Generalized Disappointment Aversion, Long-Run Volatility Risk and Asset Prices

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Motivation

- The CCAPM has been recently revived by models of long-run risks.
- Bansal and Yaron (2004) (BY) explain several asset market stylized facts by a model with
  - a small long-run predictable component driving consumption and dividend growth
  - persistent economic uncertainty measured by consumption volatility
  - Kreps-Porteus (Epstein and Zin 1989) preferences (KP).
    - With expected utility, only short-run risks are compensated.
    - With KP, preferences for early resolution of uncertainty generate positive risk premium for long-run risks.

Motivation

- The existence of a long-run risk component in expected consumption growth is a source of debate:
  - it is hard to detect statistically by univariate methods - consumption resembles a random walk;
  - the effect on asset prices depends on investors detecting it;
  - it makes (counterfactually) consumption growth predictable by the price-dividend ratio.
- A more recent calibration Bansal, Kiku and Yaron (2007) (BKY) shifts the weight towards the second source of long-run risk - persistent volatility - reducing the predicatility of consumption growth.
  - In their model, the two sources interact, but the volatility risk is not priced when expected consumption growth is not persistent (BKY 2009).

This Paper - Model

- We propose a model with following ingredients:

  preferences generalized disappointment aversion (GDA) preferences embedded in Epstein-Zin recursive preferences.

  endowment process a randow walk with persistent stochastic volatility.

This Paper - Preferences

- GDA Preferences:
  - a generalization of Gul's (1991) disappointment aversion preferences introduced by Routledge and Zin (2009).
    - it overweights outcomes below a threshold - κ times the certainty equivalent.
  - it has some implications similar to those of loss aversion preferences (Barberis, Huang and Santos, 2001), but they are built from rational axioms.
  - embedded in Epstein-Zin recursive utility utility framework - potential to price long-run risks.
  - the kink makes it specially sensitive to volatility risks.
- Interaction of long-run volatility risks and GDA preferences generates interesting asset pricing dynamics.

This Paper - Solution Method

- Technical problem: the kink prevent us from using Campbell-Shiller approximation method used on BY and BKY

- Solution method: we approximate the endowment process with a Markov switching model.

- We derive closed formula solutions for all returns moments, coefficients and R^2 of predictability regressions.

- We produce graphs with the effects of continuous variations of endowment and preference parameters of interest on asset pricing statistics.

This Paper - Results

- We are able to generate asset returns moments and predictability in line with the data. Compared to BKY we generate:
  - more predictability of excess returns by price-dividend ratios.
  - less predictability of consumption growth rates by price-dividend ratios
- Differently from BY model, our results do not depend on IES being greater than one.
- Our results are not due to overparametrization of preferences:
  - Simple DA preferences with two parameters, where risk aversion comes only from disappointment aversion generates similar results.

Epstein-Zin Recursive Framework

- Epstein and Zin recursive utility framework:

$$V_t = \left\{ (1-\delta)\, C_t^{1-\frac{1}{\psi}} + \delta \left[\mathcal{R}_t\left(V_{t+1}\right) \right]^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \quad \psi > 0, \quad 0 < \delta < 1 \quad (1)$$

δ is the time preference discount factor

ψ is the elasticity of intertemporal substitution

$\mathcal{R}_t\left(V_{t+1}\right)$ is the certainty equivalent of the random future utility

Kreps-Porteus Preferences

- If the certainty equivalent is that of $CRRA$ expected utility, then Kreps-Porteus utility:

$$\frac{\mathcal{R}^{1-\gamma}}{1-\gamma} = \int_{(-\infty,\infty)} \frac{V^{1-\gamma}}{1-\gamma} dF\left(V\right) \tag{2}$$

- Then, the recursive utility equation becomes:

$$V_t = \left\{ \left(1-\delta\right) C_t^{\frac{1-\gamma}{\theta}} + \delta \left[E_t \left(V_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \quad \psi > 0, \quad 0 < \delta < 1 \tag{3}$$

where

$$\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$$

- BY propose:
  - $\theta > 0$, that is $\gamma > \frac{1}{\psi}$ preference for earlier resolution of uncertainty.
  - $\psi > 1$, $IES > 1$ - substitution effect stronger than income effect.
    - positive innovation in consumption growth has a more important effect than increase in discount rate.

Generalized Disappointment Aversion

Introduced by Routledge and Zin (2009), generalizing Gul (1991):

$$\frac{\mathcal{R}^{1-\gamma}}{1-\gamma} = \int_{(-\infty,\infty)} \frac{V^{1-\gamma}}{1-\gamma} dF(V) + \left(1-\alpha^{-1}\right) \int_{(-\infty,\kappa\mathcal{R})} \left(\frac{V^{1-\gamma}}{1-\gamma} - \frac{(\kappa\mathcal{R})^{1-\gamma}}{1-\gamma}\right) dF(V) \quad \kappa \leq 1 \tag{4}$$

where

- α^{-1} measures the intensity of disappointment aversion
  - disappointment aversion: $\alpha < 1$
  - Kreps-Porteus: $\alpha = 1$.
- κ measures the place of the kink in terms of percentage of the certainty equivalent $\mathcal{R}$.

GDA SDF

$$M_{t,t+1} = z_{t+1}^{1-\gamma} \left(R_{t+1}^{m}\right)^{-1} \left(\frac{1 + \left(\alpha^{-1} - 1\right) I \left(z_{t+1} < \kappa\right)}{1 + \kappa^{1-\gamma} \left(\alpha^{-1} - 1\right) E_t I \left(z_{t+1} < \kappa\right)} \right). \tag{5}$$

where

$$z_{t+1} = \left(\delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} R_{t+1}^{m} \right)^{\frac{1}{1-\frac{1}{\psi}}}$$

R^m is the return on the portfolio that generates the flow of aggregate consumption.

Simple GDA

- Particular case: $\gamma = 0$ and $\psi = \infty$.

$$M_{t,t+1} = \delta \left(\frac{1 + \left(\alpha^{-1} - 1\right) I \left(R^m_{t+1} < \frac{\kappa}{\delta}\right)}{1 + \kappa \left(\alpha^{-1} - 1\right) E_t I \left(R^m_{t+1} < \frac{\kappa}{\delta}\right)} \right)$$

- For each state in t the sdf has only two possible values: one for non-disappointing outcomes and another α^{-1} times greater for disappointing outcomes
  - it generates variability in the sdf - necessary to produce sizeable risk premia.

- The probability of disappointing outcomes may differ for different states.
  - it generates state-dependent risk premium.

State-dependent risk premium: how it works

- Since $E_t\left[R^e_{t+1} - R_f\right] = -\frac{Cov_t\left(M_{t,t+1}, R^e_{t+1}\right)}{E_t\left[M_{t,t+1}\right]}$, and stock market returns are procyclical:
  - in states where disappointing and non-disappointing outcomes have sizeable probabilities:
    - when return on the market portfolio is low, R^e is low and M is high (disappointment). Thus, $Cov_t\left(M_{t,t+1}, R^e_{t+1}\right) << 0$, generating sizeable equity premia.
  - in states where the probability of disappointing outcomes is very small:
    - M is almost a constant and $Cov_t\left(M_{t,t+1}, R^e_{t+1}\right)$ is small.

BY model of long-run risks

$$\Delta c_{t+1} = x_t + \sigma_t \epsilon_{c,t+1} \tag{6}$$
$$\Delta d_{t+1} = (1-\phi_d)\,\mu_x + \phi_d x_t + \nu_d \sigma_t \epsilon_{d,t+1} \tag{7}$$
$$x_{t+1} = (1-\phi_x)\,\mu_x + \phi_x x_t + \nu_x \sigma_t \epsilon_{x,t+1} \tag{8}$$
$$\sigma^2_{t+1} = (1-\phi_\sigma)\,\mu_\sigma + \phi_\sigma \sigma^2_t + \nu_\sigma \epsilon_{\sigma,t+1} \tag{9}$$

ϕ_x, ϕ_σ parameters of special interest.

- BKY assume:

$\phi_x = 0.975$ and $\phi_\sigma = 0.999$ long-run risks

$\phi_d = 2.5$ and $\nu_d = 6.5$ $\nu_x = 0.038$

- We assume $\phi_\sigma = 0.995$ (half life of 11.5 years) as in Lettau, Ludvigson and Wachter (2008), instead of the 0.999 (half life of 58 years) in BKY. The other parameters are as in BKY.

Our model of long-run risk

$$\Delta c_{t+1} = \mu_c + \sigma_t \epsilon_{c,t+1} \tag{10}$$
$$\Delta d_{t+1} = \mu_c + \nu_d \sigma_t \epsilon_{d,t+1} \tag{11}$$
$$\sigma^2_{t+1} = (1 - \phi_\sigma)\, \mu_\sigma + \phi_\sigma \sigma^2_t + \nu_\sigma \epsilon_{\sigma,t+1} \tag{12}$$

Same calibration above with $\phi_x = 0$ and $\nu_x = 0$

LRR and GDA

- Expected utility: $LRRs$ do not matter

$$M_{t,t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}}$$

- Kreps-Porteus: $LRRs$ and $\mathcal{R}_t \left(V_{t+1} \right)$ - depends on $\gamma > \frac{1}{\gamma}$:

$$M_{t,t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}}{\mathcal{R}_t \left(V_{t+1} \right)} \right)^{\frac{1}{\psi} - \gamma}$$

- GDA: additional channel: $LRRs$ and the **kink** - does not depend on ψ.

$$\begin{aligned} M_{t,t+1} &= \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}}{\mathcal{R}_t \left(V_{t+1} \right)} \right)^{\frac{1}{\psi} - \gamma} \\ &\quad \times \left(\frac{1 + \left(\alpha^{-1} - 1 \right) I \left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} < \kappa \right)}{1 + \kappa^{1-\gamma} \left(\alpha^{-1} - 1 \right) E_t I \left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} < \kappa \right)} \right) \end{aligned}$$

Approximating endowment process

- It is not possible to rely on the usual solution techniques based on log linearization used in BY because of the kink in the SDF.

- We sidestep this problem with the following procedure:
  - we approximate the LRR process for consumption and dividends using a Markov Switching process.
  - We derive analytical formulas for:
    - the population moments of asset returns
    - coefficients and R^2 of predictability regressions.

Approximating endowment process

- Let s_t be the Markov state at time t. For BKY process we combine two states in mean and in volatility to obtain four states, $s_t \in \{\mu_L\sigma_L, \mu_L\sigma_H, \mu_H\sigma_L, \mu_H\sigma_H\}$.

$$\Delta c_{t+1} = \mu_c(s_t) \;+\; (\omega_c(s_t))^{1/2}\,\varepsilon_{c,t+1} \tag{13}$$

$$\Delta d_{t+1} = \mu_d(s_t) \;+\; (\omega_d(s_t))^{1/2}\varepsilon_{d,t+1}, \tag{14}$$

where $\varepsilon_{c,t+1}$ and $\varepsilon_{d,t+1}$ follow a bivariate normal process with mean zero and correlation ρ. The states evolve according to the 4 by 4 transition probability matrix P.

- For random walk in mean the process above is reduced to two states in volatility.

Benchmark preference parameters

- $\psi = 1.25$. BY and Lettau, Ludvigson and Watcher (2008) adopt 1.5.

- $\gamma = 3$ and $\alpha = 0.3$ are consistent with estimation of Epstein and Zin (2001).

- $\kappa = 0.989$ as in Routledge and Zin (2009).

	Data	GDA	50%	PV
δ		0.9989		
γ		2.5		
ψ		1.5		
α		0.3		
κ		0.989		
Panel A. Asset Pricing Implications				
$E[R-R_f]$	7.25	7.21	6.14	0.61
$\sigma[R]$	19.52	19.33	16.90	0.45
$E[R_f]-1$	1.21	0.93	1.39	0.62
$\sigma[R_f]$	4.10	2.34	1.84	1.00
$E[P/D]$	30.57	23.30	24.20	1.00
$\sigma[D/P]$	1.52	1.38	1.07	0.79

	Data	GDA	50%	PV
δ		0.9989		
γ		2.5		
ψ		1.5		
α		0.3		
κ		0.989		
Panel B. Predictability of Excess Returns				
$R^2\,(1)$	7.00	12.04	7.44	0.48
$[b\,(1)]$	3.12	5.05	6.25	0.20
$R^2\,(3)$	14.67	28.35	17.27	0.46
$[b\,(3)]$	7.05	14.30	16.91	0.18
$R^2\,(5)$	27.26	38.00	22.47	0.56
$[b\,(5)]$	12.34	22.49	23.14	0.25

	Data	GDA	50%	PV
δ		0.9989		
γ		2.5		
ψ		1.5		
α		0.3		
κ		0.989		
Panel C. Predictability of Consumption Growth				
$R^2\,(1)$	0.06	0	0.76	0.16
$[b\,(1)]$	-0.02	0	0.02	0.47
$R^2\,(3)$	0.09	0	1.67	0.13
$[b\,(3)]$	-0.05	0	0.07	0.46
$R^2\,(5)$	0.24	0	2.23	0.18
$[b\,(5)]$	-0.11	0	0.04	0.47
Panel D. Predictability of Dividend Growth				
$R^2\,(1)$	0.00	0	0.71	0.00
$[b\,(1)]$	0.04	0	0.11	0.49
$R^2\,(3)$	0.20	0	1.44	0.21
$[b\,(3)]$	-0.48	0	0.17	0.46
$R^2\,(5)$	0.08	0	1.75	0.14
$[b\,(5)]$	-0.37	0	-0.48	0.51

Sensitivity to preferences

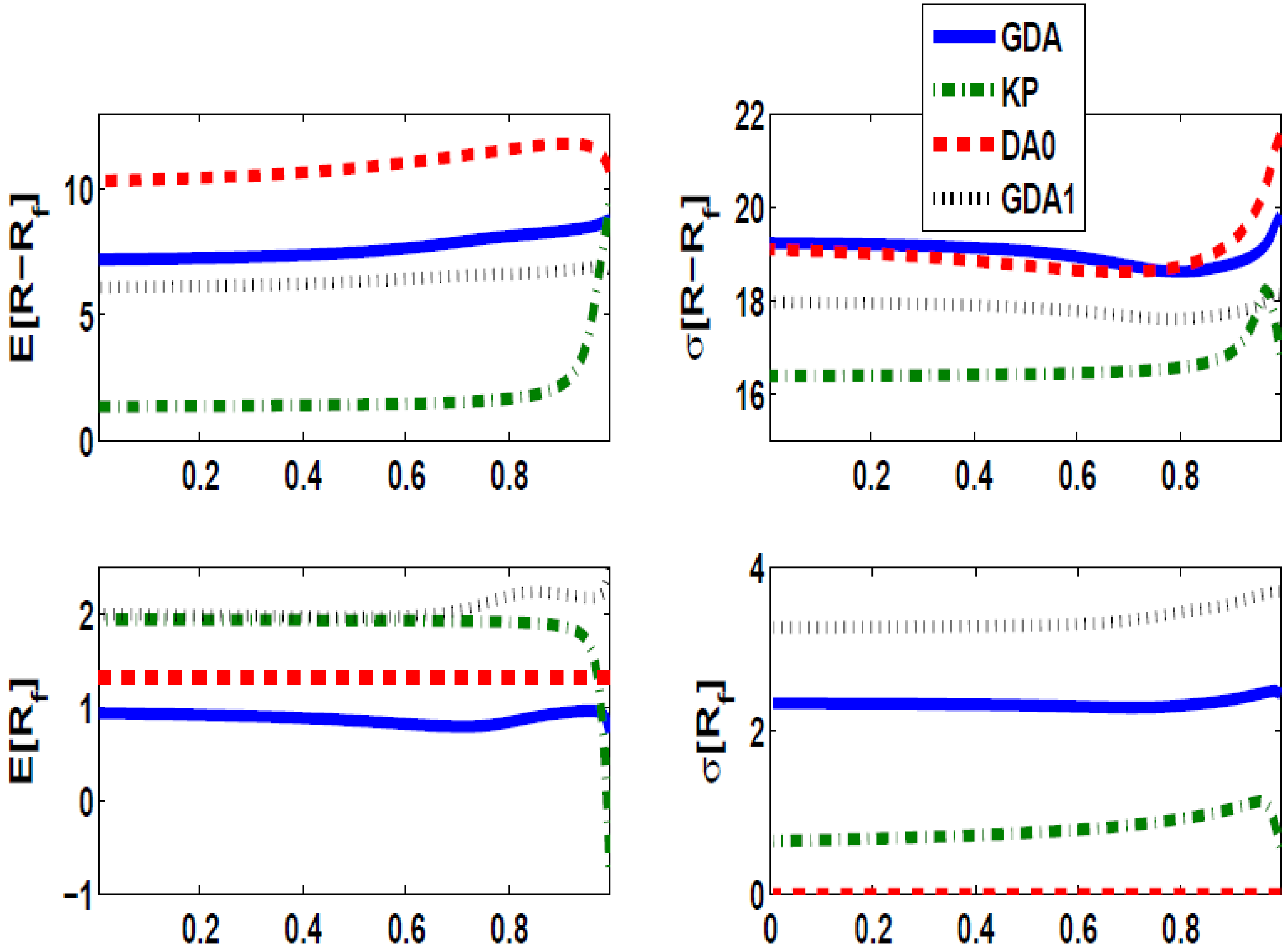
- $GDA1$: $\psi = 0.75 < 1$
  - Does sliglty better than GDA for moments and slightly worse for predictability.
- $DA0$: $\gamma = 0$, $\psi = \infty$, $k = 1$
  - Does surprisingly well for a two parameter preference, specially for predictability.
- KP : $\alpha = 1$, $\gamma = 10$ and $\psi = 1.5$ as BY
  - Too low equity premium and volatility of the risk-free rate. Price-dividend too high.
  - Predictability has wrong signs, but pass statistical tests.

	Data	GDA1	50%	PV	DA0	50%	PV	KP	50%	PV
δ		0.9989			0.9989			0.9989		
γ		2.5			0			10		
ψ		0.75			∞			1.5		
α		0.3			0.3			1		
κ		0.989			1			1		
Panel A. Asset Pricing Implications										
$E[R - R_f]$	7.25	6.12	5.00	0.69	10.32	9.56	0.12	1.42	1.16	0.98
$\sigma[R]$	19.52	18.04	15.75	0.27	19.14	16.94	0.00	16.38	13.96	0.05
$E[R_f] - 1$	1.21	1.97	2.60	0.68	1.32	1.32	0.61	1.93	2.04	0.75
$\sigma[R_f]$	4.10	3.25	2.55	1.00		0.00	1.00	0.59	0.46	1.00
$E[P/D]$	30.57	22.05	22.74	1.00	13.10	13.59	1.00	470.66	467.82	0.00
$\sigma[D/P]$	1.52	1.04	0.81	1.00	2.32	1.80	0.44	0.01	0.00	1.00

	Data	GDA1	50%	PV	DA0	50%	PV	KP	50%	PV
δ		0.9989			0.9989			0.9989		
γ		2.5			0			10		
ψ		0.75			∞			1.5		
α		0.3			0.3			1		
κ		0.989			1			1		
Panel B. Predictability of Excess Returns										
$R^2\,(1)$	7.00	13.53	7.78	0.47	8.00	4.54	0.61	1.29	0.87	0.88
$[b\,(1)]$	3.12	6.70	7.98	0.18	2.38	3.08	0.51	-294.98	-253.63	0.65
$R^2\,(3)$	14.67	30.54	17.33	0.46	19.88	11.10	0.57	3.33	1.69	0.87
$[b\,(3)]$	7.05	18.94	20.94	0.18	6.73	8.43	0.42	-834.28	-712.09	0.65
$R^2\,(5)$	27.26	39.72	21.94	0.56	27.78	14.63	0.67	4.81	2.26	0.92
$[b\,(5)]$	12.34	29.79	28.89	0.24	10.58	11.35	0.54	-1312.45	-807.13	0.61
Panel C. Predictability of Consumption Growth										
$R^2\,(1)$	0.06	0	0.76	0.16	0	0.76	0.16	0	0.75	0.17
$[b\,(1)]$	-0.02	0	0.02	0.47	0	0.01	0.45	0	-3.39	0.52
$R^2\,(3)$	0.09	0	1.68	0.13	0	1.66	0.13	0	1.65	0.13
$[b\,(3)]$	-0.05	0	0.09	0.47	0	0.04	0.45	0	-14.53	0.52
$R^2\,(5)$	0.24	0	2.23	0.18	0	2.23	0.18	0	2.24	0.18
$[b\,(5)]$	-0.11	0	0.05	0.48	0	0.02	0.46	0	-9.98	0.51

BY LRRs

- We compare all preferences used above in BY environment:
  - GDA preferences do at least as well as above.
  - KP has good performance for moments, as we know from BY.
  - KP has difficulties in generating predictability for excess returns and tends to generate excess predictability for consumption growth, although it is not always rejected in small sample test.



GDA
KP
DA0
GDA1
E[R−R_f]
σ[R−R_f]
E[R_f]
σ[R_f]
0
5
10
16
18
20
22
-1
1
2
4
0.2
0.4
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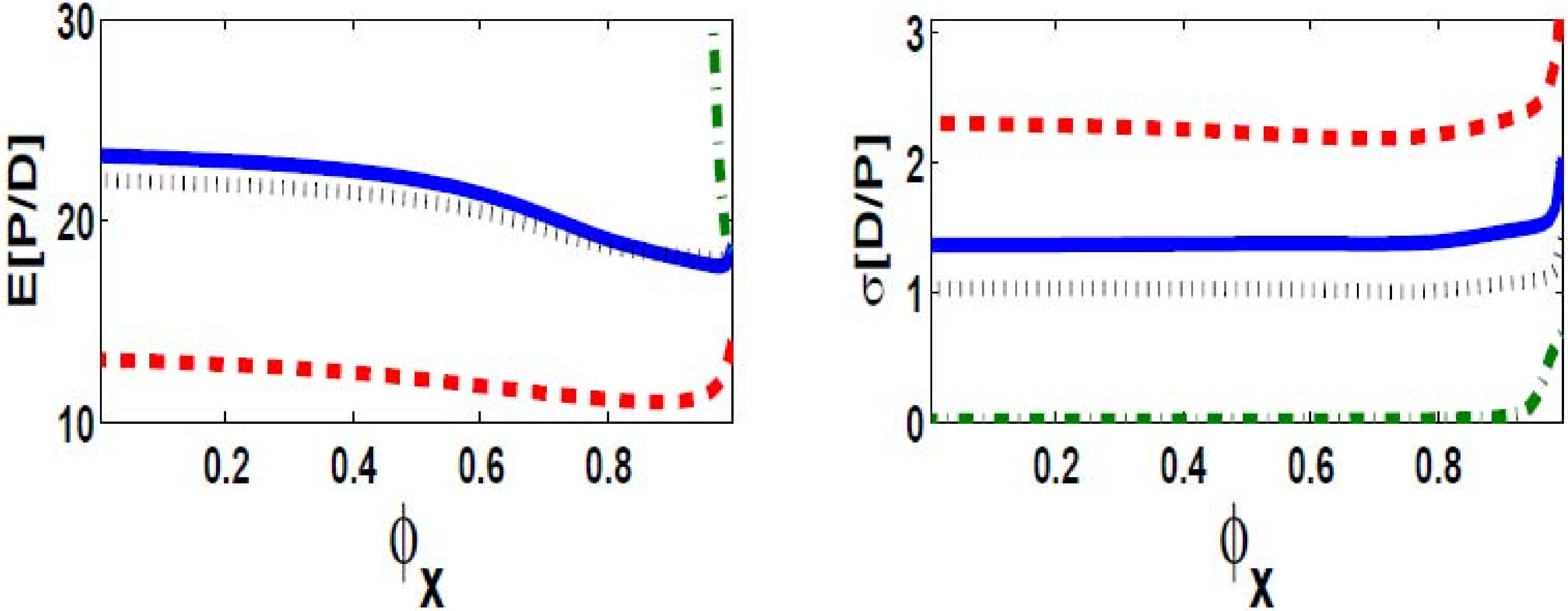
0.8



30
20
10
E[P/D]
0.2
0.4
0.6
0.8
ϕ_X
3
2
1
0
σ[D/P]
0.2
0.4
0.6
0.8
ϕ_X

Conclusions

- We provide a new model for asset pricing.

- It has a simple endowment process with only one source of long-run risk: persistent volatility.

- It uses generalized disappointment averse preferences.

- Disappointment averse preferences were shown to add realism in other settings: Allais paradox, asset allocation problem (Ang, Bekaert and Liu 2005).

- We have shown that, when combined with persistent volatility it helps to reproduce asset prices moments and predictability patterns as well.