

# How Does Illiquidity Affect Delegated Portfolio Choice?

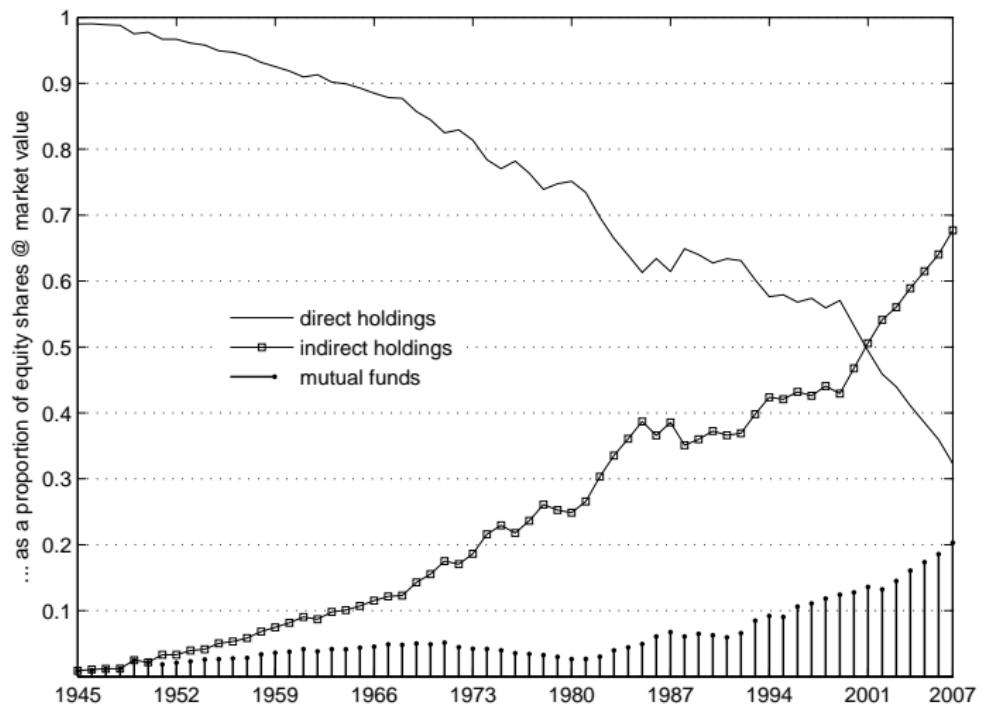
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# U.S. Households and Nonprofit Organizations: Direct and Indirect Equity Holdings (1945-2007)



Looks Like The  
MARKET'S LIQUIDITY  
PROBLEM IS WORSE  
THAN ORIGINALLY THOUGHT...



## Research Questions:

Existing models of delegated portfolio choice have assumed the possibility of continuous and frictionless trading on risky assets.

1. How does asset liquidity (or the lack of it) affect money managers' asset selection and incentives?
2. How does it affect investors' welfare costs due to portfolio delegation and misalignment of objectives?
3. Is liquidity more valuable in a direct or in a delegated investment setting?

## Preview of the Results:

1. Optimal portfolio strategies under liquidity constraints:
  - ▶ flight-to-quality
  - ▶ portfolio liquidity-shifting over the investment period
  - ▶ risk-shifting more likely to happen with more liquid assets
2. Investor's utility costs due to delegation and misalignment of incentives less severe with asset illiquidity
3. Shadow costs of illiquidity likely larger (smaller) in a delegated rather than a direct investing context when using a rebalancing (buy-and-hold) benchmark

## Related Literature:

### 1. Portfolio delegation and benchmarking:

- ▶ Browne (JFS 1999)
- ▶ Basak, Pavlova, and Shapiro (RFS 2007, JBF 2008)
- ▶ Binsbergen, Brandt, and Koijen (JF 2008)

### 2. Portfolio choice and trading restrictions:

- ▶ Longstaff (JF 1995, RFS 2001)
- ▶ Kahl, Liu and Longstaff (JFE 2003)
- ▶ Isaenko (WP 2008)

### 3. Mutual funds and liquidity:

- ▶ Falkenstein (JF 1996)
- ▶ Massa and Phalippou (WP 2005)
- ▶ Cao, Simin, and Wang (WP 2007)
- ▶ Huang (WP 2008)

# This Paper: Setup

## 1. The financial market:

- ▶ Black-Scholes-Merton nominal economy
- ▶ 1 risk-free asset and 2 illiquid risky assets [illiquidity modeled as in Longstaff (2001)]

## 2. The agents:

- ▶ an investor and a fund manager, both risk-averse and price-takers (partial equilibrium)
- ▶ a passive investor who exogenously decides to delegate her portfolio decisions to the fund manager
- ▶ a manager rewarded for increasing the value of his fund's assets, and whose investment horizon coincides with the date of the fund's (non-tradable) flows
- ▶ a peer group consisting of a large number of competing funds

# The Fund Manager's Problem:

Choose  $N_i(0)$  and  $\eta_i(t)$ , for  $i \in \{1, 2\}$ , so as to maximize

$$E_t \left[ \frac{(W(T)\phi(T))^{1-\gamma_M}}{1-\gamma_M} \right]$$

subject to

$$dW(t) = rW(t)dt + \sum_{i=1}^2 N_i(t)S_i(t)[(\mu_i - r)dt + \sigma_i dZ_i(t)]$$

$$dN_i(t) = \eta_i(t)dt, \text{ for } -\alpha_i \leq \eta_i(t) \leq \alpha_i$$

with  $\phi(T)$  driven by the benchmark payoff  $Y(T)$

$$\phi(T) = \frac{1}{Y(T)} \text{ for } Y(t) = \left(1 - \sum_{i=1}^2 \beta_i(t)\right) B(t) + \sum_{i=1}^2 \beta_i(t) S_i(t)$$

## Optimal Unconstrained Portfolio Policies:

1. WITHOUT benchmarking [ $\phi(T) = 1$ ]:

$$\omega_i^*(t) = \frac{1}{\gamma_M} \frac{\mu_i - r}{\sigma_i^2}$$

2. WITH benchmarking [ $\phi(T) = 1/Y(T)$ ]:

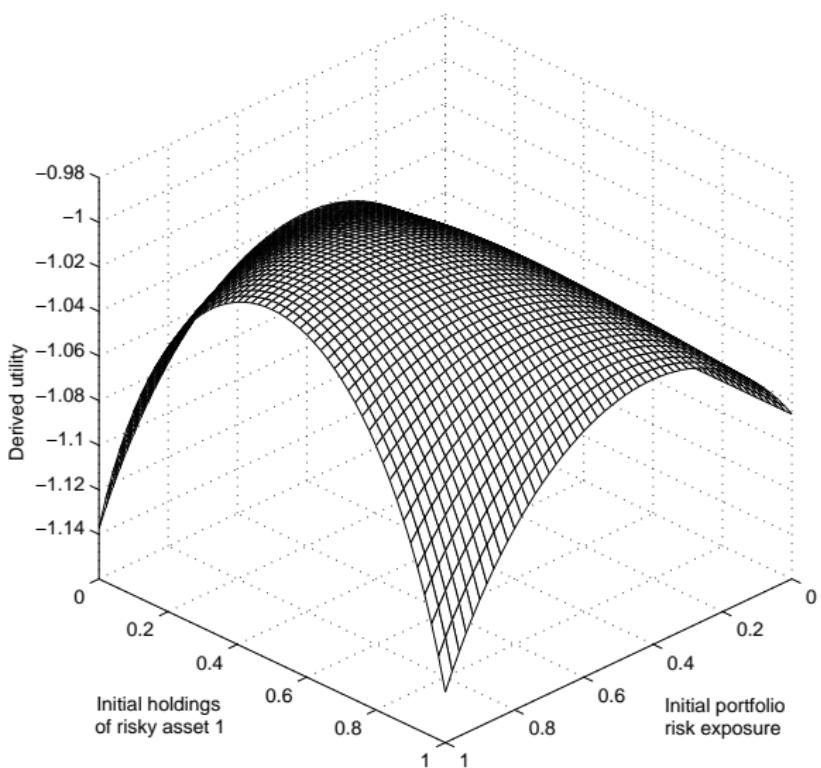
$$\omega_i^{**}(t) = \frac{1}{\gamma_M} \frac{\mu_i - r}{\sigma_i^2} + \left(1 - \frac{1}{\gamma_M}\right) \beta_i(t)$$

for  $i \in \{1, 2\}$ , and  $\rho = 0$  [ $dZ_1(t)dZ_2(t) = \rho dt$ ]

## Optimal Portfolio Policies Under Liquidity Constraints:

1. No analytical solution
2. Regression-based MC methodology: Longstaff (2001)  
application of the Least-Squares Monte Carlo algorithm by  
Longstaff and Schwartz (2001)
3. Optimal stopping/trading rule: buy or sell as much as possible  
whenever possible

# Liquidity Constraints: Optimal Holdings and Derived Utility



# INVESTOR: Non-Trading Assets [ $\alpha_1 = \alpha_2 = 0$ ], No Divergence of Incentives [ $\phi(T) = 1$ , $\gamma_I = \gamma_M$ ]

**TABLE A:**  $\alpha_i = 0$ ,  $T_j=1$ ,  $i \in \{1, 2\}$ ,  $j \in \{I, M\}$

$\rho$	RA( $\gamma_j$ )	$\sigma_i = 0.3$				$\sigma_i = 0.5$			
		$\omega_i^u(0)$	$\omega_i^c(0)$	CSV $_i$	ID(bp)	$\omega_i^u(0)$	$\omega_i^c(0)$	CSV $_i$	ID(bp)
-0.5	1	1.778	0.500	0.1197	704.03	0.640	0.500	0.1838	83.99
	2	0.889	0.500	0.1197	149.17	0.320	0.268	0.1111	44.48
	5	0.356	0.328	0.0833	20.29	0.128	0.103	0.0499	19.34
	10	0.178	0.165	0.0464	10.71	0.064	0.050	0.0261	9.99
0	1	0.889	0.500	0.0996	136.55	0.320	0.305	0.1075	12.82
	2	0.444	0.440	0.0891	5.73	0.160	0.153	0.0649	8.93
	5	0.178	0.175	0.0444	3.54	0.064	0.060	0.0298	4.39
	10	0.089	0.085	0.0243	2.15	0.032	0.030	0.0158	2.40
0.5	1	0.593	0.500	0.0718	11.70	0.213	0.210	0.0713	5.76
	2	0.296	0.293	0.0527	1.61	0.107	0.100	0.0430	4.92
	5	0.119	0.115	0.0291	1.92	0.043	0.040	0.0199	2.61
	10	0.059	0.058	0.0164	1.23	0.021	0.020	0.0105	1.41

NOTE: 100,000 simulation runs, 20 time steps / year,  $S_i(0) = 1$  ( $i = \{0, 1, 2\}$ ),  $\mu_1 = \mu_2 = 0.10$ , and  $r = 0.02$

# INVESTOR: Trading Asset 2 [ $\alpha_1 = 0$ , $\alpha_2 = 0.1$ ], No Divergence of Incentives [ $\phi(T) = 1$ , $\gamma_I = \gamma_M$ ]

**TABLE B:**  $\alpha_1 = 0$ ,  $\alpha_2 = 0.1$ ,  $\sigma_i = 0.5$ ,  $T_j=1$ ,  $i \in \{1, 2\}$ ,  $j \in \{I, M\}$

$\rho$	RA( $\gamma_j$ )	IC(bp)	CSV_W	Asset $i$	$\omega_i^u(0)$	$\omega_i^c(0)$	E[ $\omega_i^c(T)$ ]	CSV_i
-0.5	1	84.70	0.3195	1	0.640	0.500	0.500	0.1838
				2	0.640	0.500	0.500	0.1838
	2	52.27	0.1660	1	0.320	0.276	0.279	0.1100
				2	0.320	0.184	0.287	0.1230
	5	28.61	0.0671	1	0.128	0.105	0.110	0.0505
				2	0.128	0.045	0.147	0.0693
	10	23.63	0.0389	1	0.064	0.054	0.057	0.0278
				2	0.064	0.006	0.106	0.0493
	1	16.18	0.2583	1	0.320	0.285	0.284	0.1039
				2	0.320	0.285	0.376	0.1271
0	2	12.45	0.1266	1	0.160	0.150	0.154	0.0641
				2	0.160	0.100	0.200	0.0837
	5	11.58	0.0508	1	0.064	0.059	0.062	0.0291
				2	0.064	0.007	0.107	0.0501
	10	17.70	0.0368	1	0.032	0.030	0.032	0.0157
				2	0.032	0	0.097	0.0435

NOTE: 100,000 simulation runs, 20 time steps / year,  $S_i(0) = 1$  ( $i = \{0, 1, 2\}$ ),  $\mu_1 = \mu_2 = 0.10$ , and  $r = 0.02$

# MANAGER: Non-Trading Assets [ $\alpha_1 = \alpha_2 = 0$ ], with Benchmarking [ $\phi(T) = 1/Y(T)$ , $\gamma_I = \gamma_M$ ]

**Table C1: REBALANCING** Benchmark with  $\beta_i = 0.5$ ,  $\alpha_i = 0$ ,  $T_M=1$ ,  $\rho = 0$  ( $CSV_Y=0.404$ )

RA( $\gamma_M$ )	$\omega_i^u(0)$	$\omega_i^c(0)$	CSV <sub>c</sub>	TE <sub><math>\beta</math></sub>	ID(bp)	CSV <sub>W</sub>	TE <sub>Y</sub>	P[W < Y]
1	0.320	0.305	0.1075	19.71	12.82	0.2543	3.09	0.5173
2	0.410	0.395	0.1289	12.60	39.27	0.3294	2.03	0.5344
5	0.464	0.450	0.1424	10.07	108.95	0.3752	1.56	0.5864
10	0.482	0.470	0.1478	9.75	194.03	0.3919	1.46	0.6248

**Table C2: BUY-AND-HOLD** Benchmark with  $\beta_i = 0.5$ ,  $\alpha_i = 0$ ,  $T_M=1$ ,  $\rho = 0$  ( $CSV_Y=0.4169$ )

RA( $\gamma_M$ )	$\omega_i^u(0)$	$\omega_i^c(0)$	CSV <sub>u</sub>	CSV <sub>c</sub>	TE <sub><math>\beta</math></sub>	ID(bp)	CSV <sub>W</sub>	TE <sub>Y</sub>	P[W < Y]
1	0.320	0.305	0.1021	0.1075	19.27	2.24	0.2543	2.97	0.5147
2	0.410	0.405	0.1309	0.1313	9.50	2.05	0.3377	1.46	0.5147
5	0.464	0.465	0.1481	0.1464	3.54	1.13	0.3878	0.55	0.5146
10	0.482	0.480	0.1538	0.1505	2.03	0.63	0.4003	0.31	0.5146

NOTE: 100,000 simulation runs, 20 time steps / year,  $S_i(0) = 1$  ( $i = \{0, 1, 2\}$ ),  $\mu_1 = \mu_2 = 0.10$ , and  $r = 0.02$

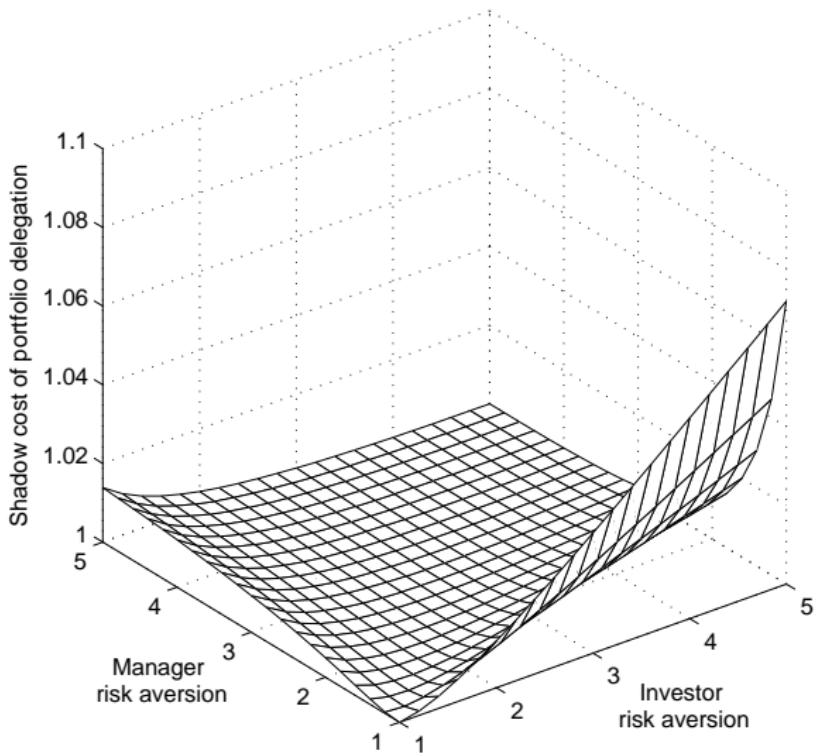
MANAGER: **Trading** Asset 2 [ $\alpha_1 = 0, \alpha_2 = 0.2$ ],  
 with **Benchmarking** [ $\phi(T) = 1/Y(T), \gamma_I = \gamma_M$ ]

**TABLE D: REBALANCING**  $\beta_i = 0.5, \alpha_1 = 0, \alpha_2 = 0.2, T_j=1, \rho = 0$  ( $CSV_Y=0.404$ )

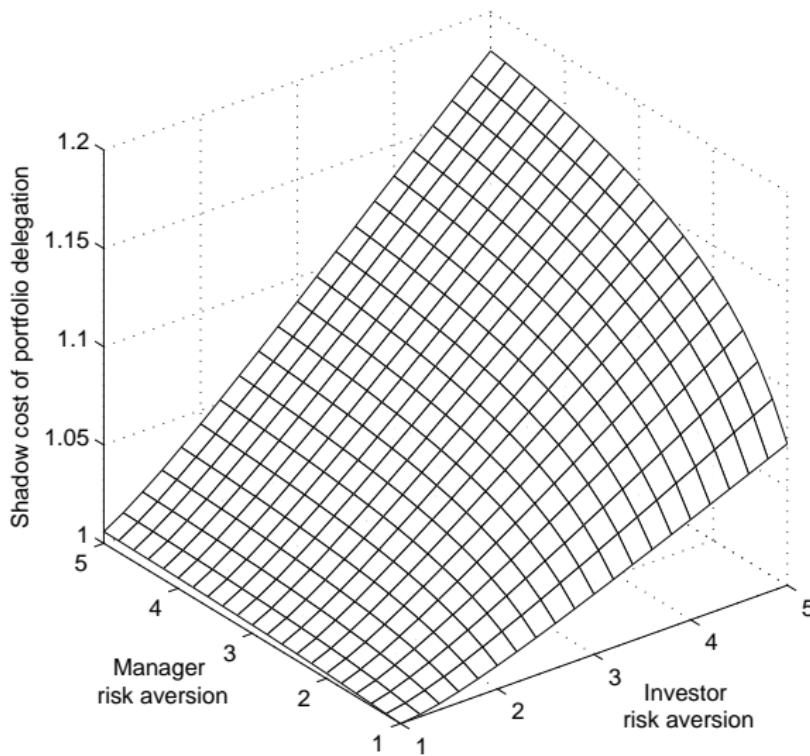
$\gamma_M$	IC(bp)	$CSV_W$	$TE_Y$	$P[W < Y]$	$i$	$\omega_i^u(0)$	$\omega_i^c(0)$	$E[\omega_i^c(T)]$	$CSV_i$	$TE_\beta$
1	19.32	0.2572	3.22	0.5215	1	0.320	0.306	0.306	0.1073	19.63
					2	0.320	0.204	0.395	0.1409	21.57
2	54.18	0.3318	2.21	0.5367	1	0.410	0.414	0.411	0.1293	11.29
					2	0.410	0.276	0.465	0.1575	16.39
5	138.21	0.4061	1.51	0.6476	1	0.464	0.455	0.457	0.1514	10.28
					2	0.464	0.455	0.539	0.1529	10.28

NOTE: 100,000 simulation runs, 20 time steps / year,  $S_i(0) = 1$  ( $i = \{0, 1, 2\}$ ),  $\mu_1 = \mu_2 = 0.10$ , and  $r = 0.02$

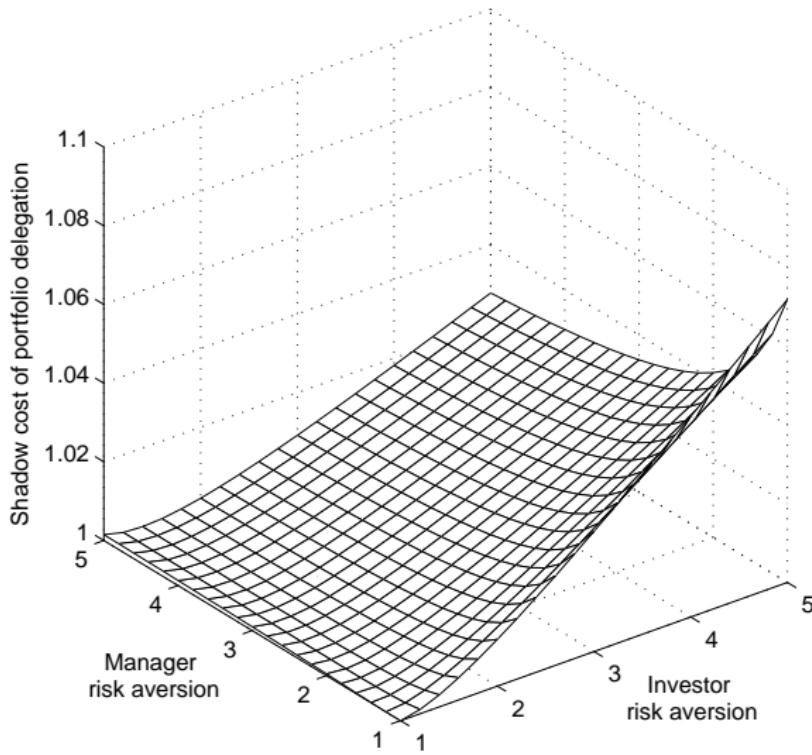
## Welfare Costs: $\phi(T) = 1$ and $\gamma_I \neq \gamma_M$



**Welfare Costs:**  $\phi(T) = 1/Y(T)$  [ $\beta_j = 0.5$ ] and  $\gamma_I \neq \gamma_M$



**Welfare Costs:**  $\phi(T) = 1/Y(T)$  [ $\beta_j = 0.2$ ] and  $\gamma_I \neq \gamma_M$



# Welfare Costs on the Investor due to Benchmarking: $\phi(T) = 1/Y(T)$ and $\gamma_I = \gamma_M$

**Table E1:** Unconstrained liquidity

RA( $\gamma_i$ )	$\beta_j = 0.2$		$\beta_j = 0.5$	
	$T_i = 1$	$2$	$1$	$2$
1	0	0	0	0
2	0.51	1.02	3.20	6.50
5	3.26	6.64	22.25	50.31
10	8.39	17.78	67.11	188.76

NOTE: 100,000 simulation runs, 20 time steps/year,  
 $S_i(0) = 1$  ( $i = \{0, 1, 2\}$ ),  $\mu_1 = \mu_2 = 0.10$ , and  
 $r = 0.02$

**Table E2:** Rebalancing bench.,  $\alpha_j = 0$

RA( $\gamma_i$ )	$\beta_j = 0.2$		$\beta_j = 0.5$	
	$T_i = 1$	$2$	$1$	$2$
1	0	0	0	0
2	0.44	0.85	3.00	5.82
5	2.81	4.82	20.32	41.28
10	6.44	10.71	58.25	136.21

**Table E3:** Buy-and-hold bench.,  $\alpha_j = 0$

RA( $\gamma_i$ )	$\beta_j = 0.2$		$\beta_j = 0.5$	
	$T_i = 1$	$2$	$1$	$2$
1	0	0	0	0
2	0.49	1.04	3.25	6.62
5	2.99	5.80	22.46	46.84
10	6.81	11.94	63.86	160.21

## Conclusions and Implications:

- ▶ The presence of an illiquid asset can significantly affect the optimal portfolio allocation on a liquid one.
- ▶ The value of liquidity is likely to increase (decrease) with the rise of financial intermediation when using equal-weight (value-weight) benchmarking.
- ▶ The propensity to delegate portfolio decisions is likely to be higher when in the presence of more illiquid assets and in more illiquid periods.
- ▶ The presence of illiquid assets can influence money managers' risk-shifting incentives.