Securitization, Structuring and Pricing of Longevity Risk

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Motivation

1. Longevity risk and financing
   a. Ageing populations, improving mortality and decreasing fertility
   b. Funding of retirement through DC funds and lack of longevity insurance
   c. Trends and volatility in longevity (“Toxic” for reinsurers)

2. Modelling and risk management
   a. product design and costing, risk management, securitization
   b. longevity guarantee products (life annuities, life time withdrawal guarantees)

3. Mortality models
   a. actuarial life table models (deterministic, projections, participating products)
   b. demographic (stochastic projections, age parameters, stochastic trend)
   c. financial (trend and volatility, flexibility for pricing – price of risk and dependence)

Research Overview

1. Demographic and financial models for longevity risk
   a. Demographic models – e.g. Lee-Carter (1992) and extensions
   b. Financial models – e.g. Milevsky and Promislow (2001), Dahl (2004), Biffis (2005), Schrager (2006)

2. Simple model for Australian data
   a. Australian population data
   b. Lives over age 60
   c. Financial framework to calibrate price of mortality risk
   d. Dependence between ages, cohort trends and volatility (PCA)

3. Application of the model
   a. Securitization of longevity risk using Tranche structure
   b. Multiple age portfolio and age dependence
   c. Calibration of price of risk to Insurance Linked Security Market
   d. Price of risk for different tranches and effect of age dependence

Mortality Models

a. **Demographic models - Lee Carter (1992) and Extensions:**

$$\boxed{\ln[m(x,t)] = a_x + b_x k_t + \varepsilon_{x,t}}$$

- Age based parameters
- Linear (stochastic) trend in k plus volatility (usually trend stationary)
- Age dependence in volatility not usually considered
- Difficult to allow for risk neutral pricing

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b. **Financial models - Dahl (2004) and Extensions:**

- Derived from financial models for interest rate risk (Vasicek, 1977; Cox et al, 1985)

$$\boxed{d\mu(t,x) = \alpha^{\mu}\left(t,x,\mu(t,x)\right)dt + \sigma^{\mu}\left(t,x,\mu(t,x)\right)dB_t}$$

- Model trend and volatility (usually difference stationary)
- Incorporate risk neutral pricing
- Extensive research and applications of term structure interest rate models

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Data

Australian Population Mortality Data, ages 50-99, 1971-2004. Human Mortality Database (www.mortality.org)

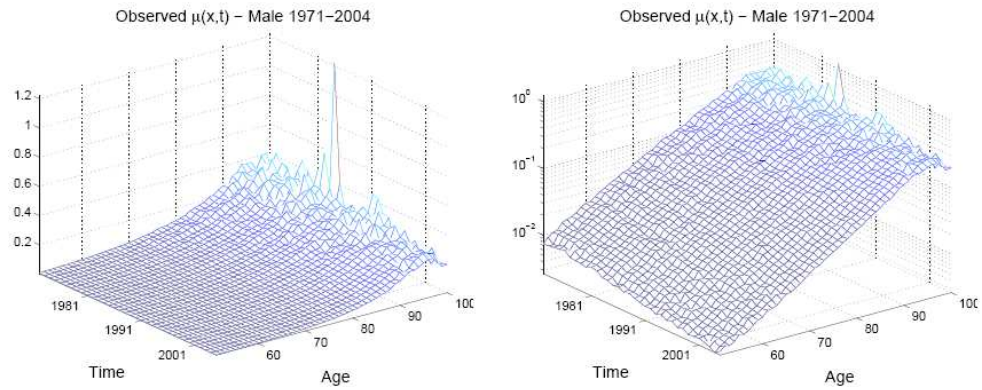

Figure 1: Observed Australian male mortality $\hat{\mu}(x, t)$: 1971-2004, on linear (left) and log (right) scales.

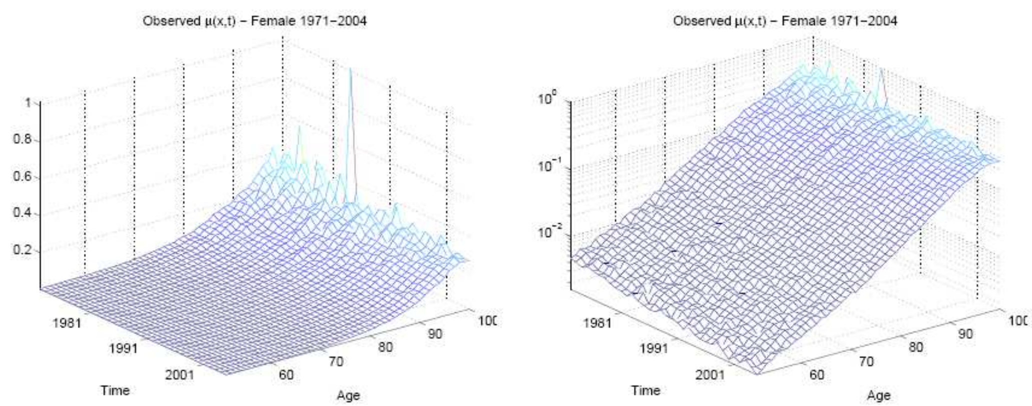

Figure 2: Observed Australian female mortality $\hat{\mu}(x, t)$: 1971-2004, on linear (left) and log (right) scales.

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Longevity Risk Securitisation

Securitisation is a vehicle for risk transfer

- CDOs - late 1980s
- Insurance-Linked Securitization – USD 5.6b issued in 2006*
  - Insurance-Linked Bonds
  - Industry Loss Warranties
  - Sidecars
- Mortality Bond Issues (Vita I-III, Tartan, Osiris, 2003-2007)
- Survivor Bond Issues (BNP Paribas/EIB, 2004)

…with a number of benefits

- Improved capacity for risk transfer as tranching broadens appeal to investors
- Issue can be tailored to manage basis risk, moral hazard and information asymmetry
- Manage credit risk through collateralization
- Diversification benefits for investors

*Lane and Beckwith (2007)

Mortality Model Structure

Fitted mortality model – general structure

Trends varying with age (cohort) and time

Volatility from dependent shocks….

$$d\mu(x,t) = \Big(a(x+t)+b\Big)\mu(x,t)dt + \sigma\mu(x,t)dW(x,t) \text{ for all } x.$$

$$\begin{bmatrix} dW(x_1,t) \\ \vdots \\ dW(x_N,t) \end{bmatrix} = \begin{bmatrix} \delta_{11} & \dots & \delta_{1N} \\ \vdots & \ddots & \vdots \\ \delta_{N1} & \dots & \delta_{NN} \end{bmatrix} \begin{bmatrix} dZ_1(t) \\ \vdots \\ dZ_N(t) \end{bmatrix}$$

…expressed as a combination of independent shocks

Models multi-age portfolios, incorporating age dependence

Easily adapted to pricing mortality-linked securities (Vasicek, 1977; Cox et al, 1985)

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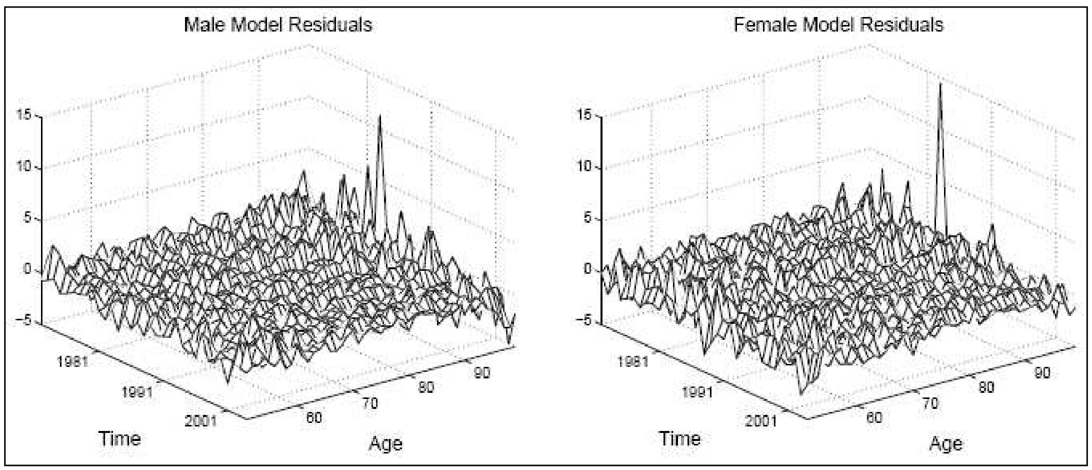
The Mortality Model – Estimated using Australian Population data

Analysis of fitted model

Parameter	MLE: Male	MLE: Female
$\hat{a}$	-9.4398E-04	2.6993E-04
$\hat{b}$	0.1347	0.0608
$\hat{\sigma}$	0.0906	0.0873



Fitted residuals normally distributed, mean zero, standard error 1, without trends across age or time

Dependence and Principal Components Analysis (PCA)

Remove trends and analyse standardised residuals

Analysis of covariance matrix of stochastic mortality factors - $\underline{dW}(x,t)$ - Σ

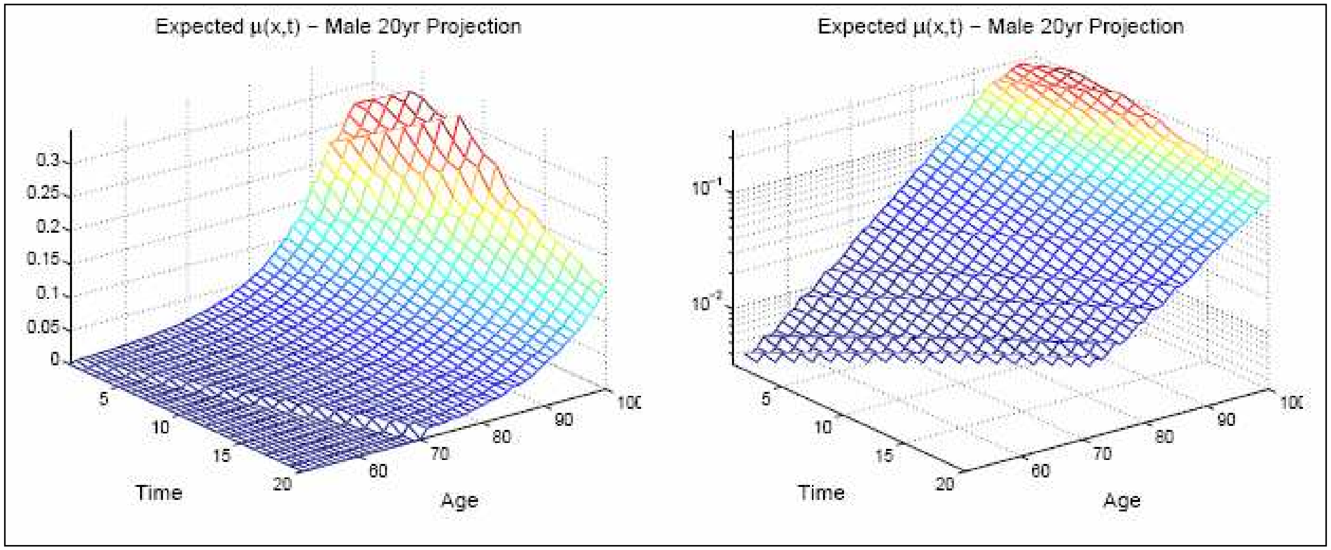
Using PCA, decompose Σ into its eigenvectors (*V*), and eigenvalues (diagonal matrix *T*):

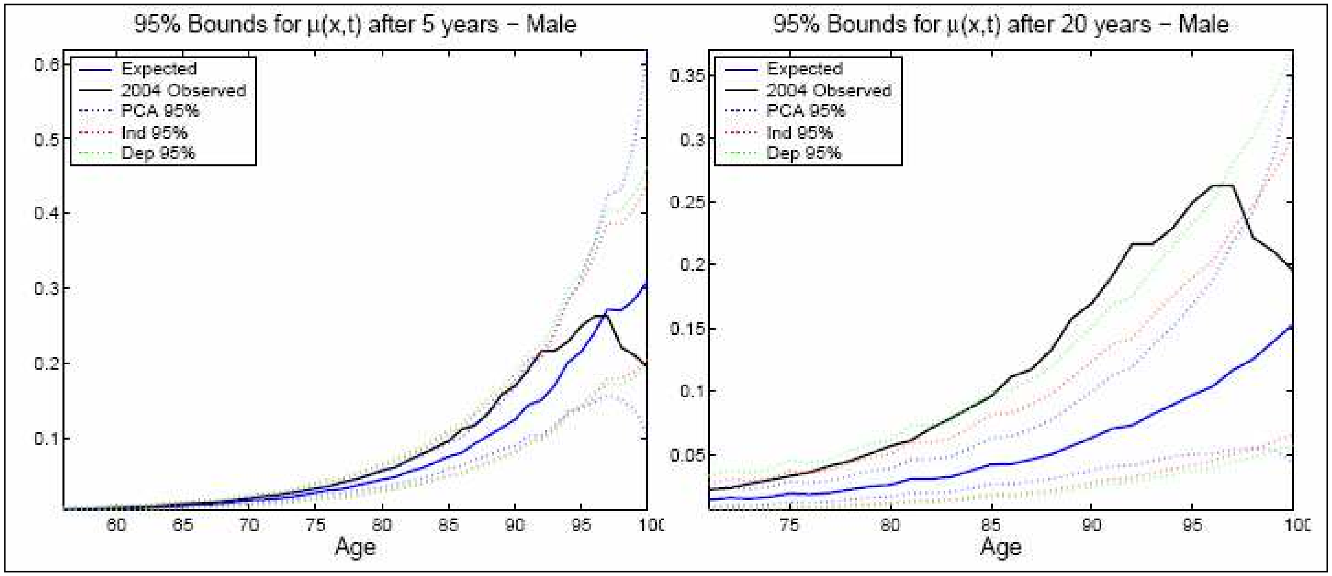
Cholesky decomposition of Σ

# of Eigenvectors	% of Observed Variation
1	29.3%
5	69.8%
10	85.1%
15	92.4%
20	96.5%
25	97.1%
30	99.1%
31	99.5%
32	100.0%

15 vectors explain 92% of variation

The Mortality Model - Projections





- Mortality projected to continue improving over the next 20 years (except older ages)

- Passage of cohort through time can be noted

- Volatility highest under perfect dependence, except at the oldest ages

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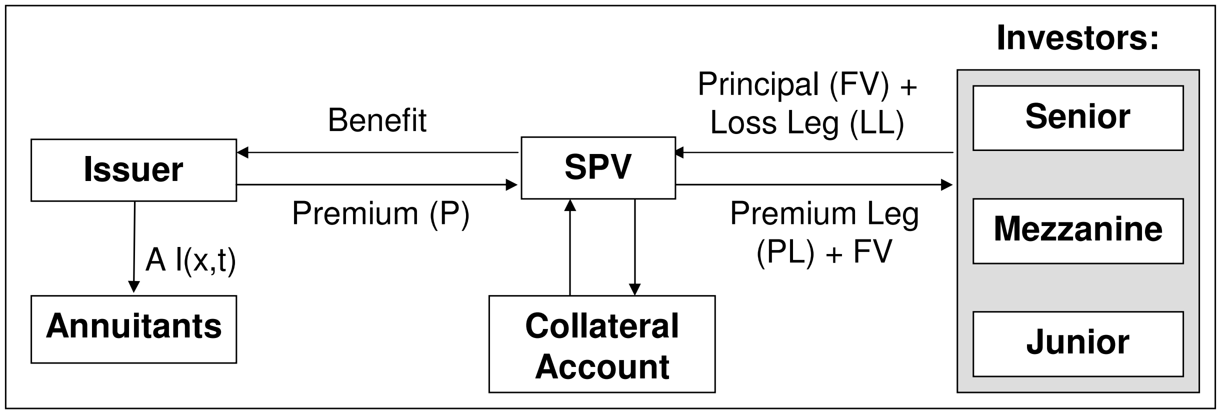
Longevity Bond

Longevity Bond Structure



- Both the PL and the LL are based on the percentage cumulative losses incurred on an underlying annuity portfolio:

$$CL(t) = \frac{\sum_{s=1}^{t} L(s)}{FV}$$

- Where the loss on the portfolio in each period is:

$$\begin{aligned} L(t) &= \left(A \sum_{\text{all x}} l(x,t) - E\left[A \sum_{\text{all x}} l(x,t) \right] \right)^{+} \\ &\approx \left(A \sum_{\text{all x}} l(x,0)\,{}_{t}p_{x} - A \sum_{\text{all x}} l(x,0)\,{}_{t}\bar{p}_{x} \right)^{+} \end{aligned}$$

Bond Structure

Proposed Longevity Bond Assumptions	
Bond Face Value:	$FV = \$750,000,000$.
Term to Maturity:	$T = 20$ years.
Payment Frequency:	Annually, for both premium and loss payments.
Number of Tranches:	$J = 3$.
Initial Age of Annuitants:	$x = 50, \ldots, 79$.
Initial No. of Annuitants:	$n(x, 0) = 60,000$. We assume this is evenly distributed between the 30 ages, with $l(x, 0) = 2,000 \forall x$.
Annuity Payments:	$A = \$50,000$ paid at the end of each year to each living annuitant.

Longevity Bond Tranching

Tranching

- Tranche losses are allocated by the cumulative loss on the portfolio. The cumulative tranche loss is then:

$$CL_j(t) = \begin{cases} 0 & \text{if } L(t) < K_{A,j}; \\ CL(t) - K_{A,j} & \text{if } K_{A,j} <= L(t) < K_{D,j}; \\ K_{D,j} - K_{A,j} & \text{if } L(t) >= K_{D,j}, \end{cases}$$

where

$$CL(t) = \sum_{j=1}^{J} CL_j(t).$$

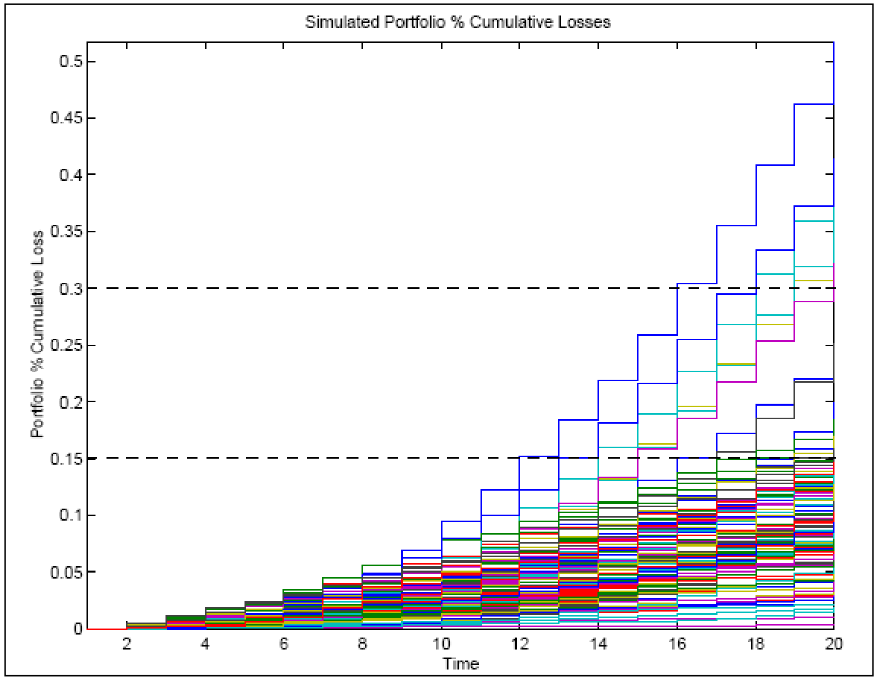
- The tranche loss as a percentage of its prescribed principal is:

$$TCL_j(t) = \frac{E[CL_j(t)]}{K_{D,j} - K_{A,j}}.$$

- The assumed tranche thresholds are:

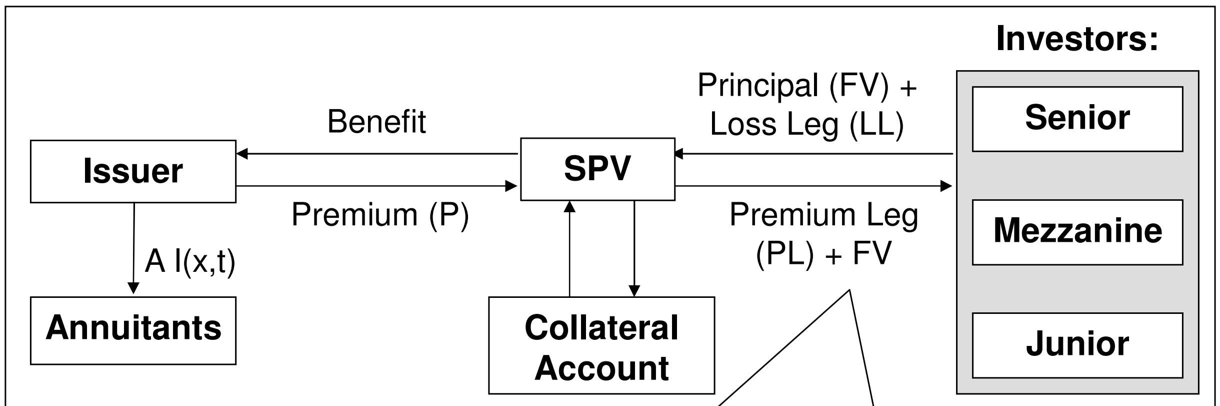
Tranche j	$K_{A,j}$	$K_{D,j}$
1	0%	15%
2	15%	30%
3	30%	100%

Portfolio cumulative loss simulations



Pricing Model

Tranche prices



The price for each tranche $P_j{}^*$ is set so that

$PL_j\ (P^*) = LL_j\ (P^*)$

$$PL_j = \sum_{t=1}^{T} P_j B(0,t-1)[1 - TCL_j(t-1)]$$

$$LL_j = \sum_{t=1}^{T} B(0,t)[TCL_j(t) - TCL_j(t-1)]$$

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Pricing Model

Risk adjustment

- Premiums valued under a risk-adjusted ***Q*** mortality measure.

$$\boxed{\begin{aligned} dW^{\mathbb{Q}}(x,t) &= \sum_{i=1}^{N} \delta_{xi}\left(dZ_i(t) + \lambda_i(t)dt\right) \\ &= dW(x,t) + \sum_{i=1}^{N} \delta_{xi}\lambda_i(t)dt. \end{aligned}}$$

or:

$$\boxed{\underline{dW}^{\mathbb{Q}}(t) = \underline{dW}(t) + \Delta\underline{\lambda}(t)dt}$$

where $\Delta\underline{\lambda}(t)$ is a 'price of risk adjustment'

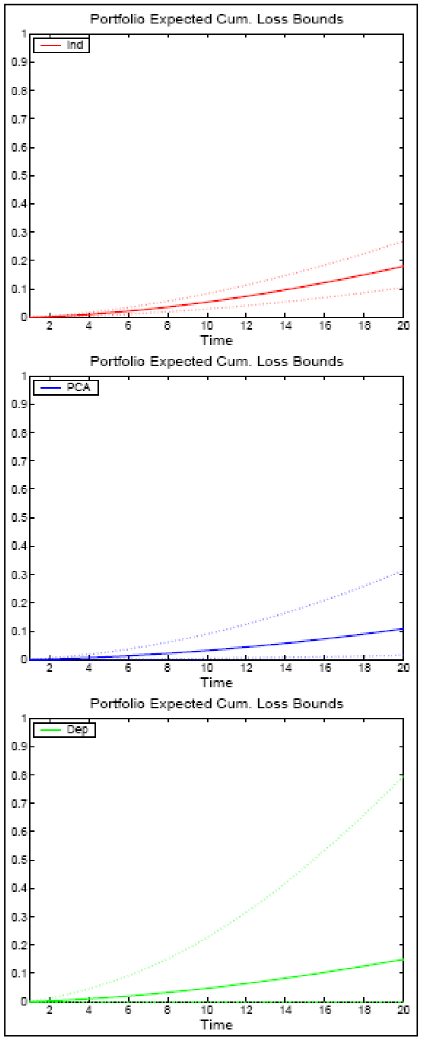
and the risk adjusted mortality process is:

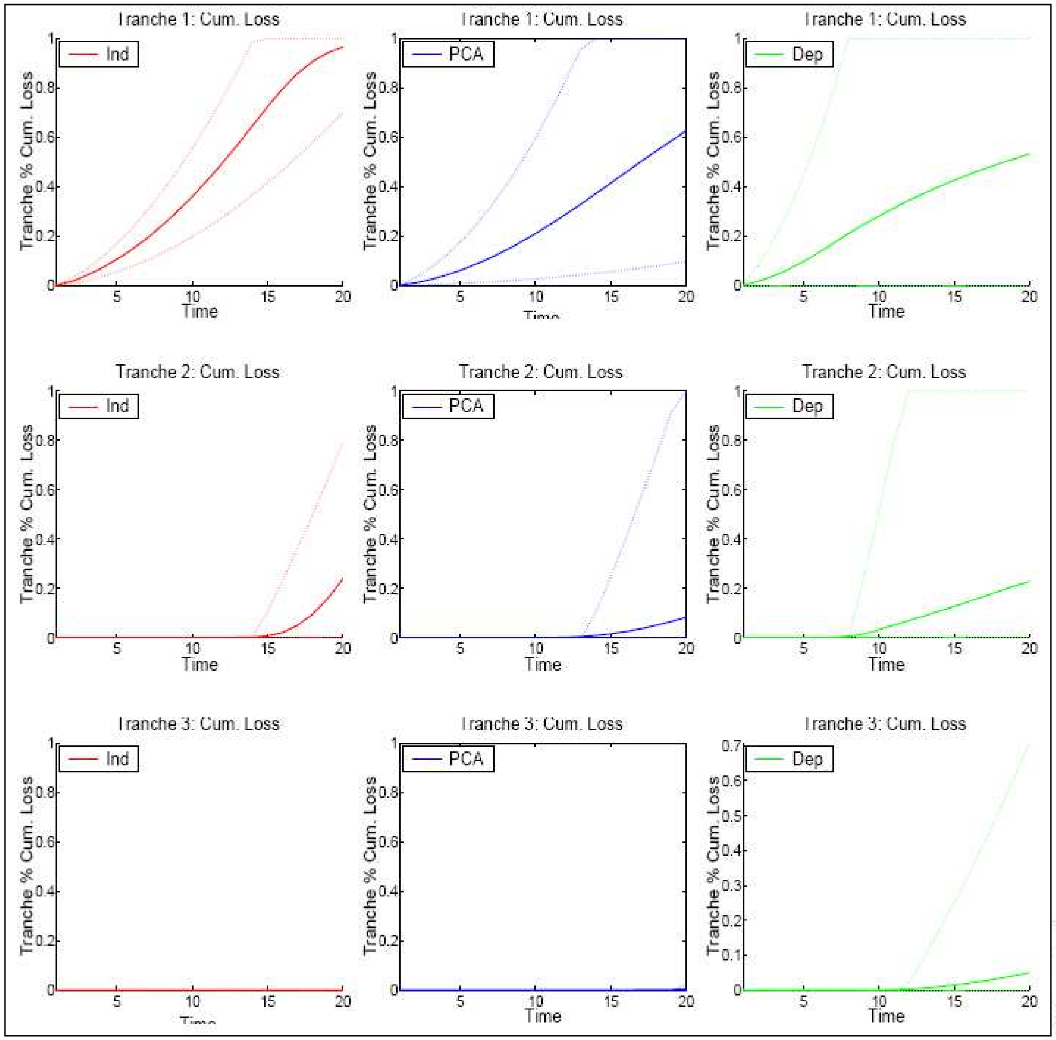
$$\boxed{d\mu^{\mathbb{Q}}(x,t) = \left(a(x+t) + b + \sum_{i=1}^{N} \delta_{xi}\lambda_i(t)\right)\mu^{\mathbb{Q}}(x,t)dt + \sigma\mu^{\mathbb{Q}}(x,t)dW(x,t)}$$

Results – Cumulative Losses

Portfolio expected cum. loss and 95% bounds

Tranche expected cum. loss and 95% bounds under 3 age-dependence assumptions.





- Variability of portfolio loss increases with age dependence

- Expected loss higher under dep., due to option-like payoff

- Tranches losses are over/under-estimated due to dependence

- Dependence has a strong impact on the size of tranche expected losses

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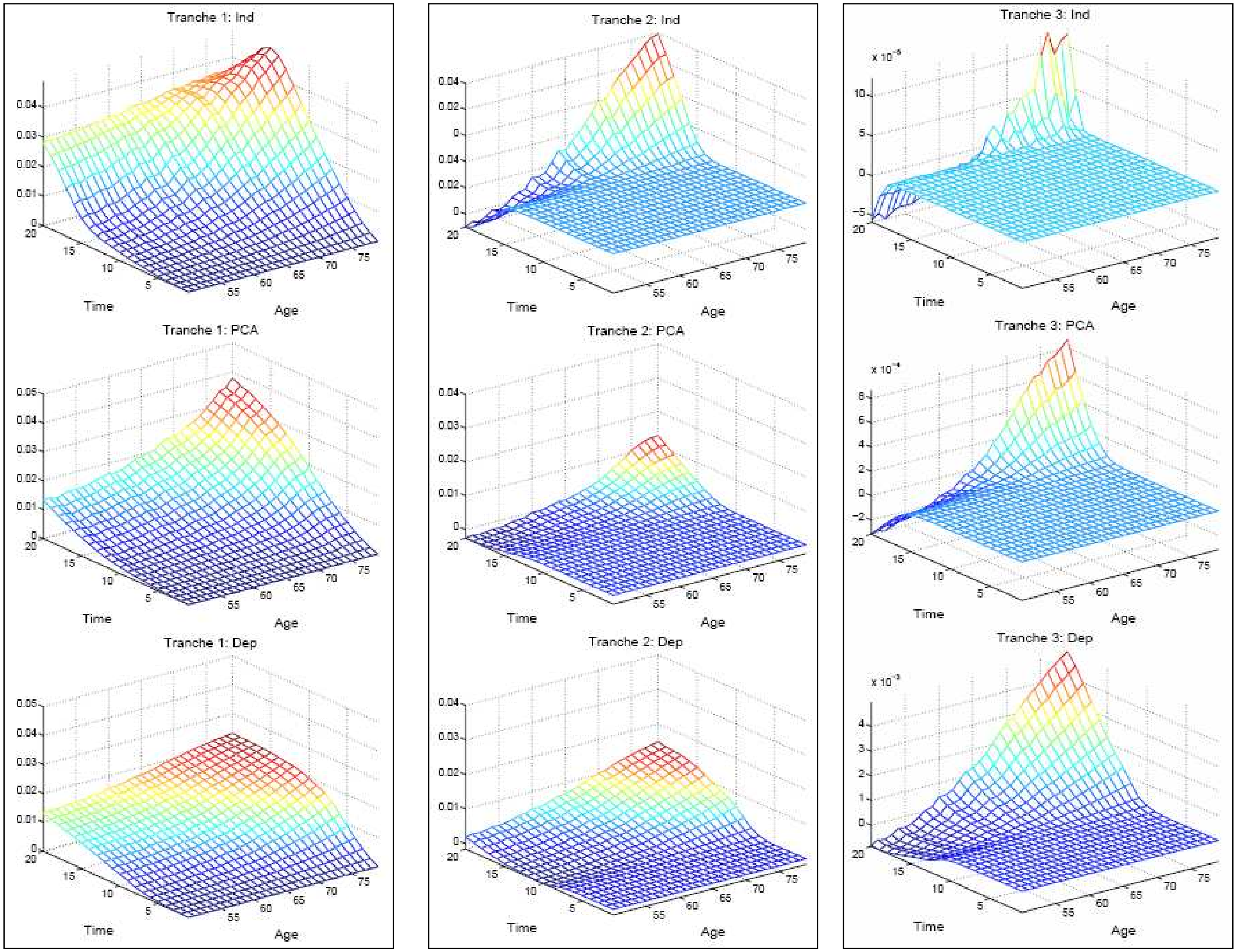
Tranche Losses by age

Tranche cumulative losses, disaggregated by age.



- Tranche losses are not equally incurred across all ages

- Lower losses in young cohorts offset high losses in old cohorts

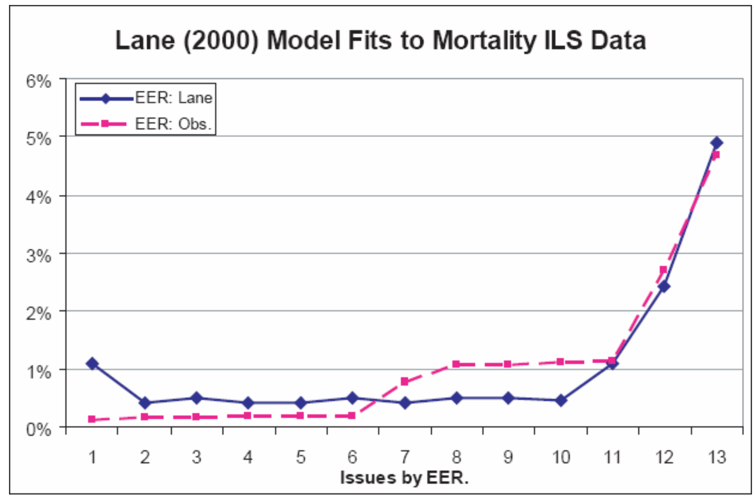
Calibration for Pricing

Model is calibrated using Lane (2000) risk premium model and 2007 mortality bond issues using non-linear least squares:

$$\hat{P}_j^L = EL_j + EER_j$$

$$EER_j = \gamma (PFL_j)^{\alpha} (CEL_j)^{\beta}$$

Parameter	2006-07 Mortality Bonds
γ	0.9980
α	0.8965
β	0.5034
X^2	0.04
χ_9^2 at 99%	2.09



- Price of risk for the model

$$\boxed{\Delta \underline{\lambda}(t) = \underline{\lambda}^* \text{ where } \underline{\lambda}^* = [\lambda^*, \ldots, \lambda^*]'}$$

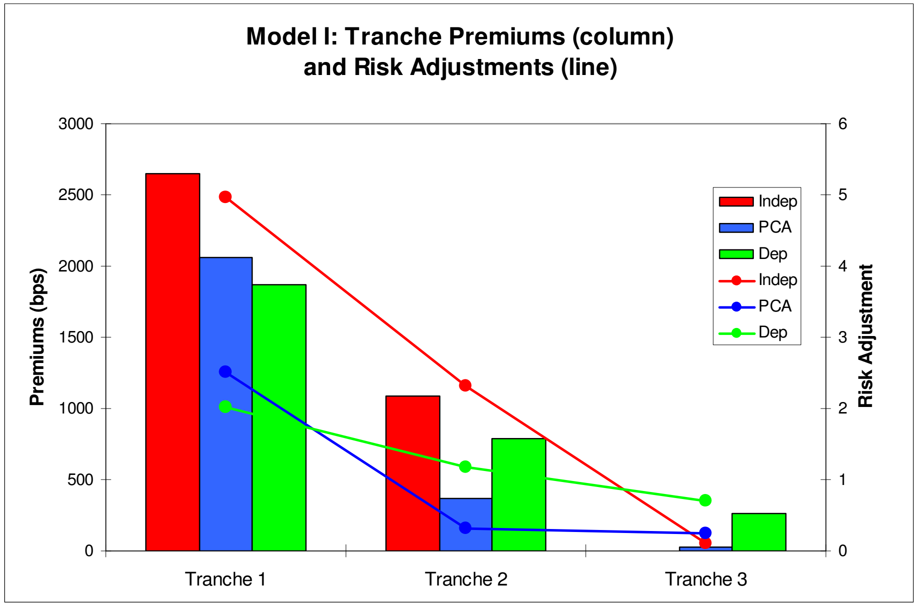
So that for each *x* and *t*:

λ^* is chosen so that: $\boxed{P_j^{\lambda^*} = P_j^L}$

Price of risk $\boxed{\lambda_j^* = f(P\hat{F}L_j, C\hat{E}L_j, \gamma, \alpha, \beta)}$

Tranche risk premiums

$$d\mu^{\mathbb{Q}}(x,t) = \Big(a(x+t) + b - \sigma\lambda^*\Big)\mu^{\mathbb{Q}}(x,t)dt + \sigma\mu^{\mathbb{Q}}(x,t)dW(x,t).$$



Tranche premiums calibrated using the Lane model and 'prices of risk' λ are implied market values

Market insurance-linked security data: 2007 issues. Drawn from Lane and Beckwith (2007).

λ sensitivities:

(observed dependence)

$$\lambda_j^* = f(P\hat{F}L_j, C\hat{E}L_j, \gamma, \alpha, \beta)$$

	Premium	λ_j^*	Sensitivities				
			$P\hat{F}L_j$	$C\hat{E}L_j$	γ	α	β
Tranche 1	2058	2.52	-	1.39	2.04	-	-3.52
Tranche 2	371	0.31	1.24	0.76	1.21	-1.76	-2.33
Tranche 3	31	0.25	0.29	0.17	0.31	-0.95	-0.79

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Tranche Premiums for Differing Age Dependence

	Premium	λ_j^*	Sensitivities				
			$P\hat{F}L_j$	$C\hat{E}L_j$	γ	α	β
Tranche 1	2652	4.97	-	1.38	1.91	-	-2.89
Tranche 2	1085	2.32	2.48	1.52	2.43	-0.60	-5.02
Tranche 3	3	0.11	0.27	0.16	0.30	-1.38	-0.97

Table 9: Tranche premiums, risk adjustments and sensitivities of λ_j^* under perfect age independence.

	Premium	λ_j^*	Sensitivities				
			$P\hat{F}L_j$	$C\hat{E}L_j$	γ	α	β
Tranche 1	2058	2.52	-	1.39	2.04	-	-3.52
Tranche 2	371	0.31	1.24	0.76	1.21	-1.76	-2.33
Tranche 3	31	0.25	0.29	0.17	0.31	-0.95	-0.79

Table 10: Tranche premiums, risk adjustments and sensitivities of λ_j^* under age co-dependence using PCA.

	Premium	λ_j^*	Sensitivities				
			$P\hat{F}L_j$	$C\hat{E}L_j$	γ	α	β
Tranche 1	1870	2.03	-	1.41	2.11	-	-3.81
Tranche 2	789	1.18	1.83	1.15	1.72	-1.72	-2.88
Tranche 3	261	0.70	0.76	0.46	0.76	-1.26	-1.55

Table 11: Tranche premiums, risk adjustments and sensitivities of λ_j^* under perfect age dependence.

Summary

Securitization and Longevity Bond

- Longevity-linked securitization assessed based on portfolio of multiple ages
- Pricing and pay-offs of tranching is assessed, under a range of age dependence assumptions

The Mortality Model

- Mortality model includes trend (cohort, age) and risk factor dependence by age
- Importance of age-dependence is assessed. Implications for modelling mortality-linked securities on multi-age portfolios.

The Pricing Model

- Financial model framework adapted to longevity modelling and calibrated to Australian population data
- Mortality model allows for ‘price of risk’ to vary by age and time.
- Price of risk calibrated to market data for insurance linked securities.

Conferences in 2010

- **World Risk and Insurance Economics Congress – at Singapore Management University 25-29 July 2010**
  - Asia-Pacific Risk and Insurance Association (APRIA), American Risk and Insurance Association (ARIA), European Group of Risk and Insurance Economists (EGRIE), The Geneva Association
  - Proposal Submission Deadline **1 February 2010** papers by **1 June 2010**
- **Longevity 6: 6th International Longevity Risk and Capital Markets Solutions Conference – hosted by Australian Institute of Population Ageing Research, UNSW, 9-10 September 2010 Bondi Beach, Sydney**
  - Focus on "Reinsurance and Financial Markets" and "Government Role, Public and Private Market Solutions"
  - Proposed date for submission of papers – **30 April 2010**

Selected References

- Cairns, A.J.G., Blake, D., and Dowd, K., 2006, Pricing Death: Frameworks for the Valuation and Securitization of Mortality Risk, *ASTIN Bulletin*, 36, 79-120
- Cox, J., Ingersoll, J., and Ross, S., 1985, A Theory of the Term-Structure of Interest Rates, *Econometrica*, 53, 385-408.
- Denuit, M., Devolder, P., and Goderniaux, A.-C., 2007, Securitization of Longevity Risk: Pricing Survivor Bonds with Wang Transform in the Lee-Carter Framework, *The Journal of Risk and Insurance*, 74(1), 87-113.
- Dahl, M., 2004, Stochastic Mortality in Life Insurance: Market Reserves and Mortality-Linked Insurance Contracts, *Insurance: Mathematics and Economics*, 35, 113-136.
- Lane, M.N., and Beckwith, R., 2007, That was the Year that was! The 2007 Review of the Insurance Securitization Market, Lane Financial L.L.C. (Available at http://lanefinanciallc.com/)
- Lee, R.D., and Carter, L.R., 1992, Modeling and Forecasting U.S. Mortality, *Journal of the American Statistical Association*, 87, 659-675.
- Milevsky M. A. and Promislow S.D., 2001, Mortality Derivatives and the Option to Annuitize, *Insurance: Mathematics and Economics,* 29, 299-318.
- Sherris, M., and Wills, S., 2007, Financial Innovation and the Hedging of Longevity Risk, *Presented at the Third International Longevity Risk and Capital Market Solutions Symposium*, Taipei.
- Schrager, D. F., 2006, Affine Stochastic Mortality, *Insurance: Mathematics and Economics*, 38, 81-97.
- Vasicek, O., 1977, An Equilibrium Characterisation of the Term Structure, *Journal of Financial Economics*, 5, 177-188.
- Wills, S. and Sherris, M. 2008, Securitization, Structuring and Pricing of Longevity Risk, *UNSW Actuarial Studies Working Paper (to appear IME)*

Questions and Discussion