



Computational Model of CA3 Hippocampal Network

Muhammad Dur-e-Ahmad
Centre for Mathematical Medicine, Field's Institute

Advisors:

Sue Ann Campbell

University of Waterloo

Frances Skinner

TWRI and University of Toronto

Facts About Human Brain



Holds only 2% of our body's weight but consume 20% of the energy. This energy is enough to light a 25 watt bulb.

There are about 100 billion neurons in the human brain.

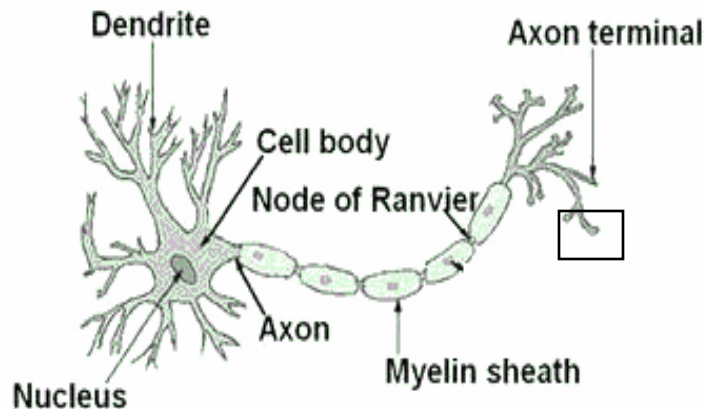
More electrical impulses are generated in one day by a single human brain than by all the telephones in the world.

An average human brain produces about 70,000 thoughts per day.

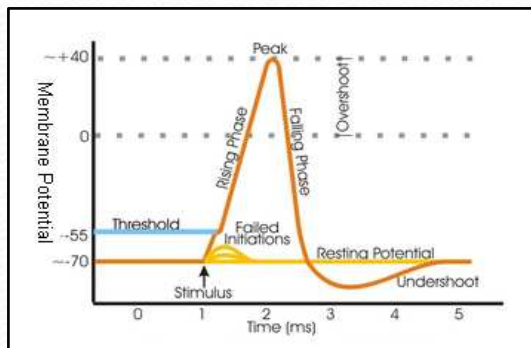
About 1000 ml of blood flow through the brain every minute. In that minute the brain will consume 46 cm³ of oxygen from that blood.

Neuron-Building Block of Nervous System

Structure of a Typical Neuron



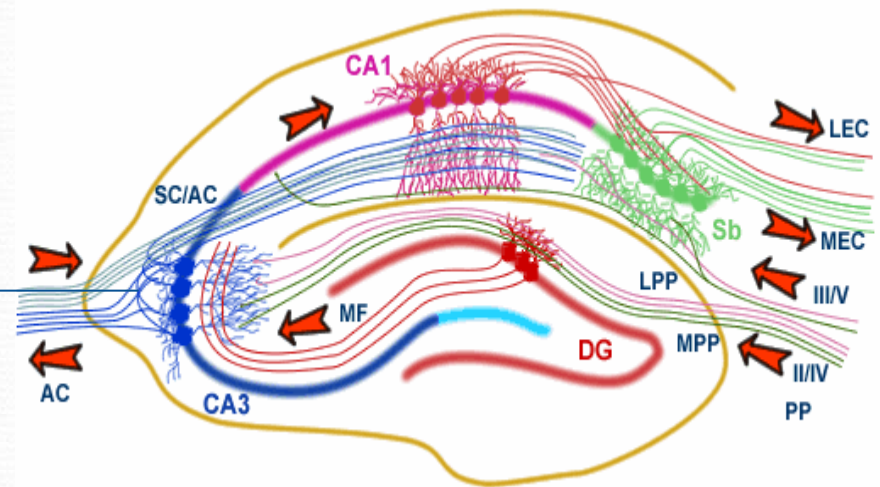
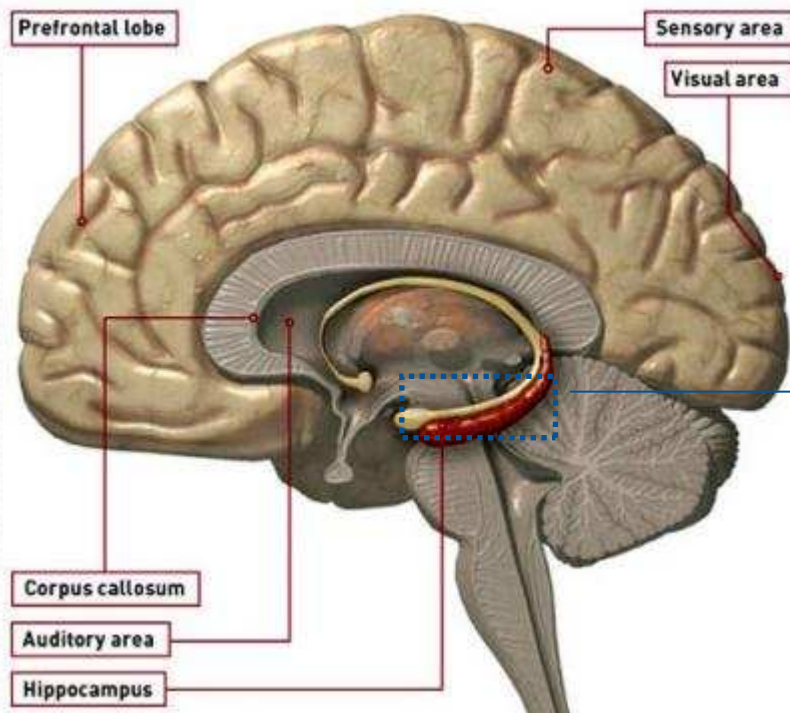
*Input:
Synaptic Input to
Dendrite from
Other Neurons*



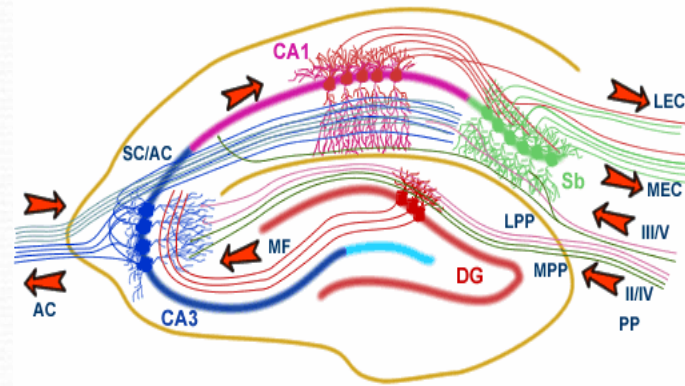
*Output:
Action Potential
to Axon Terminal*

HIPPOCAMPUS

Key player behind the formation long term memories and spatial navigation



CA3 Region



CA3 region holds a strategic position in the hippocampus because it receives sensorial information from the external and internal environment via two main pathways: the mossy fibres and the perforant path.

It is essential for spatial memory processes and specifically in memory consolidation of spatial information.

CA3 pyramidal neurons (excitatory) communicate mainly through bursts of spikes rather than trains of regular firing action potentials.

Key Features

No spontaneous firing in the absence of current.

Characteristics of Spikes

Half width = $1.0 \pm 0.1 \text{ ms}$

$dV / dt = 101.6 \pm 5.4 \text{ mV / ms}$

Amplitude = $97.6 \pm 1.9 \text{ mV}$

$fAHP = 10.2 \pm 0.5 \text{ mV}$

Weak input current (Rheobase) ----- 50-400 pA for 400 ms pulse

Strong input current (above rheobase) ----- 100-1500 pA for 400 ms pulse

Early and late Spike: ----- 50-400 pA for 400 ms pulse

Hemmond et al: Hippocampus (2008)

Behaviour of the CA3 Neurons

3 Different Firing patterns were observed.*

Regular firing with strong Spike frequency adaptation
(approx 8-10 spikes)-----37%

Weak or no Spike frequency adaptation
(approx 8-10 spikes) ----- 46%

Rapid burst
(5-6 spike train of > 50Hz in initial 100 ms of 400 ms pulse) ----- 17%

Hemmond et al: Hippocampus (2008)

CA3 Neuron Model

$$C\dot{V} = k(V - V_r)(V - V_t) - u + I \quad \text{if } V \geq V_{peak}$$

$$\dot{u} = a[b(V - V_r) - u] \quad V \leftarrow c, u \leftarrow u + d$$

Parameter Descriptions and their Values

Parameter	Description	Value
C	Capacitance	352.5 pF
V_r	Resting Membrane Potential	-65 mV
V_t	Threshold Membrane Potential	-45 mV
V_{peak}	Peak Potential	35 mV

Hippocampal Parameters

$$C \dot{V} = k (V - V_r)(V - V_t) - u + I \quad \text{if } V \geq V_{peak}$$

$$\dot{u} = a [b (V - V_r) - u] \quad V \leftarrow c, u \leftarrow u + d$$

a- *time scale of the recovery variable. Smaller values means slow recovery (ms^{-1}).*

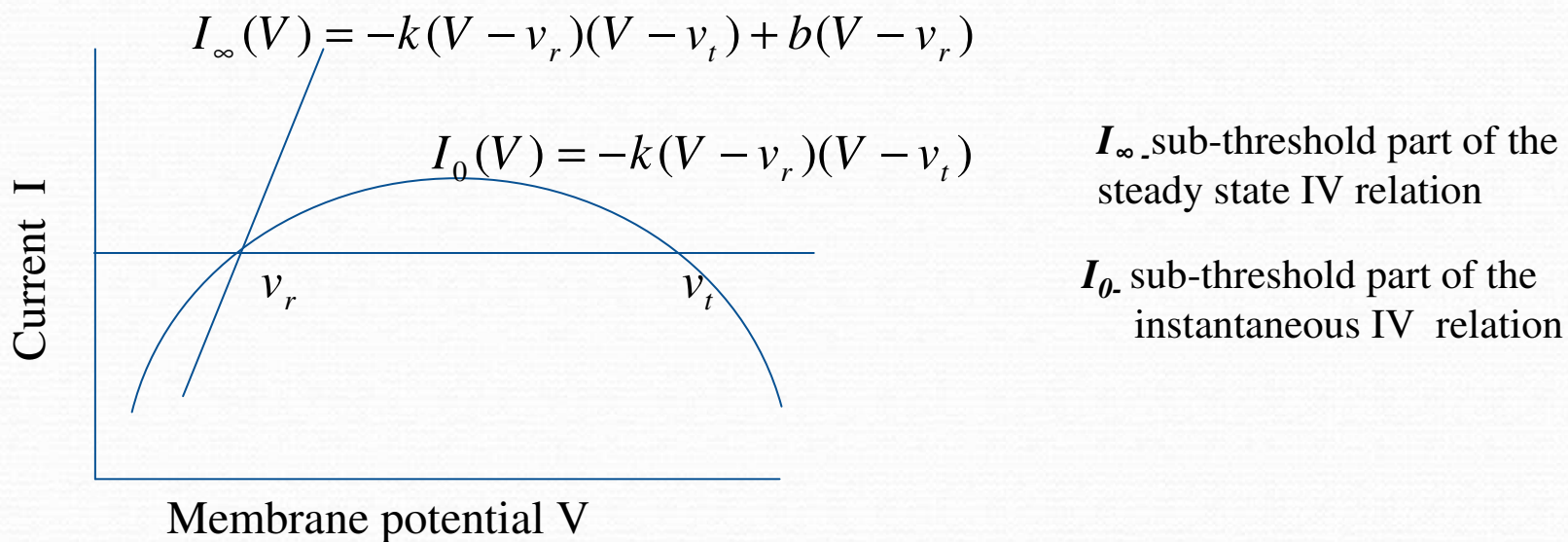
b- *Sensitivity of the recovery variable u to the sub-threshold fluctuation of the membrane potential V . (nS).*

c- *After spike rest value of membrane potential (mV)*

d- *After spike reset of the recovery variable (pA)*

k- *Scaling factor for the membrane potential V (pA/(mV)²)*

General Method of Finding the Parameter Values



b and k : Maximum of I_{∞} approx. rheobase current for $b < 0$ and its derivative wrt V at v_r corresponds to input conductance. Knowing both the rheobase and input conductance, we can solve them to find b and k .

a - is the time scale of the recovery variable, which is equivalent to inactivation of K^+ current, I_K . Therefore $a \approx 1/\text{inactivation time constant of } I_K$.

d - describes the total amount of outward minus inward currents activated during the spikes and affecting the after spike behavior. Therefore, for each model we use different values of d to obtain a reasonable behavior

Summary of our Models

$$352.5 \dot{V} = k(V + 65)(V + 45) - u + I \quad \text{if } V \geq 35$$

$$\dot{u} = a[b(V + 65) - u] \quad V \leftarrow c \quad u \leftarrow u + d$$

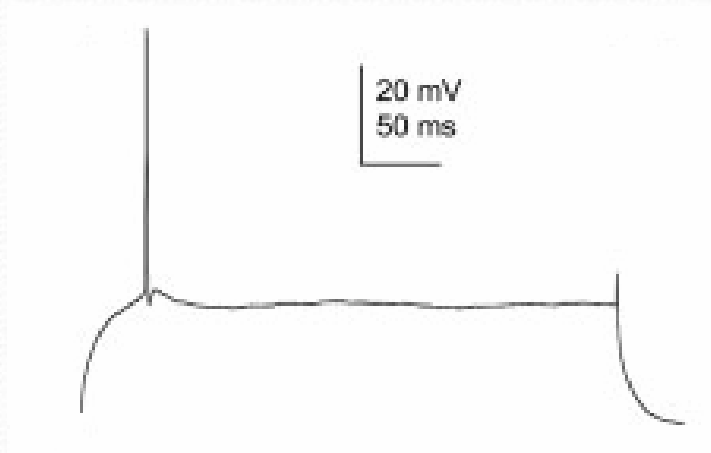
MODEL	PARAMETER VALUES	Figures Matched from Hemond Paper	Comments
1	$a=0.005$ $b=-2$ $c=-55$ $d=350$ $k=2.5$	Early Spike, Strong Adapting More Spikes	I=400 pA for early spike, I=1100 pA for the strong adaptation More spikes for higher values of I.
2	$a=0.025$ $b=-2$ $c=-55$ $d=200$ $k=2.5$	Late spikes Weakly Adapting More spiking for higher current	I=240 pA for late spike, I=500 pA for the weak adapting More spikes for higher values of I
3	$a=0.005$ $b=6.5$ $c=-55$ $d=4$ $k=8$	Early Spike Burst Longer duration burst	I=390 pA Early Spike (this is rheobase current value), Burst at I=550 pA , Longer duration burst at I=470 pA

Simulation Results

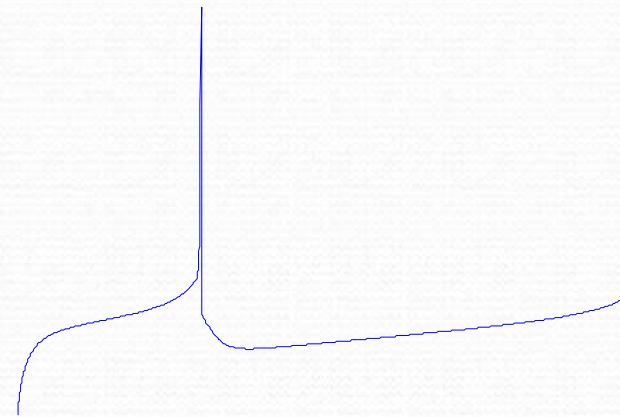
MODEL-1

Early Spike

Experimental



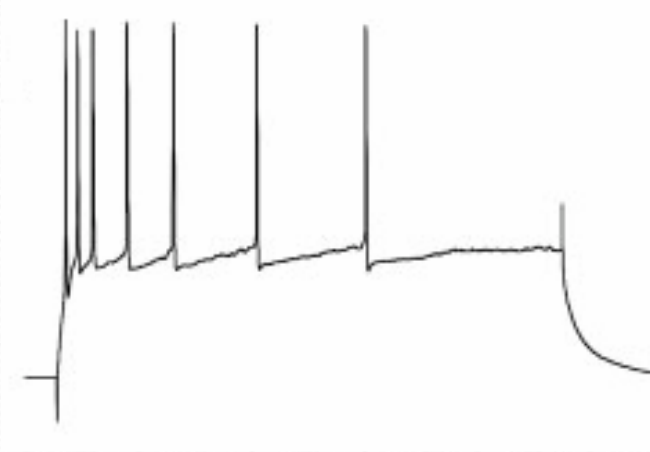
Our Model



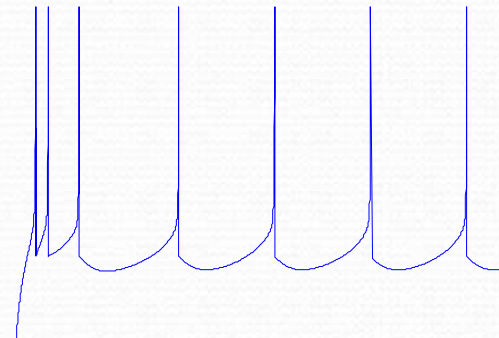
$I=400$ pA (Rheobase)

Strong Adapting

Experimental

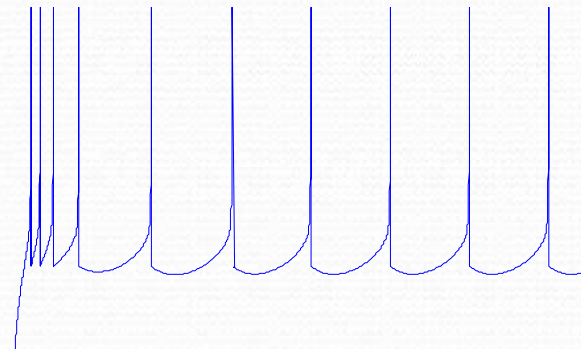
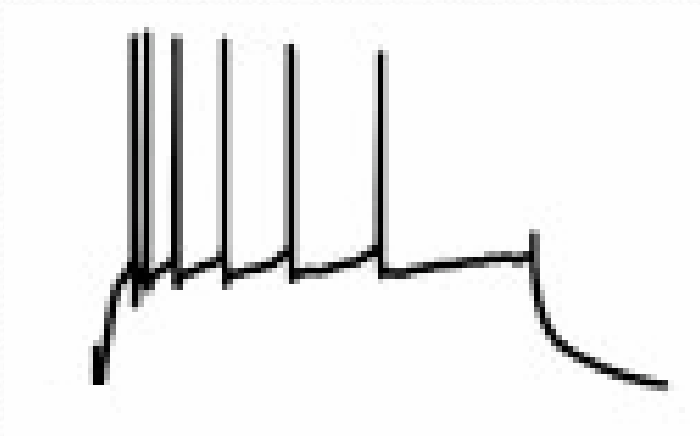


Our Model



$I=1100$ pA

More spikes for higher values of I .

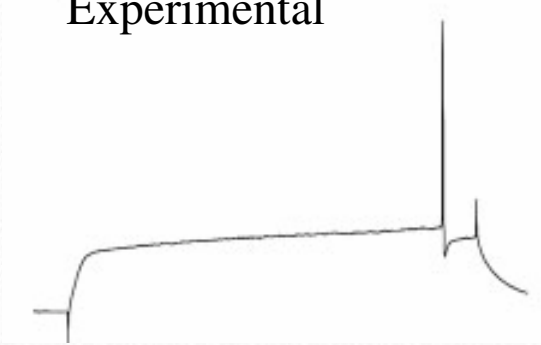


$I=1500$ pA

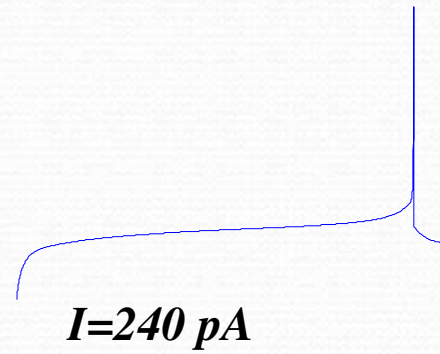
MODEL-2

Late Spike

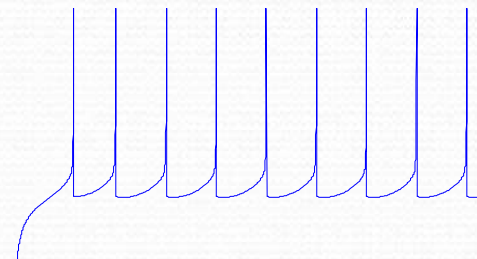
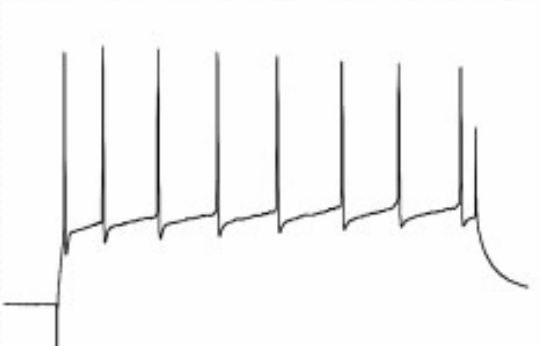
Experimental



Our Model

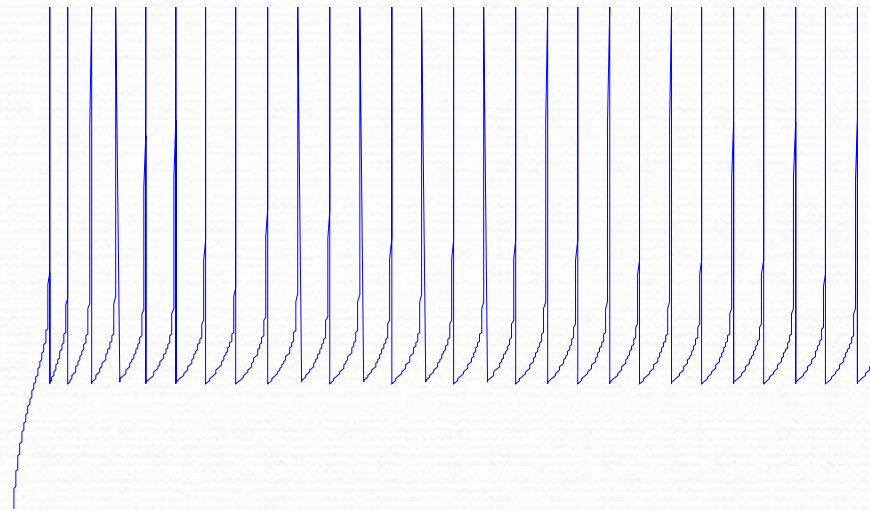


Weak Adaptation



$I=500 \text{ pA}$

More spiking for higher current

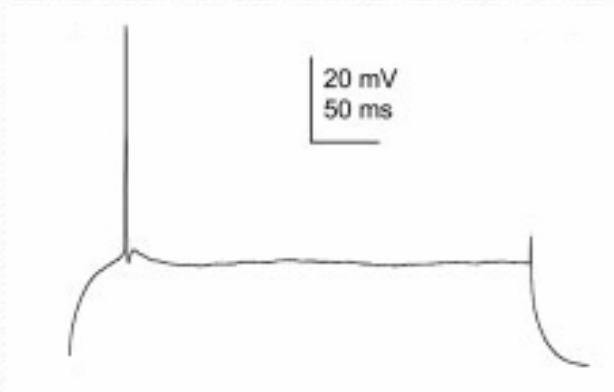


$$I=1000\text{ pA}$$

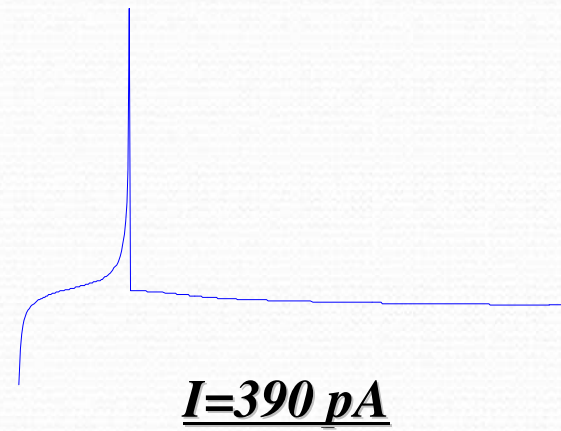
MODEL-3

Early Spike

Experimental

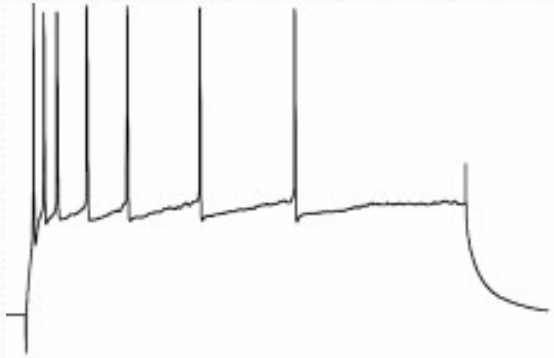


Our Model

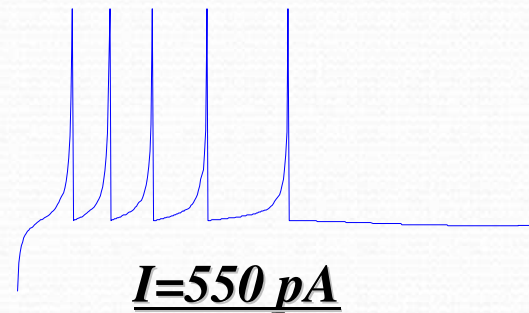


A Burst

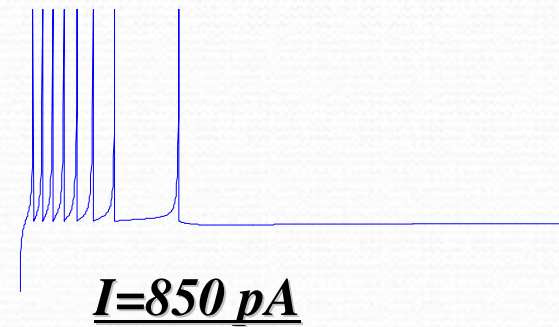
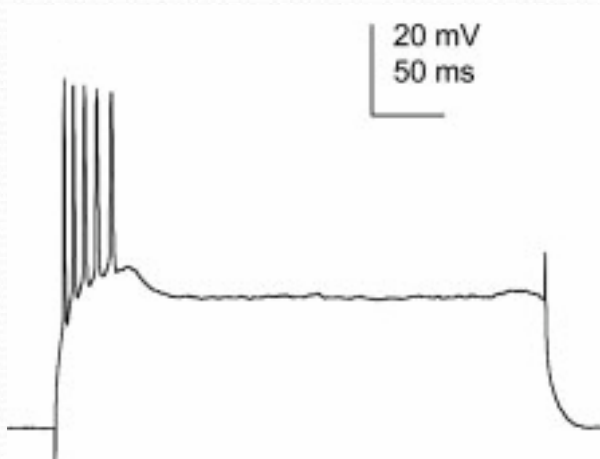
Experimental



Our Model

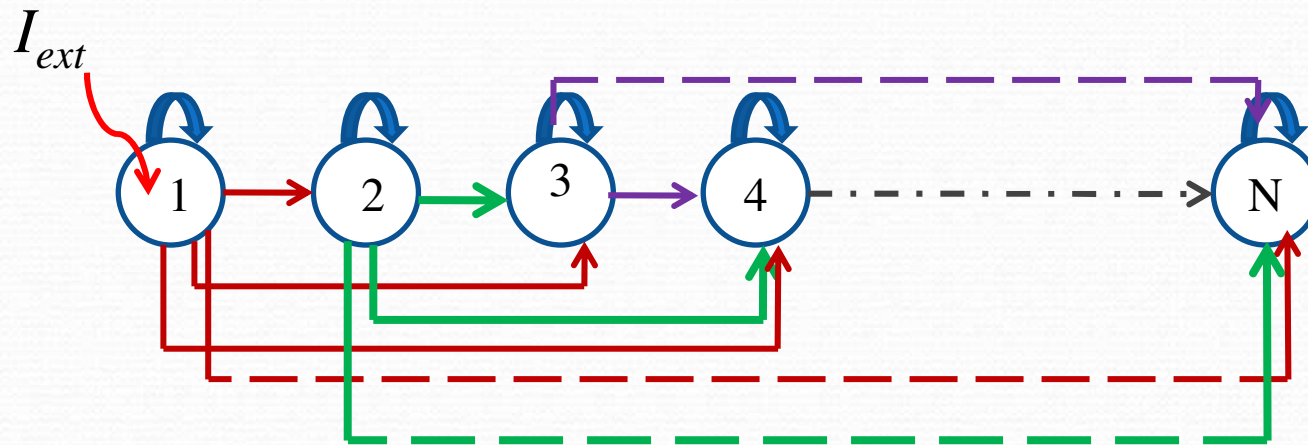


Longer Duration Burst

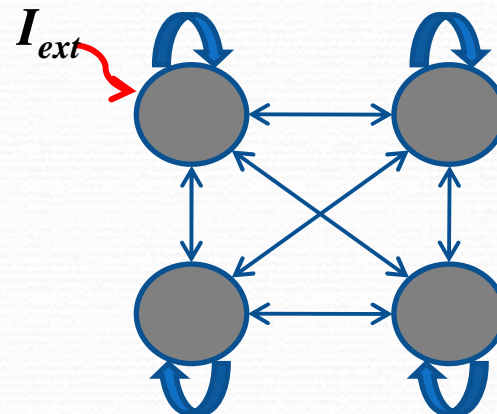


Neuronal Network

Feed-Forward Network



All-to-All Coupled



Network Model

$$C \frac{dV_j}{dt} = k(V_j - v_r)(V_j - v_t) - u_j + I_{ext} + (E - V_j) * \bar{g} \sum_{i=1}^N s_{j,i}(t, V_i)$$

$$\frac{du_j}{dt} = a \{ b(V_j - v_r) - u_j \}$$

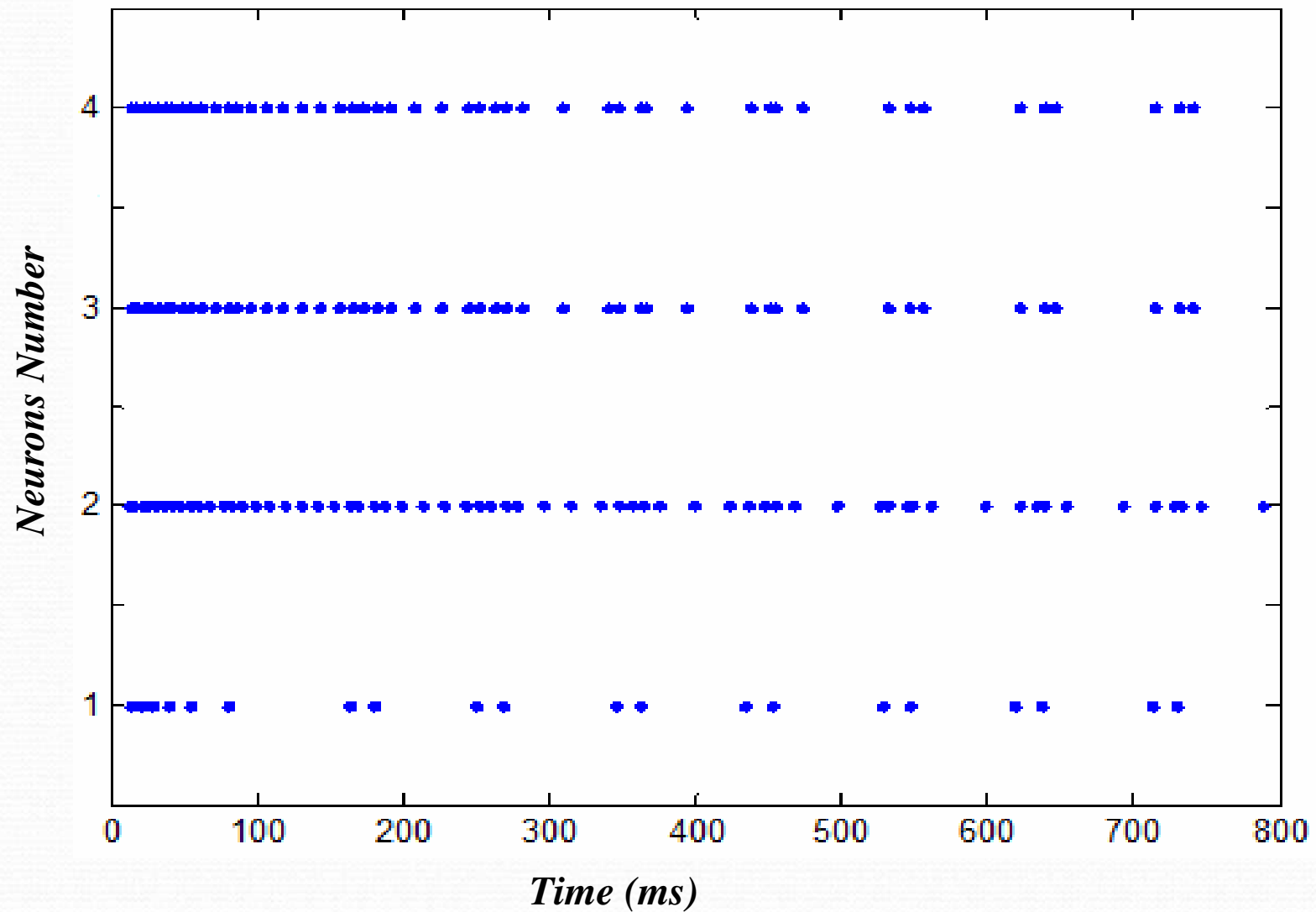
$$V_j \leftarrow c, u_j \leftarrow u_j + d \text{ whenever } V_j \geq V_{peak}, j=1,2,\dots,N$$

$$\tau_{syn} \frac{ds_{j,i}}{dt} = -s_{j,i}$$

$$s_{j,i} \leftarrow s_{j,i} + S_{max} \text{ whenever } V_i > 0 \text{ from below, for } i=1,2,\dots,N$$

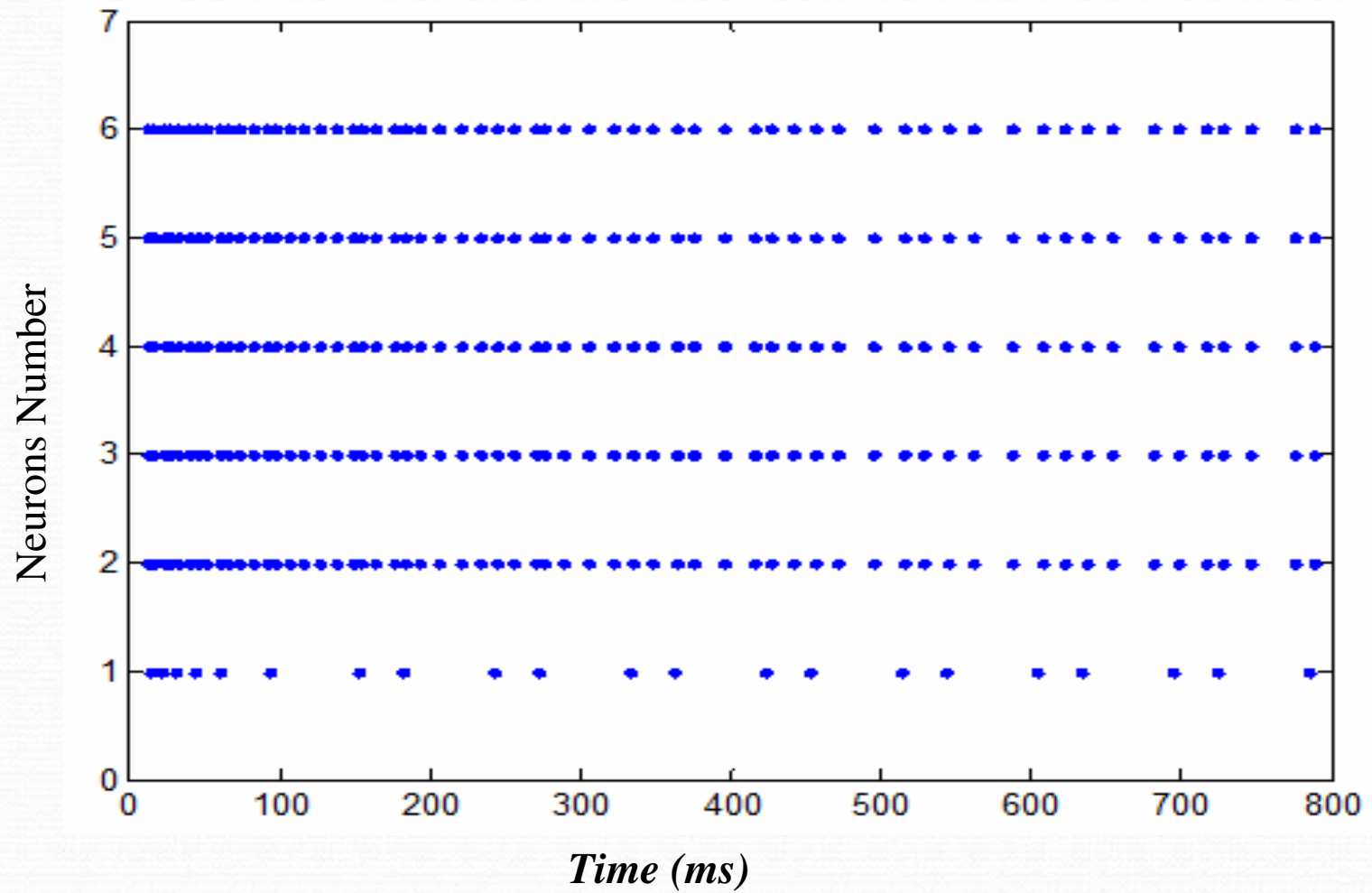
Bursting in 4-cell network

Model 1333, $I=[850,90,0,0]$ pA , $g_{syn}=110$ nS



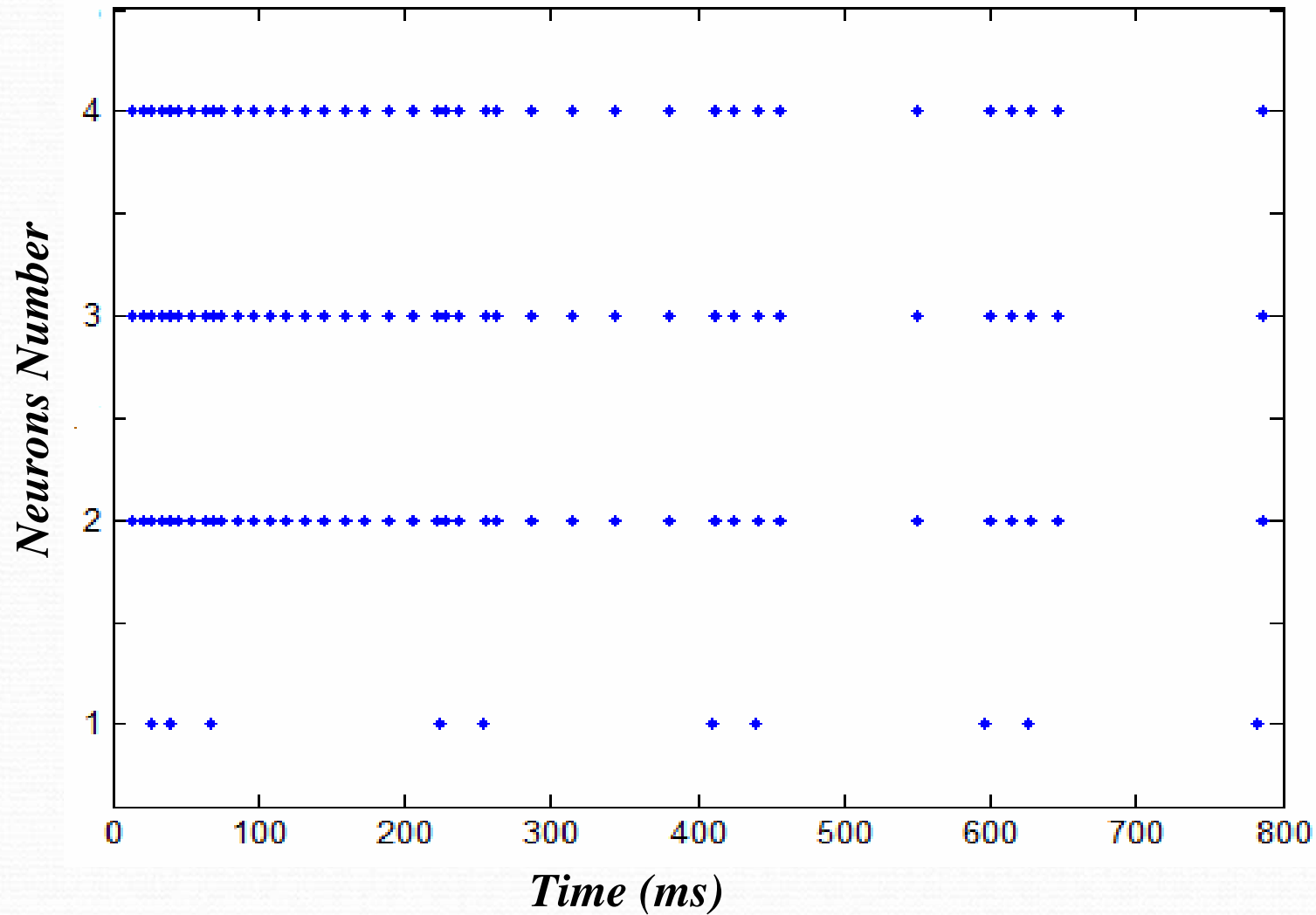
6-Cell Bursting

Model 133333, $I=[800,0,0,0,0]$ pA , $g_{syn}=75$ nS



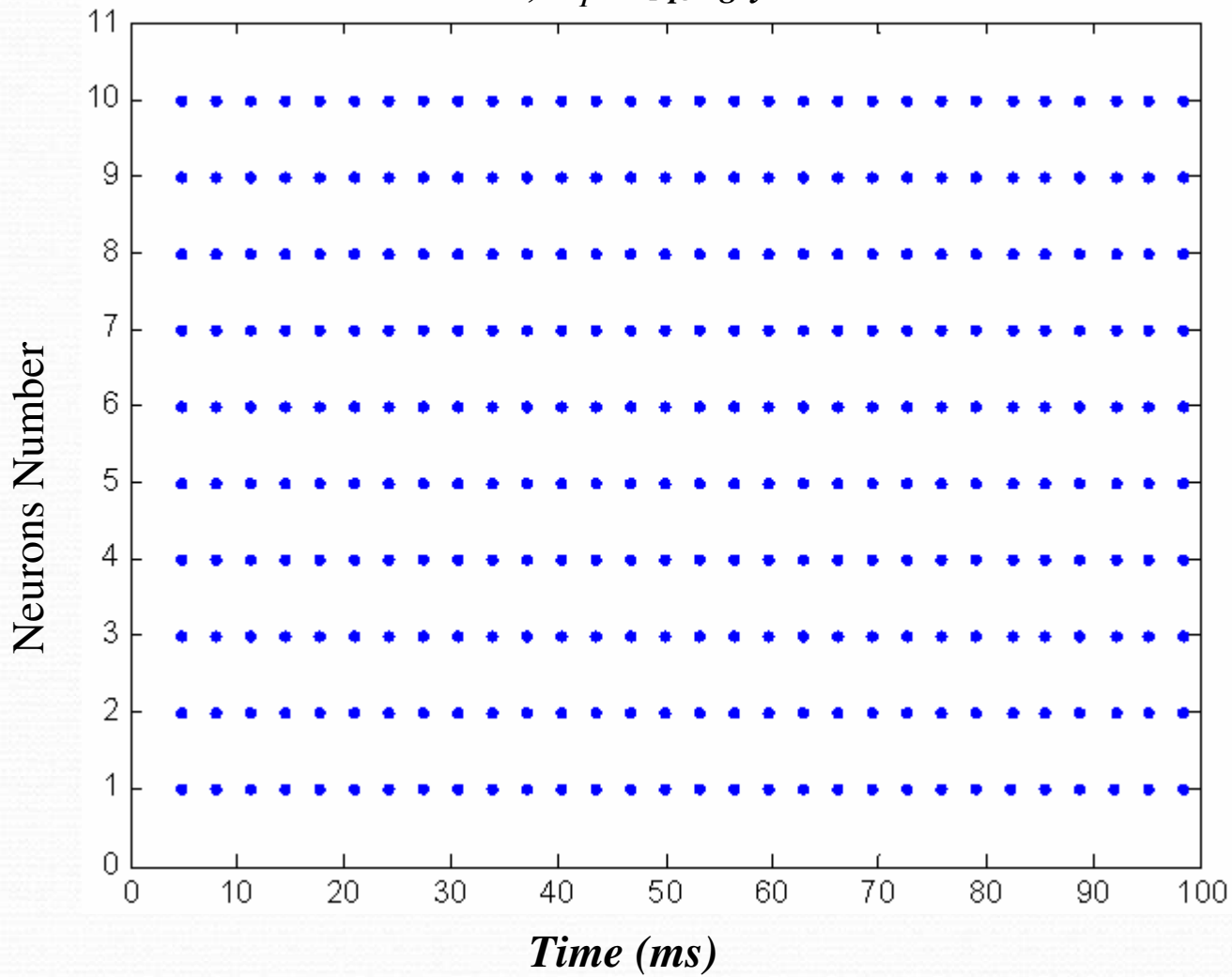
4-Cell Bursting without External Injected Current

Models 1333, $g_{syn}=110nS$



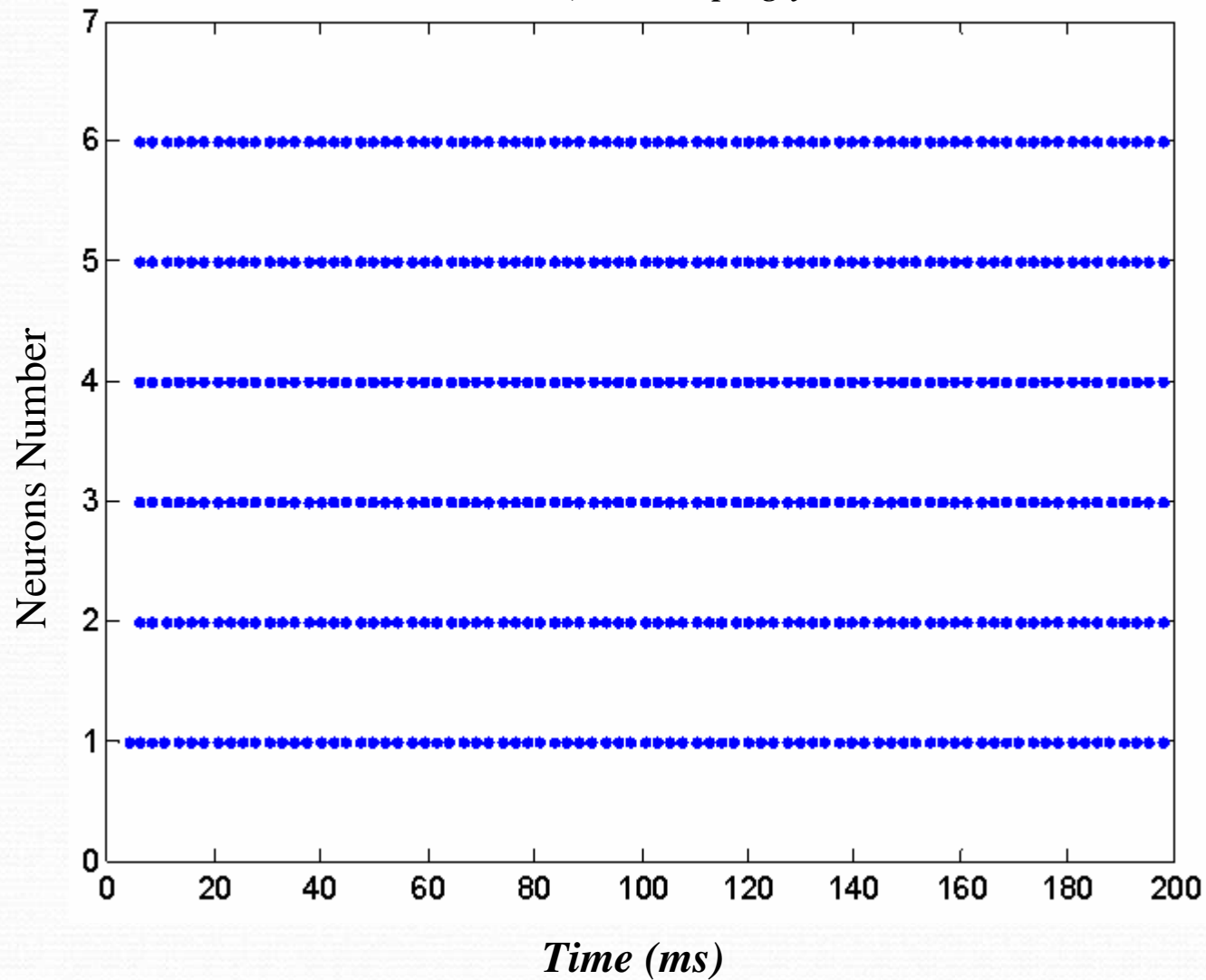
Synchronized Firing in All to All Coupled Cells

Models 13...3, $I_I=800\text{ pA}$ $g_{\text{syn}}=110\text{ nS}$



Bursting in All to All Coupled Cells

Models 133333, $I_I=800\text{ pA}$ $g_{syn}=550\text{ nS}$





Selected References

Izhikevich, EM :Simple model of spiking neurons. **IEEE Trans. Neural Network.** *14, 1569-1572 (2003).*

Izhikevich, EM: Dynamical Systems in Neuroscience. Cambridge, *MA, MIT Press.(2007)*

Peter Hemond et al.: Distinct Classes of Pyramidal Cells Exhibit Mutually Exclusive Firing Patterns in Hippocampal Area CA3b. *Hippocampus, 18:411-424 (2008).*

Mark S Goldman: Memory without Feedback in Neural Network. *Neuron 61: 621-634, 2009*

Traub RD et al: Model of the Origin of Rhythmic Population Oscillations in the Hippocampal Slice. Science, Vol 243, Issue 4896, 1319-1325, 1989

Latham LE et al: Intrinsic Dynamics in Neuronal Networks. I. Theory . *J Neurophysiol 83: 808-827, 2000;*