Information Percolation

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based on joint work with Darrell Dufife, Gaston Giroux, and Semyon Malamud

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Information Transmission in Markets

Informational Role of Prices: Hayek (1945), Grossman (1976), Grossman and Stiglitz (1981).

- Centralized Exchanges:
  - Wilson (1977), Townsend (1978), Milgrom (1981), Vives (1993), Pesendorfer and Swinkels (1997), and Reny and Perry (2006).
- Over-the-Counter Markets:
  - Wolinsky (1990), Blouin and Serrano (2002), Golosov, Lorenzoni, and Tsyvinski (2009).

Information Percolation

- Closed-form solutions for the evolution of the cross-sectional distribution of beliefs.
  - Duffie and Manso (2007).
- Rates of convergence to REE price under different market structures:
  - Duffie, Giroux, and Manso (2008), Duffie, Malamud, and Manso (2009).
- Value of information/connectivity in a segmented market:
  - Duffie, Malamud, and Manso (2010).

Outline of the Talk

1. Information Percolation
2. Double Auction
3. Connectedness and Information

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Model Primitives

Duffie and Manso (2007) and Duffie, Giroux, and Manso (2010):

- Continuum of agents
- Two possible states of nature $Y \in \{0, 1\}$.
- Common prior $\mathrm{P}(Y = 0)$ and $\mathrm{P}(Y = 1)$ over Y.
- Each agent j is initially endowed with signals $S = \{s_1, \dots, s_n\}$ s.t. $\mathrm{P}_j(s_i = 1 \mid Y = 1) \geq \mathrm{P}_j(s_i = 1 \mid Y = 0)$
- For every pair agents, their initial signals are Y-conditionally independent
- Random matching, intensity λ.

Initial Information Endowment

After observing signals $S = \{s_1, \dots, s_n\}$, the logarithm of the likelihood ratio between states $Y = 0$ and $Y = 1$ is by Bayes' rule:

$$\log \frac{\mathrm{P}(Y = 0 \,|\, s_1, \dots, s_n)}{\mathrm{P}(Y = 1 \,|\, s_1, \dots, s_n)} = \log \frac{\mathrm{P}(Y = 0)}{\mathrm{P}(Y = 1)} + \sum_{i=1}^{n} \log \frac{\mathrm{P}(s_i \,|\, Y = 0)}{\mathrm{P}(s_i \,|\, Y = 1)}.$$

We say that the "type" θ associated with this set of signals is

$$\theta = \sum_{i=1}^{n} \log \frac{\mathrm{P}(s_i \,|\, Y = 0)}{\mathrm{P}(s_i \,|\, Y = 1)}.$$

Bargaining Protocol: Double Auction

- Upon meeting, agents participate in a double auction.
- If bids are strictly increasing in the type associated with the signals agents have collected, then bids reveal type.

Information is Additive in Type Space

Proposition: Let $S = \{s_1, \ldots, s_n\}$ and $R = \{r_1, \ldots, r_m\}$ be independent sets of signals, with associated types θ and ϕ. If two agents with types θ and ϕ reveal their their types to each other, then both agents achieve the posterior type $\theta + \phi$.

This follows from Bayes' rule, by which

$$
\begin{aligned}
\log \frac{\mathrm{P}(Y = 0 \,|\, S, R, \theta + \phi)}{\mathrm{P}(Y = 1 \,|\, S, R, \theta + \phi)} &= \log \frac{\mathrm{P}(Y = 0)}{\mathrm{P}(Y = 1)} + \theta + \phi, \\
&= \log \frac{\mathrm{P}(Y = 0 \,|\, \theta + \phi)}{\mathrm{P}(Y = 1 \,|\, \theta + \phi)}
\end{aligned}
$$

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This follows from Bayes' rule, by which

$$\begin{aligned} \log \frac{P(Y = 0 \,|\, S, R, \theta + \phi)}{P(Y = 1 \,|\, S, R, \theta + \phi)} &= \log \frac{P(Y = 0)}{P(Y = 1)} + \theta + \phi, \\ &= \log \frac{P(Y = 0 \,|\, \theta + \phi)}{P(Y = 1 \,|\, \theta + \phi)} \end{aligned}$$

By induction, this property holds for all subsequent meetings.

Solution for Cross-Sectional Distribution of Information

The Boltzmann equation for the cross-sectional distribution μ_t of types is

$$\frac{d}{dt}\mu_t = -\lambda\,\mu_t + \lambda\,\mu_t * \mu_t.$$

with a given initial distribution of types μ_0.

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Proposition: The unique solution of (10) is the Wild sum

$$\mu_t = \sum_{n \geq 1} e^{-\lambda t}(1 - e^{-\lambda t})^{n-1} \mu_0^{*n}.$$

Proof of Wild Summation

Taking the Fourier transform of μ_t of the Boltzmann equation

$$\frac{d}{dt}\mu_t = -\lambda\,\mu_t + \lambda\,\mu_t * \mu_t.$$

we obtain the following ODE

$$\frac{d}{dt}\hat{\mu}_t = -\lambda\,\hat{\mu}_t + \lambda\,\hat{\mu}_t^2.$$

whose solution is

$$\hat{\mu}_t = \frac{\hat{\mu}_0}{e^{\lambda t}(1-\hat{\mu}_0)+\hat{\mu}_0}.$$

This solution can be expanded as

$$\hat{\mu}_t = \sum_{n\geq 1} e^{-\lambda t}(1-e^{-\lambda t})^{n-1}\hat{\mu}_0^n,$$

which is the Fourier transform of the Wild sum (10).

Convergence Rate

We let π_t be the cross-sectional distribution of posteriors.

We say that the rate of convergence of π_t to π_∞ is exponential at the rate $\alpha > 0$ if there are constants κ_0 and κ_1 such that, for any b in $(0, 1)$,

$$e^{-\alpha t}\kappa_0 \leq |\pi_t(0, b) - \pi_\infty(0, b)| \leq e^{-\alpha t}\kappa_1.$$

Convergence Rate in Two-Agent Meetings

Proposition: Convergence of the cross-sectional distribution of beliefs to that of complete information is exponential at the rate λ.

Proof of the Proposition

The evolution of cross-sectional types is given by:

$$\mu_t = \sum_{n\geq 1} e^{-\lambda t}(1-e^{-\lambda t})^{n-1}\mu_0^{*n}. \tag{1}$$

1. Lower bound: $\mu_t(-\infty, a) \geq e^{-\lambda t}\mu_0(-\infty, a)$.
2. Upper bound: large deviation result.

   If X is a random variable with a finite strictly positive mean and a moment generating function that is finite on $(-c, 0]$ for some $c > 0$, then $\mathrm{P}(X \leq 0) \leq \inf_{-c<s<0} E[e^{sX}] < 1$.

3. From type to posterior:

$$\pi_t(0, b) = \mu_t\left(-\infty, \log\frac{b}{(1-b)} - \log\frac{\nu}{(1-\nu)}\right).$$

Multi-Agent Meetings

The Boltzmann equation for the cross-sectional distribution μ_t of types is

$$\frac{d}{dt}\mu_t = -\lambda\,\mu_t + \lambda\,\mu_t^{*m}.$$

Taking the Fourier transform, we obtain the ODE,

$$\frac{d}{dt}\hat{\mu}_t = -\lambda\,\hat{\mu}_t + \lambda\,\hat{\mu}_t^{m}.$$

whose solution satisfies

$$\hat{\mu}_t^{m-1} = \frac{\hat{\mu}_0^{m-1}}{e^{(m-1)\lambda t}(1-\hat{\mu}_0^{m-1}) + \hat{\mu}_0^{m-1}}. \tag{2}$$

Wild Summation Solution

The unique solution of the Boltzmann equation for m-at-a-time matching is

$$\mu_t = \sum_{n \geq 1} a_{[(m-1)(n-1)+1]} e^{-\lambda t} (1 - e^{-(m-1)\lambda t})^{n-1} \mu_0^{*[(m-1)(n-1)+1]},$$

where $a_1 = 1$ and, for $n > 1$,

$$a_{(m-1)(n-1)+1} = \frac{1}{m-1} \left(1 - \sum_{\left\{ \substack{i_1, \ldots, i_{(m-1)} < n \\ \sum i_k = n+m-2} \right\}} \prod_{k=1}^{m-1} a_{[(m-1)(i_k-1)+1]} \right).$$

Invariance of the Convergence Rate

Proposition: For any group size m, the cross-sectional distribution π_t of posteriors converges to a common posterior distribution exponentially at the rate λ.

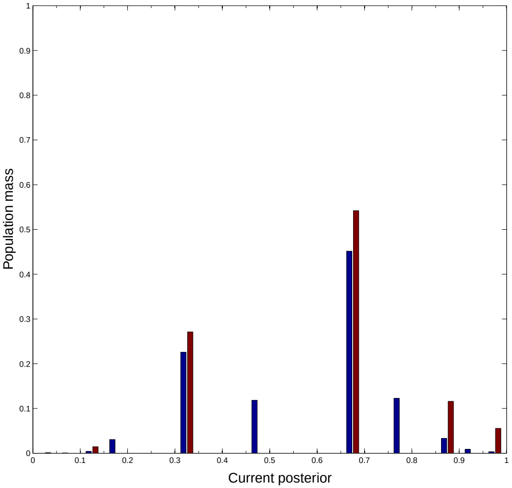
Proof of the Proposition

The cross-sectional distribution of types evolves according to:

$$\mu_t = \sum_{n \geq 1} a_{[(m-1)(n-1)+1]} e^{-\lambda t} (1 - e^{-(m-1)\lambda t})^{n-1} \mu_0^{*[(m-1)(n-1)+1]}.$$

1. Lower bound: $\mu_t(-\infty, a) \geq e^{-\lambda t} \mu_0(-\infty, a)$.
2. Upper bound: follows from the previous upper bound result.

Groups of 2 (blue) versus Groups of 3 (red)



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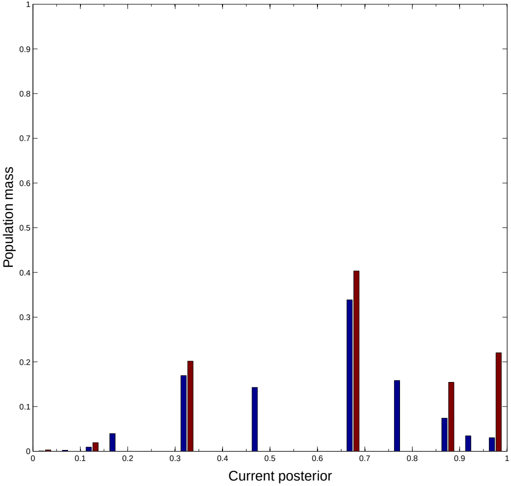

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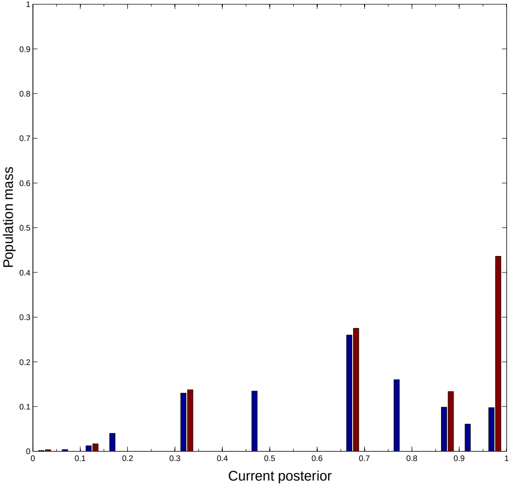

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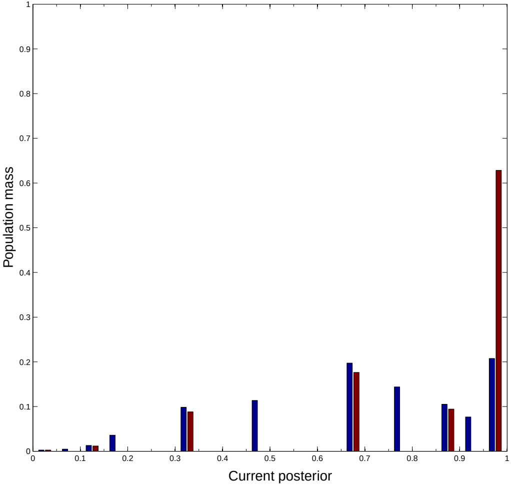

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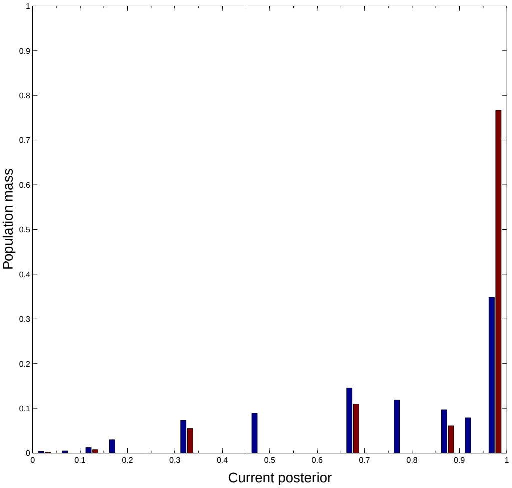

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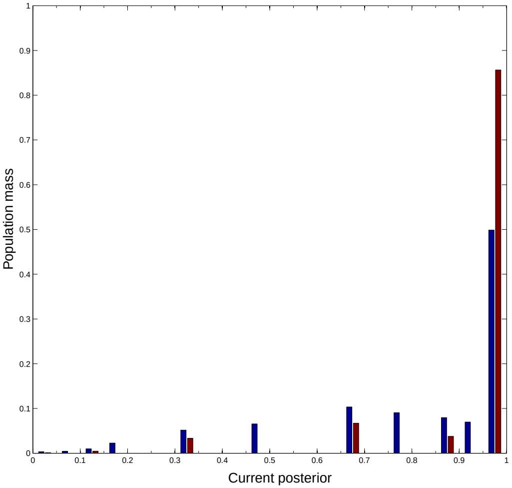

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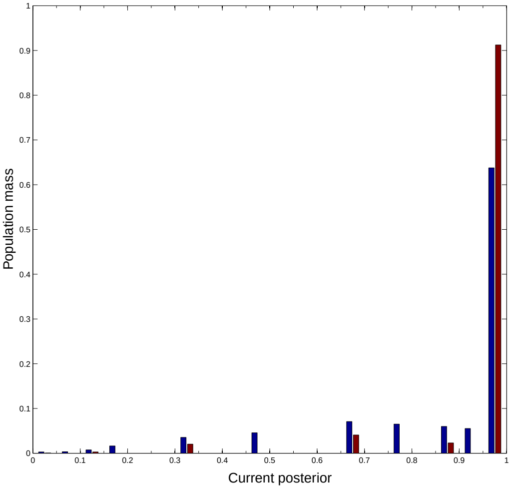

New Private Information

Suppose that, independently across agents as above, each agent receives, at Poisson mean arrival rate ρ, a new private set of signals whose type outcome y is distributed according to a probability measure ν. Then the evolution equation is extended to

$$\frac{d}{dt}\mu_t = -(\lambda + \rho)\,\mu_t + \lambda\,\mu_t * \mu_t + \rho\,\mu_t * \nu.$$

Taking Fourier transforms, we obtain the following ODE

$$\frac{d}{dt}\hat{\mu}_t = -(\lambda + \rho)\,\hat{\mu}_t + \lambda\,\hat{\mu}_t^2 + \rho\,\hat{\mu}_t\,\hat{\nu}.$$

whose solution satisfies

$$\hat{\mu}_t = \frac{\hat{\mu}_0}{e^{(\lambda+\rho(1-\hat{\nu}))t}(1-\hat{\mu}_0) + \hat{\mu}_0}$$

Other Extensions

- Public information releases
  - Duffie, Malamud, and Manso (2010).

- Endogenous search intensity
  - Duffie, Malamud, and Manso (2009).

Segmented Markets

Duffie, Malamud, and Manso (2010). Same as the previous model except that:

- N classes of investors.
- Agent of class i has matching intensity λ_i.
- Upon meeting, the probability that a class-j agent is selected as a counterparty is κ_{ij}.

Evolution of Type Distribution

The evolution equation is given by:

$$\frac{d}{dt}\psi_{it} \;=\; -\lambda_i\,\psi_{it} \;+\; \lambda_i\,\psi_{it} * \sum_{j=1}^{N} \kappa_{ij}\,\psi_{jt}\,, \quad i \in \{1,\ldots,N\},$$

Taking Fourier transforms we obtain:

$$\frac{d}{dt}\hat{\psi}_{it} \;=\; -\lambda_i\,\hat{\psi}_{it} \;+\; \lambda_i\,\hat{\psi}_{it} \sum_{j=1}^{N} \kappa_{ij}\,\hat{\psi}_{jt}\,, \quad i \in \{1,\ldots,N\},$$

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Taking Fourier transforms we obtain:

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General Case: Wild Sum Representation

Theorem: There is a unique solution of the evolution equation, given by

$$\psi_{it} = \sum_{k \in \mathbb{Z}_+^N} a_{it}(k)\, \psi_{10}^{*k_1} * \cdots * \psi_{N0}^{*k_N},$$

where ψ_{i0}^{*n} denotes n-fold convolution,

$$a'_{it} = -\lambda_i\, a_{it} + \lambda_i\, a_{it} * \sum_{j=1}^{N} \kappa_{ij}\, a_{jt}, \quad a_{i0} = \delta_{e_i},$$

$$(a_{it} * a_{jt})(k_1, \ldots, k_N) = \sum_{l=(l_1,\ldots,l_N) \in \mathbb{Z}_+^N,\, l<k} a_{it}(l)\, a_{jt}(k-l),$$

and

$$a_{it}(e_i) = e^{-\lambda_i t}\, a_{i0}(e_i).$$

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Double Auction

- At some time T, the economy ends and the utility realized by an agent of class i for each additional unit of the asset is

$$U_i = v_i Y + v^H (1 - Y),$$

measured in units of consumption, for strictly positive constants v^H and $v_i < v^H$, where Y is a non-degenerate 0-or-1 random variable whose outcome will be revealed at time T.

- If $v_i = v_j$, no trade (Milgrom and Stokey (1982), Serrano-Padial (2010)), so that $\kappa_{ij} = 0$.
- Meeting between two agents $v_i > v_j$, then i is buyer and j is seller.
- Upon meeting, participate in a double auction. If the buyer's bid β is higher than the seller's ask σ, trade occurs at the price σ.

Equilibrium

The prices (σ, β) constitute an equilibrium for a seller of class *i* and a buyer of class *j* provided that, fixing β, the offer σ maximizes the seller's conditional expected gain,

$$E\left[(\sigma - E(U_i \mid \mathcal{F}_S \cup \{\beta\}))1_{\{\sigma<\beta\}} \mid \mathcal{F}_S\right],$$

and fixing σ, the bid β maximizes the buyer's conditional expected gain

$$E\left[(E(U_j \mid \mathcal{F}_B \cup \{\sigma\}) - \sigma)1_{\{\sigma<\beta\}} \mid \mathcal{F}_B\right].$$

Counterexample: Reny and Perry (2006)

Equilibrium

The prices (σ, β) constitute an equilibrium for a seller of class i and a buyer of class j provided that, fixing β, the offer σ maximizes the seller's conditional expected gain,

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$$E\left[(E(U_j \mid \mathcal{F}_B \cup \{\sigma\}) - \sigma)1_{\{\sigma<\beta\}} \mid \mathcal{F}_B\right].$$

Counterexample: Reny and Perry (2006)

Restriction on the Initial Information Endowment

Lemma: Suppose that each signal Z satisfies

$$\mathbb{P}(Z = 1 \mid Y = 0) + \mathbb{P}(Z = 1 \mid Y = 1) = 1.$$

Then, for each agent class i and time t, the type density ψ_{it} satisfies the hazard-rate order condition as well as the property

$$\psi_{it}^{H}(x) = e^{x}\psi_{it}^{H}(-x), \qquad \psi_{it}^{L}(x) \;=\; \psi_{it}^{H}(-x) \quad x \in \mathbb{R}.$$

Bidding Strategies

Lemma: For any $V_0 \in \mathbb{R}$, there exists a unique solution $V_2(\cdot)$ on $[v_i, v^H)$ to the ODE

$$V_2'(z) = \frac{1}{v_i - v_j}\left(\frac{z - v_i}{v^H - z}\frac{1}{h_{it}^H(V_2(z))} + \frac{1}{h_{it}^L(V_2(z))}\right), \quad V_2(v_i) = V_0.$$

This solution, also denoted $V_2(V_0, z)$, is monotone increasing in both z and V_0. Further, $\lim_{v \to v^H} V_2(v) = +\infty$. The limit $V_2(-\infty, z) = \lim_{V_0 \to -\infty} V_2(V_0, z)$ exists. Moroever, $V_2(-\infty, z)$ is continuously differentiable with respect to z.

Bidding Strategies

Proposition: Suppose that (S, B) is an absolutely continuous equilibrium such that $S(\theta) \leq v^H$ for all $\theta \in \mathbb{R}$. Let $V_0 = B^{-1}(v_i) \geq -\infty$. Then,

$$B(\phi) = V_2^{-1}(\phi), \quad \phi > V_0,$$

Further, $S(-\infty) = \lim_{\theta \to -\infty} S(\theta) = v_i$ and $S(+\infty) = \lim_{\theta \to -\infty} S(\theta) = v^H$, and for any θ, we have $S(\theta) = V_1^{-1}(\theta)$ where

$$V_1(z) = \log \frac{z - v_i}{v^H - z} - V_2(z), \quad z \in (v_i, v^H).$$

Any buyer of type $\phi < V_0$ will not trade, and has a bidding policy B that is not uniquely determined at types below V_0.

Tail Condition

Definition: We say that a probability density $g(\cdot)$ on the real line is of exponential type α at $+\infty$ if, for some constants $c > 0$ and $\gamma > -1$,

$$\lim_{x \to +\infty} \frac{g(x)}{x^{\gamma} e^{\alpha x}} = c$$

In this case, we write $g(x) \sim \mathrm{Exp}_{+\infty}(c, \gamma, \alpha)$.

Exponential Tails in Percolation Models

Suppose $N = 1$, and let $\lambda = \lambda_1$ and $\psi_t = \psi_{1t}$. The Laplace transform $\hat{\psi}_t$ of ψ_t is given by

$$\hat{\psi}_t(z) \ = \ \frac{e^{-\lambda t}\,\hat{\psi}_0(z)}{1-(1-e^{-\lambda t})\hat{\psi}_0(z)}$$

and $\psi_t(x) \ \sim \ \mathrm{Exp}_{+\infty}(c_t, 0, -\alpha_t)$ in t, where α_t is the unique positive number z solving

$$\hat{\psi}_0(z) \ = \ \frac{1}{1-e^{-\lambda t}},$$

and where

$$c_t \ = \ \frac{e^{-\lambda t}}{(1-e^{-\lambda t})^2 \ \frac{d}{dz}\hat{\psi}_0(\alpha_t)}.$$

Furthermore, α_t is monotone decreasing in t, with $\lim_{t\to\infty} \alpha_t \ = \ 0$.

Strictly Monotone Equilibrium

Proposition: Suppose that, for all t in $[0, T]$, there are $\alpha_i(t)$, $c_i(t)$, and $\gamma_i(t)$ such that

$$\psi_{it}^H(x) \sim \text{Exp}_{+\infty}(c_i(t), \gamma_i(t), -\alpha_i(t)).$$

If $\alpha_i(T) < 1$, then there is no equilibrium associated with $V_0 = -\infty$. Moreover, if $v_i - v_j$ is sufficiently large and if $\alpha_i(T) > \alpha^*$, where α^* is the unique positive solution to $\alpha^* = 1 + 1/(\alpha^* 2^{\alpha^*})$ (which is approximately 1.31), then there exists a unique strictly monotone equilibrium associated with $V_0 = -\infty$. This equilibrium is in undominated strategies, and maximizes total welfare among all continuous equilibria.

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Class-i Agent Utility

The expected future profit at time t of a class-i agent is

$$\mathcal{U}_i(t, \Theta_t) \; = \; E\left[\sum_{\tau_k > t} \sum_j \kappa_{ij} \, \pi_{ij}(\tau_k, \Theta_{\tau_k}) \;\Big|\; \Theta_t \right],$$

where τ_k is this agent's k-th auction time and $\pi_{ij}(t, \theta)$ is the expected profit of a class-i agent of type θ entering an auction at time t with a class-j agent.

Agents may be able to disguise the characteristics determining their information at a particular auction. In this case, we denote the expected future profit at time t of a class-i agent as $\hat{\mathcal{U}}_i(t, \Theta_t)$.

The Value of Initial Information and Connectivity When Trades Can be Disguised

Theorem: Suppose that $v_1 = v_2$. If $\lambda_2 \geq \lambda_1$ and if the initial type densities ψ_{10} and ψ_{20} are distinguished by the fact that the density p_2 of the number of signals received by class-2 agents has first-order stochastic dominance over the density p_1 of the number of signals by class-1 agents, then

$$\frac{E[\hat{\mathcal{U}}_2(t, \Theta_{2t})]}{\lambda_2} \geq \frac{E[\hat{\mathcal{U}}_1(t, \Theta_{1t})]}{\lambda_1}, \quad t \in [0, T].$$

The above inequality holds strictly if, in addition, $\lambda_2 > \lambda_1$ or if p_2 has strict dominance over p_1.

What if Characteristics are Commonly Observed?

- trade-off between adverse selection and gains from trade.
- more informed/connected investor may achieve lower profits than less informed/connected investor.

What if Characteristics are Commonly Observed?

- If $v_1 = v_2 = 0.9$, $v_3 = 0$, $v^H = 1.9$,

$$\psi_{10}(x) = 12\,\frac{e^{3x}}{(1+e^x)^5},$$

and $\psi_{20}(x) \; = \psi_{10} * \psi_{10}$.

Then,

$$E[\mathcal{U}_2(t, \Theta_{1t})] \; < \; E[\mathcal{U}_1(t, \Theta_{2t})]$$

and

$$E[\hat{\mathcal{U}}_1(t, \Theta_{1t})] \; < \; E[\mathcal{U}_1(t, \Theta_{2t})].$$

Even If Characteristics are Commonly Observed Connectivity May be Valuable

Proposition: Suppose that $\kappa_1 = \kappa_2$ and $\lambda_1 < \lambda_2$, and suppose that class-1 and class-2 investors have the same initial information quality, that is, $\psi_{10} = \psi_{20}$, and assume the exponential tail condition $\psi_{it}^H \sim \text{Exp}_{+\infty}(c_{it}, \gamma_{it}, -\alpha_{it})$ for all i and t, with $\alpha_{10} > 3$,

$$\alpha_{30} > \frac{\alpha_{10} - 1}{3 - \alpha_{10}},$$

and

$$\frac{\alpha_{1t} + 1}{\alpha_{1t} - 1} > \alpha_{3t}, \quad t \in [0, T].$$

If $\bar{v} - v_3$ is sufficiently large, then for any time t we have

$$\frac{E[\mathcal{U}_2(t, \Theta_{2t})]}{\lambda_2} > \frac{E[\hat{\mathcal{U}}_2(t, \Theta_{2t})]}{\lambda_2} > \frac{E[\hat{\mathcal{U}}_1(t, \Theta_{1t})]}{\lambda_1} > \frac{E[\mathcal{U}_1(t, \Theta_{1t})]}{\lambda_1}.$$

Subsidizing Order Flow

- Investors i and j with $v_i = v_j$ meet at time t.
- Enter a swap agreement by which the amount

$$k\left[(p_j(t) - Y)^2 - (p_i(t) - Y)^2\right],$$

  will be paid by investor i to investor j at time T.
- Increase connectivity of class i investors.
- When would investors want to subsidize order flow?

Concluding Remarks

- tractable model of information diffusion in over-the-counter markets.
- rates of convergence for different market structures.
- initial information and connectivity may or may not increase profits:
  - more informed/connected investors attain higher profits than less informed connected investors when investors can disguise trades.
  - more informed/connected investors may attain lower profits than less informed connected investors when investors' characteristics are commonly observed.

Future Research

- Endogenous information acquisition and convergence.
- Market design

Other Applications

- centralized exchanges, decentralized information transmission
- bank runs
- knowledge spillovers
- social learning
- technology diffusion