Continuously monitored barrier options under Markov processes

Martijn Pistorius

Department of Mathematics
Imperial College London

Workshop on Financial Derivatives and Risk Management

Fields Institute, Toronto

May 28, 2010

Based on joint work with **Aleksandar Mijatović**

Overview of the talk

1. Introduction
2. Barrier contracts and Markov processes
3. Continuous-time Markov chains and the pricing algorithm
4. Approximation of the generator and numerical examples
5. Convergence and rates
6. Conclusion

1. Motivation: valuing barrier contracts

- Barrier options are among the most liquid exotic options, esp. in **foreign exchange, equity**
- Commonly traded types: double-no-touch, double/single knock-out and knock-in call/put options.
- For valuation and hedging there is interest in efficient calculation methods of first-passage time distributions and their sensitivities (Greeks).
- **Credit risk**: structural models in terms of first passage over a boundary

1. Motivation: valuing barrier contracts

A *barrier-type contract* with expiry $T > 0$ pays the random cash flow

$$g(S_T)\mathbf{I}_{\{\tau_A > T\}} + h(S_{\tau_A})\mathbf{I}_{\{\tau_A \leq T\}}, \quad \text{where} \quad \tau_A = \inf\{t \geq 0 : S_t \in A\}.$$

where

- $S = \{S_t\}_{t \geq 0}$ is the asset price;
- Knock-out set $A = (0, \ell] \cup [u, \infty), \quad 0 \leq \ell < u \leq \infty$;
- $g, h : (0, \infty) \to \mathbb{R}_+$ payoff and rebate functions respectively.

Examples:

- knock-out double barrier ($0 < \ell, u < \infty, h \equiv 0$);
- down-and-out ($u = \infty$, $h \equiv 0$), up-and-out ($\ell = 0$, $h \equiv 0$);
- rebate ($g \equiv 0$), European ($0 = \ell, u = \infty$).

1. Motivation: valuing barrier contracts

If S is under a risk neutral measure and r is the risk-free rate, the arbitrage-free price of the barrier contract is

$$\mathbb{E}_x\left[\mathrm{e}^{-rT} g(S_T)\mathbf{I}_{\{\tau_A > T\}}\right] + \mathbb{E}_x\left[\mathrm{e}^{-r\tau_A} h(S_{\tau_A})\mathbf{I}_{\{\tau_A \le T\}}\right], \quad S_0 = x.$$

It is of interest to efficiently evaluate these expectations.

1. Motivation: valuing barrier contracts

Examples of models of interest:

- diffusion models (e.g. CEV (Davydov and Linetsky));
- jump-diffusion models (e.g. Kou model, Merton model);
- Lévy models (Carr et al., Barndorff-Nielsen, Eberlein et al.);
- generalised OU models (Barndorff-Nielsen et al.).

The underlying risky asset in all of these models is driven by a **Markov process** with state-dependent volatility and/or jumps.

1. Literature review

In a given model the valuation of barrier options is more challenging than that of European type options. Methods that have been developed for specific models include:

- Spectral decompositions for diffusions (Linetsky et al., Lipton)
- Transform approaches for Lévy proc. using Wiener-Hopf fact. (Boyarchenko & Levendorskii, Kou & Wang, Jeannin & P.)

General approaches are:

- PDE/PIDE methods (discretisation/finite element/tree methods)
- Monte Carlo (Euler scheme used to approximate the SDE)

1. The Markov chain approach

The Markov chain approach has the following properties:

- first developed by Kushner in optimal control setting with discrete time chains
- works for general Markov processes;
- it is based on **approximation of the target process by a continuous-time Markov chain.**
- approximation yields **arbitrage-free model prices** at any stage

We next introduce the modelling framework and describe the pricing algorithm.

2. Definition of the model

$S = \{S_t\}_{t\geq 0}$ **Markov process** on state-space $E = [0, \infty)$ with *semigroup* $(P_t)_{t\geq 0}$, where $P_t f(x) := \mathbb{E}_x[f(S_t)]$ s.t.

$$\{\mathrm{e}^{-(r-d)t} S_t\}_{t\geq 0} \text{ is a } \textbf{martingale}$$

where r and d are the risk-free interest rate and the dividend yield.

Regularity assumptions:

Assumption 1. The semigroup $(P_t)_{t\geq 0}$ is a **Feller semigroup**, i.e.

(i) if $f \in C_0(E)$, then $P_t f \in C_0(E)$ for any $t > 0$;

(ii) $\lim_{t\to 0} P_t f(x) = f(x)$ for any $x \in E$ and $f \in C_0(E)$.

Assumption 2. $\mathbb{P}_x(\tau_A = \tau_{A^o}) = 1$ where $A^o = (0, \ell) \cup (u, \infty)$.

2. Infinitesimal generator

$(P_t)_{t\geq 0}$ is a strongly continuous semigroup. Define a **dense** subspace $\mathcal{D}$ of $C_0(E)$ by

$$\mathcal{D} \quad := \quad \left\{ f \in C_0(E) \;:\; \exists \lim_{t\downarrow 0} \tfrac{1}{t}(P_t f - f) \in C_0(E) \right\}$$

and a possibly unbounded linear operator $\mathcal{L} : \mathcal{D} \to C_0(E)$, known as the *infinitesimal generator*, by

$$\mathcal{L}f(x) \quad := \quad \lim_{t\downarrow 0} \tfrac{1}{t}(P_t f - f)(x).$$

The operator $\mathcal{L}$ on the domain $\mathcal{D}$ determines the semigroup $(P_t)_{t\geq 0}$ uniquely.

2. Stopped process

General barrier contract is a ‘vanilla’ option on the stopped and discounted process $\widetilde{S}^A$ where $\tau_A = \inf\{t \geq 0 : S_t \in A\}$:

$$\begin{aligned}\widetilde{P}_T^A f(x) &:= \mathbb{E}_x[f(\widetilde{S}_{T\wedge\tau_A})] = \mathbb{E}_x[\mathrm{e}^{-r(\tau_A\wedge T)} f(S_{T\wedge\tau_A})] \\ &= \mathbb{E}_x\left[\mathrm{e}^{-rT} g(S_T)\mathbf{I}_{\{\tau_A > T\}}\right] + \mathbb{E}_x\left[\mathrm{e}^{-r\tau_A} h(S_{\tau_A})\mathbf{I}_{\{\tau_A \leq T\}}\right]\end{aligned}$$

where the function f is defined as

$$f(x) = \begin{cases} h(x), & x \in A, \\ g(x), & x \notin A. \end{cases}$$

Generator $\widetilde{\mathcal{L}}^A$ of semigroup $(\widetilde{P}_t^A)_{t\geq 0}$ can be expressed explicitly in terms of the generator $\mathcal{L}$ of S.

2. Killed process

Knock-out barrier contract is a ‘vanilla’ option on the killed process

$$\widehat{S}^A = \{S_t \mathbf{I}_{\{t<\tau_A\}} + \partial \mathbf{I}_{\{t\geq\tau_A\}}\}$$

where ∂ is an absorbing “graveyard state”, as

$$\begin{aligned}\widehat{P}_T^A f(x) &:= \mathbb{E}_x[g(\widehat{S}_{T\wedge\tau_A})] \\ &= \mathbb{E}_x\left[g(S_T)\mathbf{I}_{\{\tau_A>T\}}\right]\end{aligned}$$

with $g(\partial) = 0$

Generator $\widehat{\mathcal{L}}^A$ of semigroup $(\widehat{P}_t^A)_{t\geq 0}$ can be expressed explicitly in terms of the generator $\mathcal{L}$ of S.

3. Markov chains: European options

Notation

- $\mathbb{G} = \{x_1, \ldots, x_N\} \subset \mathbb{R}_+$ finite set with $x_1 < \ldots < x_N$.
- For any matrix $\mathcal{A} \in \mathbb{R}^{N \times N}$ and vector $\phi \in \mathbb{R}^N$ identify

  $$\mathcal{A}(x, y) := e_x' \mathcal{A} e_y \quad \text{and} \quad \phi(x) := e_x' \phi \quad x, y \in \mathbb{G},$$

  where e_x, e_y are the standard basis vectors of $\mathbb{R}^N$.
- $X = \{X_t\}_{t \geq 0}$ continuous-time Markov chain with generator matrix Λ that approximates the generator $\mathcal{L}$ of the Markov process S.

3. Markov chains: European options

The price of a European option with vanilla payoff

$$\phi : \mathbb{G} \to \mathbb{R}$$

is given by

$$\mathbb{E}_x[\phi(X_T)] \quad = \quad P_T\phi(x) = \Big(\exp\big(T\Lambda\big)\phi \Big)(x).$$

for $X_0 = x \in \mathbb{G}$.

3. Markov chains: Barrier contracts

Define matrices $\widetilde{\Lambda}$ and $\widehat{\Lambda}$ as:

$$\Lambda = \begin{array}{cc} & \overbrace{\phantom{\Lambda_{12}}}^{\widehat{\mathbb{G}}} \\ \widehat{\mathbb{G}}\left\{\right. & \left(\begin{array}{c|c} \Lambda_1 & \Lambda_{12} \\ \hline \Lambda_{21} & \Lambda_2 \end{array}\right) \end{array}$$

$$\widetilde{\Lambda} = \left(\begin{array}{c|c} 0 & 0 \\ \hline \Lambda_{21} & \Lambda_2 \end{array}\right)$$

$$\widehat{\Lambda} = \Lambda_2$$

The subset $\widehat{\mathbb{G}} \subset \mathbb{G}$ consists of the elements of $\mathbb{G}$ between ℓ and u.

3. Markov chains: barrier contracts

Theorem. For any $T > 0$, $r \geq 0$ and any functions $g : \widehat{\mathbb{G}} \to \mathbb{R}$ and $h : \mathbb{G} \to \mathbb{R}$ with $h(y) = 0$ for $y \in \widehat{\mathbb{G}}$ we have that

$$\begin{aligned} \mathbb{E}_x \left[g(X_T) \mathbf{I}_{\{\tau > T\}} \right] &= \left(\exp\left(T \widehat{\Lambda} \right) g \right)(x), \quad x \in \widehat{\mathbb{G}}, \\ \mathbb{E}_x \left[\mathrm{e}^{-r\tau} h(X_\tau) \mathbf{I}_{\{\tau \leq T\}} \right] &= \left(\exp\left(T(\widetilde{\Lambda} - r\widetilde{I}) \right) h \right)(x), \quad x \in \mathbb{G}, \end{aligned}$$

where X is the chain generated by Λ and

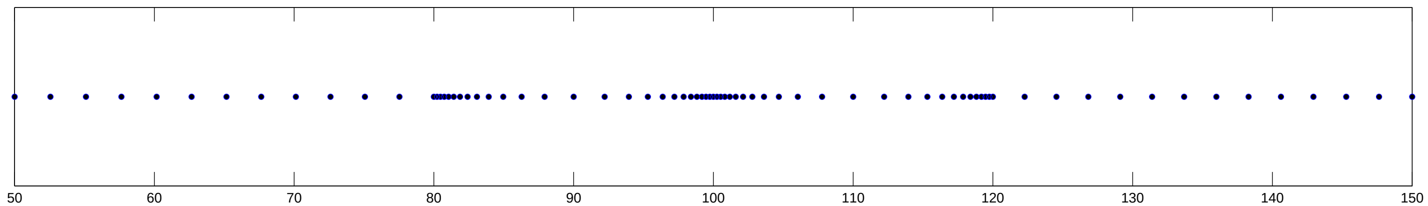
$$\tau := \inf\{ t \geq 0 : \; X_t \notin (\ell, u) \}.$$

3. The algorithm

Let S be a Markov process with state-space $E = (0, \infty)$ and generator $\mathcal{L}$. The algorithm for the pricing of barrier contracts is:

(1) Construct the approximating Markov chain by specifying a finite state-space $\mathbb{G} \subset E$ and a generator matrix Λ that approximates the operator $\mathcal{L}$ on $\mathbb{G}$.

(2) To value knock-out and rebate options, obtain $\widehat{\Lambda}$ and $\widetilde{\Lambda}$ from Λ, exponentiate and multiply with pay-off vector.



4. Convergence

Theorem Let S be a Feller process with state-space E and infinitesimal generator $\mathcal{L}$ satisfying Assumption 2, and $X^{(n)}$ a sequence of Markov chains with generator matrices $\Lambda^{(n)}$ such that

$$\max_{x \in \mathbb{G}^{(n)}} \left| \Lambda^{(n)} f_n(x) - \mathcal{L} f(x) \right| \to 0 \qquad \text{as } n \to \infty$$

for any function f in a core of $\mathcal{L}$. If $\tau_A^{(n)} = \inf\{t \geq 0 : X_t^{(n)} \notin A\}$ we have

$$\begin{aligned} \mathbb{E}_x \left[g\left(X_T^{(n)} \right) \mathbf{I}_{\{\tau_A^{(n)} > T\}} \right] &\longrightarrow \mathbb{E}_x \left[g(S_T) \mathbf{I}_{\{\tau_A > T\}} \right], \\ \mathbb{E}_x \left[\mathrm{e}^{-r\tau_A^{(n)}} h\left(X_{\tau_A^{(n)}}^{(n)} \right) \mathbf{I}_{\{\tau_A^{(n)} \leq T\}} \right] &\longrightarrow \mathbb{E}_x \left[\mathrm{e}^{-r\tau_A} h(S_{\tau_A}) \mathbf{I}_{\{\tau_A \leq T\}} \right], \end{aligned}$$

as $n \to \infty$ for any bounded continuous functions $g, h : E \to \mathbb{R}$.

4. Error estimates

Consider the (non-uniform) time and spatial grids $\mathbb{G}^{(n)} = \{x_i^{(n)}\}$ and $\mathbb{T}^{(m)} = \{T_j^{(m)}\}$ and denote by $k = k(n)$ the tail mass of the jump-measure, and let

$$h = h(n) = \max_i \left| x_{i+1}^{(n)} - x_i^{(n)} \right|, \qquad \delta = \delta(m) = \max_j \left| T_{j+1}^{(m)} - T_j^{(m)} \right|.$$

Theorem 1 *Under suitable regularity assumptions on the coefficients, there exists a sequence of Markov chains $X^{(n)}$ and constants C_1, C_2, C_3, such that for all n sufficiently large and $x \in \mathbb{G}^{(n)}$*

$$\begin{aligned} \left| \mathbb{E}_{0,x}\left[\mathrm{e}^{-\int_0^{T\wedge\tau_A^{(n)}} r^{(n)}(t)\mathrm{d}t} f\left(X^{(n)}_{T\wedge\tau_A^{(n)}} \right) \right] \right. & - \left. \mathbb{E}_{0,x}\left[\mathrm{e}^{-\int_0^{T\wedge\tau_A} r(t)\mathrm{d}t} f\left(S_{T\wedge\tau_A} \right) \right] \right| \\ & \leq C_1 h + C_2 k + C_3 \delta \end{aligned}$$

5. Diffusion models

Let $S = \{S_t\}_{t\geq 0}$ be an asset price process that evolves under a risk-neutral measure according to the SDE

$$\frac{dS_t}{S_t} = \gamma \mathsf{d}t + \sigma\left(S_t\right)\mathsf{d}W_t$$

where W is a Wiener process, and $\sigma : \mathbb{R}_+ \to \mathbb{R}_+$ is locally Lipschitz and such that $\{\mathrm{e}^{-\gamma t} S_t\}_{t\in[0,T]}$ is a martingale.

For $f \in C_0^2$ the infinitesimal generator of S is given by

$$\mathcal{L}f(s) = \frac{\sigma^2(s)s^2}{2} f''(s) + \gamma s f'(s).$$

5. Diffusion models

For a given finite state-space $\mathbb{G}$, the generator matrix Λ of the continuous-time Markov chain $X = \{X_t\}_{t\geq 0}$ is defined via the instantaneous moment matching conditions (set $X_0 := S_0 \in \mathbb{G}$):

$$\mathbb{E}_{S_0}\left[(S_{\Delta t} - S_0)^j\right] \quad = \quad \mathbb{E}_{X_0}\left[(X_{\Delta t} - X_0)^j\right] + o(\Delta t), \quad \text{for } j \in \{1,2\}.$$

The entries of Λ thus have to satisfy the system for each $x \in \mathbb{G}$:

$$\begin{aligned} \sum_{y\in\mathbb{G}} \Lambda(x,y) &= 0 \quad \text{and} \quad \Lambda(x,y) \geq 0 \ \forall y \in \mathbb{G}\backslash\{x\}, \\ \sum_{y\in\mathbb{G}} \Lambda(x,y)(y-x) &= \gamma x, \qquad (1) \\ \sum_{y\in\mathbb{G}} \Lambda(x,y)(y-x)^2 &= \sigma(x)^2 x^2 \end{aligned}$$

5. Diffusion models: CEV model

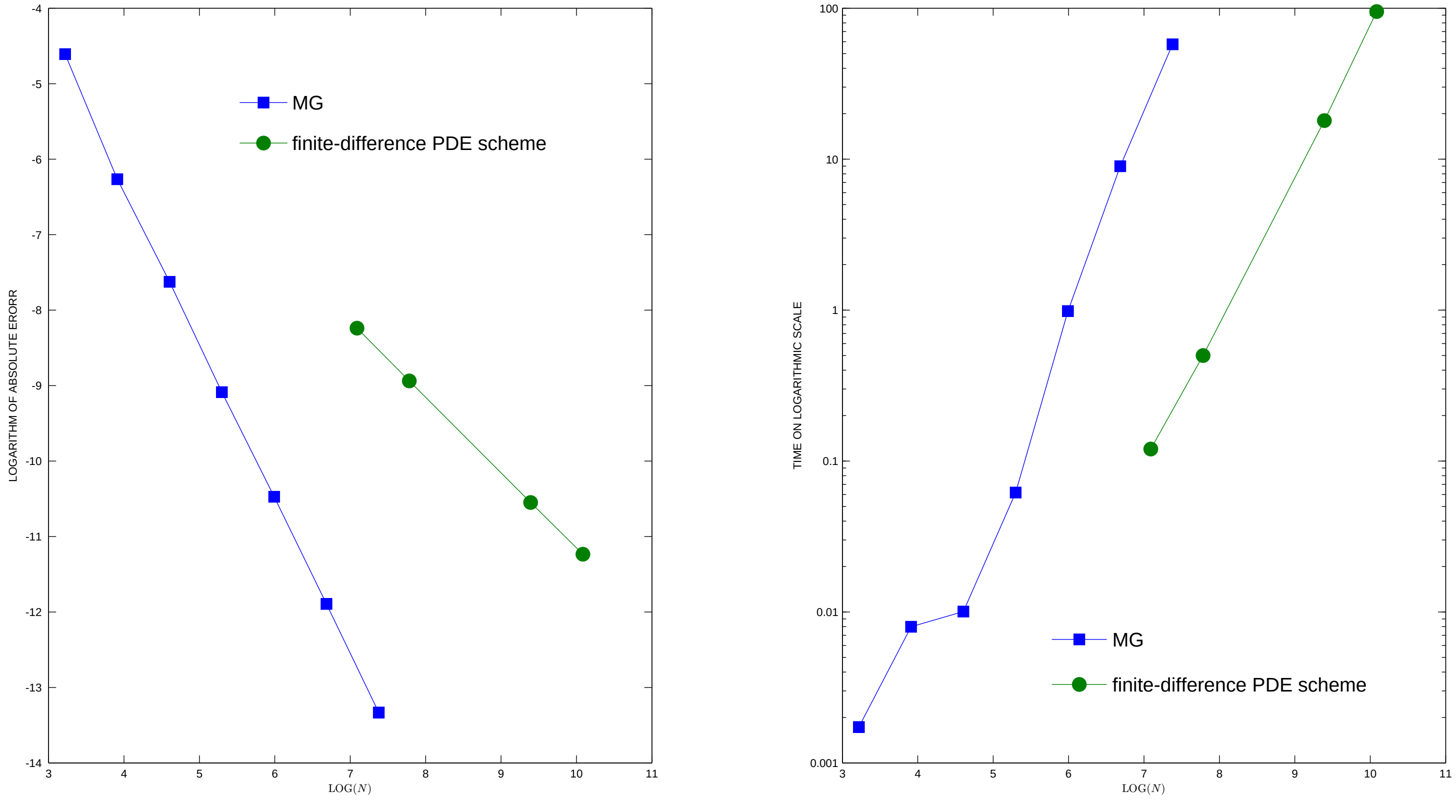


Figure 1: Blue is the MG algorithm and the green finite difference Crank-Nicholson PDE scheme.

5. State-dependent jump measure

General form of the generator $\mathcal{L}$ of a Feller process S:

$$\begin{aligned}\mathcal{L}f(x) &= \frac{\sigma^2(x)x^2}{2}\Delta f(x) + (r-d-\mu(x))x\nabla f(x) \\ &+ \int_{-1}^{\infty}[f(x(1+y)) - f(x) - \nabla f(x)xy\mathbf{I}_{\{|y|<1\}}]\nu(x,\mathsf{d}y),\end{aligned}$$

where $\mu,\sigma : E \to \mathbb{R}$ and for $x \in E$, $\nu(x,\mathsf{d}y)$ is a (Lévy) measure supported in $(-1,\infty)$ s.t. $\int_{-1}^{\infty} y^2\nu(x,\mathsf{d}y) < \infty$.

The discounted process $\{\mathrm{e}^{-(r-d)t}S_t\}_{t\geq 0}$ is a local martingale if

$$\mu(x) = \int_1^{\infty} y\nu(x,\mathsf{d}y) < \infty \qquad \forall x \in E.$$

5. State-dependent jump measure

To approximate this $\mathcal{L}$ we define a matrix $\Lambda = \Lambda_J + \Lambda_c$ as follows.

$$\begin{aligned}
\Lambda_J(x, x(1+y_i)) &:= \nu\left(x, (\alpha_x(y_{i-1}), \alpha_x(y_i))\right), \;\; y_i \neq 0, \\
\Lambda_J(x, x) &:= -\sum_{z \in \mathbb{G} \setminus \{x\}} \Lambda_J(x, z). \\
\sum_{z \in \mathbb{G}} \Lambda_c(x, z) &= 0 \quad \text{and} \quad \Lambda_c(x, z) \geq 0 \quad \forall z \in \mathbb{G} \setminus \{x\}, \\
\sum_{z \in \mathbb{G}} \Lambda_c(x, z)(z - x) &= (r - d)x - \sum_{z' \in \mathbb{G}} \Lambda_J(x, z')(z' - x), \\
\sum_{z \in \mathbb{G}} \Lambda_c(x, z)(z - x)^2 &= x^2 \left[\sigma\left(x\right)^2 + \int_{-1}^{\infty} y^2 \nu(x, \mathrm{d}y)\right] \\
&\quad - \sum_{z' \in \mathbb{G}} \Lambda_J(x, z')(z' - x)^2
\end{aligned}$$

5. Numerical example: CGMY

The price process S is modelled as

$$S_t = S_0 \mathrm{e}^{-(r-d)t} \frac{\mathrm{e}^{X_t}}{\mathbb{E}[\mathrm{e}^{X_t}]}$$

where X is a CGMY process, i.e. a Lévy process without a Gaussian component, with Lévy density

$$k(x) = 1_{\{x<0\}} C \frac{\mathrm{e}^{-G|x|}}{|x|^{Y+1}} + 1_{\{x>0\}} C \frac{\mathrm{e}^{-M|x|}}{|x|^{Y+1}}.$$

5. Numerical example: CGMY subordinator

In Madan and Yor (2006) it is shown that X has the same law as the process

$$X'_t = W_{Y_t} + \theta Y_t$$

where $\theta = (G-M)/2$ and Y is a subordinator that has Laplace exponent ψ

$$\mathbb{E}[\mathrm{e}^{-\lambda Y_t}] = \mathrm{e}^{t\psi(\lambda)} = \exp\left(tC\Gamma(-Y)\right)\left[2r^Y\cos(\eta Y) - M^Y - G^Y\right]$$

where

$$r(\lambda) := \sqrt{2\lambda + GM} \quad \text{and} \quad \eta(\lambda) := \arctan\left(\frac{\sqrt{2\lambda - \theta^2}}{\frac{G+M}{2}}\right).$$

5. Numerical example: CGMY

By the Philips theorem, the infinitesimal generator $\mathcal{L}$ of X' satisfies

$$\mathcal{L} = \psi(-\mathcal{G}), \tag{2}$$

where $\mathcal{G}$ is the infinitesimal generator of a Brownian motion with drift θ, that acts on $f \in C_0^2$ as

$$\mathcal{G}f = \frac{1}{2}f'' + \theta f'.$$

- Construct a Markov chain approximating the BM with drift by solving the related system (1).
- Subsequently use the relation (2) to obtain the generator matrix of the approximating chain for X.

5. Numerical example: CGMY

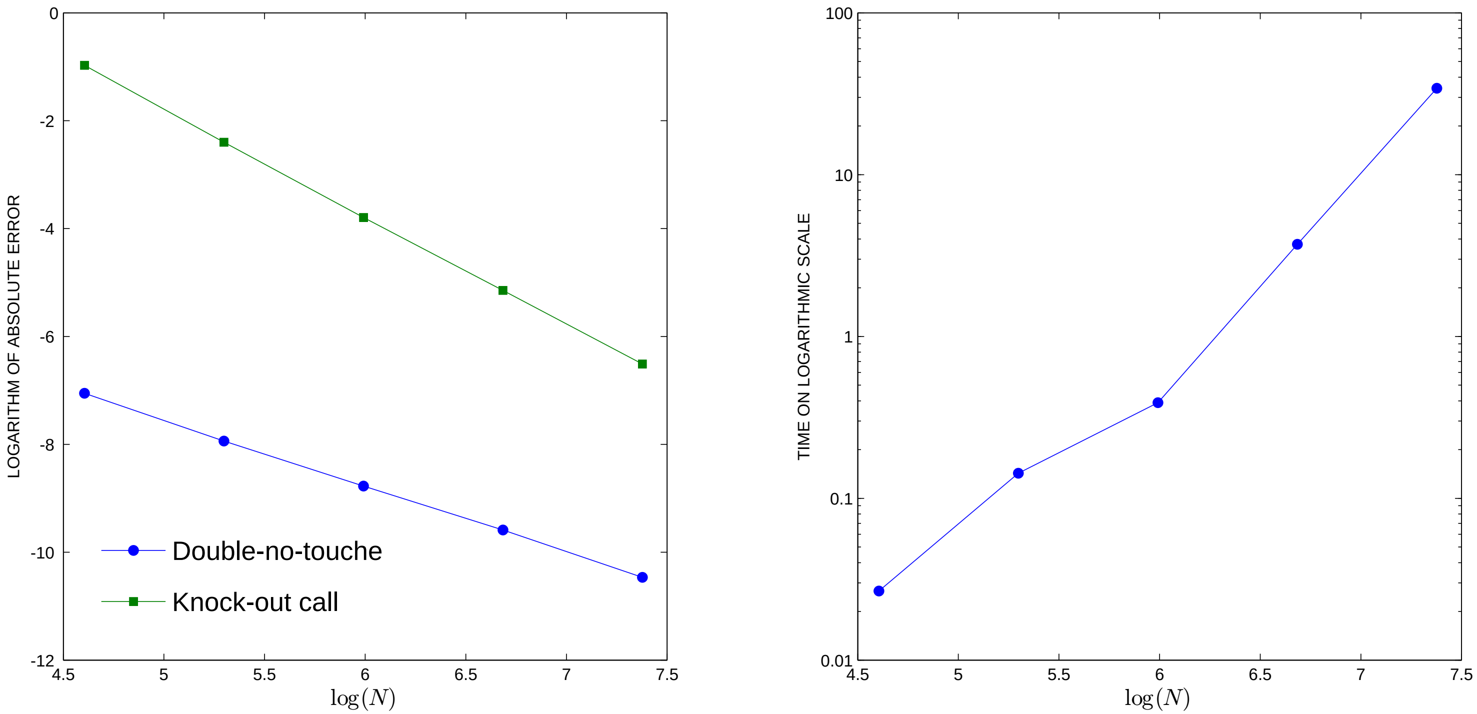


Figure 2: Slopes on the left are approximately -1.2 and -2.

5. Local Lévy model – Example of a Lévy driven SDE

$$\frac{\mathrm{d}S_t}{S_{t-}} = (r - d - \lambda\zeta(S_{t-}/S_0)^\beta)\mathrm{d}t + (S_{t-}/S_0)^\beta \mathrm{d}L_t, \qquad \text{where}$$

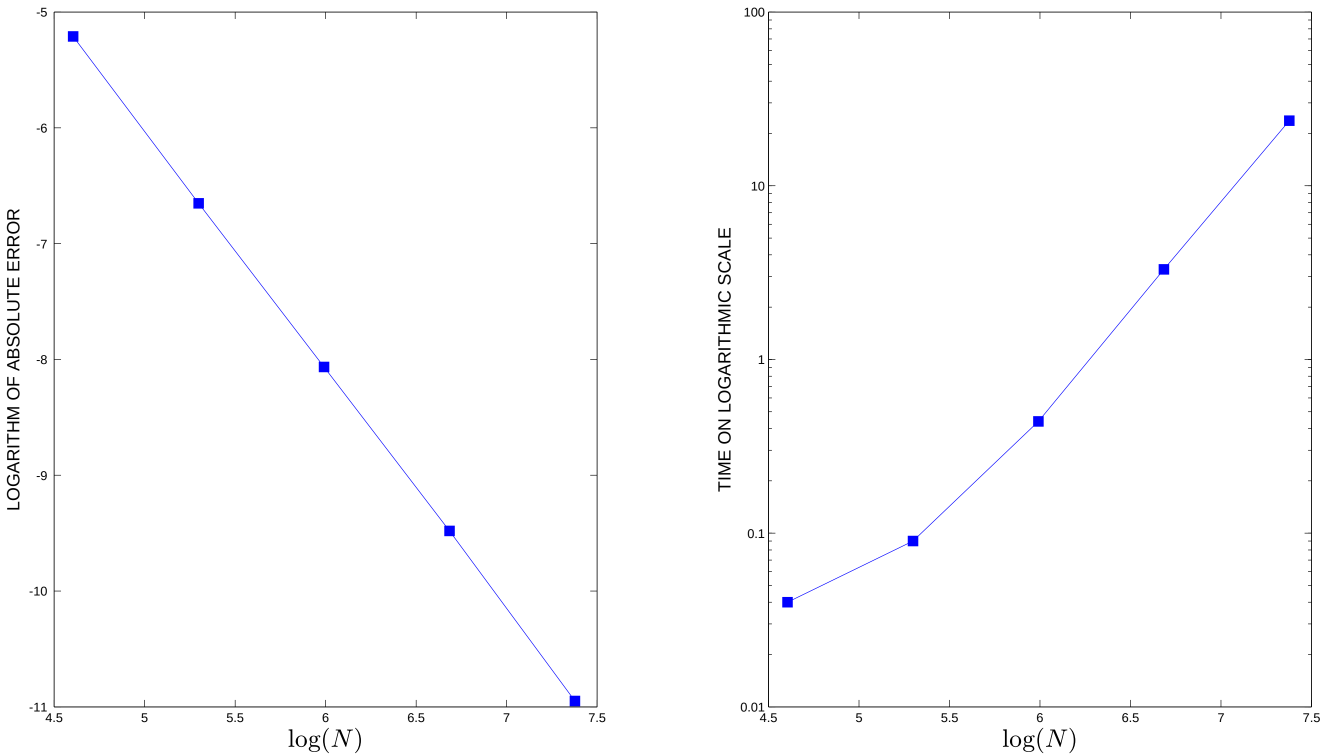
$$L_t := \sigma_0 W_t + \sum_{i=1}^{N_t} \left(\mathrm{e}^{K_i} - 1\right), \quad \sigma_0 \in (0,\infty) \quad \text{and} \quad \beta \in \mathbb{R}.$$

with state-dependent jump measure

$$\nu(x,\mathrm{d}y) = (x/S_0)^\beta \lambda \left[p\eta_1 (y+1)^{-1-\eta_1} \mathbf{I}_{\{y>0\}} + (1-p)\eta_2 (y+1)^{\eta_2 - 1} \mathbf{I}_{\{-1<y<0\}}\right] \mathrm{d}$$

- Brownian motion W, Poisson process N, double-exponential K_i, $i \in \mathbb{N}$, are all independent.
- The genrator of S is as described above.
- If $\beta = 0$ we get the Kou model.

5. Local Lévy model – numerical results



5. Time-inhomogeneous Markov chains

- State-space E, $N := |E|$, times $0 =: T_0 < T_1 < \cdots < T_n := T$ and $T_{n+1} = \infty$.
- X a continuous-time Markov chain on E with generator

$$\mathcal{L}_t := \sum_{i=1}^{n+1} \mathcal{L}_i 1_{[T_{i-1},T_i)}(t), \qquad t \geq 0,$$

  where $\mathcal{L}_i$, for $i \in \{1, \ldots, n+1\}$, is a generator matrix.
- Then for each $x \in E$ we have

$$\mathbb{E}_x \left[1_{\{\tau > T\}} \phi(X_T) \right] = \left(\exp\left(\Delta T_1 \widehat{\mathcal{L}}_1 \right) \cdots \exp\left(\Delta T_n \widehat{\mathcal{L}}_n \right) \phi \right)(x),$$

  where τ is the first passage time.

5. Numerical example: Sato process

CGMY (2007) introduced into financial modelling the process

$$S_t = S_0 \mathrm{e}^{(r-d)t} \frac{\mathrm{e}^{Y_t}}{\mathbb{E}_0[\mathrm{e}^{Y_t}]}$$

where Y is an additive process which is:

- *self-similar*: $Y_t \sim t^\gamma Y_1$ for some constant $\gamma > 0$ and all $t > 0$,
- the law of Y_1 is *self decomposable*.

CGMY (2007) prove that, if Y has bounded variation, the characteristic function of Y_t is of the form

$$\Phi_Y(u,t) = \mathbb{E}_0\left[\mathrm{e}^{iuY_t}\right] \quad = \quad \exp\left(\int_{\mathbb{R}} \left(\mathrm{e}^{iuy} - 1\right) \frac{h(y/t^\gamma)}{|y|} \mathrm{d}y\right).$$

5. Numerical example: VG-Sato process

VG-Sato: $h(x) = C\exp(-G|x|)1_{\{x<0\}} + C\exp(-Mx)1_{\{x>0\}}$.

Approximate Y by a **time-inhomogeneous** Markov process X^n with a piecewise constant generator on $0 = t_0 < t_1 < \ldots < t_n = T$:

- On the time interval (t_i, t_{i+1}), X^n is a *forward Variance Gamma* process, with the characteristic exponent

$$(t_{i+1} - t_i)^{-1} \log(\Phi_Y(u, t_{i+1})/\Phi_Y(u, t_i)).$$

- We have $(Y_{t_1}, \ldots, Y_{t_n}) \sim (X^n_{t_1}, \ldots, X^n_{t_n})$.

5. Numerical example: VG-Sato process

Sato process $\nu = 26.4, \gamma = -0.53, \theta = 0, \sigma = 1$		$n = 5$	$n = 50$	$n = 100$	$n = 500$
N=600	KO Call:	0.4534	0.4604	0.4605	0.4605
	Double-no-touch:	0.1459	0.1481	0.1481	0.1481
N=1200	KO Call:	0.4628	0.4699	0.4700	0.4700
	Double-no-touch:	0.1483	0.1504	0.1504	0.1504
N=1800	KO Call:	0.4652	0.4722	0.4722	0.4722
	Double-no-touch:	0.1489	0.1510	0.1510	0.1510
N=2400	KO Call:	0.4662	0.4732	0.4732	0.4732
	Double-no-touch:	0.1491	0.1512	0.1512	0.1512
N=3000	KO Call:	0.4668	0.4737	0.4737	0.4737
	Double-no-touch:	0.1493	0.1513	0.1513	0.1513

The prices of the double barrier knock-out call option and the double-no-touch option in the Sato VG model.
Market data: $S_0 = 100$, $r = 0.02$, $d = 0$ and $T = 0.1$.
Contracts: $K = 100$, $L = 80, U = 120$.
MC approx: N number of states, n the number of time-steps.

6. Conclusion

- General class of models: Markov processes
- Consistent pricing: European and barrier options
- Easy, robust implementation
- Convergence and error estimates

Preprint: A. Mijatovic & M. Pistorius, Continuously monitored barrier options under Markov processes.

available at **http://ssrn.com/abstract=1462822**

6. Conclusion

Possible future work:

- Sharp rates of convergence (under weaker smoothness conditions)
- Extension to moderate dimensions: efficient moderate-dimensional grids