one of the following..

-

$$\widehat{\sigma(t_i)}^2 \equiv \frac{1}{(K-2)c^2} \sum_{j=i-K+2}^{i-1} |\log S(t_j)/S(t_{j-1})| |\log S(t_{j-1})/S(t_{j-2})|,$$

where u is a standard normal random variable, $K = b\Delta t^a$ with $-1 < a < -1/2$ for some constant b, and $c = E|u| \approx 0.7979$.

- For any $g > 0$ and $0 < \widetilde{\omega} < 1/2$,

$$\widehat{\sigma(t_i)}^2 \equiv \frac{\Delta t^{-1}}{K} \sum_{j=i-K}^{i-1} \left(\log S(t_j)/S(t_{j-1}) \right)^2 I_{\{|\log S(t_j)/S(t_{j-1})| \leq g\Delta t^{\bar{\omega}}\}},$$

where $K = b\Delta t^a$ with $-1 < a < 0$, for some constant b.

Properties of Admissible Tests
Proposition 1

- $\mathcal{L}(i)$ be as in **Definition 1** and **Assumption C** is satisfied. Then, the following statements hold, as $\Delta t \to 0$.
- A. If there is no jump in $(t_{i-1}, t_i]$, i.e. $dJ(t_i) = J(t_i) - J(t_{i-1}) = 0$, then,

$$\mathcal{L}(i) \xrightarrow{\mathcal{D}} \mathcal{N}(0, 1),$$

  where $\mathcal{N}(0, 1)$ denotes a standard normal random variable.
- B. If there is a jump at τ within $(t_{i-1}, t_i]$, i.e. $dJ(t_i) = J(t_i) - J(t_{i-1}) = 1$, then, $\mathcal{L}(i) \to \infty$.
- C. Let the rejection region for a chosen test be $\mathcal{R}_n(\alpha_n)$. Then,

$$d\hat{J}(t_i) = \hat{J}(t_i) - \hat{J}(t_{i-1}) = I(\mathcal{L}(i) \in \mathcal{R}_n(\alpha_n)) \xrightarrow{P} dJ(t_i) = 1,$$

  for any $(t_{i-1}, t_i]$ with jump and

$$d\hat{J}(t_i) = \hat{J}(t_i) - \hat{J}(t_{i-1}) = I(\mathcal{L}(i) \in \mathcal{R}_n(\alpha_n)) \xrightarrow{P} dJ(t_i) = 0,$$

  for any $(t_{i-1}, t_i]$ without jump.

Three Likelihoods

$J(t)$ Doubly stochastic Poisson process with $\Lambda_\theta(t) = \gamma(t, X(t); \theta)$

- A. True Likelihood of Continuous-time Jump Model

  $$\widetilde{L(\theta|\mathcal{F}_T)} = \widetilde{\prod_{s \in [0,T]}} d\Lambda_\theta(s)^{dJ(s)} \widetilde{\prod_{s \in [0,T]}} (1 - d\Lambda_\theta(s))^{1 - dJ(s)}$$

- B. Full Likelihood of Pure Jump Models without Diffusion

  $$L_n(\theta|\mathcal{F}_T) = \prod_{1 \le i \le n} d\Lambda_\theta(t_i)^{dJ(t_i)} \prod_{1 \le i \le n} (1 - d\Lambda_\theta(t_i))^{1 - dJ(t_i)}$$

- C. Partial Likelihood of Sub-Jump Models in Jump-Diffusion

  $$PL_n(\theta|\mathcal{F}_T) = \prod_{1 \le i \le n} d\hat{\Lambda}_\theta(t_i)^{d\hat{J}(t_i)} \prod_{1 \le i \le n} (1 - d\hat{\Lambda}_\theta(t_i))^{1 - d\hat{J}(t_i)},$$

  where $d\hat{\Lambda}_\theta(t_i) = E[I_{\{\mathcal{L}(i) \in \mathcal{R}_n(\alpha_n)\}}]$ and $d\hat{J}(t_i) = I_{\{\mathcal{L}(i) \in \mathcal{R}_n(\alpha_n)\}}$, with $\mathcal{L}(i)$, $\mathcal{R}_n(\alpha_n)$, and α_n as in **Proposition 1**.

- *Suppose that **Assumptions C** and **D** hold.*

- *Let $L_n(\theta|\mathcal{F}_T)$ and $PL_n(\theta|\mathcal{F}_T)$ be as in **Definition 3.B** and **3.C** with $\mathcal{F}_T$ being the information filtration up to time T. The test used in Stage I satisfies the properties of admissible tests.*

- *Then, as $\Delta t \to 0$ and $\alpha_n \to 0$,*

$$\frac{PL_n(\theta|\mathcal{F}_T)}{L_n(\theta|\mathcal{F}_T)} \xrightarrow{P} 1,$$

when there are the finite number of jumps during the time horizon $[0, T]$.

Partial Likelihood is Sufficient!
Proposition 3

- *Suppose **Assumptions C** and **D** hold.*

- *Let $\widetilde{L(\theta|\mathcal{F}_T)}$ and $PL_n(\theta|\mathcal{F}_T)$ be as in **Definition 3.A** and **3.C** with $\mathcal{F}_T$ being the information filtration up to time T. The test used in Stage I satisfies the properties of admissible tests in **Proposition 1**.*

- *Then, as $\Delta t \to 0$ and $\alpha_n \to 0$,*

$$\frac{PL_n(\theta|\mathcal{F}_T)}{\widetilde{L(\theta|\mathcal{F}_T)}} \xrightarrow{P} 1,$$

when there are the finite number of jumps during the time horizon $[0, T]$.

Jump Predictor Test (JPT)
Theorem 1

- *Suppose that **Assumptions C** and **D** hold.*

- $X(t) = [X_1(t), X_2(t), .., X_p(t)]$: *jump predictor that affect $\Lambda_\theta(t)$*

- $\hat{\theta} = [\hat{\theta}_1, ..., \hat{\theta}_p]$: *MLE for θ based on $PL_n(\theta|\mathcal{F}_T)$ function as in* **Definition 3.C**. *As $\Delta t \to 0$, $\hat{\theta} \xrightarrow{\mathcal{D}} \mathcal{N}(0, Var(\theta))$ under the null.*

- *A. $X_k(t)$ is a good jump predictor if $Prob(z > \frac{\hat{\theta}_k}{SE(\hat{\theta}_k)}) < \beta$, with β significance level and z is a standard normal random variable.*

- *B. The prediction error for jump intensity, $d\hat{\Lambda}_\theta(t) - d\Lambda_\theta(t)$*

$$d\hat{\Lambda}_\theta(t) - d\Lambda_\theta(t) \xrightarrow{\mathcal{D}} \mathcal{N}\left(0, \triangledown d\Lambda_\theta' \mathcal{Z}^{-1}(\theta) \triangledown d\Lambda_\theta\right),$$

where $\triangledown d\Lambda_\theta$ is the partial derivatives of $d\Lambda_\theta(t)$ with respect to θ.

Simulation Analysis
$d \log S(t) = \mu(t)dt + \sigma(t)dW(t) + Y(t)dJ(t)$

- $\sigma(t) = \sigma = 30\%$, Jump size standard deviation $\sigma_y(t)$
- $X(t)$ Monthly information release
- data sampled every 15 minute over 1 year horizon
- $d\Lambda_\theta(t) = \frac{1}{1+\exp(-\theta_0 - \theta_1 X(t))}$ with $\theta_0 = -4$ and $\theta_1 = 6$

	Stage I by **Definition 1.A**				Stage I by **Definition 1.B**			
σ_y	$\hat{\theta}_0$	$SE(\hat{\theta}_0)$	z_{θ_0}	p-value	$\hat{\theta}_0$	$SE(\hat{\theta}_0)$	z_{θ_0}	p-value
3σ	-4.04	0.04	-81.94	0.00	-4.01	0.04	-82.64	0.00
2σ	-4.05	0.04	-81.77	0.00	-4.02	0.04	-82.45	0.00
1σ	-4.07	0.05	-81.36	0.00	-4.04	0.04	-82.15	0.00
σ_y	$\hat{\theta}_1$	$SE(\hat{\theta}_1)$	z_{θ_1}	p-value	$\hat{\theta}_1$	$SE(\hat{\theta}_1)$	z_{θ_1}	p-value
3σ	5.69	0.82	6.91	2.35e-010	5.83	0.86	6.72	3.88e-010
2σ	5.67	0.81	6.93	2.25e-010	5.80	0.85	6.77	3.47e-010
1σ	5.58	0.79	7.04	1.82e-010	5.74	0.83	6.84	2.83e-010

- Data Source: Trade and Quote(TAQ) database, transaction prices
- Time span: 16 years from January 4, 1993 to December 31, 2008
- U.S. large equities: DJIA component stocks
- Most actively traded in NYSE (active trade filter: 50 trades /day)
- Observation frequency: 15 minutes during NYSE trading hours
- Total 4,017 trading days
- 5% significance level for Stage I
- Outcome: jump arrival date and time (jump size and signs)

- Stock jump predictors derived using about 40 different real-time macroeconomic and firm-specific information
- MARKET : detected jumps in S&P 500 index
- FOMC : Fed's announcement : every 6 weeks
- NONFARM : report on # of jobs at business and government : monthly
- JOBLESS : report on # of people filed for unemployment benefits : weekly
- EARNINGS : earnings release and revisions : quarterly
- ANALYST : all types of recommendation changes by all analysts
- CLUSTER : own jumps in the past
- DIVIDEND : ex-dividend date

Empirical Analysis
Table 4: Data for Stage II

- Total: number of observations (e.q. $\int_0^T$ FOMC$(s) = 134$)
- For firm specific variables, cross-sectional averages of total number of observations and standard errors

Macroeconomic	Total	Times	Data Source
MARKET	446	Irregular	Trade and Quote (TAQ)
FOMC	134	$14{:}15^{\diamond}$	Federal Reserve Board & Bloomberg
NONFARM	192	8:30	Bureau of Labor Statistics & Bloomberg
JOBLESS	834	8:30	Employment and Training Administration &
Firm-specific	Total§	Times	Data Source
EARNINGS	70 (10.14)	Irregular	First Call Historical Database
ANALYST	519 (129.85)	Irregular	First Call Historical Database
CLUSTER	348 (45.30)	Irregular	Trade and Quote (TAQ)
DIVIDEND	190 (23.79)§§	Irregular	Center for Research in Security Prices (CRSP

- **Macroeconomic information**: GDP advance, GDP preliminary, GDP final, retail sales, industrial production, capacity utilization, personal income, consumer credit (15:00), personal consumption expenditures, new home sales, durable goods orders, construction spending, factory orders, business inventory, government budget deficit (14:00), trade balance, producer price index, consumer price index, consumer confidence, housing starts, NAPM Index, leading indicator

- **Firm-specific information**: earnings estimates issued by brokers contributing to First Call Historical Database, dividend announcement, date of record, and payout dates and stock split related dates from CRSP database

- Some cases were significant but not broad enough

Empirical Analysis

A Parsimonious jump intensity model for a company c

- $d\Lambda_\theta(t) = \frac{1}{1+\exp(-\theta_0 - \sum_{j=1}^{10} \theta_j X_j(t))}$
- $X_1(t) = I(9:30 \leq h(t) < 10:00)$
- $X_2(t) = I(10:00 \leq h(t) < 11:00)$
- $X_3(t) = I(\int_{t-30min}^{t} \textbf{MARKET}(s) > 0)$
- $X_4(t) = I(\int_{t-30min}^{t} \textbf{FOMC}(s) > 0)$
- $X_5(t) = I(\int_{t-30min}^{t} \textbf{NONFARM}(s) > 0)$
- $X_6(t) = I(\int_{t-30min}^{t} \textbf{JOBLESS}(s) > 0)$
- $X_7(t) = I(\int_{t}^{t+1day} \textbf{EARNINGS}_c(s) > 0)$
- $X_8(t) = I(\int_{t-30min}^{t} \textbf{ANALYST}_c(s) > 0)$
- $X_9(t) = I(\int_{t-3hour}^{t} \textbf{CLUSTER}_c(s) > 0)$
- $X_{10}(t) = I(\textbf{DIVIDEND}_c(t) \times (X_1(t) + X_2(t)) > 0)$
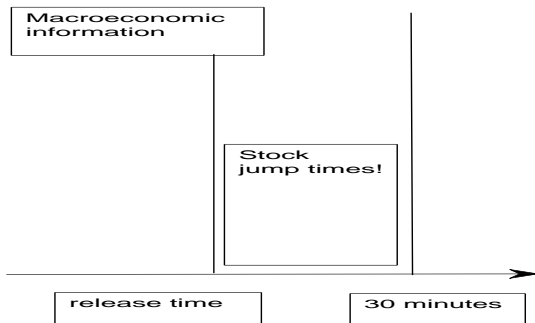
Jump Predictor Test Results

Table 5: $d\Lambda_\theta(t) = \frac{1}{1+\exp(-\theta_0 - \sum_{j=1}^{10} \theta_j X_j(t))}$

Ticker	MARKE	FOMC	NONFA	JOBLE	EARNI	ANALY	CLUST	DIVID
AA	1.532***	3.650***	1.233***	0.988***	1.403***	1.298***	0.953***	-0.529
AXP	1.956***	3.166***	1.487***	0.765***	1.493***	1.122***	0.928***	1.199***
BA	1.108***	2.191***	1.061***	0.541***	2.227***	1.146***	0.785***	0.199
CAT	1.687***	3.820***	1.234***	0.971***	2.069***	1.296***	0.664***	0.492*
CVX	1.876***	2.984***	0.516	1.063***	1.019***	1.002***	1.032***	2.042***
DD	1.249***	3.925***	1.624***	1.049***	2.364***	1.190***	0.352	1.210***
DIS	1.369***	1.690***	0.982***	1.444	1.304***	1.436***	1.473***	0.691
GE	0.819**	3.690***	0.739**	-0.494	1.654***	0.849***	1.086***	1.200***
HD	1.420***	4.012***	1.372***	0.944***	2.321***	0.849***	0.288	1.094***
HPQ	1.125***	3.382***	0.823***	0.769***	1.429***	1.197***	0.641***	-0.381
IBM	0.794***	3.314***	1.139***	0.588***	1.785***	1.460***	0.069	0.218
JNJ	1.568***	3.326***	1.126***	1.119***	2.183***	0.921***	0.555**	1.008***
JPM	1.679***	3.802***	1.238***	0.948***	1.943***	0.937***	0.292	1.237***
KO	1.654***	1.938***	1.385***	0.654***	2.285***	1.494***	0.137	0.991**
MCD	1.520***	2.634***	1.006***	0.549***	1.554***	1.021***	1.142***	0.851*
MMM	1.763***	2.391***	1.610***	0.448**	2.399***	1.464***	0.176	0.681
MRK	1.817***	2.426***	1.278***	1.148***	1.824***	0.955***	0.587***	0.902**
PFE	1.314***	2.553***	1.455***	0.919***	1.723***	0.804***	1.033***	1.467***
PG	1.637***	2.798***	0.622	0.686***	2.307***	0.983***	0.762***	0.492
T	2.110***	2.085***	0.988***	0.800***	1.798***	0.949***	0.944***	1.848***
UTX	1.753***	2.735***	1.442***	0.814***	2.225***	1.340***	0.487**	0.561
WMT	1.625***	3.581***	1.456***	1.299***	2.570***	1.153***	-0.464	0.062
XOM	1.934***	3.840***	0.939***	0.512**	1.937***	0.653**	0.156	1.575***
Average	1.535	3.041	1.114	0.764	1.905	1.109	0.568	0.744

- Likely to have jump arrival within first 30 trading minutes after Fed's announcements, overall market jump, employment report and unemployment claim

Distinguishing Systematic/Macro Jumps

Systematic Jump Extraction

- Important implication of having jump predictor test: understanding systematic jump dynamics
- Extracting systematic jump component from estimated model

$$\widehat{\Lambda}_\theta^{systematic}(T) = \int_0^T d\Lambda_\theta(s)|_{\theta_i=\hat{\theta}_i} \text{ for all } i$$

$$- \int_0^T d\Lambda_\theta(s)|_{\theta_i=0} \text{ for } i=3,4,5,6, \text{ and } \theta_i=\hat{\theta}_i \text{ for } i=1,2,7,8,9,10$$

where the instantaneous jump intensity is set up as

$$d\Lambda_\theta(t) = \frac{1}{1 + \exp(-\theta_0 - \sum_{j=1}^{10} \theta_j X_j(t))}$$

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Distinguishing Systematic/Macro Jumps

Table 7 using DJIA stock transaction prices from 1993 to 2008

Sample Period	1993-2008		1993-2000		2001-2008	
Ticker	Systematic	Idiosyncratic	Systematic	Idiosyncratic	Systematic	Idiosyncratic
AA	0.1719	0.8281	0.1659	0.8341	0.1777	0.8223
AXP	0.1788	0.8212	0.1648	0.8352	0.1920	0.8080
BA	0.0898	0.9102	0.0873	0.9127	0.0921	0.9079
CAT	0.1763	0.8237	0.1662	0.8338	0.1861	0.8139
CVX	0.1695	0.8305	0.1636	0.8364	0.1753	0.8247
DD	0.1738	0.8262	0.1641	0.8359	0.1832	0.8168
DIS	0.0794	0.9206	0.0704	0.9296	0.0885	0.9115
GE	0.0274	0.9726	0.0170	0.9830	0.0369	0.9631
HD	0.1609	0.8391	0.1542	0.8458	0.1673	0.8327
HPQ	0.1155	0.8845	0.1067	0.8933	0.1241	0.8759
IBM	0.0932	0.9068	0.0872	0.9128	0.0989	0.9011
JNJ	0.1693	0.8307	0.1589	0.8411	0.1794	0.8206
JPM	0.1729	0.8271	0.1561	0.8439	0.1889	0.8111
KO	0.1382	0.8618	0.1288	0.8712	0.1476	0.8524
MCD	0.1082	0.8918	0.1036	0.8964	0.1127	0.8873
MMM	0.1294	0.8706	0.1258	0.8742	0.1329	0.8671
MRK	0.1822	0.8178	0.1785	0.8215	0.1860	0.8140
PFE	0.1465	0.8535	0.1402	0.8598	0.1527	0.8473
PG	0.1206	0.8794	0.1188	0.8812	0.1224	0.8776
T	0.1700	0.8300	0.1604	0.8396	0.1789	0.8211
UTX	0.1539	0.8461	0.1470	0.8530	0.1606	0.8394
WMT	0.2028	0.7972	0.1882	0.8118	0.2167	0.7833
XOM	0.1424	0.8576	0.1290	0.8710	0.1551	0.8449
Average	0.1423	0.8577	0.1340	0.8660	0.1503	0.8497

Summary

- New jump predictor test and its inference theory
- Unique empirical evidence uncovered by the powerful test
- Macroeconomic predictors are found to be in general more effective in equity markets.
- Best equity jump predictors related to macroeconomic and firm specific information
- We need to take into account increased systematic jumps in recent years.