



Population dynamics of central place foragers

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Central Place Foraging

Consumers (crickets, ants, seabirds, cave bats, ...)

- ⌚ central place (cave, mound, colony, den, ...)
- ⌚ spatially distributed resource
- ⌚ foraging strategies in space and time
- ⌚ discrete-time reproductive events

Consumer-Resource Models

- ⑥ Non-spatial: differential/difference equations
homogeneous distribution: intermediate spatial scale,
- ⑥ Spatial: reaction-diffusion or integrodifference
dispersal events: large spatial scale

The scale of a single foraging patch

Overview

1. Fixed foraging distribution
 - ⌚ Effects of patch size
 - ⌚ Effects of forager distribution
2. Adaptation of forager distribution to resources
 - ⌚ group-level optimization
 - ⌚ individual movement model

Non-spatial Model (Hassell, Kot)

Resource

$$f_{t+1} = G(f_t)(1 - P(c_t))$$

Total consumption

$$e_t = G(f_t)P(c_t)$$

Consumer

$$c_{t+1} = s_c c_t + \beta e_t$$

G : Beverton-Holt

P : probability of finding resource $P(c) = 1 - \exp(-c)$

s_c : Survival of consumers to next generation

β : conversion coefficient

Spatial Model

Immobile resource

$$f_{t+1}(x) = G(f_t(x))(1 - P(c_t k_t(x)))$$

Forager distribution: $k_t(x)$

Total consumption, Patch size L

$$e_t = \int_{-L/2}^{L/2} G(f_t(x))P(c_t k_t(x))dx$$

Consumer

$$c_{t+1} = s_c c_t + \beta e_t$$

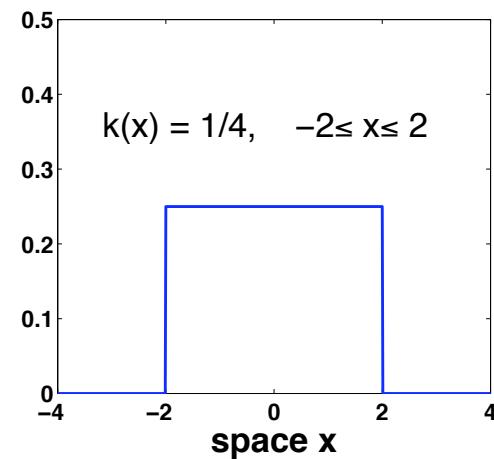
No resource outside patch, no foraging-related death

Foraging Pattern

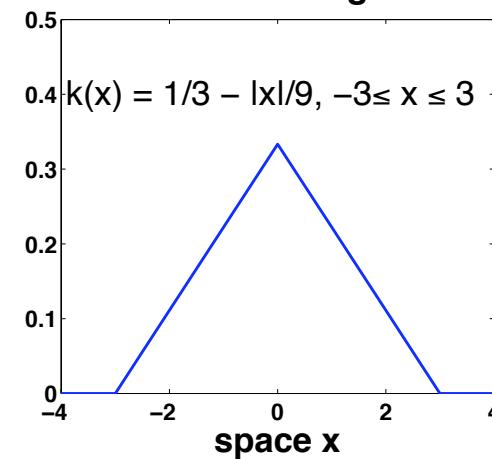


$k(x)$: distribution of foraging locations, $\int |x|k(x)dx = 1$

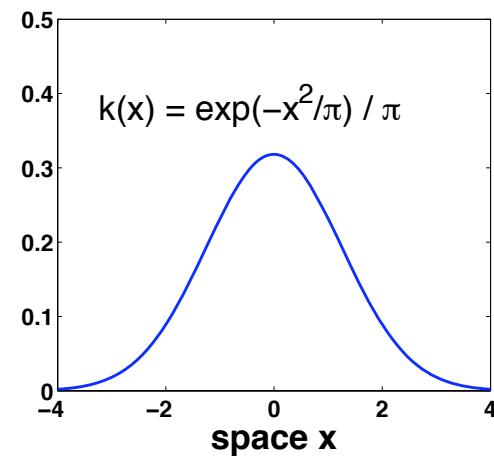
Panel A: uniform



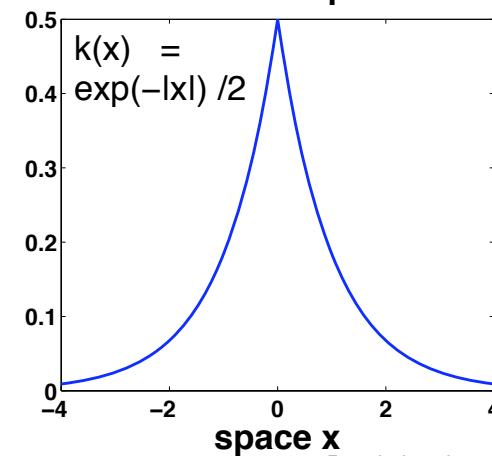
Panel B: triangular



Panel C: Gaussian



Panel D: Laplace



Part I

- ⑥ Fixed foraging distribution
- ⑥ Effect of shape of distribution
- ⑥ Effect of patch size

Critical Patch Size

- ⑥ The steady state $f^* = 1, c^* = 0$ is unstable if

$$\int_{-L/2}^{L/2} k(x)dx > \frac{1 - s_c}{\beta}$$

- ⑥ Extinction if $s_c + \beta < 1$.
- ⑥ If $s_c + \beta > 1$, critical value: L^*

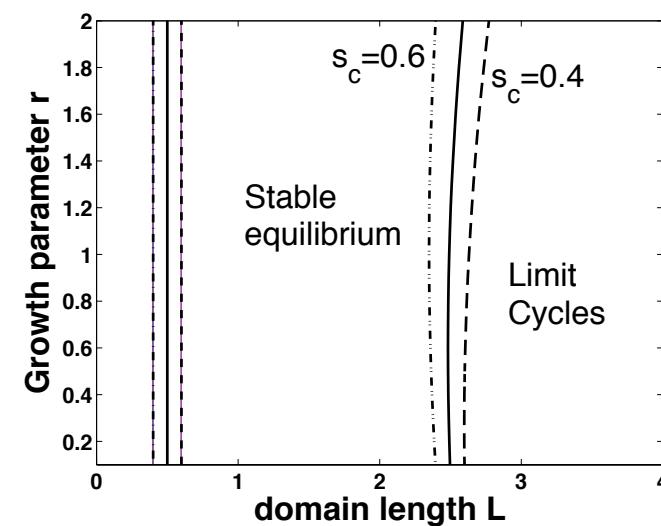
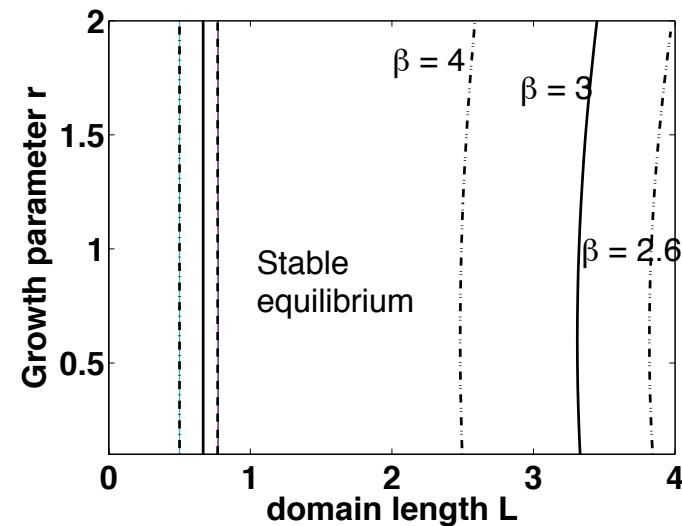
$$L_{\text{Laplace}}^* < L_{\text{Gauss}}^* < L_{\text{triangle}}^* < L_{\text{top-hat}}^*$$

Top-hat Kernel

Reduction to non-spatial model

$$f_{t+1} = G(f_t) \exp(-c_t/4)$$

$$c_{t+1} = s_c c_t + L\beta G(f_t)(1 - \exp(-c_t/4))$$



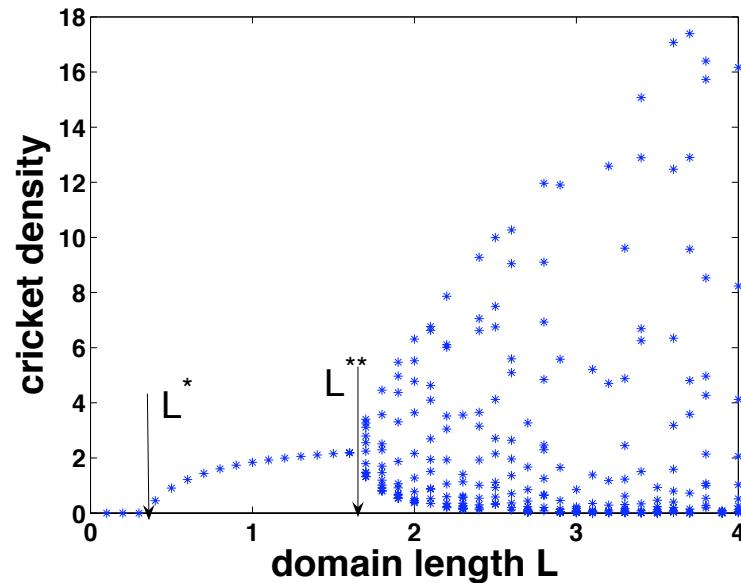
Bifurcations with patch size L

Transcritical bifurcation at $L^* > 0$

Hopf bifurcation at $L^{**} > L^*$

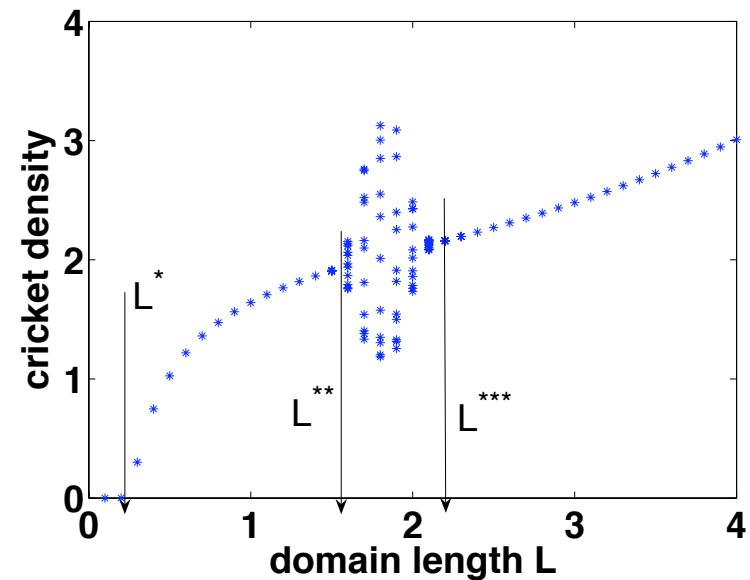
Collapse of cycles at $L^{***} > L^{**}$

Panel A: uniform



Top-hat

Panel B: triangular



Triangular

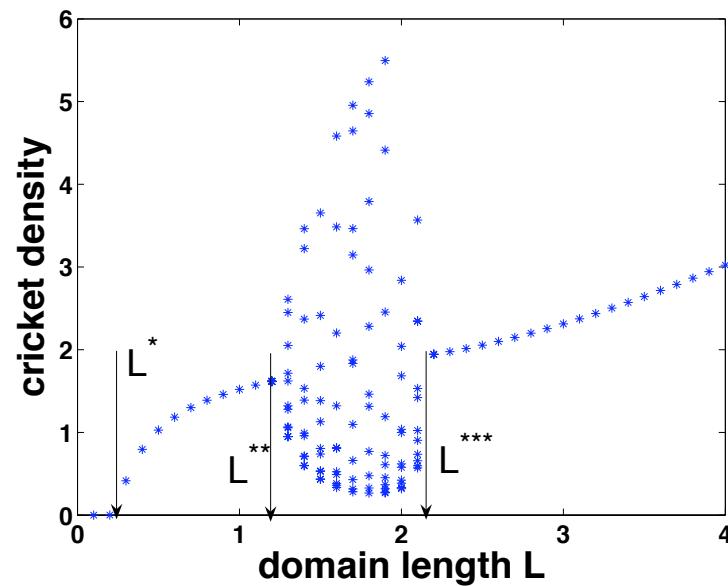
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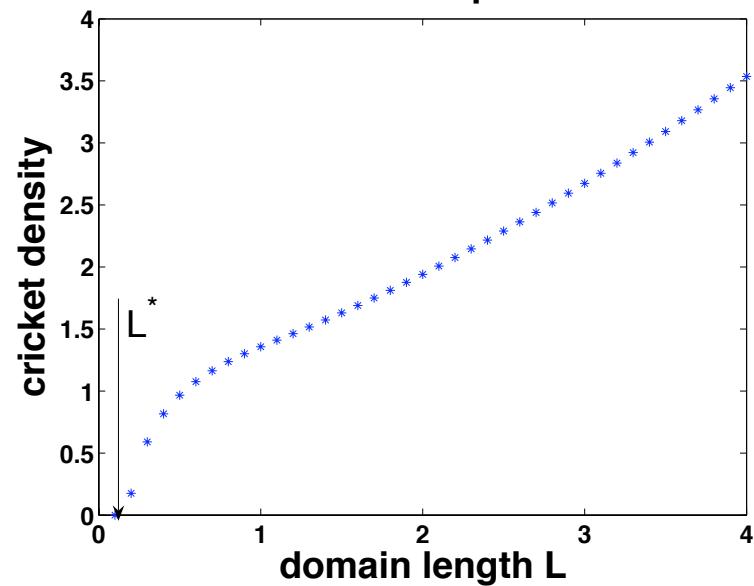
Collapse of cycles at $L^{***} > L^{**}$

Panel C: Gaussian



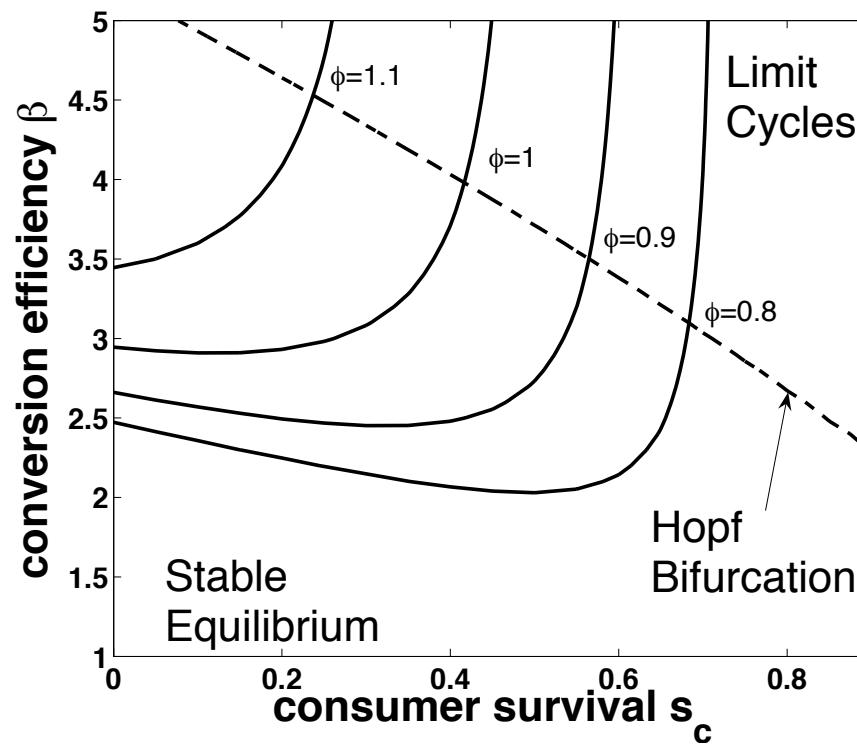
Gauss

Panel D: Laplace



Laplace

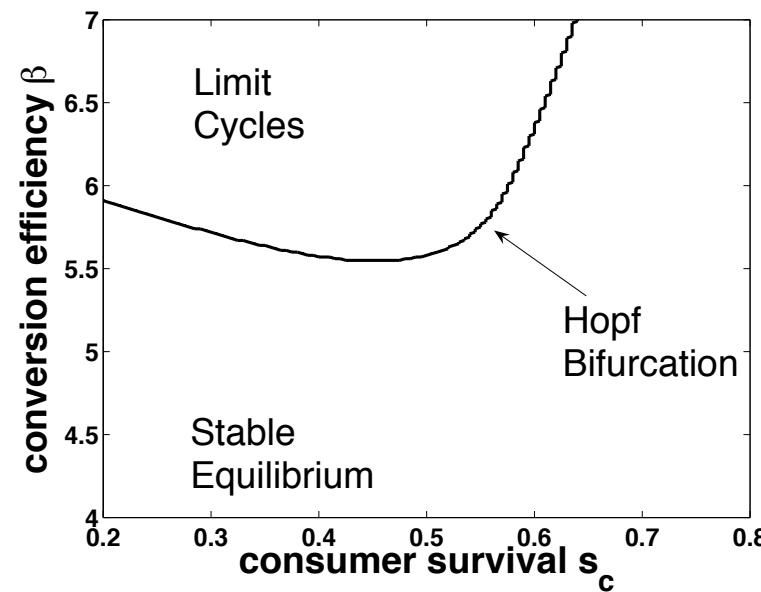
Dependency on s_c and β



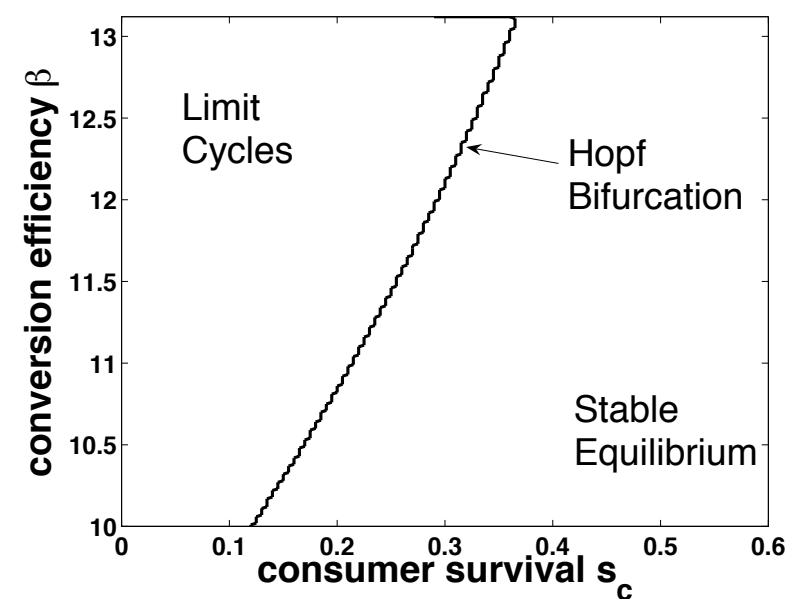
Top-hat kernel

Period $T = 2\pi/\Phi$

Dependency on s_c and β



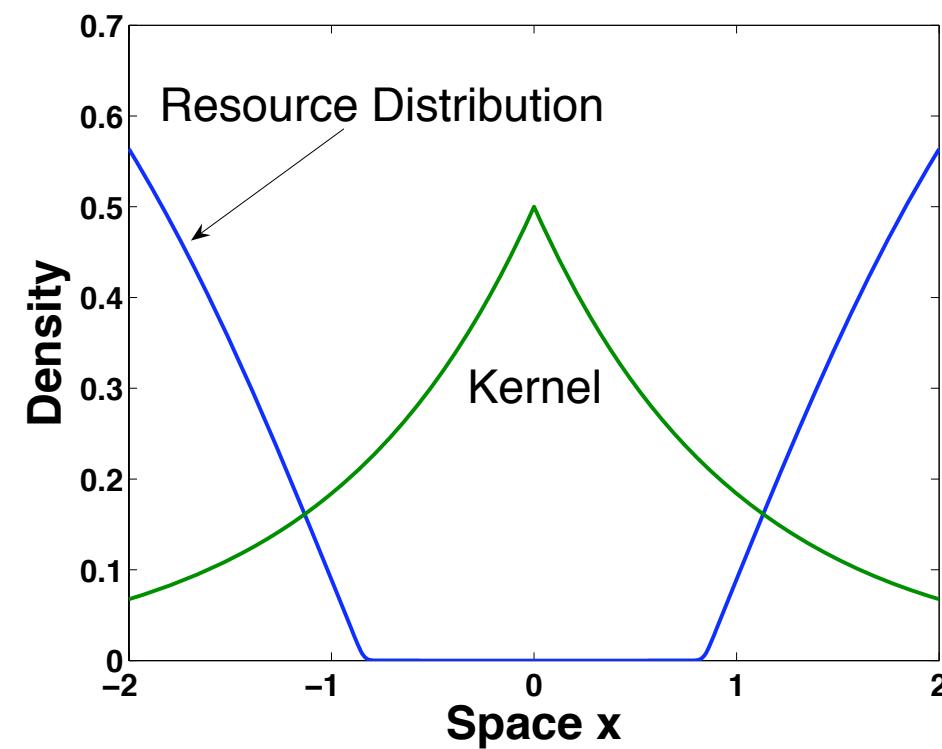
Triangular



Laplace

Foraging pattern changes the Hopf-bifurcation line

Steady State Profile



Resource depletion near central place

Part II

- ⌚ Adaptive foraging distribution
- ⌚ Group level optimization
- ⌚ Individual movement model

Group level optimization

- ➊ Foraging kernel with parameter α (e.g., variance)
- ➋ In each time step, choose a as to maximize food intake

$$e_t = \max_{\alpha > 0} \int_{-L/2}^{L/2} G(f_t(x)) P(c_t k(x; \alpha)) dx$$

- ➌ Update $f_t(x)$ accordingly

Results

- ⑥ The steady state $f^* = 1, c^* = 0$ is unstable if

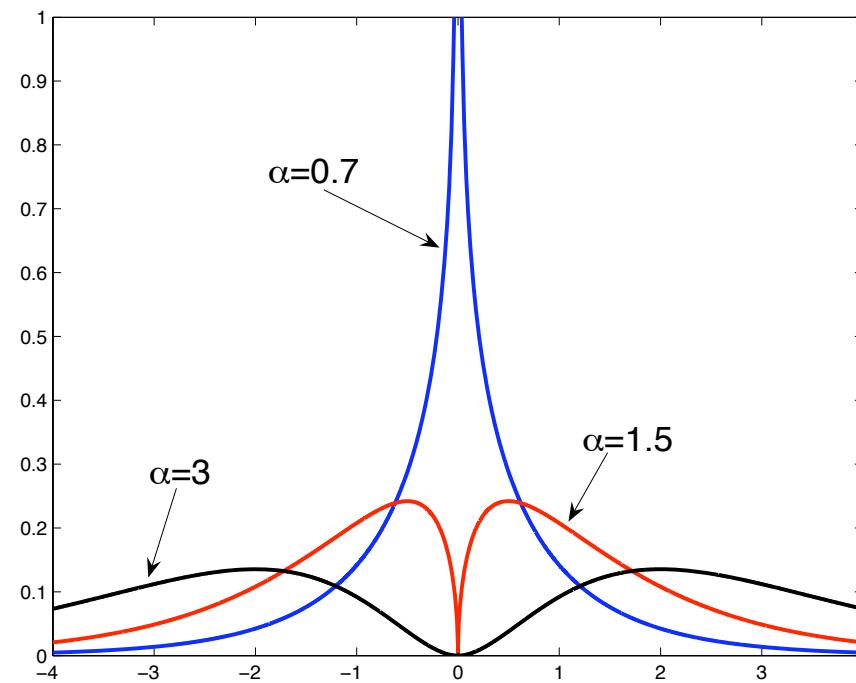
$$\sup_{\alpha > 0} \left\{ \int_{-L/2}^{L/2} k(x; \alpha) dx \right\} > \frac{1 - s_c}{\beta}$$

- ⑥ Extinction if $s_c + \beta < 1$.
- ⑥ If $s_c + \beta > 1$, critical patch size becomes zero

$$L^* = 0 \quad \text{if} \quad k(\cdot; \alpha) \rightarrow \delta \quad \text{as} \quad a \rightarrow 0$$

- ⑥ Increased stability

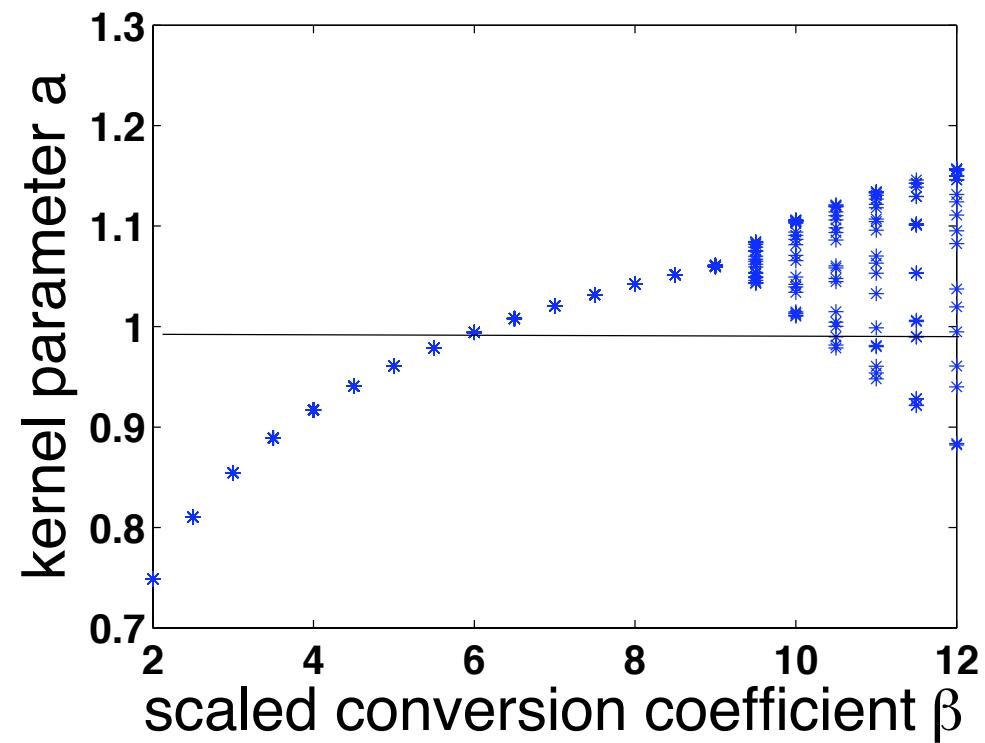
Gamma Kernel



$$k(x; \alpha) = |x|^{\alpha-1} \exp(-|x|)/(2\Gamma(\alpha))$$

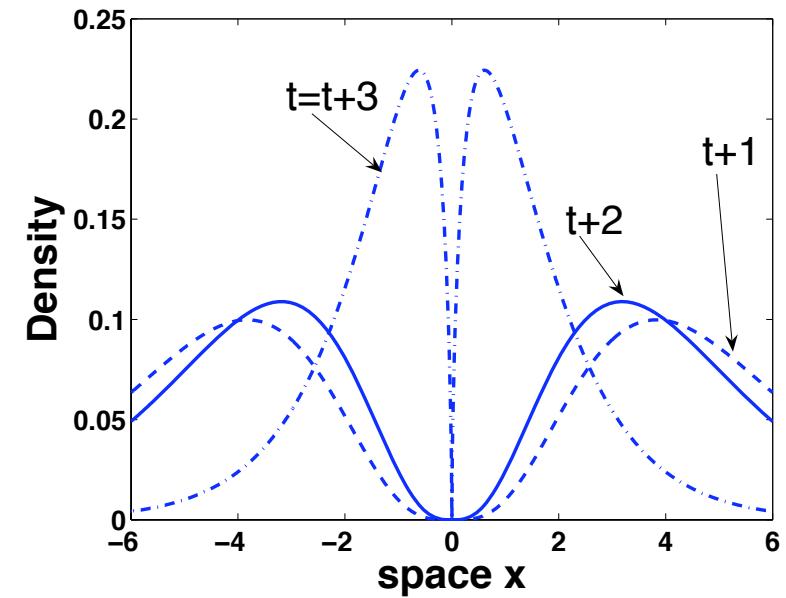
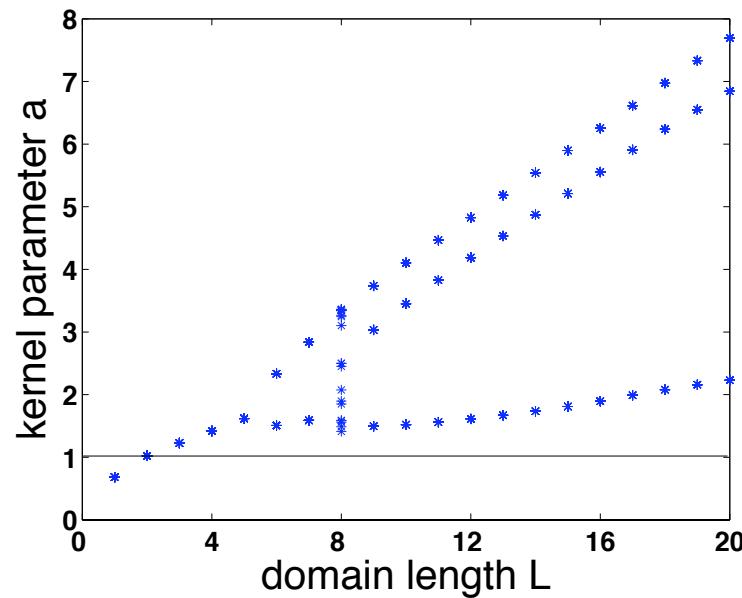
If $\alpha > 1$: forage away from home

Hopf bifurcation with β



Gamma kernel: $\alpha < 1$: peak at zero
 $\alpha > 1$: peak away from zero

Bifurcations with L



3-cycle

Gamma kernel: $\alpha > 1$: peaks away from zero

T

Individual Movement Model



Random walk plus resource-dependent settling

$$\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2} - a(f(x))u, \quad u(0, x) = \delta(x)$$

Foraging distribution

$$k(x) = \lim_{t \rightarrow \infty} \int_0^t a(f(x))u(t, x)dt$$

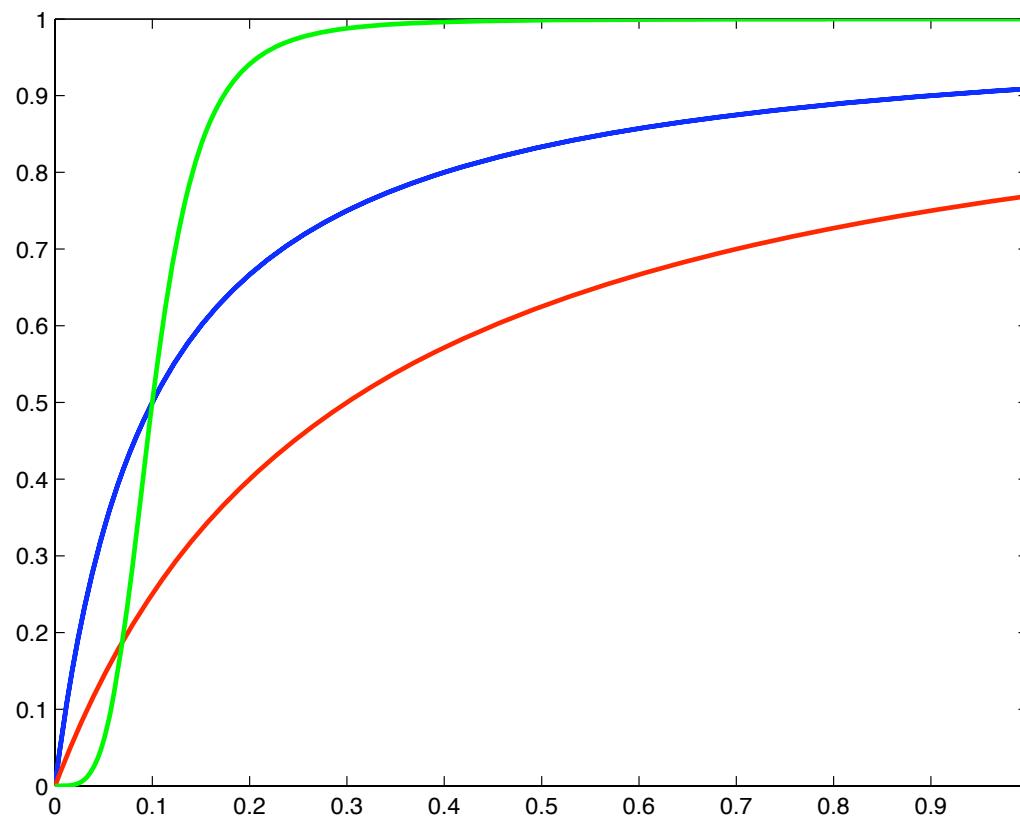
Hill's function

$$a(f) = a^* \frac{f^q}{b^q + f^q}$$

For $b = 0$ or $q = 0$ the Laplace kernel results.

L

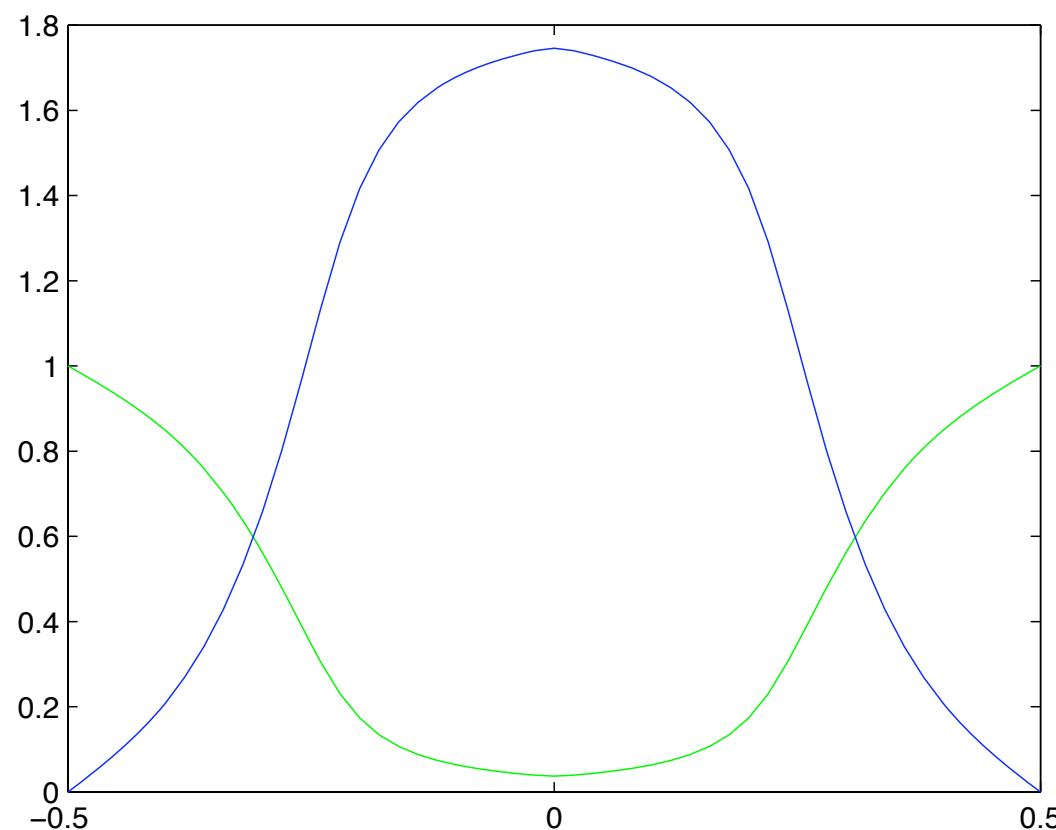
Settling rate



b : choosiness

q : precision

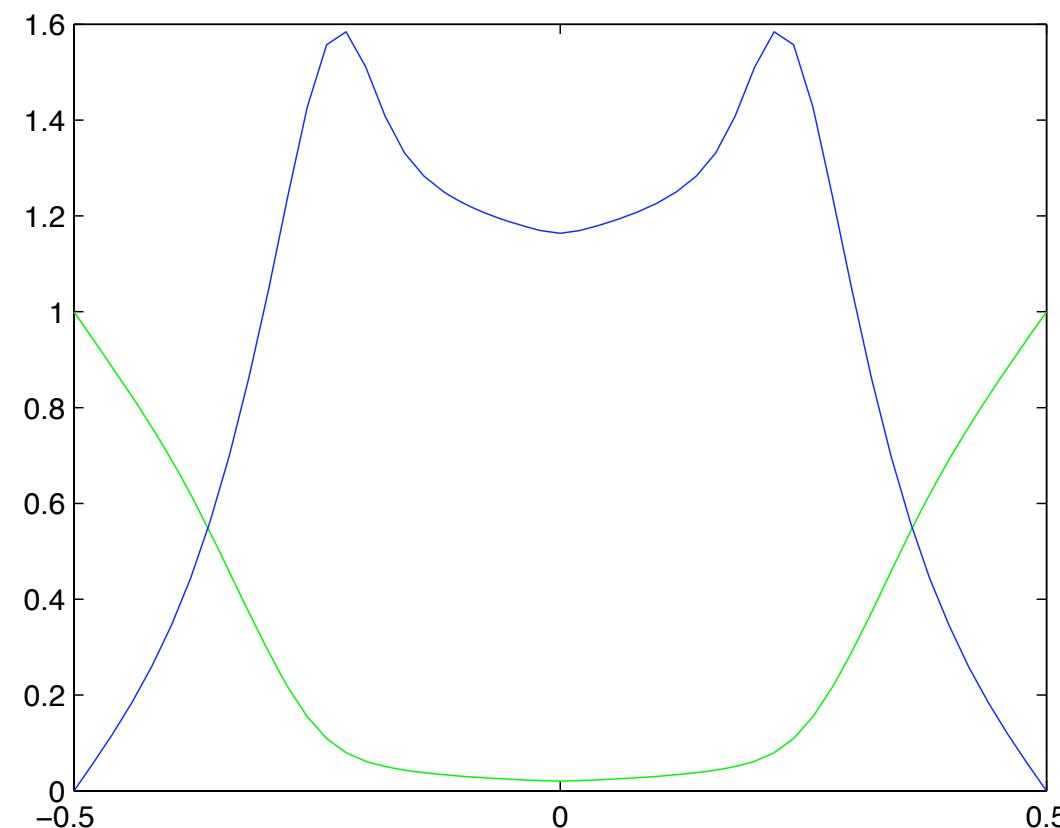
Foraging distribution I



resource density

consumer distribution

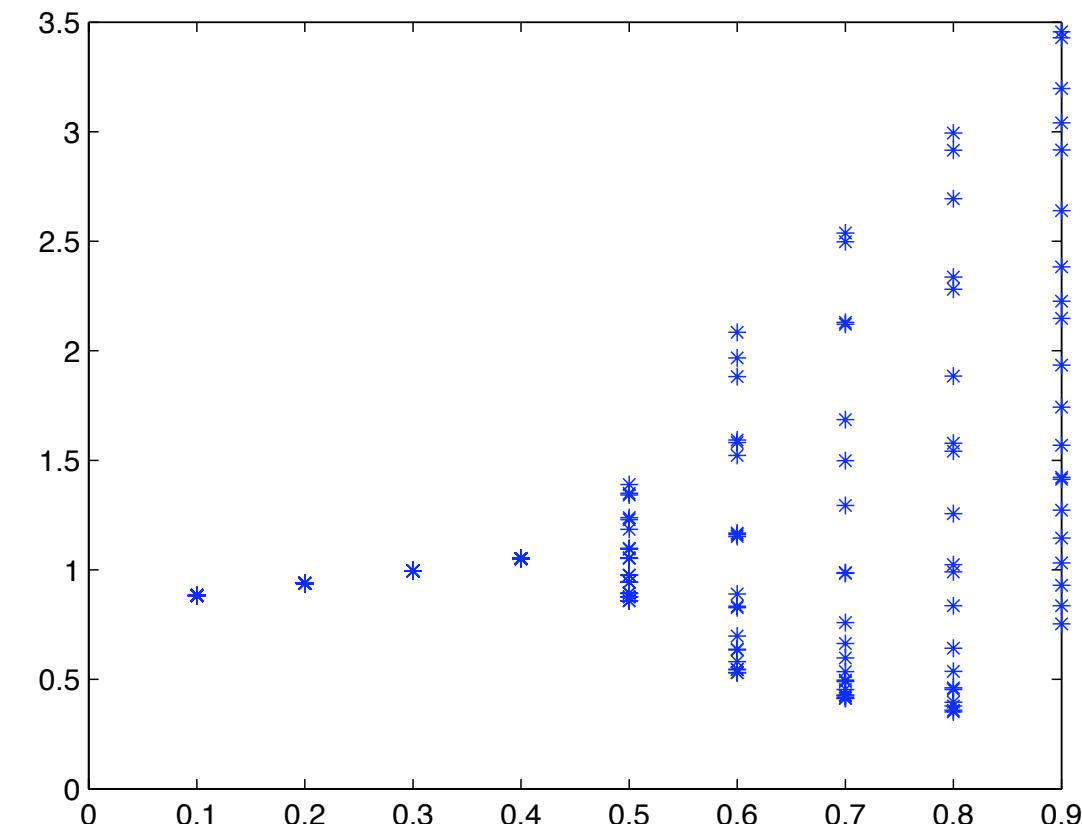
Foraging distribution II



resource density

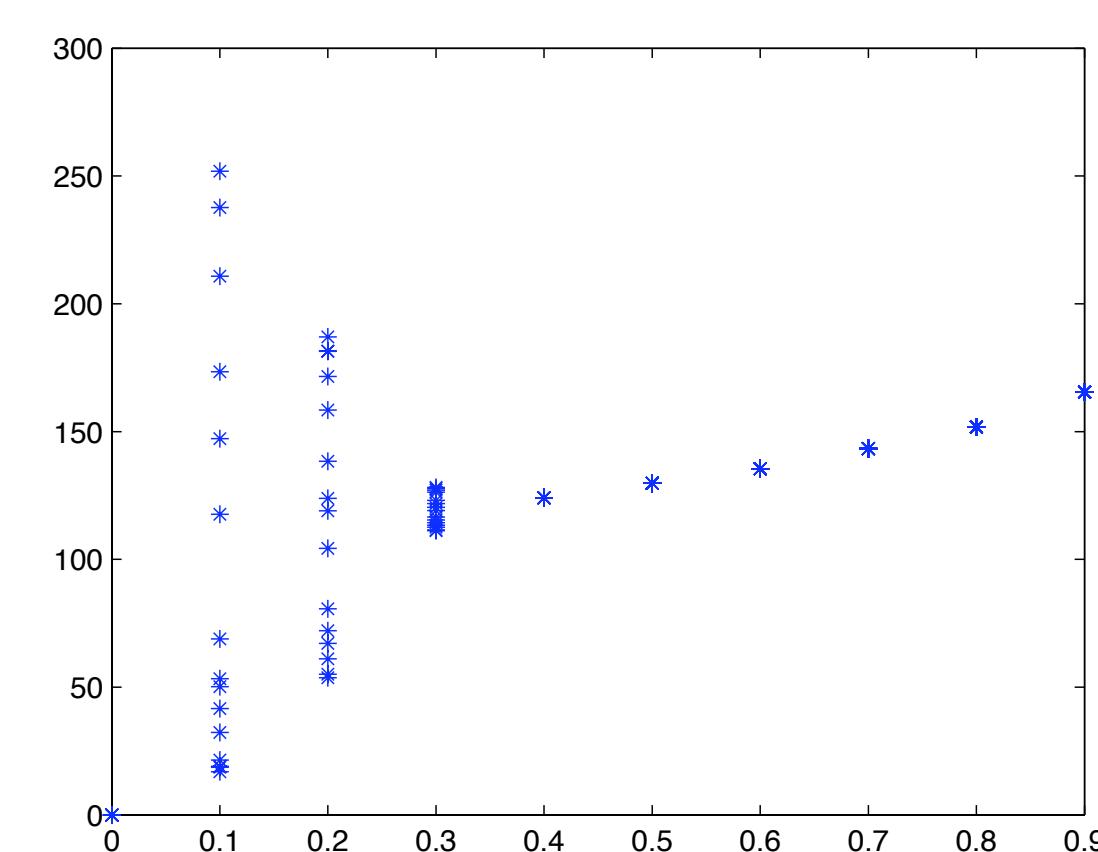
consumer distribution

Dynamics - uniform



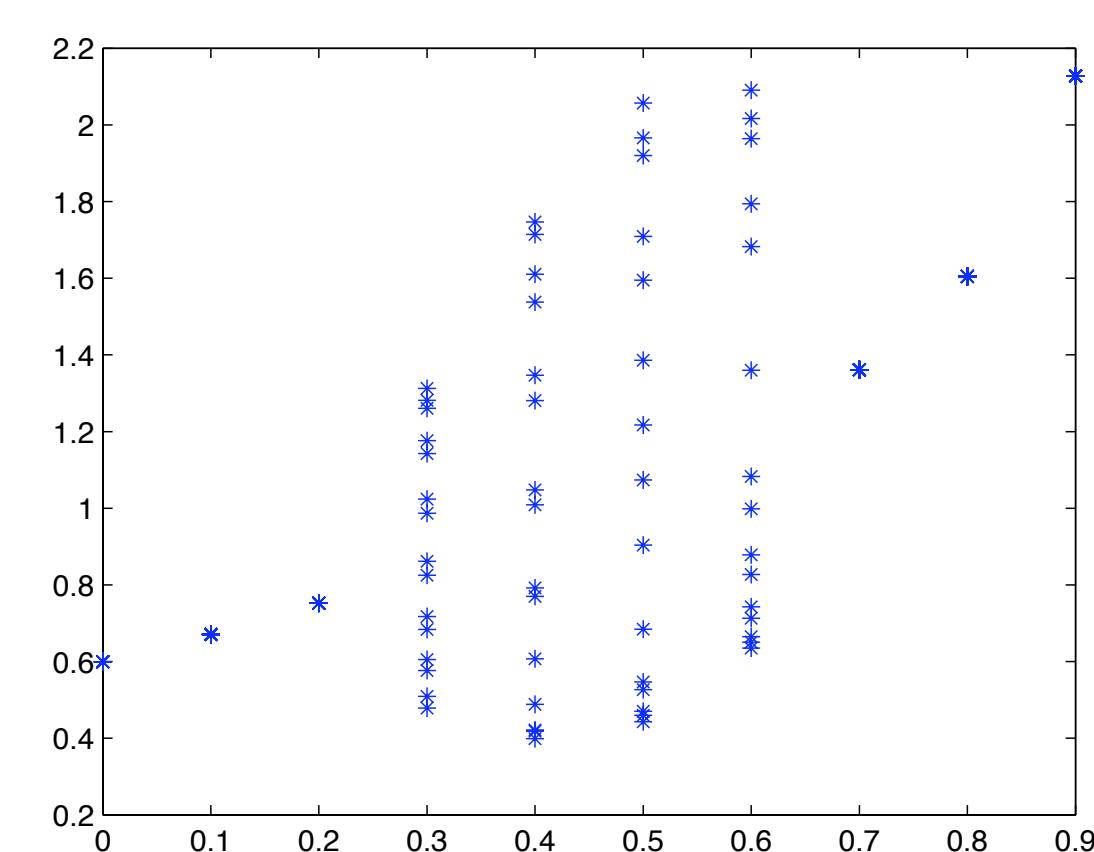
bifurcation parameter s : consumer survival

Dynamics - Laplace



bifurcation parameter s : consumer survival

Dynamics - Movement model



bifurcation parameter s : consumer survival

Conclusions

- ⌚ Central place foraging:
 - ⚠ consumer home base
 - ⚠ spatial resource
 - ⚠ discrete reproductive events
- ⌚ Foraging pattern determines stability
- ⌚ Group-level optimization
- ⌚ Movement model

Future

- ⑥ Energy budget for distance traveled
- ⑥ "Optimal" foraging strategies
- ⑥ Resource redistribution
- ⑥ Causes and effects of consumer death