Likelihood-based inference on the cause of multiple waves in the 1918 influenza pandemic in London, UK

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Joint work with

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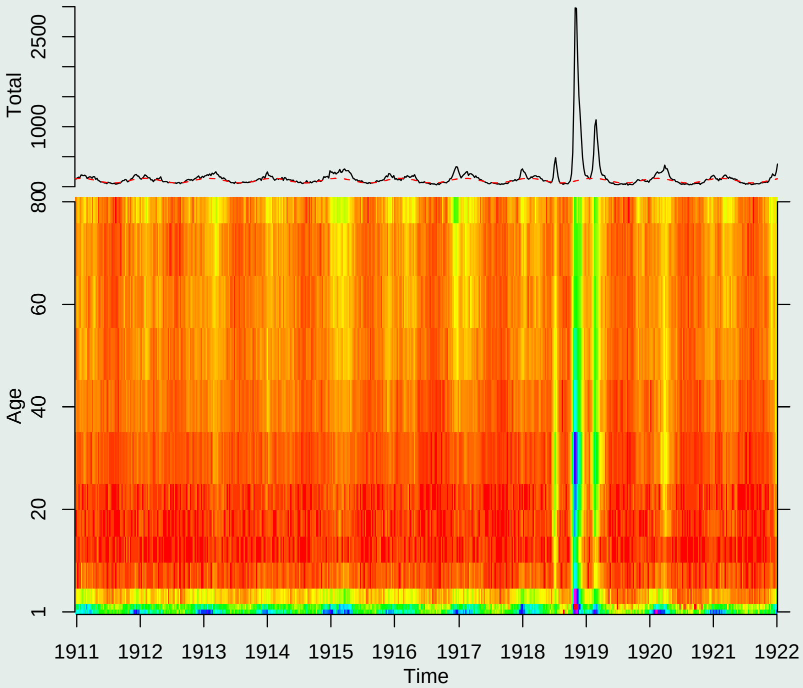
Pneumonia & Influenza Mortality in London (UK)



Kermack-McKendrick model with vital

(Susceptible→Infectious→Recovered)

$$\begin{aligned}
\dot{S} &= \nu N - \frac{\beta}{N} S I - \mu S \\
\dot{I} &= \frac{\beta}{N} S I - \gamma I - \mu I \\
\dot{R} &= \gamma I - \mu R
\end{aligned}$$

- β = transmission rate
- γ = recovery rate (γ^{-1} = mean infectious period)
- ν, μ, birth and death rate *per capita*

The scaled SIR model without vital

$$\begin{aligned} \dot{S} &= -\beta S I \\ \dot{I} &= \beta S I - \gamma I \\ \dot{R} &= \gamma I \end{aligned}$$

The reproductive number

$$\mathcal{R}_{\mathrm{e}} = \frac{\beta}{\gamma} S$$

The basic reproductive number

$$\mathcal{R}_0 = \frac{\beta}{\gamma} = \frac{\mathcal{R}_{\mathrm{e}}}{S}$$

The SIRS model with immunity decay

$$
\begin{aligned}
\dot{S} &= -\beta SI \quad + \delta R \\
\dot{I} &= \beta SI - \gamma I \\
\dot{R} &= \gamma I \quad - \delta R
\end{aligned}
$$

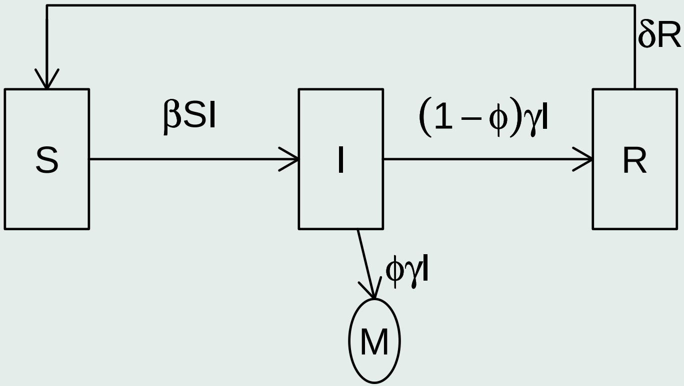
- δ = immunity decay rate (due to viral evolution),
  δ^{-1} = mean immunity duration
- β = transmission rate
- γ = recovery rate,
  γ^{-1} = mean infectious period

The SIRS model with mortality

$$
\begin{aligned}
\dot{S} &= -\beta SI \quad + \delta R \\
\dot{I} &= \beta SI - \gamma I \\
\dot{R} &= (1-\phi)\gamma I - \delta R \\
\dot{M} &= \phi\gamma I
\end{aligned}
$$

- ϕ = Case Fatality Proportion (CFP)

The SIRS model flow chart



Likelihood of a model with parameters θ

(Burnham and Anderson, 2002; Ionides et al., 2006)

- Data: observed time series $C_{1:N_\mathrm{s}}$
- "**Likelihood of the model (θ) given the data ($C_{1:N_\mathrm{s}}$)**" is defined to be the "**probability of the data given the model:**"

$$f(C_{1:N_\mathrm{s}}|\theta) = \prod_{n=1}^{N_\mathrm{s}} f(C_n|C_{1:n-1},\theta)$$

  where $f(C_n|C_{1:n-1},\theta)$ is conditional probability of C_n given the earlier data $C_{1:n-1}$

- Log likelihood function is $\ell(\theta) = \sum_{n=1}^{N_\mathrm{s}} \log f(C_n|C_{1:n-1},\theta)$

Compare models using Akaike Information Criterion

(Akaike, 1974; Burnham and Anderson, 2002; King et al., 2008)

The second-order Akaike Information Criterion (AIC_{c}) is

$$AIC_{\mathrm{c}} = -2 \max_{\theta} \ell(\theta) + \frac{2N_{\mathrm{p}}N_{\mathrm{s}}}{N_{\mathrm{s}} - N_{\mathrm{p}} - 1}$$

where

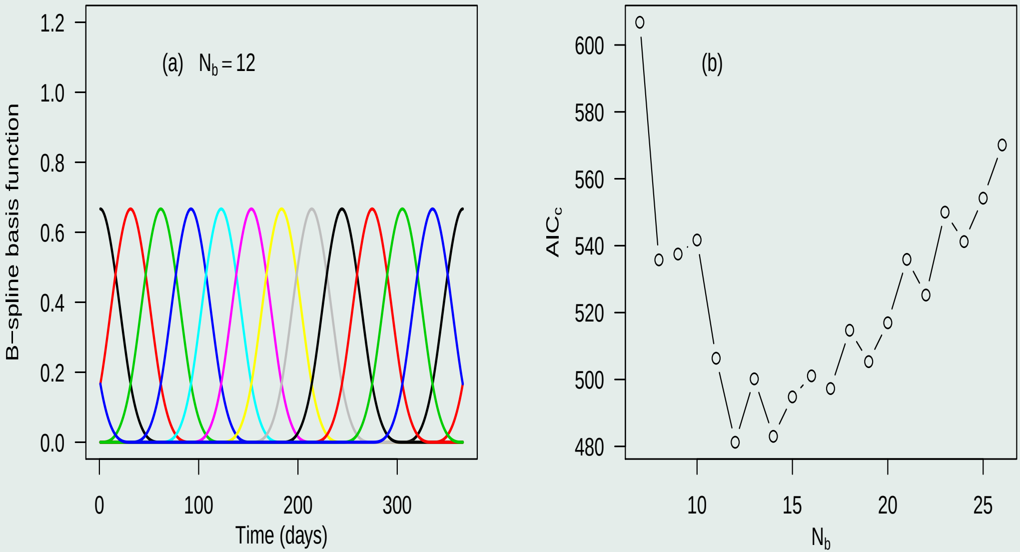
N_{p} is the number of parameters in the model
N_{s} is the sample size ($N_{\mathrm{s}} = 52$ here).

Calculate and maximize likelihood using Particle Filtering, MIF, and POMP

- Calculate $\ell(\theta)$ via Particle Filtering (Doucet et al., 2001)
- Maximize $\ell(\theta)$ via MIF (maximization via iterated filtering) (Ionides et al., 2006)
- An R package POMP (partially observed Markov processes) (King et al., 2009)

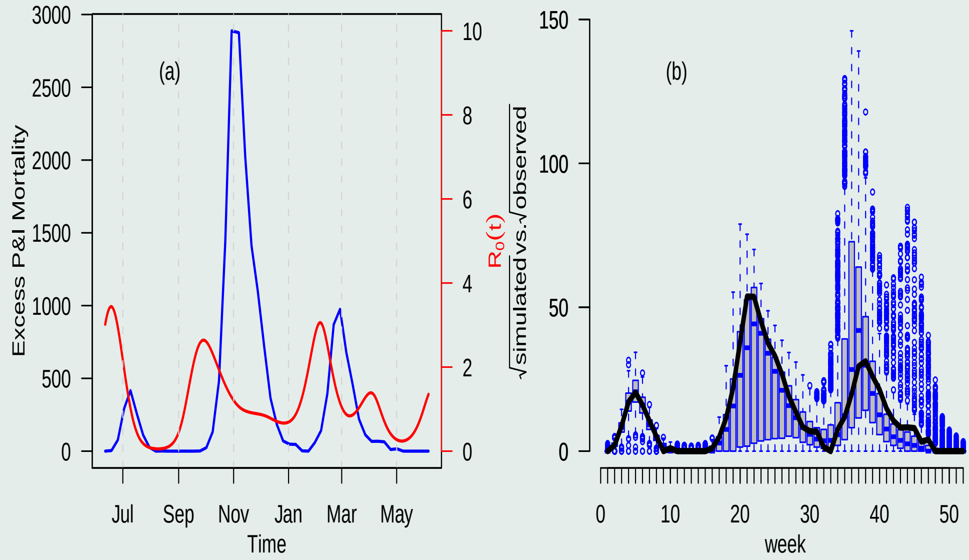
Application to excess P&I mortality in 1918-1919

B-spline function and AIC_c as a function of N_b



Application to excess P&I mortality in 1918-1919

Reconstructed $\mathcal{R}_0(t)$ and simulations



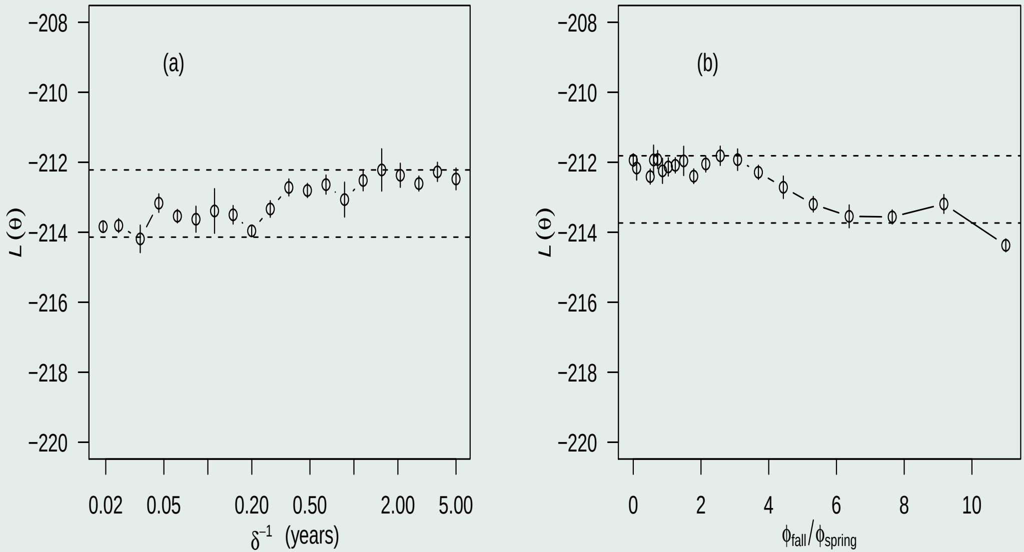
Application to excess P&I mortality in 1918-1919

Parameter Estimates

Parameter	Our Estimates	Our 95% CI	Previous Results
Infectious Period (days)	4.5	(3.4, 5.8)	3 to 6
Overall R_0	1.39	(1.09, 1.86)	1.3 to 1.9 (Chowell et al, 2008)
CFP (ϕ)	0.0092	(0.0068, 0.0231)	0.02 (Mills et al, 2004)

Application to excess P&I mortality in 1918-1919

Effects of immunity decay and CFP changes



Conclusions

- Likelihood method using POMP allows us to compare hypotheses for the origin of multiple waves in pandemics
- Transmission rate changes ‘alone’ could have been responsible for the multiple waves.
- Our estimates of mean infectious period, basic reproductive number ($\mathcal{R}_0$) and case fatality proportion (CFP) are comparable to conventionally accepted values.
- The changes in transmission rate we infer are intuitively consistent with climatic variation and human behavioral changes.
- We did not find evidence to support either a constant immunity decay or changes in CFP.

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