Longitudinal Mixed-Membership Models for Survey Data on Disability

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Carnegie Mellon University

DATA ANALYSIS AND STATISTICAL FOUNDATIONS 3—DASF(3)
April 30-31, 2010

University of Toronto—1960-1964

- My UofT Teachers (note overlap with DB and APD)
  - Coxeter (yrs. 1, 2, 3, 4)
  - DeLury (yrs. 2, 4)
  - Tutte (yr. 2)
  - DASF (yrs. 3, 4)
  - Wormleighton (yr. 3 lab)
  - Weber
- The Princeton Connection
  - The end of the line
- Other UofT Links
  - Dempster (Harvard)
  - Brillinger (Tukey's NBC Election Night ForecastingTeam)
    - Hierarchical Bayesian modeling
- Tukey-Mosteller collaboration

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- Learning about inference and interval estimation.

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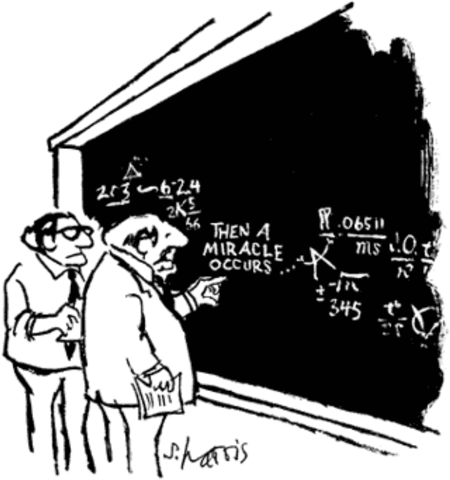
- Was DASF frequentist, fiducial, Bayesian, or structural?



"I think you should be more explicit here in step two."

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- Even though the meeting was in Mexico, all the talks were in English!

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- Morrie DeGroot often commented on the problem of deciding what was prior and what was likelihood, and the fact that the likelihood is often more subjective than the prior.
- As I will illustrate, these issues are exacerbated in hierarchical model settings, but at some level gentle priors are all we require.
- *My Approach:* Pragmatic subjectivism or perhaps "constructive realistic" subjectivism.

My Only Collaboration With DASF

STATISTICS AT YORK

Prepared by:

Stephen E. Fienberg
D.A.S. Fraser
John Fox
Peter Peskun
Helene Massam

Draft: March 18, 1993

PREAMBLE

Several statisticians within the Department of Mathematics and Statistics continue to believe that the time has come for greater autonomy for statistics at York. The new divisional structure of the department has clearly allowed for some important dimensions of curricular control and has allowed the statisticians to organize activities in a more productive fashion. But the fact remains that the applied nature of statistics is far from flourishing within the department and that the formal link to mathematics is a deterrent to possible developments for statistics at York.

Many in the current department and elsewhere at York are unaware of the context in which statistics has developed as an independent discipline, albeit linked in various important ways to mathematics, and especially probability. In this document we have attempted to provide a brief overview of the development of the field of statistics and its institutionalization in Ontario universities. We have also identified some features that bind statistics with mathematics, e.g., computing and probability and others that draw statistics towards other disciplines and activities at York, especially in the social and behavioral sciences. We would also like to add an extensive discussion of the role of probability at the interface of Mathematics and Statistics.

Our intent is to begin a dialogue with colleagues in the Department of Mathematics and Statistics, as well as with interested colleagues outside the Department, about the future of statistics at York and how the aspirations for autonomy of statistics can be fulfilled. We are mindful that the probabilists in the current Department have strong academic interests in both mathematics and statistics and that any divided structure would need to have them in a bridging role.

We have explicitly avoided suggesting an administrative arrangement that would entail the necessary autonomy for the statisticians, although a separate department is obviously one possible model. We have not precluded other options, especially ones involving share resources and facilities. We have also not suggested any particular timetable for action.

We would welcome edits and additions to the following materials, and hope that some revised and expanded document might serve as the focus for a series of meetings on the development of statistics at York.

Outline for Remainder of My Talk

1. Motivation - Disability and long-term care in the U.S.
2. Data—The NLTCS
3. Proposed Approach—Trajectory GoM models
   1. General Construction
   2. Basic Model
   3. Estimation
4. Example Computations
5. Extensions
6. Discussion

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Answers to these questions require a longitudinal view that also takes into account the heterogeneity of the population.

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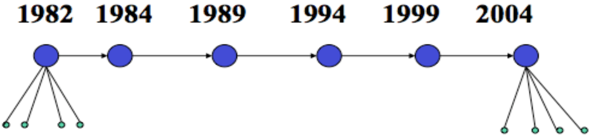
- Each individual that enters the survey is reinterviewed in all subsequent waves until death.
- Approx. $20k$ individuals per wave. $45,009$ unique individuals sampled in all six waves together. Each wave incorporates $\approx 5k$ new subjects to replace those who have died.

Longitudinal Modeling of NLTCS—Overview

- Sequential measurements on the same individuals allow to assess *individual* disability trajectories over time.



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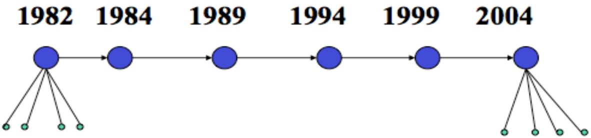

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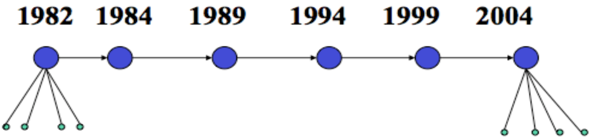

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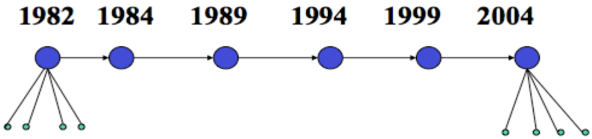

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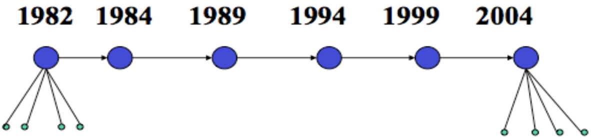

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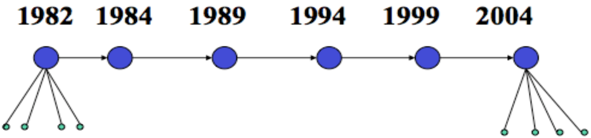

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- Understand evolution over time:
  - Individuals
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- Identify 'typical' evolutions over time
- Account for and understand individual variability

Data—NLTCS Longitudinal View and Notation

		$t=1$	$t=2$	$t=3$	$t=4$
$i=1$	$j=1$	0	0	1	1
	$j=2$	0	0	0	1
			$\vdots$		
	$j=J$	0	0	0	0
	Age	67	69	74	79
	Sex		F		

$\vdots$

		$t=1$	$t=2$	$t=3$	$t=4$
$i=N$	$j=1$	0	0	1	–
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  - Aspects of the spread of malaria in the Amazon.
- The specific longitudinal models we describe have potential application to the study of other phenomena measured at discrete points in time.

Modeling—Construction of a trajectory GoM model (1)

- Assume the existence of K “ideal classes” or “extreme profiles”
- Assign each individual a *Membership Vector*:

$$g_i = (g_{i1}, g_{i2}, ..., g_{iK})$$

with $g_{ik} > 0$ and $\sum_{k=1}^{K} g_{ik} = 1$ ($g_i \in \Delta_{K-1}$).
- For the “ideal” individuals, specify the marginal distribution of response j, at measurement time t, as a function of some time-dependent covariates.

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Modeling—Construction of a trajectory GoM model (1)

- Assume the existence of K "ideal classes" or "extreme profiles"
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- Mixed Membership: For a generic individual *i*, we model

$$\Pr\left(Y_{ijt} = y_{ijt}\,|g_i, X_i, \theta\right) = \sum_{k=1}^{K} g_{ik} f_{\theta_{j|k}}(y_{ijt}|X_{it})$$

- Assuming conditional independence,

$$\Pr\left(Y_i = y_i\,|g_i, X_i, \theta\right) = \prod_{j=1}^{J}\prod_{t=1}^{N_i}\sum_{k=1}^{K} g_{ik} f_{\theta_{j|k}}(y_{ijt}|X_{it})$$

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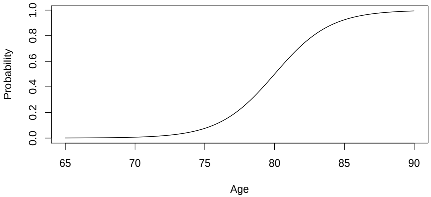
Basic Model—Extreme profile Trajectories

- For each extreme profile ($g_k = 1$) specify trajectories of probability of disability in ADLs as a monotone function of Age:



$$
\begin{aligned}
y_{ijt} &\sim \text{Bernoulli}\left[\lambda_{j|k}(Age_{it})\right] \\
\lambda_{j|k}(X_{it}) &= \text{logit}^{-1}\left[\beta_{0j|k} + \beta_{1j|k} \times Age_{it}\right]
\end{aligned}
$$

(Connor, 2006)

Basic Model—Distribution for g_i ($\sim G_\alpha$)

- Membership vectors from a Dirichlet distribution

$$g_i \overset{iid}{\sim} \text{Dirichlet}(\alpha_0 \times \xi)$$

with $\alpha_0 > 0$ and $\xi = (\xi_1, \xi_2, ..., \xi_K) \in \Delta_{K-1}$.

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Basic Model—Priors

- For the membership distribution

$$g_i \overset{iid}{\sim} \text{Dirichlet}(\alpha)$$

We use the same priors as Erosheva (2002):

$$\begin{aligned} \alpha &= \alpha_0 \times \xi \\ \alpha_0 &\sim \text{Gamma}(1, 5) \\ \xi &\sim \text{Uniform}[\Delta_{K-1}] \end{aligned}$$

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- And complete the specification with

$$\begin{aligned} \beta_{0j|k} &\overset{iid}{\sim} N(0,100) \\ \beta_{1j|k} &\overset{iid}{\sim} N(0,100) \end{aligned}$$

Estimation—MCMC sampling

- MCMC algorithm based on a method from Erosheva (2002) for fitting GoM model to cross sectional data. Using an equivalent latent class representation for the GoM model.
- Difficult to run
  - Huge latent space.
  - Nonstandard distributions.
  - Numerical problems.
- 40,000 long chains (using the “improved” algorithm). $5 \sim 7$h runs

Test Computations—Data

- Tested for six ADLs:

ADL (j-index)	Abbrv	Description
1	EAT	Eating
2	BED	Getting in and out of bed
3	MOB	Inside mobility
4	DRS	Dressing
5	BTH	Bathing
6	TLT	Toileting

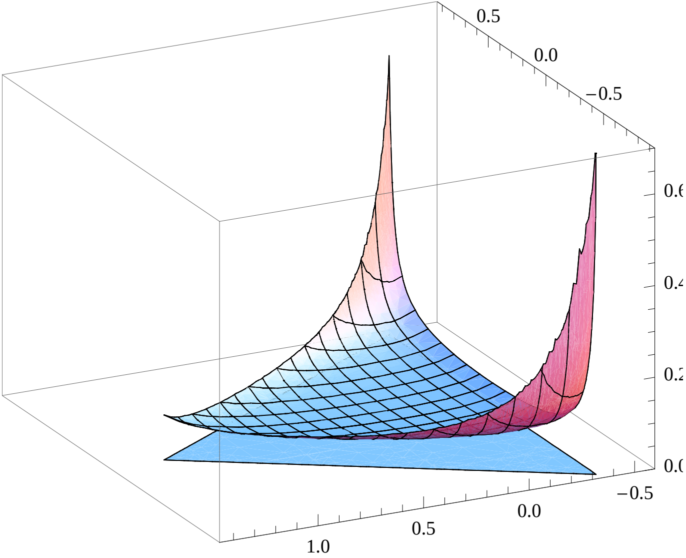
- Data from 6 waves (1982 - 2004).
- Individuals from 2004 are only those that were already in the 1999 sample.
- $N \approx 40K$

Test Computations - Posterior Summaries ($K = 3$)

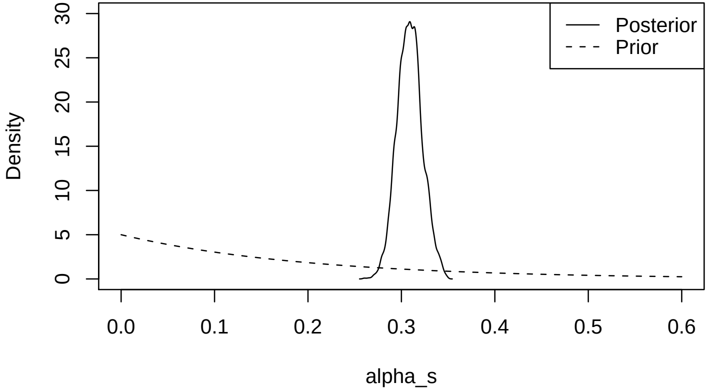
Parameter	Estimate (sd)
α_0	0.264 (0.00489)

Parameter	ADL(j)	Estimate Extreme Profile-k (sd)					
		$k = 1$		$k = 2$		$k = 3$	
ξ	—	0.645	(0.004)	0.252	(0.003)	0.104	(0.002)
β_0	1 (EAT)	-8.845	(0.313)	-3.103	(0.057)	-0.066	(0.044)
	2 (BED)	-7.02	(0.144)	-1.739	(0.053)	3.581	(0.142)
	3 (MOB)	-5.339	(0.093)	-0.759	(0.044)	5.803	(0.277)
	4 (DRS)	-7.912	(0.216)	-2.256	(0.051)	2.042	(0.082)
	5 (BTH)	-4.458	(0.075)	-0.23	(0.035)	6.257	(0.28)
	6 (LTL)	-6.59	(0.148)	-1.768	(0.047)	2.506	(0.098)
β_1	1 (EAT)	0.357	(0.017)	0.347	(0.008)	0.105	(0.006)
	2 (BED)	0.394	(0.01)	0.551	(0.013)	0.29	(0.012)
	3 (MOB)	0.348	(0.007)	0.52	(0.012)	0.426	(0.022)
	4 (DRS)	0.392	(0.013)	0.463	(0.011)	0.203	(0.008)
	5 (BTH)	0.295	(0.006)	0.426	(0.009)	0.445	(0.022)
	6 (LTL)	0.337	(0.009)	0.475	(0.011)	0.234	(0.009)
$Age_{1/2}$	1 (EAT)	104.82	(0.46)	88.945	(0.163)	80.641	(0.444)
	2 (BED)	97.838	(0.167)	83.154	(0.089)	67.657	(0.173)
	3 (MOB)	95.338	(0.137)	81.458	(0.083)	66.389	(0.151)
	4 (DRS)	100.212	(0.231)	84.869	(0.104)	69.934	(0.192)
	5 (BTH)	95.118	(0.151)	80.54	(0.082)	65.927	(0.16)
	6 (LTL)	99.553	(0.222)	83.725	(0.092)	69.3	(0.175)

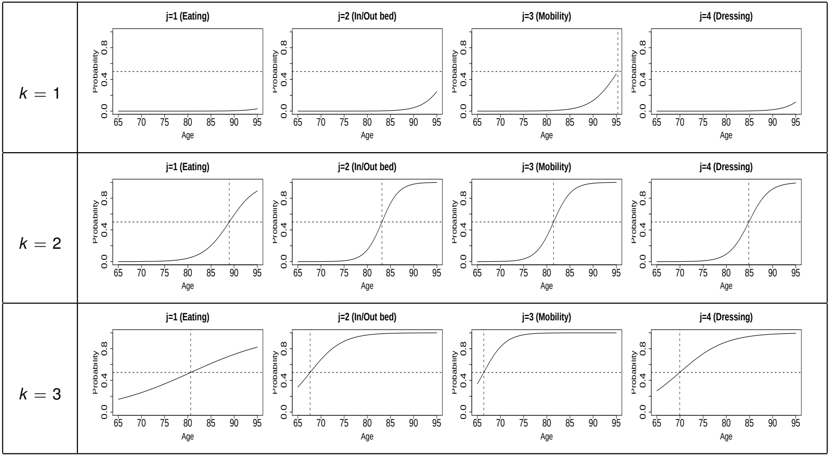
Computations - "Posterior density" for g_i (K=3)



Computations - prior/posterior for α_0 ($K = 3$)

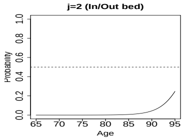


Test Computations—Profiles for $K = 3$

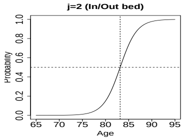


Test Computations—From profiles to Individuals

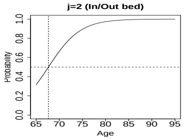
$k = 1$



$k = 2$



$k = 3$



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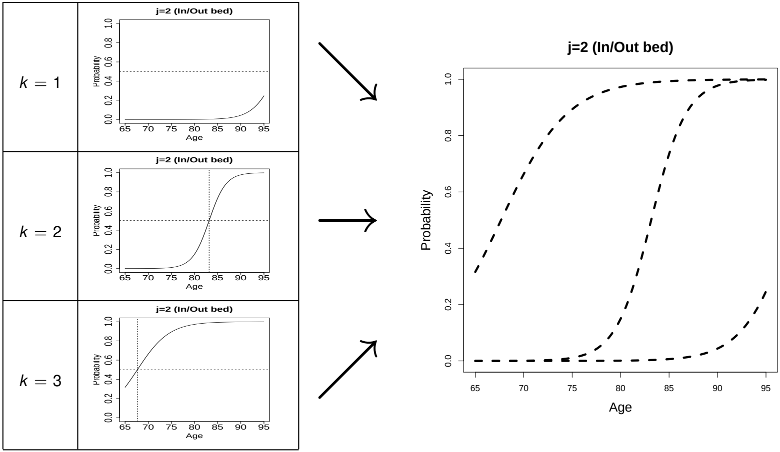

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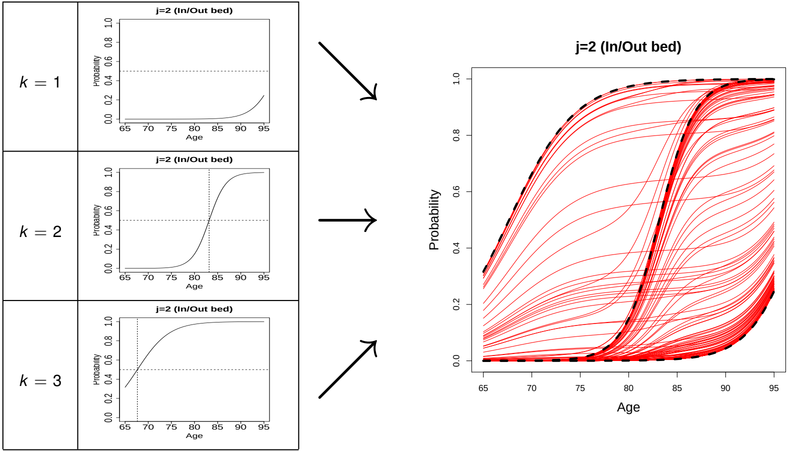
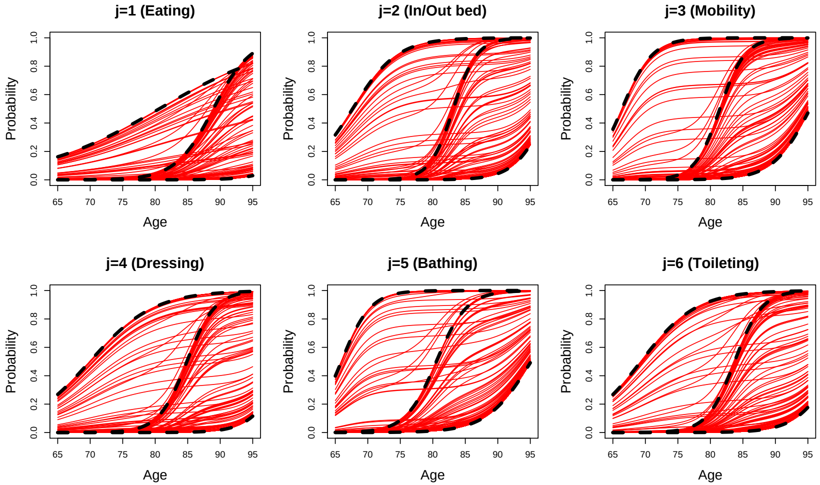


Test Computations—Individual Trajectories



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- Study generations (i.e. people that were born in the same period) to understand changes in how people acquire disabilities.
- Longitudinal data allows us to compare whole aging life trajectories for different individuals from different generations.

Extensions (2) - Modeling Generational differences

- Approach: make group membership dependent on the generation to which the individual belongs, keeping the extreme trajectories fixed:

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- This way we can assess the distribution of membership scores for different generational groups.

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- 5 (rather arbitrary) “Generational Groups”. Born...
  1. ...before 1906 (up to 1873)
  2. between 1906 and 1914,
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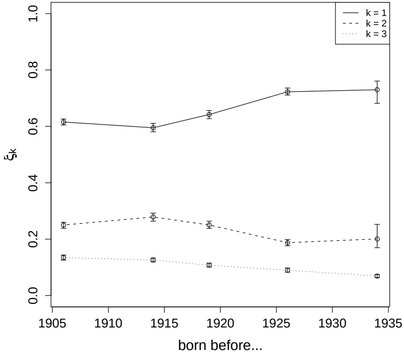

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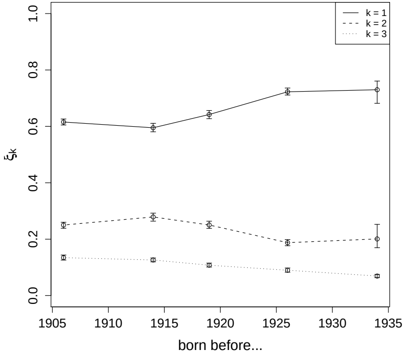

Individuals form younger generations tend to be closer to "healthy" trajectory profiles.

Other Extensions

1. Joint modeling of survival times and disability acquisition.
   - Use survival information to achieve better classification.
   - Understand the relationship between disability and mortality.
2. Other trajectory functions (e.g. step functions)
   - Test the constraints imposed by the selection of disability trajectory curves.
3. Using full database,
4. Model choice—picking the value of K.
5. Incorporating fuller set of covariates.
6. Adapting all of these models and methods for other surveys—e.g., HRS and NHAT.

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  - Making individual membership scores dependent on the individual's generation allows to assess changes on the ways of aging.
- General methodology. Can be applied in other settings!

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The End