

Richard Lockhart

Outline

Conclusions

IHC & Higg

Mode

D-4-

Rayes Power

Simulation

Summary

The Large Hadron Collider and Goodness of Fit

Richard Lockhart

Simon Fraser University

DASF III, April 30 - May 1, 2010

Fraser (Trans Roy Soc Can, 1967)

Page # -24

LHC & GO

Richard Lockhart

Outline

Conclusion

LHC & His

Model

iviouei

Dayes I ow

Summar

"The statistical model has not always been a black box."

Richard Lockhart

Outline

Conclusion

LHC & Hig

Model

Rayes Boyes

. . . .

. . . .

- "The statistical model has not always been a black box."
- "And it need not be a black box now."



Richard Lockhart

Outline

1 Conclusions

2 The Higgs particle and LHC

3 The Statistical Model

Data

Bayes Power of Tests

6 Simulations

Real Data

Richard Lockhart

Outlin

Conclusions

LHC & Higg

Model

Bayes Powe

Simulation

Real Data

Summary

Physics problems are hard.



LHC & GOI

Richard Lockhart

Outline

Conclusions

LHC & Higg

Model

Dutu

- Physics problems are hard.
- As is not usual you really don't want the raw data.



LHC & GO

Richard Lockhart

Outline

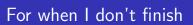
Conclusions

Model

Bayes Powe

Simulation

- Physics problems are hard.
- As is not usual you really don't want the raw data.
- The scientific null hypothesis is a point null.





Richard Lockhart

Outline

Conclusions

Conclusion

N4 - J - I

iviouei

. .

Simulation

Real Data

- Physics problems are hard.
- As is not usual you really don't want the raw data.
- The scientific null hypothesis is a point null.
- But the data analytic null is more nuanced.



LHC & GOI

Richard Lockhart

Outline

Conclusions

Conclusion

. . . .

ivioaei

Pausa Paus

Simulation

Real Data

Physics problems are hard.

- As is not usual you really don't want the raw data.
- The scientific null hypothesis is a point null.
- But the data analytic null is more nuanced.
- Particle detection can be mixture of GOF with more traditional tests.



LHC & GOF

Richard Lockhart

Outline

Conclusions

Mod

Data

Bayes Powe

Roal Data

- Physics problems are hard.
- As is not usual you really don't want the raw data.
- The scientific null hypothesis is a point null.
- But the data analytic null is more nuanced.
- Particle detection can be mixture of GOF with more traditional tests.
- Non-parametric priors lead to Neyman Pearson tests.



LHC & GO

Richard Lockhart

Outline

Conclusions

Conclusion

. .

Data

Bayes Powe

Roal Data

- Physics problems are hard.
- As is not usual you really don't want the raw data.
- The scientific null hypothesis is a point null.
- But the data analytic null is more nuanced.
- Particle detection can be mixture of GOF with more traditional tests.
- Non-parametric priors lead to Neyman Pearson tests.
- Gaussian priors lead to quadratic tests.



LHC & GOI

Richard Lockhart

Outline

Conclusions

Mode

Data

Bayes Powe

Real Data

- Physics problems are hard.
- As is not usual you really don't want the raw data.
- The scientific null hypothesis is a point null.
- But the data analytic null is more nuanced.
- Particle detection can be mixture of GOF with more traditional tests.
- Non-parametric priors lead to Neyman Pearson tests.
- Gaussian priors lead to quadratic tests.
- Sensible prior: decision won't be obvious a priori sample size dependence.





Richard Lockhart

Outline

Conclusions

Mode

Data

Bayes Powe

Real Data

- Physics problems are hard.
- As is not usual you really don't want the raw data.
- The scientific null hypothesis is a point null.
- But the data analytic null is more nuanced.
- Particle detection can be mixture of GOF with more traditional tests.
- Non-parametric priors lead to Neyman Pearson tests.
- Gaussian priors lead to quadratic tests.
- Sensible prior: decision won't be obvious a priori sample size dependence.
- The real problems are strikingly hard and the physicists want and expect smart solutions.







Richard Lockhart

Outlin

Conclusion

LHC & Higgs

Bayes Powe

- Elementary particle; one of 5 elementary bosons.
- Existence predicted by Standard Model of particle physics.
- Not yet observed.
- Major target of experiments at Large Hadron Collider at CERN.







LHC

-20

LHC & GOI

Richard Lockhart

0.....

Conclusions

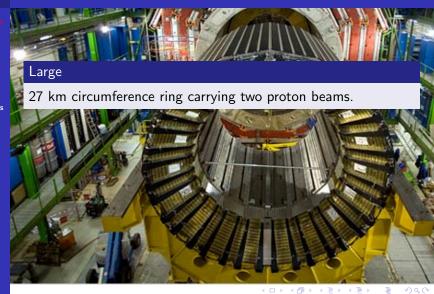
LHC & Higgs

Model

Model

Dayes Pow

David David



Richard Lockhart

Outlin

Conclusions

LHC & Higgs

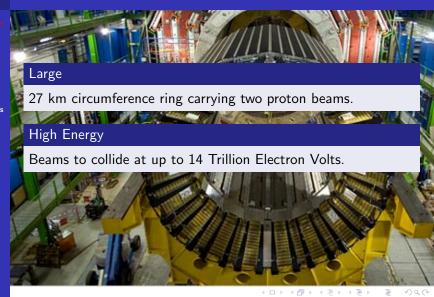
....

....

Bayes Pow

Simulation

Real Data



Richard Lockhart

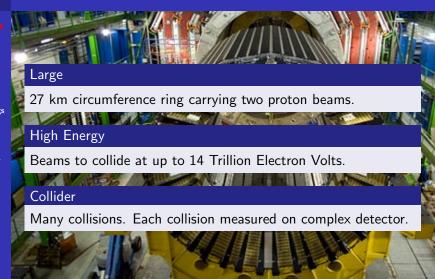
Outlin

Conclusions

LHC & Higgs

Model

. . . .







Richard Lockhart

Outlin

Conclusion

LHC & Higg

Model

Simulation

Real Data

Summary

Model data as Poisson Process of events in time.





IC & GO

Richard Lockhart

Outlin

Conclusion

Conclusion

.

Model

Simulation

Pool Data

- Model data as Poisson Process of events in time.
- lacksquare At each event measure a response X the marks.





1C & GO

Richard Lockhart

Outlin

Conclusion

Conclusion

. . . .

Model

Rausa Daus

_ ._

- Model data as Poisson Process of events in time.
- lacksquare At each event measure a response X the marks.
- The event space is huge, huge, huge.





Richard Lockhart

Outline

Conclusion

Conclusion

Model

iviouei

Bayes Powe

Simulatio

Real Data

- Model data as Poisson Process of events in time.
- lacksquare At each event measure a response X the marks.
- The event space is huge, huge, huge.
- Given times of events, marks are nearly iid.





1C & GO

Richard Lockhart

Outlin

Conclusion

Conclusion

Model

Data

Bayes Powe

J....a.a.c.o.

- Model data as Poisson Process of events in time.
- lacktriangle At each event measure a response X the marks.
- The event space is huge, huge, huge.
- Given times of events, marks are nearly iid.
- Collapse data over time to get sample of N values of X_i .



A Marked Poisson Process Model

Richard

Outline

Conclusion

LUC 0. U:

Model

Bayes Powe

Simulation

Real Data

- Model data as Poisson Process of events in time.
- lacktriangle At each event measure a response X the marks.
- The event space is huge, huge, huge.
- Given times of events, marks are nearly iid.
- Collapse data over time to get sample of N values of X_i .
- Poisson process on the mark space.



Richard Lockhart

Outlin

Conclusion

LHC & Higg

Model

Cimulation

_ ._

Summary

Null hypothesis is





Richard Lockhart

Outlin

Conclusion

LHC & Hig

Model

C:......

Summary

Null hypothesis is

There is no such thing as a Higgs particle





Richard Lockhart

Outlin

Conclusion

Model

Bayes Powe

Simulatio

Real Data

Null hypothesis is

There is no such thing as a Higgs particle

More general null hypothesis is standard physics "The Standard Model".



Richard Lockhart

Outlin

Conclusion

Model

Bayes Powe

Jiiilalatioi

Summar

Null hypothesis is

There is no such thing as a Higgs particle

- More general null hypothesis is standard physics "The Standard Model".
- Alternative hypothesis is some other model of physics.



Richard Lockhart

Outlin

Conclusion

Model

iviou

. .

D--1 D-4-

Summa

■ Null hypothesis is

There is no such thing as a Higgs particle

- More general null hypothesis is standard physics "The Standard Model".
- Alternative hypothesis is some other model of physics.
- or perhaps



Richard Lockhart

Outline

Conclusion

Model

Data

Bayes Powe

Summa

■ Null hypothesis is

There is no such thing as a Higgs particle

- More general null hypothesis is standard physics "The Standard Model".
- Alternative hypothesis is some other model of physics.
- or perhaps

The standard model is wrong



Richard Lockhart

Outline

Conclusion

....

Model

Data

Bayes Powe

.

Summa

■ Null hypothesis is

There is no such thing as a Higgs particle

- More general null hypothesis is standard physics "The Standard Model".
- Alternative hypothesis is some other model of physics.
- or perhaps

The standard model is wrong

or perhaps





Richard Lockhart

Outline

Conclusion

Model

Bayes Powe

Summa

Null hypothesis is

There is no such thing as a Higgs particle

- More general null hypothesis is standard physics "The Standard Model".
- Alternative hypothesis is some other model of physics.
- or perhaps

The standard model is wrong

or perhaps

The Higg's particle is not produced at LHC energies.

Statistical Translation of No Higgs

LHC & GOI

Richard Lockhart

Outline

Conclusion

LHC & Hig

Model

_ _

Simulation

Real Data

Summary

■ Null hypothesis is N has Poisson(Λ) distribution and given N the X_i are iid with some density f.





Richard Lockhart

Outline

Conclusion

Model

Model

Payer Pour

Simulation

Roal Data

Summar

Null hypothesis is N has Poisson(Λ) distribution and given N the X_i are iid with some density f.

■ Alternative is N has Poisson $(\Lambda + M)$ distribution and given N the X_i are iid with some density g given by

$$g = \frac{\Lambda}{\Lambda + M} f + \frac{M}{\Lambda + M} f^*$$

with $f^* \neq f$.



Richard Lockhart

Model

- Null hypothesis is N has Poisson(Λ) distribution and given N the X_i are iid with some density f.
- Alternative is N has Poisson($\Lambda + M$) distribution and given N the X_i are iid with some density g given by

$$g = \frac{\Lambda}{\Lambda + M} f + \frac{M}{\Lambda + M} f^*$$

with $f^* \neq f$.

■ The density f^* is the density of the marks in events which produce Higgs particles.



Richard Lockhart

Outline

Conclusion

Conclusion

Model

iviouei

Simulatio

. . . .

Summar

• With f^* , g and M known use Neyman Pearson:

$$\ell \equiv N \log(1 + M/\Lambda) + \sum_{i} \log\left(\frac{g(X_i)}{f(X_i)}\right).$$



Richard Lockhart

Outline

Conclusion

Conclusion

Model

iviouei

_ ____

Roal Data

Summar

• With f^* , g and M known use Neyman Pearson:

$$\ell \equiv N \log(1 + M/\Lambda) + \sum_{i} \log \left(\frac{g(X_i)}{f(X_i)} \right).$$

Not much to discuss but none of the things you "know" is known exactly.



Richard Lockhart

Outline

Conclusion

Conclusion

Model

iviouci

D------ D-----

. . . .

Real Data

Summar

• With f^* , g and M known use Neyman Pearson:

$$\ell \equiv N \log(1 + M/\Lambda) + \sum_{i} \log\left(\frac{g(X_i)}{f(X_i)}\right).$$

- Not much to discuss but none of the things you "know" is known exactly.
- And there is too much data.



One Data Point

LHC & GOF

Richard

Outline

Conclusion

I HC & Him

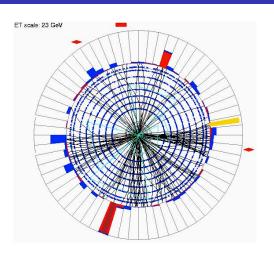
Mode

Data

Bayes Power

Simulatio

Real Data







Richard Lockhart

Outline

Conclusion

LHC & Hig

Mode

. . . .

Simulation

_ ._

Summary

The LHC experiments represent about 150 million sensors delivering data 40 million times per second. After filtering there will be about 100 collisions of interest per second.

Outline

Conclusions

IHC & Higg

Mode

Data

Bayes Power

Simulation

Real Data

Summar

The LHC experiments represent about 150 million sensors delivering data 40 million times per second. After filtering there will be about 100 collisions of interest per second.

For a year's worth of data

$$\Lambda\approx 10^{15}$$

and

$$M \approx 10^3$$

Richard Lockhart

Outline

Conclusions

LHC & Higg

Mode

Data

Bayes Power

Simulation

Real Data

Summa

The LHC experiments represent about 150 million sensors delivering data 40 million times per second. After filtering there will be about 100 collisions of interest per second.

For a year's worth of data

$$\Lambda\approx 10^{15}$$

and

$$M \approx 10^3$$

■ So *N* provides no information?

Richard Lockhart

Outline

Conclusions

LHC & Hig

Mode

Data

Bayes Power

Jiiiaiacion.

Summa

The LHC experiments represent about 150 million sensors delivering data 40 million times per second. After filtering there will be about 100 collisions of interest per second.

■ For a year's worth of data

$$\Lambda\approx 10^{15}$$

and

$$M \approx 10^3$$

- So N provides no information?
- No information at all unless f^* is nothing like f.





Richard Lockhart

Outline

Conclusions

LHC & His

Mod

Data

Bayes Powe

Jillalation

Summa

The LHC experiments represent about 150 million sensors delivering data 40 million times per second. After filtering there will be about 100 collisions of interest per second.

■ For a year's worth of data

$$\Lambda\approx 10^{15}$$

and

$$M \approx 10^3$$

- So *N* provides no information?
- No information at all unless f^* is nothing like f.
- So what do we really do?





Richard Lockhart

Outlin

Conclusion

LHC & High

....

Data

Bayes Powe

Simulation

Summary

■ In fact f^* is known or expected to be concentrated on 'small' region of mark space.





Richard Lockhart

Outline

Conclusion

Mode

Data

Bayes Pow

Simulation

_

- In fact f^* is known or expected to be concentrated on 'small' region of mark space.
- As events are registered 'triggering' algorithms determine whether or not the event is conceivably 'interesting'.





Richard Lockhart

Outline

Conclusions

LHC & Hi

Mode

Data

Bayes Powe

- In fact f^* is known or expected to be concentrated on 'small' region of mark space.
 - As events are registered 'triggering" algorithms determine whether or not the event is conceivably 'interesting'.
- Triggering lowers event rate from 40M per second to roughly 100 per second.





Richard Lockhart

Outline

Conclusions

LHC & Hig

Mode

Data

Bayes Powe

- In fact f^* is known or expected to be concentrated on 'small' region of mark space.
- As events are registered 'triggering" algorithms determine whether or not the event is conceivably 'interesting'.
- Triggering lowers event rate from 40M per second to roughly 100 per second.
- Now apply same model to reduced part of the mark space.





Richard Lockhart

Outline

Conclusions

LHC & H

Mode

Data

Bayes Powe

_ ._

Summary

In fact f^* is known or expected to be concentrated on 'small' region of mark space.

- As events are registered 'triggering" algorithms determine whether or not the event is conceivably 'interesting'.
- Triggering lowers event rate from 40M per second to roughly 100 per second.
- Now apply same model to reduced part of the mark space.
- Still have

$$\Lambda\approx 10^9$$

and

$$M \approx 10^3$$

or a bit less.



Richard Lockhart

Outlin

Conclusion

LHC & Higg

Data

Rayes Powe

Simulation

Roal Data

Summary

■ When serious analysis begins make further cuts.





Richard Lockhart

Outline

Conclusion

Conclusion

Mode

Data

Bayes Powe

Jiiiiaiatioi

- When serious analysis begins make further cuts.
- This amounts to trimming the mark space further to reduce Λ a lot and M hopefully only a bit.



Richard Lockhart

Outline

Conclusions

Data

_ _

Simulation

Real Data

Summar

■ When serious analysis begins make further cuts.

- This amounts to trimming the mark space further to reduce Λ a lot and M hopefully only a bit.
- So now I imagine my problem as posed above:

$$H_o: M = 0$$

versus

$$H_1: M = M_0$$
, density is g .





Richard Lockhart

Outlin

Conclusion

IHC & Hig

Model

Data

Bayes Powe

Simulation

Real Data

Summary

■ Treat f^* (so too alternative g) as imprecisely known.

Richard Lockhart

Outline

Conclusion

Conclusion

. . .

Data

Payer Pour

Simulation

Real Data

■ Treat f^* (so too alternative g) as imprecisely known.

■ Test combination of two hypotheses:

 $H_{0,\text{count}}: M = 0$ and $H_{0,\text{shape}}: g = f$



Richard Lockhart

Outline

Conclusion

Mode

Data

Bayes Powe

Summary

■ Treat f^* (so too alternative g) as imprecisely known.

■ Test combination of two hypotheses:

$$H_{0,\text{count}}: M = 0$$
 and $H_{0,\text{shape}}: g = f$

Second part is goodness-of-fit.



Richard Lockhart

Outline

Conclusion

Mode

Data

Bayes Power

Jiiilalation

Real Data

Summary

■ Treat f^* (so too alternative g) as imprecisely known.

■ Test combination of two hypotheses:

$$H_{0,\text{count}}: M = 0$$
 and $H_{0,\text{shape}}: g = f$

- Second part is goodness-of-fit.
- GOF is somewhat disreputable.

-11



Composite alternative

LHC & GO

Richard Lockhart

Outline

Conclusion

Mode

Data

Bayes Powe

Summary

■ Treat f^* (so too alternative g) as imprecisely known.

Test combination of two hypotheses:

$$H_{0,\text{count}}: M = 0$$
 and $H_{0,\text{shape}}: g = f$

- Second part is goodness-of-fit.
- GOF is somewhat disreputable.
- No really, it is.



Composite alternative

LHC & GO

Richard Lockhart

Outlin

Conclusion

Mode

Data

Bayes Powe

Summary

■ Treat f^* (so too alternative g) as imprecisely known.

■ Test combination of two hypotheses:

$$H_{0,\text{count}}: M = 0$$
 and $H_{0,\text{shape}}: g = f$

- Second part is goodness-of-fit.
- GOF is somewhat disreputable.
- No really, it is.
- Ad hoc, ad hoc, ad hoc.

Richard Lockhart

Outlin

Conclusion

Conclusion

Mad

Data

Bayes Power

Simulation

Real Data

Summary

• Must carry out fixed level α test.



....

Richard Lockhart

Outline

Conclusion

Conclusion

N4 - 4

D-4-

Bayes Power

Simulation

Real Data

- Must carry out fixed level α test.
- Must publish a protocol.



Richard Lockhart

Outline

Conclusion

Madal

iviouei

Bayes Power

Dayes Fowe

Jiiiuiation

- Must carry out fixed level α test.
- Must publish a protocol.
- Wants to reject H_o .



A Bayesian trapped in frequentist world

LHC & GO

Richard Lockhart

Outline

Conclusion

Model

Bayes Power

- Must carry out fixed level α test.
- Must publish a protocol.
- Wants to reject H_o .
- Uses prior on alternative to design Neyman-Pearson test.



A Bayesian trapped in frequentist world

LHC & GO

Richard Lockhart

Outlin

Conclusion

Model

...

Bayes Power

Simulation

Real Data

- Must carry out fixed level α test.
- Must publish a protocol.
- Wants to reject H_o .
- Uses prior on alternative to design Neyman-Pearson test.
- Maximizes expected power.



A Bayesian trapped in frequentist world

LHC & GO

Richard Lockhart

Outline

Conclusion

Model

Doto

Bayes Power

Simulations

Pool Data

Summa

■ Must carry out fixed level α test.

- Must publish a protocol.
- Wants to reject H_o .
- Uses prior on alternative to design Neyman-Pearson test.
- Maximizes expected power.

A frequentist can use the idea to design tests.



Priors on Densities

LHC & GOI

Richard Lockhart

Outline

Conclusion

Conclusion

Mod

Data

Bayes Power

Simulation

Real Data

Stochastic Process Prior

Think of unknown density g as a random function (= stochastic process) which happens to be positive and integrate to 1.



Richard Lockhart

Outline

Conclusions

Mode

Data

Bayes Power

Jiiilalation

Real Data

Stochastic Process Prior

Think of unknown density g as a random function (= stochastic process) which happens to be positive and integrate to 1.

If g is random then the joint density of $\mathbf{X} \equiv (X_1, \dots, X_n)$ at the point $\mathbf{x} \equiv (x_1, \dots, x_m)$ is

$$\Psi(\mathbf{x}) \equiv \mathrm{E}\left\{g(x_1)\cdots g(x_n)\right\}$$

where it is g not the x_i s that are being averaged over!



Richard Lockhart

Outline

Conclusions

Conclusions

Model

Data

Bayes Power

Simulation

Real Da

Stochastic Process Prior

Think of unknown density g as a random function (= stochastic process) which happens to be positive and integrate to 1.

If g is random then the joint density of $\mathbf{X} \equiv (X_1, \dots, X_n)$ at the point $\mathbf{x} \equiv (x_1, \dots, x_m)$ is

$$\Psi(\mathbf{x}) \equiv \mathrm{E}\left\{g(x_1)\cdots g(x_n)\right\}$$

where it is g not the x_i s that are being averaged over! So NP Likelihood ratio test statistic is

$$(1 + M/\Lambda)^N e^{-\Lambda \Psi(\mathbf{X})}$$
.



Richard Lockhart

Outline

Conclusions

Concidiionii

Model

- Ivioaei

Bayes Power

Simulation

Real Data

Summa

Stochastic Process Prior

Think of unknown density g as a random function (= stochastic process) which happens to be positive and integrate to 1.

If g is random then the joint density of $\mathbf{X} \equiv (X_1, \dots, X_n)$ at the point $\mathbf{x} \equiv (x_1, \dots, x_m)$ is

$$\Psi(\mathbf{x}) \equiv \mathrm{E}\left\{g(x_1)\cdots g(x_n)\right\}$$

where it is g not the x_i s that are being averaged over! So NP Likelihood ratio test statistic is

$$(1+M/\Lambda)^N e^{-\Lambda \Psi(\mathbf{X})}$$
.

Now I need computable examples.



LHC & GO

Richard Lockhart

Outline

Conclusion

1 HC 0, H:~

Mod

Data

Bayes Power

Simulation

Summary

■ Idea: $\epsilon Z(x)$ Gaussian process approximating (up to location and scale) log likelihood ratio.



LHC & GO

Richard Lockhart

Outline

Conclusion

Conclusion

. . . .

iviouci

Baves Power

Dayes . o...

Summary

■ Idea: $\epsilon Z(x)$ Gaussian process approximating (up to location and scale) log likelihood ratio.

Problem:

$$\int \exp\{\epsilon Z(x)\}f(x)dx \neq 1$$

so define our random density by

$$g(x) = \exp{\{\epsilon Z(x)\}}f(x)/\int \exp{\{\epsilon Z(x)\}}f(x)dx.$$



LHC & GO

Richard Lockhart

Outline

Conclusion

Conclusion

. . . .

Bayes Power

Jiiiuiatio

Real Data

■ Idea: $\epsilon Z(x)$ Gaussian process approximating (up to location and scale) log likelihood ratio.

Problem:

$$\int \exp\{\epsilon Z(x)\}f(x)dx \neq 1$$

so define our random density by

$$g(x) = \exp{\{\epsilon Z(x)\}} f(x) / \int \exp{\{\epsilon Z(x)\}} f(x) dx.$$

■ Additive invariance: wlog take $\int Z(x)f(x)dx = 0$.



LHC & GO

Richard Lockhart

Outline

Conclusion

Conclusion

. . .

Bayes Power

. . . .

Bool Date

Summa

■ Idea: $\epsilon Z(x)$ Gaussian process approximating (up to location and scale) log likelihood ratio.

■ Problem:

$$\int \exp\{\epsilon Z(x)\}f(x)dx \neq 1$$

so define our random density by

$$g(x) = \exp\{\epsilon Z(x)\}f(x)/\int \exp\{\epsilon Z(x)\}f(x)dx.$$

- Additive invariance: wlog take $\int Z(x)f(x)dx = 0$.
- lacktriangle Problem: denominator. Solution: choose ϵ depending on sample size.



LHC & GOF

Richard Lockhart

Outline

Conclusion

IHC & Hio

Mode

Dutu

Bayes Power

Jiiiidiatioi

Summary

1 Statistical tests are useful when 0.05 < power < 1: alternatives of interest are neither indetectably nor grossly different from the null hypothesis.



LHC & GOF

Richard Lockhart

Outlin

Conclusion

Conclusion

NA - -I

iviou

Bayes Power

Simulation

Real Data

I Statistical tests are useful when 0.05 < power < 1: alternatives of interest are neither indetectably nor grossly different from the null hypothesis.

2 Good tests are designed to be sensitive to alternatives likely to arise in practice.



LHC & GO

Richard Lockhart

Outlin

Conclusion

Conclusion

.. .

Bayes Power

Simulation

Real Data

I Statistical tests are useful when 0.05 < power < 1: alternatives of interest are neither indetectably nor grossly different from the null hypothesis.

- 2 Good tests are designed to be sensitive to alternatives likely to arise in practice.
- 3 The purpose of computing large sample limits is approximation.



LHC & GOF

Richard Lockhart

Outlin

Conclusion

Conclusion

.. .

Bayes Power

Jiiiuiatioii

Real Data

1 Statistical tests are useful when 0.05 < power < 1: alternatives of interest are neither indetectably nor grossly different from the null hypothesis.

- 2 Good tests are designed to be sensitive to alternatives likely to arise in practice.
- 3 The purpose of computing large sample limits is approximation.
- 4 Data structure and model at hand can be embedded in any convenient sequence to get approximation.



LHC & GOI

Richard Lockhart

Outlin

Conclusion

N4 - 4

Data

Bayes Power

J....a.a.c.o.

Real Data

I Statistical tests are useful when 0.05 < power < 1: alternatives of interest are neither indetectably nor grossly different from the null hypothesis.

- 2 Good tests are designed to be sensitive to alternatives likely to arise in practice.
- 3 The purpose of computing large sample limits is approximation.
- Data structure and model at hand can be embedded in any convenient sequence to get approximation.
- 5 Pick sequence to get quick convergence to computable limit!



I believe the problem will be interesting

LHC & GO

Richard Lockhart

Outlin

Conclusion

Conclusion

NA - -I

Data

Bayes Power

J....a.a.c.o..

Summa

 Principle: design tests for good properties when alternative detectably but not obviously different from null.

- Precise large sample version: take $\epsilon = a/\sqrt{M}$.
- Log-likelihood ratio given Z is approximately

$$\sum \log\{g(x_i)/f(x_i)\} \approx W \equiv \frac{a\sum Z(x_i)}{\sqrt{M}} - a^2 \int Z^2(u)f(u)du/2u$$

Now: compute marginal joint density of

$$X_1,\ldots,X_n$$

by averaging density over Z.



LHC & GC

Richard Lockhart

Outline

Conclusion

Model

Wiodei

Bayes Power

Simulation

Real Data

Summary

Test rejects for large values of

$$e^{-M}(1+M/\Lambda)^N \exp\{\mathsf{GOF}\ \mathsf{Statistic}(N,M)\}$$

where "GOF statistic" is generalization of EDF type tests.



LHC & GC

Richard Lockhart

Outline

Conclusion

NA - J - I

iviouei

Baves Power

Summary

Test rejects for large values of

$$e^{-M}(1+M/\Lambda)^N \exp\{\mathsf{GOF}\ \mathsf{Statistic}(N,M)\}$$

where "GOF statistic" is generalization of EDF type tests. Actual formula is a secret.



LHC & GO

Richard Lockhart

Outline

Conclusion

LHC & Hi

Model

Bayes Power

Simulation

Real Data

Summar

Test rejects for large values of

$$e^{-M}(1+M/\Lambda)^N \exp\{\mathsf{GOF}\ \mathsf{Statistic}(N,M)\}$$

where "GOF statistic" is generalization of EDF type tests. Actual formula is a secret.

Because it is hideous.



LHC & GO

Richard Lockhart

Outline

Conclusion

Model

Bayes Power

C:----I---

Real Data

Summ

Test rejects for large values of

$$e^{-M}(1+M/\Lambda)^N \exp\{\mathsf{GOF}\ \mathsf{Statistic}(N,M)\}$$

where "GOF statistic" is generalization of EDF type tests. Actual formula is a secret.

Because it is hideous.

But I do have a formula in terms of eigenvalues, eigenfunctions, non-centrality parameters and data.



LHC & GO

Richard Lockhart

Outline

Conclusion

Conclusion

Model

Bayes Power

_. . .

Real Data

Summ

Test rejects for large values of

$$e^{-M}(1+M/\Lambda)^N \exp\{\mathsf{GOF}\ \mathsf{Statistic}(N,M)\}$$

where "GOF statistic" is generalization of EDF type tests. Actual formula is a secret.

Because it is hideous.

But I do have a formula in terms of eigenvalues, eigenfunctions, non-centrality parameters and data. Large sample approximate *P*-values are computable.



Composite Null: One Example

Richard Lockhart

Bayes Power

Specify alternative in two pieces.

- Alternative $g(x) = f(x, \theta) \exp{\{\epsilon Z(x, \theta)\}}$.
- Apply prior π_1 to θ .
- Prior on alternative decomposed into conditional alternative given θ averaged over θ .
- Null: apply prior π_0 .



Structure of Likelihood Ratio

LHC & GO

Richard Lockhart

Outline

Conclusions

I HC & Higg

Mode

Data

Bayes Power

Summa

Neyman-Pearson likelihood ratio is product of two terms.

■ Conditional joint density of $X_1, ..., X_n$ given θ is

$$\Psi(\mathbf{x}, \theta)$$

computed as before for $Z(x, \theta)$.

First term in likelihood ratio is posterior expectation

$$\Psi(\mathbf{x}) \equiv \int \Psi\left(\mathbf{x}, heta
ight) \pi_1(d heta|\mathbf{x}).$$

where $\pi_1(d\theta|\mathbf{x})$ is posterior computed under null for prior π_1 .

Second is ratio of marginals under null:

$$\frac{\int \prod f(x_i,\theta)\pi_1(\theta)d\theta}{\int \prod f(x_i,\theta)\pi_0(\theta)d\theta}.$$

• Adjust π_0 to get level α .

LHC & GOI

Lockhar

Outline

Conclusion

LHC & Higg

Mode

Simulations

Real Data



Simulation Results

LHC & GOF

Richard Lockhart

Outline

Conclusion

I HC & Him

Model

Rayes Bayes

Simulations

Summary

"Not today, sir."

LHC & GOF

Lockhar

Outline

Conclusion

LHC & Higg

Mode

Real Data

LHC & GOF

Lockhar

Outline

Conclusion

LHC & Higg

Model

Data

Bayes Powe

Real Data

Summary

Are you kidding?



LHC & GO

Richard Lockhart

Outlin

Conclusion

LHC & Higg

ivioae

D-4-

Bayes Powe

Real Data

Summary

■ The density f^* is computed by measurement, approximation, and simulation.



LHC & GO

Richard Lockhart

Outline

Conclusion

Conclusion

Mode

Data

Bayes Powe

Real Data

- The density f^* is computed by measurement, approximation, and simulation.
- The density g is computed by measurement, approximation, and simulation – from extreme tails.



Richard

Outline

Conclusions

Mode

Date

Bayes Powe

Real Data

- The density f^* is computed by measurement, approximation, and simulation.
- The density g is computed by measurement, approximation, and simulation – from extreme tails.
- \blacksquare The intensity M is an unknown parameter of interest.



LHC & GOI

Richard Lockhart

Outline

Conclusion

Made

Bayes Powe

Real Data

- The density f^* is computed by measurement, approximation, and simulation.
- The density g is computed by measurement, approximation, and simulation – from extreme tails.
- $lue{}$ The intensity M is an unknown parameter of interest.
- In fact M is product of parameter of interest cross-section and other things — prior information available for the other things.



LHC & GO

Richard Lockhart

Outline

Conclusions

Liic d

Mod

Bayes Powe

Real Data

- The density f^* is computed by measurement, approximation, and simulation.
- The density *g* is computed by measurement, approximation, and simulation from extreme tails.
- lacksquare The intensity M is an unknown parameter of interest.
- In fact M is product of parameter of interest cross-section and other things — prior information available for the other things.
- But Bayes on cross-section not tolerable to physicists.



LHC & GO

Richard Lockhart

Outline

Conclusion:

Mode

Data

Simulations

Real Data

- The density f^* is computed by measurement, approximation, and simulation.
- The density *g* is computed by measurement, approximation, and simulation from extreme tails.
- \blacksquare The intensity M is an unknown parameter of interest.
- In fact M is product of parameter of interest cross-section and other things — prior information available for the other things.
- But Bayes on cross-section not tolerable to physicists.
- lacktriangle Null intensity Λ is computed by approximation and simulation.





LHC & GO

Richard Lockhart

Outlin

Conclusion

Conclusion

Mod

Data

Simulations

Real Data

- The density f^* is computed by measurement, approximation, and simulation.
- The density *g* is computed by measurement, approximation, and simulation from extreme tails.
- \blacksquare The intensity M is an unknown parameter of interest.
- In fact M is product of parameter of interest cross-section and other things — prior information available for the other things.
- But Bayes on cross-section not tolerable to physicists.
- Null intensity Λ is computed by approximation and simulation.
- The analysis will be carried out over several energy ranges; multiple comparisons.



LHC & GOI

Richard Lockhart

Outline

Conclusion

LHC & His

IVIO

Data

Bayes Powe

Summary

 An experimenter wanting to provide evidence convincing to others should use prior on alternative and publish a data analysis protocol.



LHC & GO

Richard Lockhart

Outline

Conclusion

LHC & Him

Mod

Data

Bayes Powe

- An experimenter wanting to provide evidence convincing to others should use prior on alternative and publish a data analysis protocol.
- A fairly natural prior leads to pooling quadratic GOF tests with a test based on *N*.



LHC & GOI

Richard Lockhart

Outline

Conclusion

LHC & Hig

Mod

Data

Bayes Power

_ ._

- An experimenter wanting to provide evidence convincing to others should use prior on alternative and publish a data analysis protocol.
- A fairly natural prior leads to pooling quadratic GOF tests with a test based on N.
- I need to find more priors.



LHC & GOF

Richard Lockhart

Outline

Conclusion

LHC & Hig

Mode

Data

Bayes Powe

Real Data

- An experimenter wanting to provide evidence convincing to others should use prior on alternative and publish a data analysis protocol.
- A fairly natural prior leads to pooling quadratic GOF tests with a test based on *N*.
- I need to find more priors.
- The data analysis null hypothesis is uncertain; is Bayes avoidable?



LHC & GO

Richard Lockhart

Outline

Conclusion

LHC & Hig

Mode

Data

Bayes Powe

Summary

An experimenter wanting to provide evidence convincing to others should use prior on alternative and publish a data analysis protocol.

- A fairly natural prior leads to pooling quadratic GOF tests with a test based on *N*.
- I need to find more priors.
- The data analysis null hypothesis is uncertain; is Bayes avoidable?
- What is the foundational status of "systematics"?



LHC & GOI

Richard Lockhart

Outline

Conclusion

LHC & Hig

Mode

Data

Bayes Powe

Summary

 An experimenter wanting to provide evidence convincing to others should use prior on alternative and publish a data analysis protocol.

- A fairly natural prior leads to pooling quadratic GOF tests with a test based on *N*.
- I need to find more priors.
- The data analysis null hypothesis is uncertain; is Bayes avoidable?
- What is the foundational status of "systematics"?
- Blind analysis?

