Default intensities implied by CDO spreads: inversion formula and model calibration

Yu Hang KAN

Joint work with Rama CONT and Romain DEGUEST

IEOR Department
Columbia University - New York

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Credit portfolio models

Bottom-up models: Model individual default rates + "default correlation" structure.

- Static (copula) models. Li (2001).
- Dynamic reduced form models: Factor model with affine processes as factors. Duffie and Gârleanu (2001).
- Multi-name structural models.

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Top-down models: Model loss process (L_t) of the portfolio as an increasing jump process by specifying its intensity (λ_t).

- Local intensity model: $\lambda_t = F(t, L_t)$. Cont and Minca (2008), Herbertsson (2008), Laurent et al (2007).
- Two factor spread/default model: $\lambda_t = F(t, L_t, X_t)$. Arnsdorff and Halperin (2008), Lopatin and Misirpashaev (2007).
- Self-exciting defaults. Giesecke and Goldberg (2008), Errais et al. (2008).

Motivation

- Although dynamic models are more realistic, they are typically more difficult to estimate. The main obstacle in their implementation has been the lack of stable calibration methods.
- Common practice to calibrate dynamic models: Black-box optimization applied to non-convex, non-linear least squares minimization.
- Problem: Convergence and stability are not guaranteed.
- Alternative: Calibration of portfolio default intensity via entropy minimization by Cont and Minca (2008).

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- Problem: Convergence and stability are not guaranteed.
- Alternative: Calibration of portfolio default intensity via entropy minimization by Cont and Minca (2008).
- We develop a simple method to recover the portfolio default intensity based on an **analytical inversion formula** and **quadratic programming** and compare it with alternative calibration methods: parametric method by Herbertsson (2008) and entropy minimization method by Cont and Minca (2008).
- Comparisons reveal a large amount of model uncertainty in pricing and hedging.

Roadmap

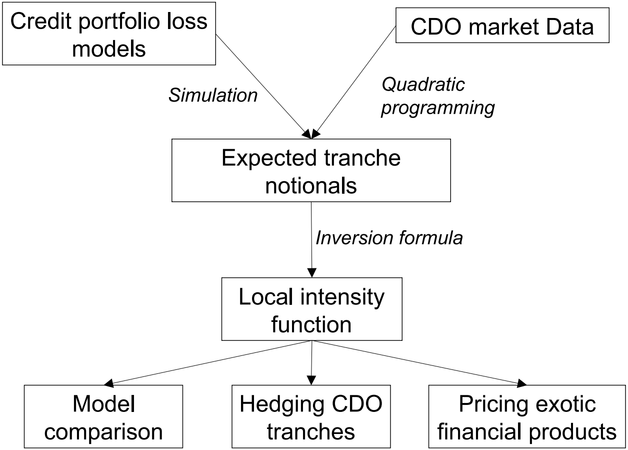
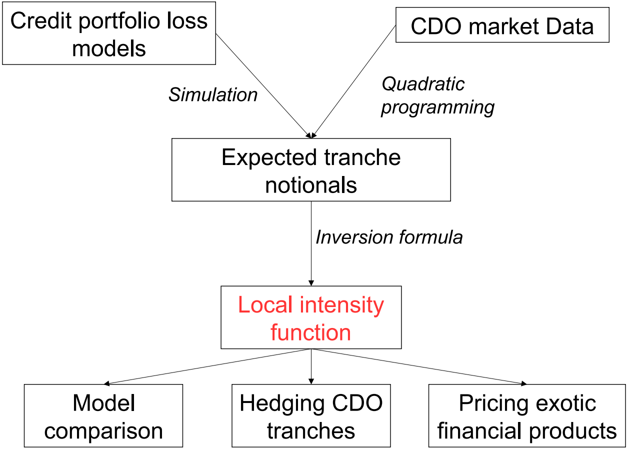


Figure 1: Application of the inversion formula to recover the local intensity function.

Roadmap



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Local intensity function and Markovian projection

- An equally weighted credit portfolio consisting of n names.
- N_t: number of defaults by time t.
- δ: loss given default, assumed to be constant.
- $L_t = \delta N_t$: credit portfolio loss at time t.
- Assumption: (N_t) admits an intensity (λ_t).
- Interest rates are independent from default times.

Definition 1

Consider a loss process satisfying the above setting with

$$\forall t \in (0, T^*], \qquad E[\lambda_t] < \infty.$$

The *local intensity function* $a : [0, T^*] \times \{0, 1, .., n\} \mapsto \mathbb{R}_+$ at $t = 0$ is defined as

$$a(t, i) := E^{\mathbb{Q}}[\lambda_t | N_{t-} = i, \mathcal{F}_0]. \tag{1}$$

If $\mathbb{Q}(N_{t-} = i | \mathcal{F}_0) = 0$, we set $a(t, i) = 0$ by convention. We call $\lambda_t^{\text{eff}} := a(t, N_{t-})$ the *effective intensity* of the loss process.

Mimicking marked point processes with Markovian jump processes

Proposition 1 (Cont and Minca (2008))

Consider any non-explosive jump process (L_t) with an intensity (λ_t) and i.i.d. jumps with distribution G. Define $(\tilde{L}_t)$ as the Markovian jump process with jump size distribution G and intensity $(a(t, \tilde{N}_{t-}))$. Then, for any $t \in [0, T^*]$, L_t and $\tilde{L}_t$ have the same distribution conditional on $\mathcal{F}_0$. In particular, the flow of marginal distributions of (L_t) only depends on the intensity (λ_t) through its conditional expectation $a(.,.)$.

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- The local intensity function is an analogue to the local volatility function

$$(\sigma^{local}(t, K))^2 = E^{\mathbb{Q}}[\sigma_t^2 | \mathcal{F}_0, S_t = K]$$

for stochastic volatility models.
- Gyöngy (1986) shows a mimicking theorem for Ito processes.
- Bentata and Cont (2009) show a more general mimicking theorem for discontinuous semimartingales.

Forward equations for marginal distribution

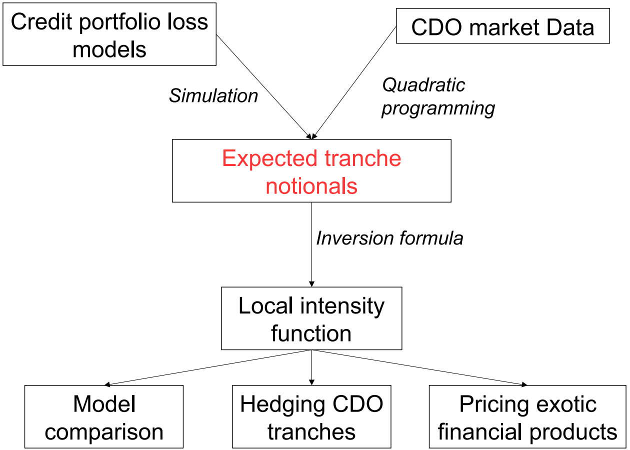
For a Markovian jump process, the transition probabilities $\mathbb{Q}(N_T = i|\mathcal{F}_0) = q(T, i)$ can be computed by solving a Fokker-Planck equation: for $T \in (0, T^*]$,

$$\begin{aligned}
\partial_T q(T,0) &= -a(T,0)q(T,0), \\
\partial_T q(T,i) &= -a(T,i)q(T,i) + a(T,i-1)q(T,i-1), \quad i = 1, ..., n-1, \\
\partial_T q(T,n) &= a(T,n-1)q(T,n-1),
\end{aligned}$$

with initial condition $q(0,0) = 1$, $q(0,i) = 0$ for $i = 1, ..., n$.

- With the transition probabilities, we can compute the prices of index default swaps and CDO tranches.

Roadmap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Expected tranche notionals

Definition 2

Consider the equity tranche of a synthetic CDO with detachment point K. The expected remaining notional value of this equity tranche at time T is equal to

$$P(T, K) := E^{\mathbb{Q}}[(K - L_T)^+ | \mathcal{F}_0].$$

We follow the notation in Cont and Savescu (2008) and call this quantity the *expected tranche notional* with maturity T and strike K.

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The mark-to-market value of a CDO tranche $[a, b]$ with upfront payment $U^{[a,b]}$ and periodic spread $s^{[a,b]}$ is equal to:

$$\begin{aligned} MTM^{[a,b]} &= U^{[a,b]}(b-a) + s^{[a,b]} \sum_{t_j > 0} D(0, t_j)(t_j - t_{j-1}) \left[P(t_j, b) - P(t_j, a)\right] \\ &\quad - \sum_{j=1}^{m} D(0, t_j) \left[P(t_j, a) - P(t_j, b) - P(t_{j-1}, a) + P(t_{j-1}, b)\right] \end{aligned}$$

which is *linear* in the expected tranche notionals.

Expected tranche notionals

Property 1 (Static arbitrage constraints)

(a) $P(T, K) \geq 0$,

(b) $P(T, 0) = 0$,

(c) $P(0, K) = K$,

(d) $K \mapsto P(T, K)$ *is convex,*

(e) $P(T_2, K_1) - P(T_1, K_1) \geq P(T_2, K_2) - P(T_1, K_2)$ *for any* $T_1 \leq T_2$, $K_1 \leq K_2$,

(f) $K \mapsto P(T, K)$ *is continuous and piecewise linear on* $[(i-1)\delta, i\delta]$, $i = 1, ..., n$.

All constraints are *linear* in the expected tranche notionals.

Expected tranche notionals - forward differential equations

Cont and Savescu (2008) show that the expected tranche notionals can be computed directly from the local intensity function by solving a system of forward differential equations: for $T \in (0, T^*]$, $i = 1, ..., n$,

$$\partial_T P(T, i\delta) = -a(T, 0)P(T, \delta) - \sum_{k=1}^{i-1} a(T, k)\nabla_K^2 P(T, (k-1)\delta)$$

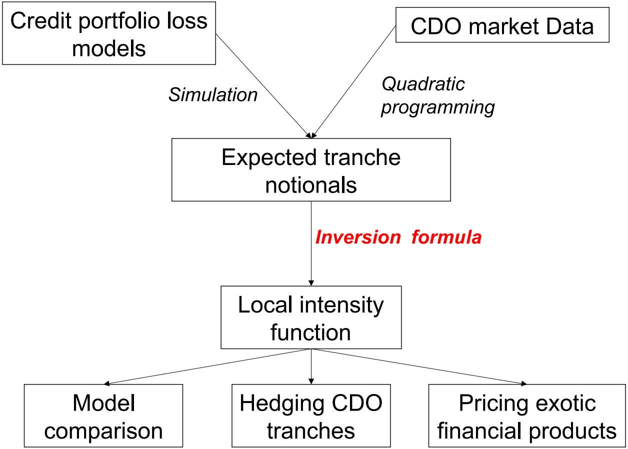
$$\text{with initial condition} \qquad P(0, i\delta) = i\delta$$

where ∇_K is the forward difference operator in strike:

$$\nabla_K F(T, i\delta) := F(T, (i+1)\delta) - F(T, i\delta)$$

for any function $F : [0, T^*] \times (i\delta)_{i=0,...,n-1} \mapsto \mathbb{R}$.

Roadmap



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Inversion formula

Theorem 3 (Inversion formula)

Consider a portfolio loss process $L_t = \delta N_t$ where (N_t) admits an intensity (λ_t) and

$$\forall t \in (0, T^*], \qquad E^{\mathbb{Q}}[\lambda_t | \mathcal{F}_0] < \infty,$$

the local intensity function defined by (1) is given by

$$a(T, i) = \begin{cases} \dfrac{-\partial_T P(T, \delta)}{P(T, \delta)}, & i = 0, \\ \dfrac{-\nabla_K \partial_T P(T, i\delta)}{\nabla_K^2 P(T, (i-1)\delta)}, & i = 1, ..., n-1, \\ 0, & i = n, \end{cases} \tag{2}$$

for all $T \in (0, T^*]$, and $P(T, i\delta) = E^{\mathbb{Q}}[(\delta i - L_T)^+ | \mathcal{F}_0]$.

Inversion formula

Theorem 4 (Local intensity implied by expected tranche notionals)

Let $\{P(T,i\delta)\}_{T\in[0,T^*],i=0,\dots,n}$ be a (complete) set of expected tranche notionals verifying Property 1 and define the function $a:(0,T^*]\times\{0,1,..,n\}$ by

$$a(T,i)=\begin{cases}\dfrac{-\partial_T P(T,\delta)}{P(T,\delta)}, & i=0,\\ \dfrac{-\nabla_K\partial_T P(T,i\delta)}{\nabla_K^2 P(T,(i-1)\delta)}, & i=1,\dots,n-1,\\ 0, & i=n,\end{cases} \tag{3}$$

for all $T\in(0,T^*]$. If $a(.,.)$ is bounded, there exists a Markovian point process (M_t) with intensity $\gamma_t=a(t,M_{t-})$ defined on some probability space $(\Omega_0,\mathcal{G},(\mathcal{G}_t),\mathbb{Q}_0)$ such that

$$\forall T\in[0,T^*],\qquad \forall i\in\{0,\dots,n\},\quad P(T,i\delta)=E^{\mathbb{Q}_0}[(\delta i-\delta M_T)^+|\mathcal{G}_0].$$

Inversion formula

- The inversion formula is an analogue to the Dupire (1994) formula for diffusion models:

$$\sigma^2(T, K) = \frac{2}{K^2} \frac{\partial_T C(T, K)}{\partial_K^2 C(T, K)}, \quad T \geq 0, K \geq 0$$

where $C(T, K)$ is the call price with maturity T and strike K.

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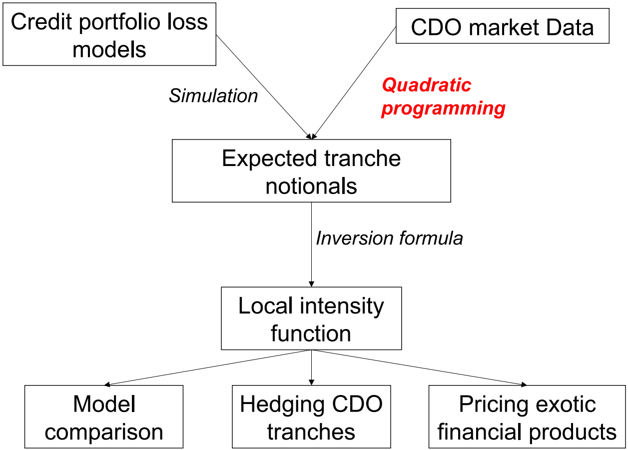
- A similar formula, but expressed in terms of the marginal distribution, has been shown by Schönbucher (2005):

$$a(T,i) = \frac{-\sum_{k=0}^{i} \partial_T \mathbb{Q}(L_T = i\delta|\mathcal{F}_0)}{\mathbb{Q}(L_T = i\delta|\mathcal{F}_0)}, \quad i = 0, ..., n-1, \quad T \in (0, T^*].$$

  However, expressing the value of CDO tranche in terms of marginal distribution is more difficult while it can be expressed in terms of a small set of expected tranche notionals.

Roadmap



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Recovery of expected tranche notionals

- Given a set of CDO tranche spreads, we want to recover expect tranche notionals $\{P(t_j, i\delta)\}_{j=1,\dots,m; i=1,\dots,n}$ which must satisfy:
  - Static arbitrage constraints
  - Mark-to-market value constraints

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  - Static arbitrage constraints
  - Mark-to-market value constraints
- Both static arbitrage and the mark-to-market value constraints are *linear* in the expected tranche notionals.
- Recovering the expected tranche notional can be achieved by solving a linear system of inequalities:

$$\begin{aligned} \mathbf{A}\,\mathbf{p} = \mathbf{b}, &\qquad \text{(Market CDO)} \\ \mathbf{B}\,\mathbf{p} \le \mathbf{e} &\qquad \text{(Static arbitrage)} \end{aligned}$$

  where $\mathbf{p}$ is a vector of expected tranche notionals.

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  where $\mathbf{p}$ is a vector of expected tranche notionals.
- However, the linear system may have infinitely many solutions.

Recovery of expected tranche notionals

- In order to guarantee a unique solution, we solve the following convex optimization problem with linear constraints:

$$\begin{aligned} \min_{\mathbf{p}} \quad & f(\mathbf{p}) & \\ s.t. \quad & \mathbf{A}\,\mathbf{p} = \mathbf{b} & \text{(Market CDO)} \\ & \mathbf{B}\,\mathbf{p} \leq \mathbf{e} & \text{(Static arbitrage)} \end{aligned}$$

  where

$$f(\mathbf{p}) \;=\; \sum_{j=0}^{m} \sum_{i=1}^{n} w_{ij} \left(P(t_j, i\delta) - \widetilde{P}(t_j, i\delta) \right)^2$$

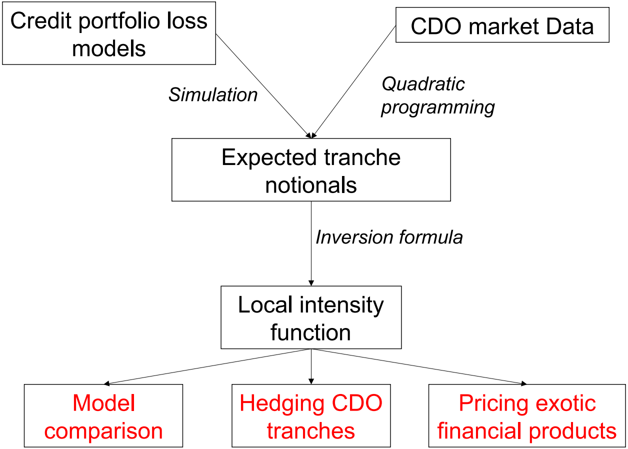
  where (w_{ij}) are weights, and $\{\widetilde{P}(t_j, i\delta)\}$ is a reference set of expected tranche notionals.
- This is a quadratic programming problem.
- The calibration algorithm is non-parametric.

Local intensity function calibration algorithm

Algorithm 1

1. *Compute matrices* $\mathbf{A}$ *and* $\mathbf{b}$ *using observed CDO tranche spreads, and matrix* $\mathbf{B}$ *and* $\mathbf{e}$ *according to static arbitrage constraints.*
2. *Solve quadratic programming problem and obtain a set of arbitrage-free expected tranche notionals which is consistent with the CDO tranche spreads.*
3. *Convert the calibrated expected tranche notionals into local intensity function using formula in Theorem 2.*

Roadmap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Application to iTraxx IG data

- We apply our algorithm to iTraxx IG S9 data on 20 September 2006 and 25 March 2008.
- We also compare the results to
  (1) Parametric model by Herbertsson (2008),
  (2) Entropy-minimization method by Cont and Minca (2008).

Tranche	0%-3%	3%-6%	6%-9%	9%-12%	12%-22%	22%-100%
Market bid	37.7%	441.6	270.2	174.4	97.4	42.8
Market ask	39.7%	466.6	290.2	189.4	110.7	46.9
QP	38.4%	451.9	279.0	181.1	103.2	44.3
Entropy	38.6%	453.3	279.5	181.2	103.4	44.6
Parametric	38.7%	454.1	280.2	181.9	104.1	44.8

Table 1: CDO tranche spreads of 5Y iTraxx Europe IG Series 9 on 25 March 2008. Quotes are given in bps except for equity tranches which are quoted as upfront in percent with 500bps periodic coupons.

- All calibrated spreads are well-within bid-ask.

Local intensity function

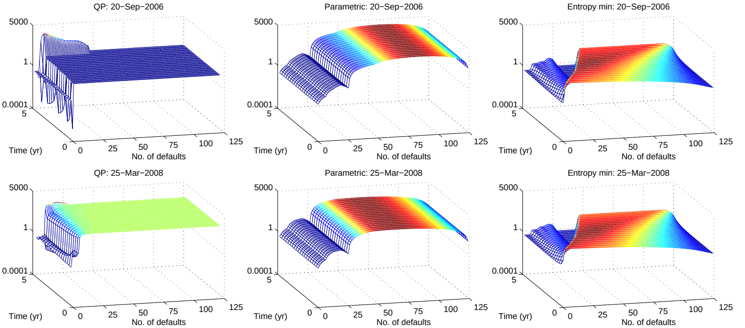


Figure 2: Local intensity functions based on different calibration approaches. Data: 5Y iTraxx Europe IG S9 on 20 September 2006 (top) and 25 March 2008 (bottom).

- Different calibration methods yield significantly different local intensity functions.
- For each method, the local intensity functions are similar for different dataset.

Stability analysis

To examine the stability of the calibration methods, we apply a 1% proportional shift to all CDO market spreads, recalibrate the local intensity function to the shifted CDO spreads and measure the magnitude of the changes using the Frobenius norm:

$$\left(\sum_{i=0}^{n} \sum_{j=0}^{q} \left| a(T_j, i) - \widehat{a}(T_j, i) \right|^2 \right)^{1/2}$$

where $\{a(T_j, i)\}$ and $\{\widehat{a}(T_j, i)\}$ are, respectively, the local intensity functions calibrated to the original and perturbed CDO tranche spreads.

	QP	Parametric	Entropy Min
20-Sep-06	56.2	32116.2	2.0×10^{-2}
25-Mar-08	673.2	728.3	2.0×10^{-1}

Table 2: Frobenius norm of the changes in the local intensity function with respect to 1% proportional increase in the CDO spreads. Data: 5Y iTraxx Europe IG S6 on 20 September 2006 and S9 on 25 March 2008.

- Non-parametric methods are more stable than the parametric method.
- Similar findings in studies using equity derivatives: Cont and Tankov (2004).

Forward starting tranche spreads

A forward tranche with attachment-detachment interval $[a, b]$ can be valued as the forward value of a tranche with adjusted interval $[a', b']$ where $a' = \min(1, a + L_t)$ and $b' = \min(1, b + L_t)$. This dependence of the payoff on the loss makes the forward tranche path dependent.

	20 September 2006			25 March 2008		
	QP	Parametric	Entropy Min	QP	Parametric	Entropy Min
0% - 3%	12.05	12.25	14.26	53.46	36.92	65.92
3% - 6%	2.72	17.89	33.62	93.79	290.65	482.23
6%- 9%	2.46	3.18	7.46	92.46	142.25	236.22
9% - 12%	2.21	0.79	4.14	91.45	63.45	170.80
12% - 22%	1.59	0.36	4.03	89.36	34.49	165.59
22% - 100%	0.03	0.15	0.69	37.99	13.38	27.60

Table 3: Spreads of forward starting tranches which start in 1 year and mature 3 years afterwards. Data: 5Y iTraxx Europe IG S6 on 20 September 2006 and S9 on 25 March 2008.

- Forward tranches spreads can be different by more than double, even the local intensity functions are calibrated to the same market CDO spreads $\Rightarrow$ Substantial model uncertainty

Hedge ratios

In the local intensity framework, the market is complete and the self-financing strategy to replicate the payoff of a CDO tranche involves trading the underlying index default swap. The corresponding hedge ratio, which is known as the *jump-to-default ratio*, is defined by:

$$\frac{v^{[a,b]}(t, N_t + 1) - v^{[a,b]}(t, N_t)}{v^{index}(t, N_t + 1) - v^{index}(t, N_t)}$$

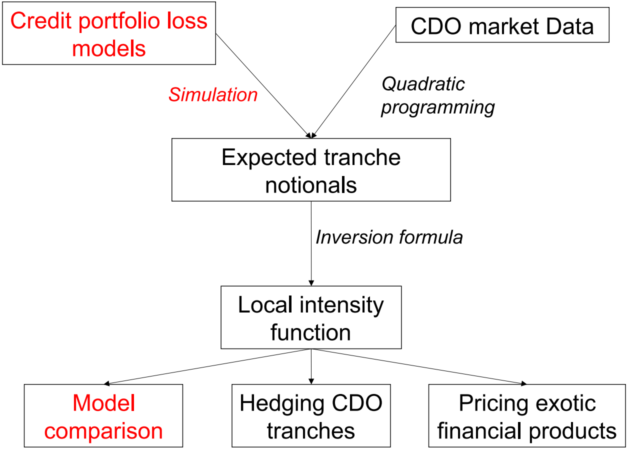
where $v^{\cdot}(t, m)$ denotes the mark-to-market value conditional on m defaults being occurred by time t.

	20 September 2006			25 March 2008		
	QP	Parametric	Entropy Min	QP	Parametric	Entropy Min
0% - 3%	6.29	20.97	6.32	1.03	3.62	1.60
3% - 6%	2.12	5.16	3.51	1.69	3.31	2.33
6%- 9%	1.63	2.00	2.23	1.68	2.65	2.15
9% - 12%	1.52	1.02	1.72	1.68	2.08	1.97
12% - 22%	1.47	0.48	1.39	1.68	1.48	1.76
22% - 100%	0.67	0.22	0.61	0.81	0.66	0.75

Table 4: Jump-to-default ratios computed from the calibrated local intensity functions. Data: 5Y iTraxx Europe IG S6 on 20 September 2006 and S9 on 25 March 2008.

- Jump-to-default ratios are also significantly different across calibration methods $\Rightarrow$ Substantial model uncertainty

Roadmap



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Comparison of credit portfolio loss models

We compare the local intensity functions of six different models:

1. **Parametric local intensity model**: Herbertsson (2008)
   - $\lambda_t = (n - N_{t-}) \sum_{k=0}^{N_{t-}} b_k$
   - $\lambda_t^{eff} = \lambda_t$

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   - $\lambda_t^{eff} = \lambda_t$
2. **Bivariate spread-loss model**: Arnsdorf and Halperin (2008)
   - $\lambda_t = e^{X_t}(n - N_{t-}) \sum_{k=0}^{N_{t-}} b_k$
     where $dX_t = \kappa(b - X_t)dt + \sigma dW_t$

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     where $dX_t = \kappa(b - X_t)dt + \sigma dW_t$
3. **Shot-noise model**: Gaspar and Schmidt (2008)
   - $\lambda_t = \eta_t + J_t$
     where (η_t) is a CIR process and (J_t) is a compound Poisson process with exponential jump size.
   - A semi-analytical expression for the local intensity function:

$$a(T, k) = \frac{\left.\frac{\partial^k}{\partial \theta^k}\right|_{\theta=-1} \frac{\partial}{\partial T} \frac{1}{\theta} S(\theta, T)}{\left.\frac{\partial^k}{\partial \theta^k}\right|_{\theta=-1} S(\theta, T)}$$

     where $S(\theta, T)$ is the Laplace transform of the cumulative portfolio default intensity.

Comparison of credit portfolio loss models

4. **Gaussian copula model**: Li (2000)
   - Given a family of marginal default time distributions $(F_i, i = 1, ..., n)$, the joint distribution of the default times τ_i is modeled by first defining latent factors $X_i = \rho Z_0 + \sqrt{1-\rho^2} Z_i$, where Z_0, Z_i are i.i.d. standard normal random variables. Defining the default times by

     $$\tau_i = F_i^{-1}(F_{X_i}(X_i)),$$

     where $F_{X_i}(.)$ denotes the distribution of X_i.

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5. **Student-t copula model**: Demarta and McNeil (2005)
   - Same as the Gaussian copula case but replacing normal latent factors by $X_i = \sqrt{\nu/V}\left(\rho Z_0 + \sqrt{1-\rho^2} Z_i\right)$ where $V \sim \chi^2_\nu$.

Comparison of credit portfolio loss models

4. **Gaussian copula model**: Li (2000)
   - Given a family of marginal default time distributions $(F_i, i = 1, ..., n)$, the joint distribution of the default times τ_i is modeled by first defining latent factors $X_i = \rho Z_0 + \sqrt{1-\rho^2} Z_i$, where Z_0, Z_i are i.i.d. standard normal random variables. Defining the default times by

     $$\tau_i = F_i^{-1}(F_{X_i}(X_i)),$$

     where $F_{X_i}(.)$ denotes the distribution of X_i.
5. **Student-t copula model**: Demarta and McNeil (2005)
   - Same as the Gaussian copula case but replacing normal latent factors by $X_i = \sqrt{\nu/V}\left(\rho Z_0 + \sqrt{1-\rho^2} Z_i\right)$ where $V \sim \chi^2_\nu$.
6. **Bottom-up affine jump-diffusion model**: Duffie and Gârleanu (2001)
   - The default intensity for obligor i follows: $\lambda_t^i = X_t^i + a_i X_t^0$ where $dX_t^i = \kappa_i(b_i - X_t^i)dt + \sigma_i \sqrt{X_t^i} dW_t^i + dJ_t^i$

Local intensity functions implied by credit portfolio loss models

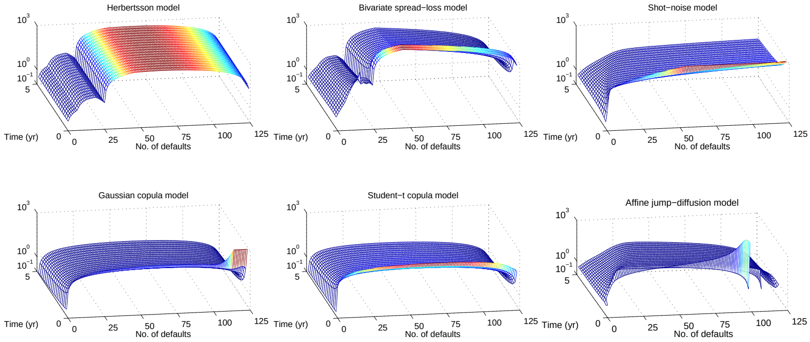

Figure 3: Local intensity functions implied by credit portfolio loss models. Data: 5Y iTraxx Europe IG S9 on 25 March 2008.

- Static copula models have similar effective intensities as the dynamic affine jump-diffusion model
  $\Rightarrow$ Market prices alone are insufficient to discriminate between these model classes.

Conclusion

- We derive an inversion formula for the local intensity function which is an analogue to the Dupire (1994) local volatility function.
- Inversion formula + QP $\Rightarrow$ a simple, efficient and stable calibration algorithm for the effective default intensity.
- Even under the *same* modeling framework, there are substantially *differences* in model-dependent quantities such as jump-to-default ratios and forward tranche prices.
  $\Rightarrow$ **Model uncertainty**
- We observe *similar* local intensity functions implied by models defined in *different* manners, e.g. static copula models vs dynamic affine jump-diffusion model.
  $\Rightarrow$ **Market prices alone are insufficient to discriminate between these model classes.**