Evolution of Conditional Dispersal in Spatially Heterogeneous Habitats

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Talk Outline

1. Random dispersal
2. Conditional dispersal
3. Ideal free dispersal
4. Some ideas of the proofs
5. Future directions

Reaction-diffusion models

- A. Hastings (TPB, 83)

$$\begin{aligned} u_t &= uf(u+v,x) \quad \text{in } \Omega \times (0,\infty), \\ v_t &= vf(u+v,x) \quad \text{in } \Omega \times (0,\infty), \end{aligned} \tag{1}$$

- $u(x,t), v(x,t)$: densities of species

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- $\mu, \nu > 0$: random dispersal rates

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- $u(x,t), v(x,t)$: densities of species
- $\mu, \nu > 0$: random dispersal rates
- No-flux boundary condition

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- The slower diffuser is the winner.

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- Kirkland et al (SIAM J. Appl. Math. 2006): discrete time and space, n-patch

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- Semi-trivial steady states: if $m > 0$, $(\tilde{u}, 0)$ and $(0, \tilde{v})$ both exist
- Coexistence state: If both semi-trivial steady states are unstable, the system has one stable coexistence state

Weak advection

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- Geometry of habitat matters

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- Chen and L. (Indiana Univ. Math. J, 08): for $\beta = 0$ and large α, if m has a unique local maximum, the species u concentrates around this maximum.

Advection-induced extinction

Chen et al. JMB 2008: Suppose that $\partial m/\partial n < 0$ on $\partial\Omega$, m has only one critical point x_0 in $\overline{\Omega}$, with $x_0 \in \Omega$ and $D^2 m(x_0) < 0$.

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- Strong biased movement of *single* species can induce the coexistence of both competing species
- Strong biased movement of *both* species can induce the extinction of the species with the stronger biased movement

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- How to estimate α^*?

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- $\alpha^* \in (1/\max_{\overline{\Omega}} m, 1/\min_{\overline{\Omega}} m)$

Ideal free distribution

Consider

$$u_t = \mu \nabla \cdot [\nabla u - \alpha \mathbf{u} \nabla \mathbf{P}(\mathbf{x})] + u(m-u) \quad \text{in} \ \ \Omega \times (0, \infty),$$

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- If $P(x) = lnm$, then $\tilde{u} \equiv m$ is the unique positive equilibrium
- The species with the dispersal strategy $P = lnm$ can perfectly match the environmental resource, which leads to its fitness equilibrated across the habitats; i.e., $m - \tilde{u} \equiv 0$.

Evolution of ideal free dispersal

(Cantrell et al, 2009)

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- Is the dispersal strategy $P = lnm$ evolutionarily stable?

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- Suppose that $P(x) = \ln m$, $Q(x) = \ln m(x) + \epsilon R(x)$, where $R \in C^2(\bar{\Omega})$. If R is non-constant, then $(0, \tilde{v})$ is unstable and $(\tilde{u}, 0)$ is globally asymptotically stable for $0 < |\epsilon| \ll 1$.

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- Suppose that $P(x) - lnm$ is non-constant. There exists some $R \in C^2(\bar{\Omega})$ such that for $Q(x) = P(x) + \epsilon R(x)$, $(\tilde{u}, 0)$ is unstable for $0 < |\epsilon| \ll 1$.

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- Suppose that $P(x) - \ln m$ is non-constant. There exists some $R \in C^2(\bar{\Omega})$ such that for $Q(x) = P(x) + \epsilon R(x)$, $(\tilde{u}, 0)$ is unstable for $0 < |\epsilon| \ll 1$.
- $P = \ln m$ is a local evolutionarily stable strategy (ESS), and no other strategy can be a local ESS.

Convergent stable strategy

(Cantrell et al. 2009) Suppose that $\mu = \nu$, $P(x) = \ln m + \alpha R$, $Q(x) = \ln m + \beta R$, $m > 0$, $\Omega = (0, 1)$ and $R_x > 0$ in $[0, 1]$.

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- If $\alpha < \beta < 0$ or $0 < \beta < \alpha$, $(\tilde{u}, 0)$ is unstable and $(0, \tilde{v})$ is stable.
- Given any $\eta > 0$, there exists $\kappa > 0$ such that if either (i) $\alpha, \beta \in [-\eta, 0]$ and $0 < \beta - \alpha < \kappa$ or (ii) $\alpha, \beta \in [0, \eta]$ and $-\kappa < \beta - \alpha < 0$, $(0, \tilde{v})$ is globally asymptotically stable.

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- If either $\alpha < 0 < \beta$ or $\beta < 0 < \alpha$, both $(\tilde{u}, 0)$ and $(0, \tilde{v})$ are unstable, and system (4) has one stable positive steady state.

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- If either $\alpha < 0 < \beta$ or $\beta < 0 < \alpha$, both $(\tilde{u}, 0)$ and $(0, \tilde{v})$ are unstable, and system (4) has one stable positive steady state.
- $P(x) = \mathit{ln}m$ is a CSS.

Stability of $(0, \tilde{v})$

- Stability of $(0, \tilde{v})$: The sign of the principal eigenvalue $\lambda(\alpha)$ of

$$\mu\nabla\cdot[\nabla\varphi-\alpha\varphi\nabla m]+\varphi(m-\tilde{v})=-\lambda\varphi \quad \text{in } \Omega,$$

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- What is the behavior of $\lambda(\alpha)$ for large α?

Asymptotic behavior

- Consider

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$$\lim_{\alpha\to\infty} \lambda(\alpha) = \min_{x\in\mathcal{M}} c(x),$$

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$$-\mu[\Delta\varphi + \alpha\nabla m \cdot \nabla\varphi] + c(x)\varphi = \lambda\varphi \quad \text{in } \Omega, \quad \frac{\partial\varphi}{\partial n}|_{\partial\Omega} = 0.$$

- Chen and L. (Indiana Math Univ. J, 08): suppose that $m \in C^2(\bar{\Omega})$ and all critical points of m are non-degenerate. Then

$$\lim_{\alpha\to\infty} \lambda(\alpha) = \min_{x\in\mathcal{M}} c(x),$$

where $\mathcal{M}$ is the set of points of local maximum of m.

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$$-\mu[\Delta\psi + \alpha\nabla m \cdot \nabla\psi] + \psi(\tilde{v} - m) = \lambda(\alpha)\psi \quad \text{in} \ \ \Omega,$$

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- It follows from previous result of Chen and L. that

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where $\mathcal{M}$=the set of points of local maximum of $m(x)$.

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- Ni (private communication): At each local maximum x_0 of $m(x)$,

$$\liminf_{\beta\to\infty} \tilde{v} \geq m(x_0);$$

  Furthermore, as $\beta \to \infty$, $\tilde{v} \to 0$ uniformly in any compact set which does not contain any local maximum of m.

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Acknowledgement

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Thank you