Dark Markets
Part 2. Search and Information Percolation

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Collaboration: Nicolae Gârleanu, Gaston Giroux, Gustavo Manso, Semyon Malamud, Lasse Pedersen, and Yeneng Sun

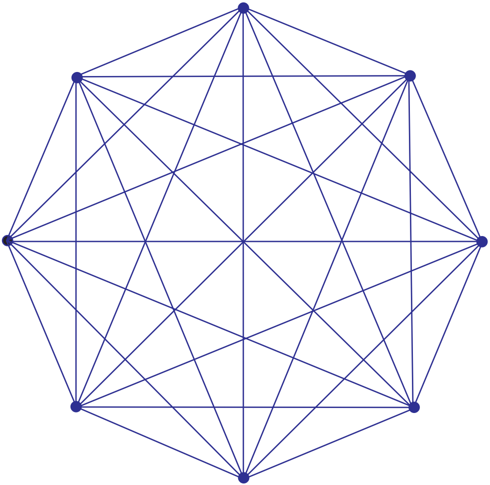

Figure: An over-the-counter market.

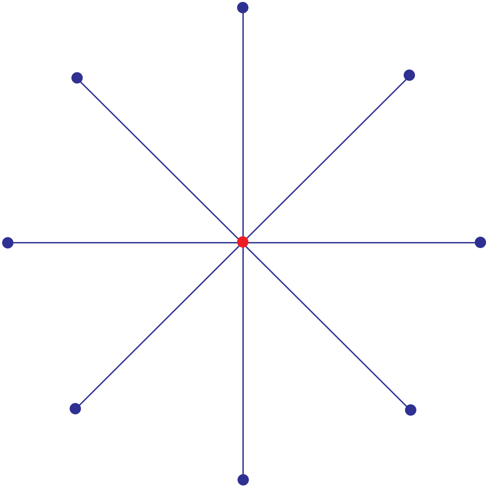

Figure: A standard-paradigm centralized market.

Search in Large Markets

- Agents: a non-atomic measure space.
- Events: a probability space $(\Omega, \mathcal{F}, P)$.
- Agents meet other agents at random and negotiate asset trades.
- Agents have heterogeneous information and motives to own the asset.
- Trading motives can change at random.
- Without dealers: the probability of re-encountering the same agent is zero, so there are no strategic effects in bargaining. Each agent has an outside option that does not depend on the other agent's strategy.
- The law of large numbers keeps the dimension of the state variable under control.

Exact Law of Large Numbers for Random Matching

From work with Yeneng Sun (2005, 2007).

- Suppose the population is 40% red, 60% blue.
- A red agent meets a blue agent with probability 0.6.
- Assuming the exact law of large numbers, the quantity of matches of red to blue is $0.4 \times 0.6 = 0.24$ a.s.
- The quantity of matches of blue to red is $0.6 \times 0.4 = 0.24$.
- Total matches of red with blue: 0.48. Red with red: 0.16. Blue with blue 0.36.
- This applies under independence assumptions and richness assumptions on the σ-algebra on agents $\times$ states.
- LLN-based dynamics of infinite-agent search models (in discrete time) follows similarly. (Boltzmann's "Stosszahlansatz").

Search-Based Asset Pricing

From work with Nicolae Gârleanu and Lasse Heje Pedersen.

- An agent has a high or low (h or ℓ) value for the asset, and owns it or not (o or n).
- A high-type agent becomes low with intensity k_u. The opposite transition intensity is k_d. Agents are matched at intensity λ.
- For each type σ in $\mathcal{T} = \{ho, hn, \ell o, \ell n\}$, $\mu_\sigma(t)$ is the fraction at time t of agents of type σ.
- With per-capita asset supply s,

$$s = \mu_{ho}(t) + \mu_{lo}(t),$$

- Example: evolution of quantity of agents of type ℓo:

$$\frac{d}{dt}\mu_{lo}(t) = -k_u\mu_{lo}(t) + k_d\mu_{ho}(t) - \lambda\mu_{hn}(t)\mu_{lo}(t).$$

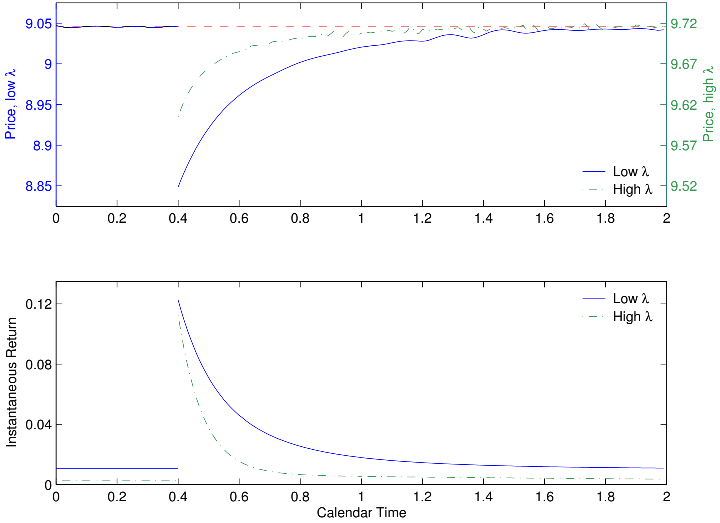

Figure: Liquidity shock at time 0.4. Low search intensity $\lambda = 125$; high search intensity $\lambda = 625$. Source: Duffie, Gârleanu, and Pedersen (2005).

- An agent's current "type" can include current preference for the asset, private information, and amount of the asset held.
- The measure μ_t at time t describing the fraction of the population at time t whose type is in a given set evolves by some variant of a Boltzmann equation

$$\frac{d\mu_t}{dt} = A\mu_t + \lambda\mu_t \circ \mu_t,$$

  where A whose interaction term $\mu_t \circ \mu_t$ depends on the nature of trade and information acquisition.
- Under conditions, as search intensities get large, efficiency is obtained (Gale (1986), McLennan and Sonnenschein (1991)).
- With search frictions, information remains heterogeneous (Wolinsky (1990)) and benefits intermediaries (Green (2007); Green, Hollifield, and Schürhoff (2007); Duffie, Malamud, and Manso (2010)).

Information Transmission in Markets

Informational Role of Prices: Hayek (1945), Grossman (1976), Grossman and Stiglitz (1981).

- Centralized Exchanges:
  - Wilson (1977), Townsend (1978), Milgrom (1981), Vives (1993), Pesendorfer and Swinkels (1997), and Reny and Perry (2006).
- Over-the-Counter Markets:
  - Wolinsky (1990), Blouin and Serrano (2002), Golosov, Lorenzoni, and Tsyvinski (2009).
  - Duffie and Manso (2007), Duffie, Giroux, and Manso (2008), Duffie, Malamud, and Manso (2009).

Model Primitives

Duffie and Manso (2007) and Duffie, Giroux, and Manso (2010):

- Two possible states of nature $Y \in \{0, 1\}$.
- Each agent is initially endowed with signals $S = \{s_1, \ldots, s_n\}$ s.t. $\mathrm{P}(s_i = 1 \mid Y = 1) \geq \mathrm{P}(s_i = 1 \mid Y = 0)$
- For every pair agents, their initial signals are Y-conditionally independent
- Random matching, intensity λ.

Initial Information Endowment

After observing signals $S = \{s_1, \ldots, s_n\}$, the logarithm of the likelihood ratio between states $Y = 0$ and $Y = 1$ is by Bayes' rule:

$$\log \frac{\mathrm{P}(Y = 0 \,|\, s_1, \ldots, s_n)}{\mathrm{P}(Y = 1 \,|\, s_1, \ldots, s_n)} = \log \frac{\mathrm{P}(Y = 0)}{\mathrm{P}(Y = 1)} + \sum_{i=1}^{n} \log \frac{\mathrm{P}(s_i \,|\, Y = 0)}{\mathrm{P}(s_i \,|\, Y = 1)}.$$

We say that the "type" θ associated with this set of signals is

$$\theta = \sum_{i=1}^{n} \log \frac{\mathrm{P}(s_i \,|\, Y = 0)}{\mathrm{P}(s_i \,|\, Y = 1)}.$$

What Happens in a Meeting?

- Upon meeting, agents participate in a double auction.
- If bids are strictly increasing in the type associated with the signals agents have collected, then bids reveal type.

Information is Additive in Type Space

Proposition: Let $S = \{s_1, \ldots, s_n\}$ and $R = \{r_1, \ldots, r_m\}$ be independent sets of signals, with associated types θ and ϕ. If two agents with types θ and ϕ reveal their types to each other, then both agents achieve the posterior type $\theta + \phi$.

This follows from Bayes' rule, by which

$$\begin{aligned}
\log \frac{\mathrm{P}(Y = 0 \,|\, S, R, \theta + \phi)}{\mathrm{P}(Y = 1 \,|\, S, R, \theta + \phi)} &= \log \frac{\mathrm{P}(Y = 0)}{\mathrm{P}(Y = 1)} + \theta + \phi, \\
&= \log \frac{\mathrm{P}(Y = 0 \,|\, \theta + \phi)}{\mathrm{P}(Y = 1 \,|\, \theta + \phi)}
\end{aligned}$$

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By induction, this property holds for all subsequent meetings.

Solution for Cross-Sectional Distribution of Information

The Boltzmann equation for the cross-sectional distribution μ_t of types is

$$\frac{d}{dt}\mu_t = -\lambda\,\mu_t + \lambda\,\mu_t * \mu_t.$$

with a given initial distribution of types μ_0.

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Proposition: The unique solution of (14) is the Wild sum

$$\mu_t = \sum_{n\geq 1} e^{-\lambda t}(1-e^{-\lambda t})^{n-1}\mu_0^{*n}.$$

Proof of Wild Summation

Taking the Fourier transform $\hat{\mu}_t$ of μ_t of the Boltzmann equation

$$\frac{d}{dt}\mu_t = -\lambda\,\mu_t + \lambda\,\mu_t * \mu_t\,,$$

we obtain the following ODE

$$\frac{d}{dt}\hat{\mu}_t = -\lambda\,\hat{\mu}_t + \lambda\,\hat{\mu}_t^2\,,$$

whose solution is

$$\hat{\mu}_t = \frac{\hat{\mu}_0}{e^{\lambda t}(1-\hat{\mu}_0)+\hat{\mu}_0}.$$

This solution can be expanded as

$$\hat{\mu}_t = \sum_{n\geq 1} e^{-\lambda t}(1-e^{-\lambda t})^{n-1}\hat{\mu}_0^n,$$

which is the Fourier transform of the Wild sum (14).

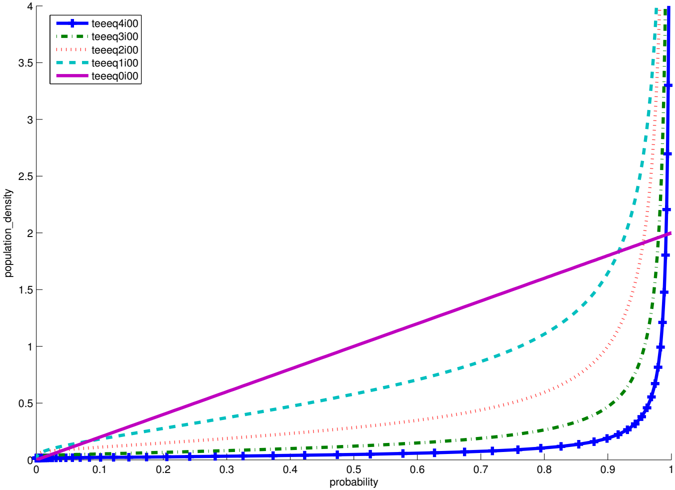


Figure: Evolution of the cross-section of posteriors. Source: Duffie, Giroux, Manso (2008).

Multi-Agent Meetings

The Boltzmann equation for the cross-sectional distribution μ_t of types is

$$\frac{d}{dt}\mu_t = -\lambda\,\mu_t + \lambda\,\mu_t^{*m}.$$

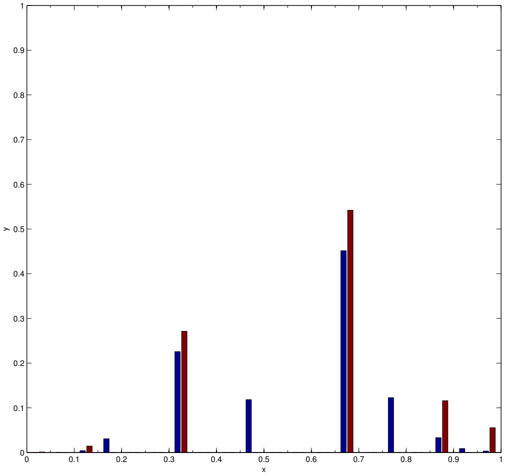
Taking the Fourier transform, we obtain the ODE,

$$\frac{d}{dt}\hat{\mu}_t = -\lambda\,\hat{\mu}_t + \lambda\,\hat{\mu}_t^{m},$$

whose solution satisfies

$$\hat{\mu}_t^{m-1} = \frac{\hat{\mu}_0^{m-1}}{e^{(m-1)\lambda t}(1 - \hat{\mu}_0^{m-1}) + \hat{\mu}_0^{m-1}}\,. \tag{1}$$

Groups of 2 (blue) versus Groups of 3 (red)



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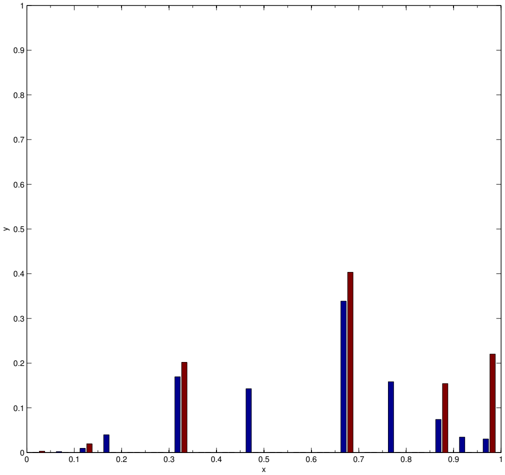

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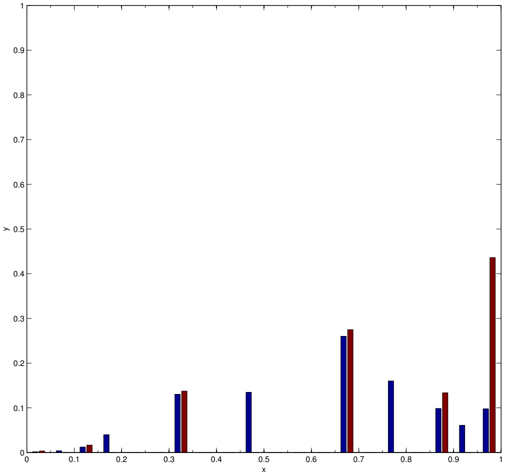

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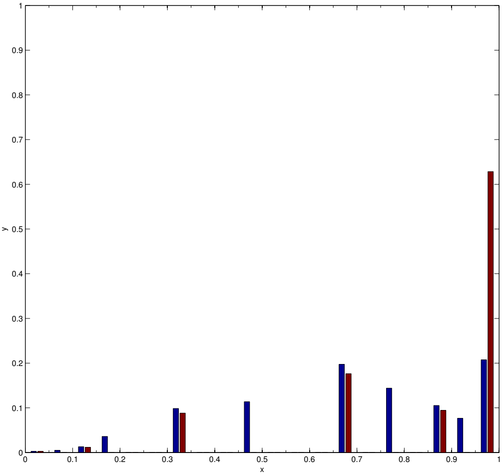

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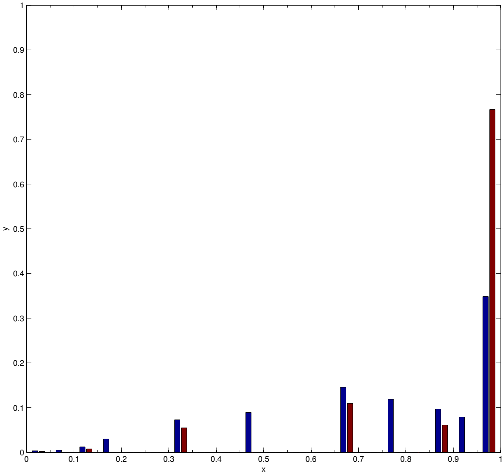

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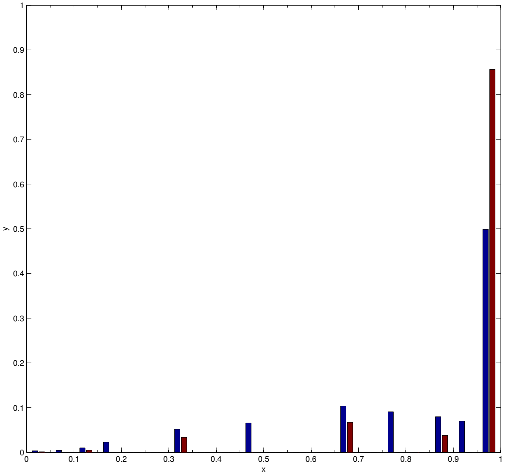
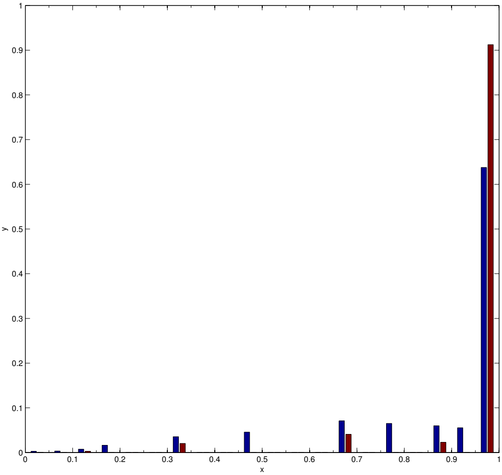


Groups of 2 (blue) versus Groups of 3 (red)



New Private Information

Suppose that, independently across agents as above, each agent receives, at Poisson mean arrival rate ρ, a new private set of signals whose type outcome y is distributed according to a probability measure ν. Then the evolution equation is extended to

$$\frac{d}{dt}\mu_t = -(\lambda + \rho)\,\mu_t + \lambda\,\mu_t * \mu_t + \rho\,\mu_t * \nu.$$

Taking Fourier transforms, we obtain the following ODE

$$\frac{d}{dt}\hat{\mu}_t = -(\lambda + \rho)\,\hat{\mu}_t + \lambda\,\hat{\mu}_t^2 + \rho\,\hat{\mu}_t\,\hat{\nu}.$$

whose solution satisfies

$$\hat{\mu}_t = \frac{\hat{\mu}_0}{e^{(\lambda+\rho(1-\hat{\nu}))t}(1-\hat{\mu}_0) + \hat{\mu}_0}$$

Other Extensions

- Public information releases
  - Duffie, Malamud, and Manso (2010).
- Endogenous search intensity
  - Duffie, Malamud, and Manso (2009).

Duffie, Malamud and Manso (2010): Public Releases of Information

1. At public information release random times $\{T_1, T_2, \ldots\}$ (Poisson arrival process with intensity η) n randomly selected agents have their posterior probabilities revealed to all agents.
2. We allow for random number of agents in each meeting and in each public information release:
   - Meeting group size m: $q_l = \mathrm{P}(m = l)$.
   - Public information release group size n: $p_k = \mathrm{P}(n = k)$.

Evolution of type distribution

Theorem. Given the variable X of common concern, the probability distribution of each agent's type at time t is $\nu_t = \alpha_t * \beta_t$, where $\alpha_t = h(\mu_0, t)$ is the type distribution in a model with no public releases of information, satisfying the differential equation

$$\frac{d\alpha_t}{dt} \; = \; \lambda \left(\sum_{l=2}^{\infty} q_l \, \alpha_t^{*l} \, - \, \alpha_t \right), \qquad \alpha_0 \; = \; \mu_0, \tag{2}$$

and where β_t is the probability distribution over types that solves the differential equation

$$\frac{d\beta_t}{dt} = -\eta\beta_t + \eta\beta_t * \sum_{k=1}^{\infty} p_k \, \alpha_t^{*k}, \tag{3}$$

with initial condition given by the Dirac measure δ_0 at zero.

The rate of Convergence

Let

$$s \mapsto M(s) = \int e^{sx}\, d\mu_0(x)$$

and

$$R = \sup_{y \in \mathbb{R}} \left(-\log M(y)\right). \tag{4}$$

and

$$\Phi(z) = \sum_{n=1}^{\infty} p_n\, z^n,$$

Theorem Convergence is exponential at the rate $\lambda + \eta$, as long as $\lambda > 0$. Otherwise, the rate

$$\rho = \eta \left(1 - \Phi\left(e^{-R}\right)\right). \tag{5}$$

is strictly less than η.

Outline

1. **Segmented Markets**

2. Double Auction

3. Connectedness and Information

Model Primitives

Same as the previous model except that:

- N classes of investors.
- Agent of class i has matching intensity λ_i.
- Upon meeting, the probability that a class-j agent is selected as a counterparty is κ_{ij}.

Evolution of Type Distribution

The evolution equation is given by:

$$\frac{d}{dt}\psi_{it} \;=\; -\lambda_i\,\psi_{it} \;+\; \lambda_i\,\psi_{it} * \sum_{j=1}^{N} \kappa_{ij}\,\psi_{jt}, \quad i \in \{1,\ldots,N\}\,.$$

Taking Fourier transforms we obtain:

$$\frac{d}{dt}\hat{\psi}_{it} \;=\; -\lambda_i\,\hat{\psi}_{it} \;+\; \lambda_i\,\hat{\psi}_{it} \sum_{j=1}^{N} \kappa_{ij}\,\hat{\psi}_{jt}, \quad i \in \{1,\ldots,N\},$$

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Special Case: $N = 2$ and $\lambda_1 = \lambda_2$

Proposition: Suppose $N = 2$ and $\lambda_1 = \lambda_2 = \lambda$. Then

$$\hat{\psi}_1 = \frac{e^{-\lambda t}\,(\hat{\psi}_{20} - \hat{\psi}_{10})}{\hat{\psi}_{20} e^{-\hat{\psi}_{20}(1-e^{-\lambda t})} - \hat{\psi}_{10} e^{-\hat{\psi}_{10}(1-e^{-\lambda t})}}\,\hat{\psi}_{10}\,e^{-\hat{\psi}_{10}(1-e^{-\lambda t})}$$

$$\hat{\psi}_2 = \frac{e^{-\lambda t}\,(\hat{\psi}_{20} - \hat{\psi}_{10})}{\hat{\psi}_{20} e^{-\hat{\psi}_{20}(1-e^{-\lambda t})} - \hat{\psi}_{10} e^{-\hat{\psi}_{10}(1-e^{-\lambda t})}}\,\hat{\psi}_{20}\,e^{-\hat{\psi}_{20}(1-e^{-\lambda t})}.$$

General Case: Wild Sum Representation

Theorem: There is a unique solution of the evolution equation, given by

$$\psi_{it} \;=\; \sum_{k \in \mathbb{Z}_+^N} a_{it}(k)\, \psi_{10}^{*k_1} * \cdots * \psi_{N0}^{*k_N},$$

where ψ_{i0}^{*n} denotes n-fold convolution,

$$a_{it}' \;=\; -\lambda_i\, a_{it} \;+\; \lambda_i\, a_{it} * \sum_{j=1}^{N} \kappa_{ij}\, a_{jt}, \quad a_{i0} = \delta_{e_i},$$

$$(a_{it} * a_{jt})(k_1, \ldots, k_N) \;=\; \sum_{l=(l_1,\ldots,l_N) \in \mathbb{Z}_+^N,\, l<k} a_{it}(l)\, a_{jt}(k-l),$$

and

$$a_{it}(e_i) \;=\; e^{-\lambda_i t}\, a_{i0}(e_i).$$

Outline

Double Auction

- At some time T, the economy ends and the utility realized by an agent of class i for each additional unit of the asset is

$$U_i = v_i Y + v^H (1 - Y),$$

measured in units of consumption, for strictly positive constants v^H and $v_i < v^H$, where Y is a non-degenerate 0-or-1 random variable whose outcome will be revealed at time T.

- If $v_i = v_j$, no trade (Milgrom and Stokey (1982)), so that $\kappa_{ij} = 0$.
- Meeting between two agents $v_i > v_j$, then i is buyer and j is seller.
- Upon meeting, participate in a double auction. If the buyer's bid β is higher than the seller's ask σ, trade occurs at the price σ.

Equilibrium

The prices (σ, β) constitute an equilibrium for a seller of class i and a buyer of class j provided that, fixing β, the offer σ maximizes the seller's conditional expected gain,

$$E\left[(\sigma - E(U_i \,|\, \mathcal{F}_S \cup \{\beta\}))1_{\{\sigma<\beta\}} \,|\, \mathcal{F}_S\right],$$

and fixing σ, the bid β maximizes the buyer's conditional expected gain

$$E\left[(E(U_j \,|\, \mathcal{F}_B \cup \{\sigma\}) - \sigma)1_{\{\sigma<\beta\}} \,|\, \mathcal{F}_B\right].$$

Counterexample: Reny and Perry (2006)

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Counterexample: Reny and Perry (2006)

Restriction on the Initial Information Endowment

Lemma: Suppose that each signal Z satisfies

$$P(Z = 1 \,|\, Y = 0) + P(Z = 1 \,|\, Y = 1) = 1\,.$$

Then, for each agent class i and time t, the type density ψ_{it} satisfies

$$\begin{aligned} \psi_{it}^{H}(x) &= e^{x}\psi_{it}^{H}(-x), \\ \psi_{it}^{L}(x) &= \psi_{it}^{H}(-x) \quad x \in \mathbb{R}\,. \end{aligned}$$

and the *hazard rate condition*

$$h_{it}^{H}(x) \stackrel{def}{=} \frac{\psi_{it}^{H}(x)}{\int_{x}^{+\infty} \psi_{it}^{H}(y)\,dy} \geq \frac{\psi_{it}^{L}(x)}{\int_{x}^{+\infty} \psi_{it}^{L}(y)\,dy} \stackrel{def}{=} h_{it}^{L}(x)\,.$$

Bidding Strategies

Lemma: For any $V_0 \in \mathbb{R}$, there exists a unique solution $V_2(\,\cdot\,)$ on $[v_i, v^H)$ to the ODE

$$V_2'(z) \;=\; \frac{1}{v_i - v_j} \left(\frac{z - v_i}{v^H - z} \frac{1}{h_{it}^H(V_2(z))} + \frac{1}{h_{it}^L(V_2(z))} \right), \quad V_2(v_i) \;=\; V_0.$$

This solution, also denoted $V_2(V_0, z)$, is monotone increasing in both z and V_0. Further, $\lim_{v \to v^H} V_2(v) = +\infty$. The limit $V_2(-\infty, z) \;=\; \lim_{V_0 \to -\infty} V_2(V_0, z)$ exists. Moroever, $V_2(-\infty, z)$ is continuously differentiable with respect to z.

Bidding Strategies

Proposition: Suppose that (S, B) is a continuous equilibrium such that $S(\theta) \le v^H$ for all $\theta \in \mathbb{R}$. Let $V_0 = B^{-1}(v_i) \ge -\infty$. Then,

$$B(\phi) \ = \ V_2^{-1}(\phi), \quad \phi \, > \, V_0,$$

Further, $S(-\infty) = \lim_{\theta \to -\infty} S(\theta) \ = \ v_i$ and $S(+\infty) = \lim_{\theta \to -\infty} S(\theta) \ = \ v^H$, and for any θ, we have $S(\theta) \ = \ V_1^{-1}(\theta)$ where

$$V_1(z) \ = \ \log \frac{z - v_i}{v^H - z} \ - \ V_2(z), \quad z \, \in \, (v_i, v^H) \, .$$

Any buyer of type $\phi < V_0$ will not trade, and has a bidding policy B that is not uniquely determined at types below V_0.

Tail Condition

Definition: We say that a probability density $g(\cdot)$ on the real line is of exponential type α at $+\infty$ if, for some constants $c > 0$ and $\gamma > -1$,

$$\lim_{x \to +\infty} \frac{g(x)}{x^{\gamma}\, e^{\alpha x}} \;=\; c$$

In this case, we write $g(x) \sim \mathrm{Exp}_{+\infty}(c, \gamma, \alpha)$.

Exponential Tails in Percolation Models

Suppose $N = 1$, and let $\lambda = \lambda_1$ and $\psi_t = \psi_{1t}$. The Laplace transform $\hat{\psi}_t$ of ψ_t is given by

$$\hat{\psi}_t(z) \;=\; \frac{e^{-\lambda t}\,\hat{\psi}_0(z)}{1-(1-e^{-\lambda t})\hat{\psi}_0(z)}$$

and $\psi_t(x) \;\sim\; \mathrm{Exp}_{+\infty}(c_t, 0, -\alpha_t)$ in t, where α_t is the unique positive number z solving

$$\hat{\psi}_0(z) \;=\; \frac{1}{1-e^{-\lambda t}},$$

and where

$$c_t \;=\; \frac{e^{-\lambda t}}{(1-e^{-\lambda t})^2\;\frac{d}{dz}\hat{\psi}_0(\alpha_t)}.$$

Furthermore, α_t is monotone decreasing in t, with $\lim_{t\to\infty}\alpha_t \;=\; 0$.

Strictly Monotone Equilibrium

Proposition: Suppose that, for all t in $[0, T]$, there are $\alpha_i(t)$, $c_i(t)$, and $\gamma_i(t)$ such that

$$\psi_{it}^{H}(x) \sim \mathrm{Exp}_{+\infty}(c_i(t), \gamma_i(t), -\alpha_i(t)).$$

If $\alpha_i(T) < 1$, then there is no equilibrium associated with $V_0 = -\infty$. Moreover, if $v_i - v_j$ is sufficiently large and if $\alpha_i(T) > \alpha^*$, where α^* is the unique positive solution to $\alpha^* = 1 + 1/(\alpha^* 2^{\alpha^*})$ (which is approximately 1.31), then there exists a unique strictly monotone equilibrium associated with $V_0 = -\infty$. This equilibrium is in undominated strategies, and maximizes total welfare among all continuous equilibria.

Outline

1. Segmented Markets

2. Double Auction

3. **Connectedness and Information**

Class-i Agent Utility

The expected future profit at time t of a class-i agent is

$$\mathcal{U}_i(t, \Theta_t) \;=\; E\left[\, \sum_{\tau_k > t} \sum_{j} \kappa_{ij}\, \pi_{ij}(\tau_k, \Theta_{\tau_k}) \;\middle|\; \Theta_t \right],$$

where τ_k is this agent's k-th auction time and $\pi_{ij}(t, \theta)$ is the expected profit of a class-i agent of type θ entering an auction at time t with a class-j agent.

Agents may be able to disguise the characteristics determining their information at a particular auction. In this case, we denote the expected future profit at time t of a class-i agent as $\hat{\mathcal{U}}_i(t, \Theta_t)$.

The Value of Initial Information and Connectivity When Trades Can be Disguised

Theorem: Suppose that $v_1 = v_2$. If $\lambda_2 \geq \lambda_1$ and if the initial type densities ψ_{10} and ψ_{20} are distinguished by the fact that the density p_2 of the number of signals received by class-2 agents has first-order stochastic dominance over the density p_1 of the number of signals by class-1 agents, then

$$\frac{E[\hat{\mathcal{U}}_2(t, \Theta_{2t})]}{\lambda_2} \geq \frac{E[\hat{\mathcal{U}}_1(t, \Theta_{1t})]}{\lambda_1}, \quad t \in [0, T].$$

The above inequality holds strictly if, in addition, $\lambda_2 > \lambda_1$ or if p_2 has strict dominance over p_1.

What if Characteristics are Commonly Observed?

- trade-off between adverse selection and gains from trade.
- more informed/connected investor may achieve lower profits than less informed/connected investor.
- If $v_1 = v_2 = 0.9$, $v_3 = 0$, $v^H = 1.9$,

$$\psi_{10}(x) = 12\,\frac{e^{3x}}{(1+e^x)^5},$$

and $\psi_{20}(x) = \psi_{10} * \psi_{10}$.

Then,

$$E[\mathcal{U}_2(t, \Theta_{1t})] < E[\mathcal{U}_1(t, \Theta_{2t})]$$

and

$$E[\hat{\mathcal{U}}_1(t, \Theta_{1t})] < E[\mathcal{U}_1(t, \Theta_{2t})].$$

Even If Characteristics are Commonly Observed Connectivity May be Valuable

Proposition: Suppose that $\kappa_1 = \kappa_2$, $v_1 = v_2$ and $\lambda_1 < \lambda_2$, and suppose that class-1 and class-2 investors have the same initial information quality, that is, $\psi_{10} = \psi_{20}$, and assume the exponential tail condition $\psi_{it}^H \sim \mathrm{Exp}_{+\infty}(c_{it}, \gamma_{it}, -\alpha_{it})$ for all i and t, with $\alpha_{10} > 3$, $\alpha_{30} < 3$ and

$$\alpha_{30} > \frac{\alpha_{10} - 1}{3 - \alpha_{10}},$$

and

$$\frac{\alpha_{1t} + 1}{\alpha_{1t} - 1} > \alpha_{3t}, \quad t \in [0, T].$$

If $\frac{v_1 - v_3}{v^H - v_1}$ is sufficiently large, then for any time t we have

$$\frac{E[\mathcal{U}_2(t, \Theta_{2t})]}{\lambda_2} > \frac{E[\hat{\mathcal{U}}_2(t, \Theta_{2t})]}{\lambda_2} > \frac{E[\hat{\mathcal{U}}_1(t, \Theta_{1t})]}{\lambda_1} > \frac{E[\mathcal{U}_1(t, \Theta_{1t})]}{\lambda_1}.$$

Remarks

- tractable model of information diffusion in over-the-counter markets.
- initial information and connectivity may or may not increase profits:
  - more informed/connected investors attain higher profits than less informed connected investors when investors can disguise trades.
  - more informed/connected investors may attain lower profits than less informed connected investors when investors' characteristics are commonly observed.

Other Applications

- centralized exchanges, decentralized information transmission
- bank runs
- knowledge spillovers
- social learning
- technology diffusion