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# Entanglement Dynamics of Qubit Systems: Markov and Non-Markovian phase Noises

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# Noise is Everywhere: Disentanglement is Both Fundamental and Technical

**Fundamental:**

Quantum and classical,  
Markovian and non-Markovian,

Pure and mixed

**Technical:**

Thermal and non-Thermal, Environmental  
Ambient Influences on Phase and/or  
Amplitude of Operating Variables

# Easy versus Difficult

I. Pure States –Easy

II. Bipartite – Easy

III. Static – Easy

V. Markov Noise -- Easy

Mixed States- Difficult

Multipartite – Difficult

Dynamic -- Difficult

Non-Markovian Noise – Difficult

Entanglement Dynamics of Mixed States

# **Entanglement and Disentanglement**

# Entanglement-A Formal Definition

- Bipartite system:  $H = H_A \otimes H_B$
- The state  $\rho$  is **separable** if and only if

$$\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i$$

←  
classical  
Correlation only

Entangled = Non-separable

# Entanglement –Concurrence

Wootters concurrence  $C(\rho)$

$$C(\rho) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4})$$

Where  $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$  are the eigenvalues of the matrix:

$$\zeta = \rho(\sigma_y^A \otimes \sigma_y^B)\rho^*(\sigma_y^A \otimes \sigma_y^B) \quad \sigma_y^{A,B} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

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Separable state  $0 \leq C(\rho) \leq 1$  Maximally entangled states

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W. K. Wootters, PRL, 80, 2245 (1997)

# Entanglement-Example of Concurrence

$$|\Psi\rangle_{AB} = a_1|++\rangle + a_2|+-\rangle + a_3|-+\rangle + a_4|--\rangle$$

Pure state

$$C(\Psi) = 2|a_1a_4 - a_2a_3|,$$

$$C(Bell\ state) = 1.$$

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Mixed state

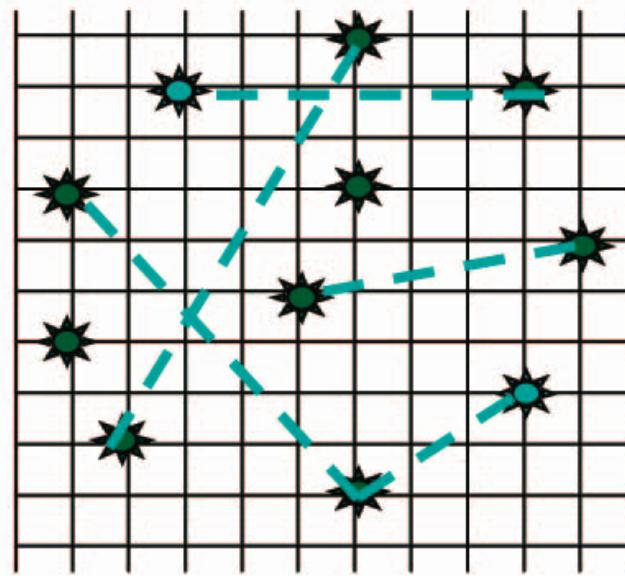
$$\rho = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix},$$

$$C(\rho) = 2\max\{|z| - \sqrt{ad}, 0\}$$

# **Non-Markovian Phase Noise: Extended Kubo Model**

**TY and JH Eberly, Quant-ph: 0906.5378**

# Entangled Web

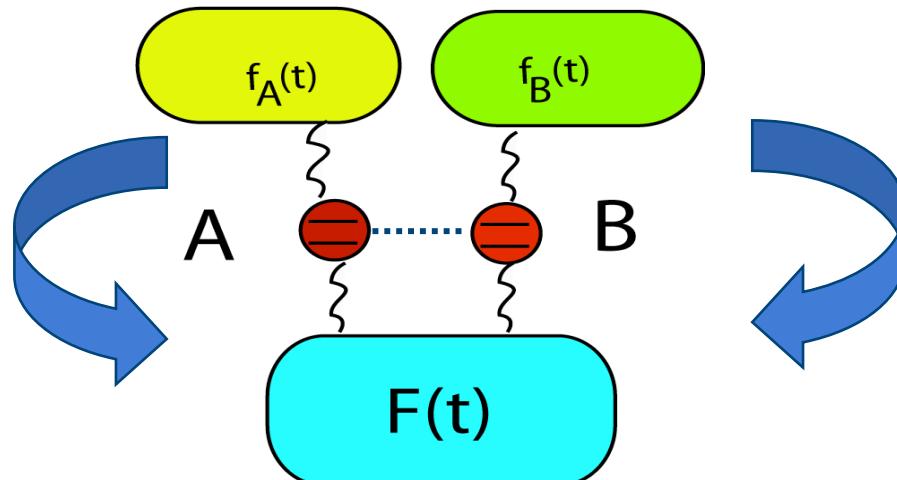


**Each qubit is affected by a non-Markovian noise**

# Two spins: A simple model

Hamiltonian:

$$H(t) = F(t)(\sigma_z^A + \sigma_z^B) + f_A(t)\sigma_z^A + f_B(t)\sigma_z^B$$



Remark:

Decoherence can be caused by either local or global noises or both

## Ornstein-Uhlenbeck Noises

$$H_{tot}(t) = [f_A(t)\sigma_z^A + f_B(t)\sigma_z^B]/2$$

$$M[f_A(t)f_A(s)] = \alpha(t-s)$$

$$M[f_B(t)f_B(s)] = \alpha(t-s)$$

$$M[f_B(t)] = M[f_B(s)] = 0$$

$$M[f_A(t)f_B(s)] = 0$$

$$\alpha(t-s) = \frac{\Gamma\gamma}{2} e^{-\gamma|t-s|}$$

## Solution of Extended Kubo Model

$$H_{tot}(t) = [f_A(t)\sigma_z^A + f_B(t)\sigma_z^B]/2$$

**Stochastic Schrodinger equation**

$$i\frac{d}{dt} |\psi(t)\rangle = H_{tot}(t) |\psi(t)\rangle$$

$$|\psi(t)\rangle = U(t, f_A, f_B) |\psi(0)\rangle$$

$$U(t, f_A, f_B) = \exp[-i \int_0^t (f_A(s)\sigma_z^A + f_B(s)\sigma_z^B)/2 ds]$$

## Reduced Density Matrix

$$\rho = M[|\psi(t)\rangle\langle\psi(t)|]$$

$$\rho(t) = \Phi(\rho) = \sum E_j^\dagger \rho(0) E_j$$

# Explicit Solutions

$$\rho(t) = \Phi(\rho) = \sum E_j^\dagger \rho(0) E_j$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma_A & 0 \\ 0 & 0 & 0 & \gamma_A \end{bmatrix} \quad E_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_A & 0 \\ 0 & 0 & 0 & \omega_A \end{bmatrix} \quad E_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \gamma_B & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \gamma_B \end{bmatrix} \quad E_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \omega_B & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_B \end{bmatrix}$$

$$\gamma_i = e^{-F(t)}, \omega_i = \sqrt{1 - \gamma_i^2} \quad (i = A, B)$$

$$F(t) = \int_0^t ds \alpha(s) = \frac{\Gamma}{2} [t + \frac{1}{\gamma} (e^{-\gamma t} - 1)]$$

## Entanglement Evolution-Markov Limit

$$F(t) = \int_0^t ds \alpha(s) = \frac{\Gamma}{2} \left[ t + \frac{1}{\gamma} (e^{-\gamma t} - 1) \right] \rightarrow \frac{\Gamma t}{2} \quad \gamma \gg 1$$

$$\alpha(t-s) = \Gamma \delta(t-s)$$

## Entanglement Evolution-Short-Time Limit

$$e^{-\gamma t} \approx 1 - \gamma t + \frac{1}{2} \gamma^2 t^2 \quad \gamma t \ll 1$$

$$F(t) = \frac{1}{4} \Gamma \gamma t^2$$

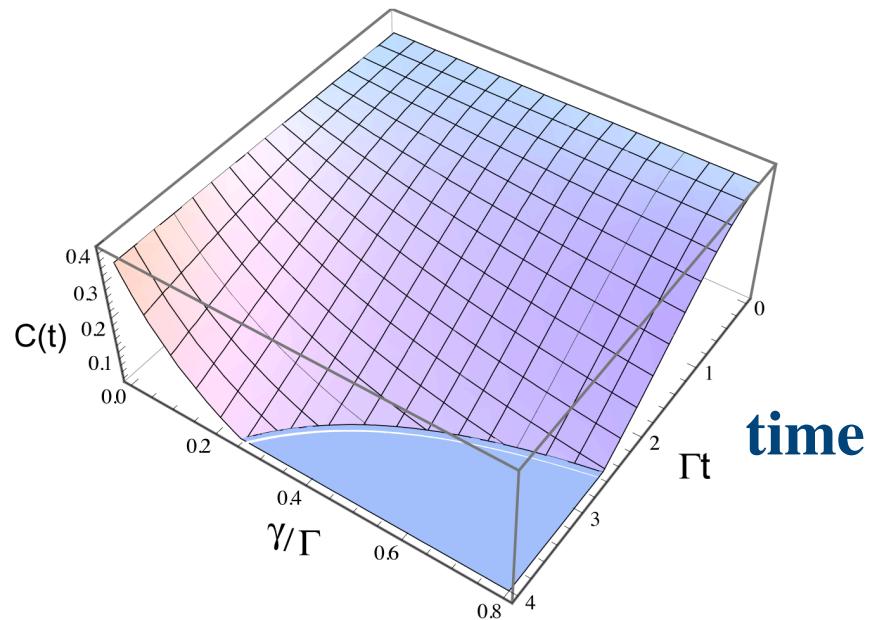
## Entanglement Evolution -- X Matrix

$$\rho = \begin{pmatrix} a & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z & c & 0 \\ w & 0 & 0 & d \end{pmatrix}$$

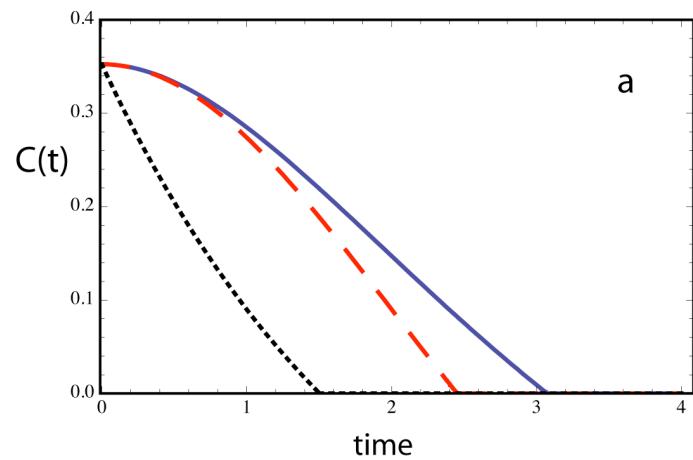
$$C(t) = 2 \max\{0, |z| - \sqrt{ad}, |w| - \sqrt{bc}\}$$

# Entanglement Evolution – I

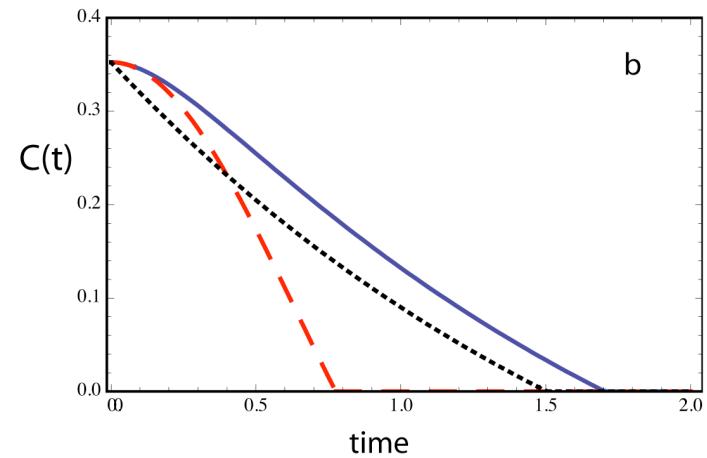
## Ornstein–Uhlenbeck noise



## Entanglement Evolution – II



a



b

Small gamma

$\gamma$

Large gamma

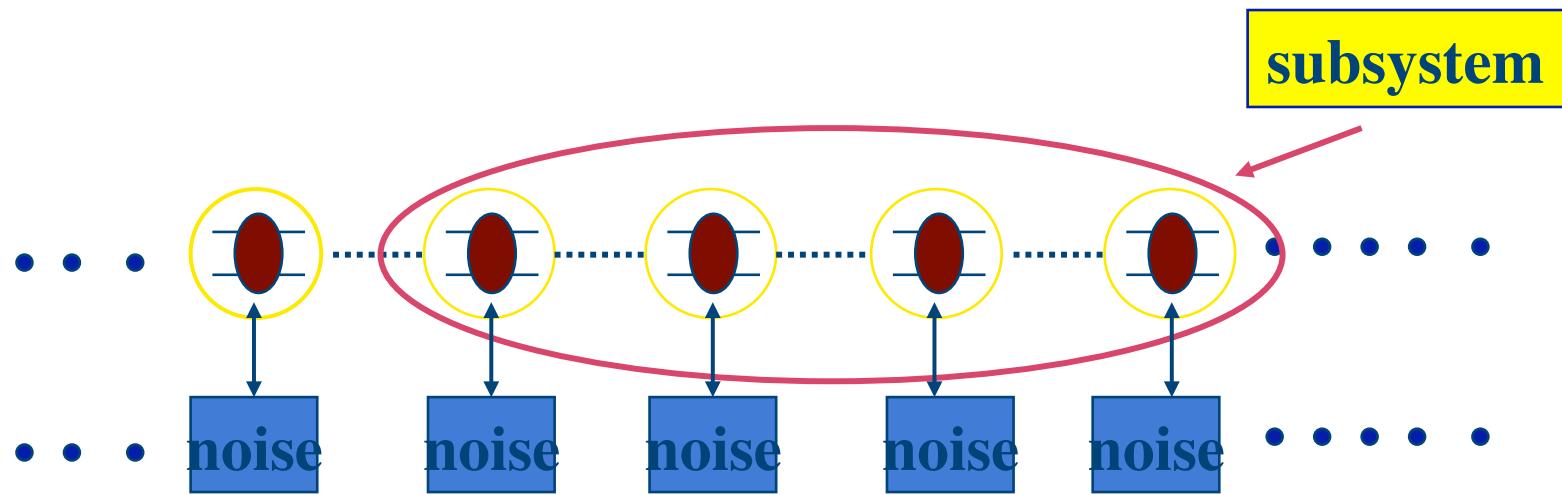
- Short-time limit
- ..... Markov limit
- \_\_\_\_\_ Exact solution

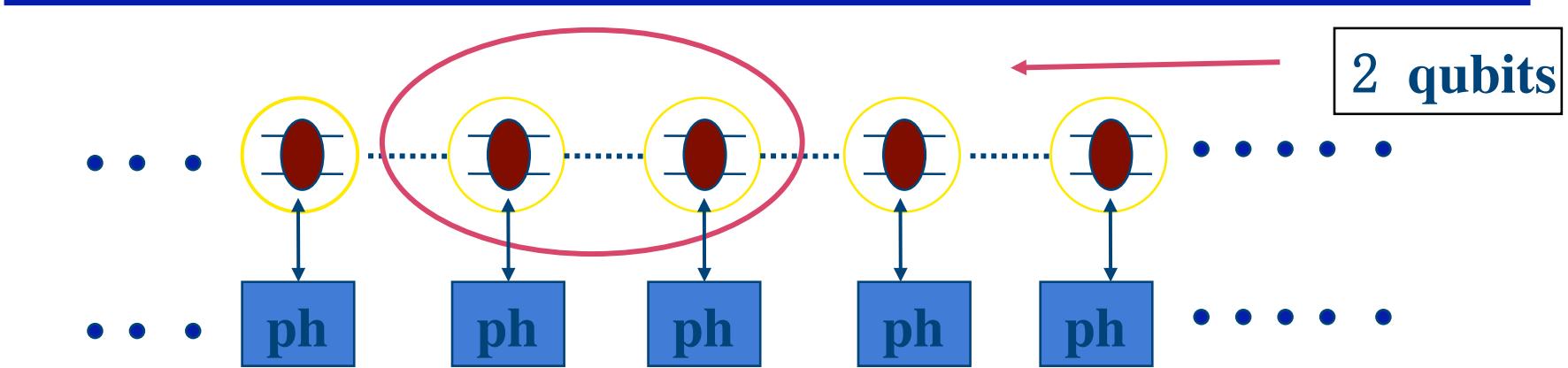
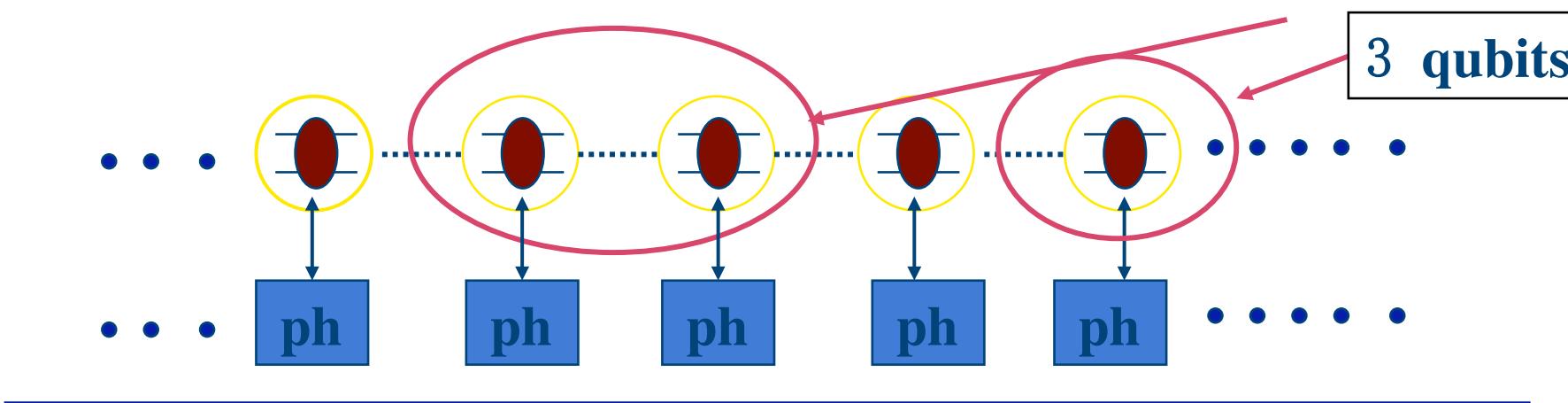
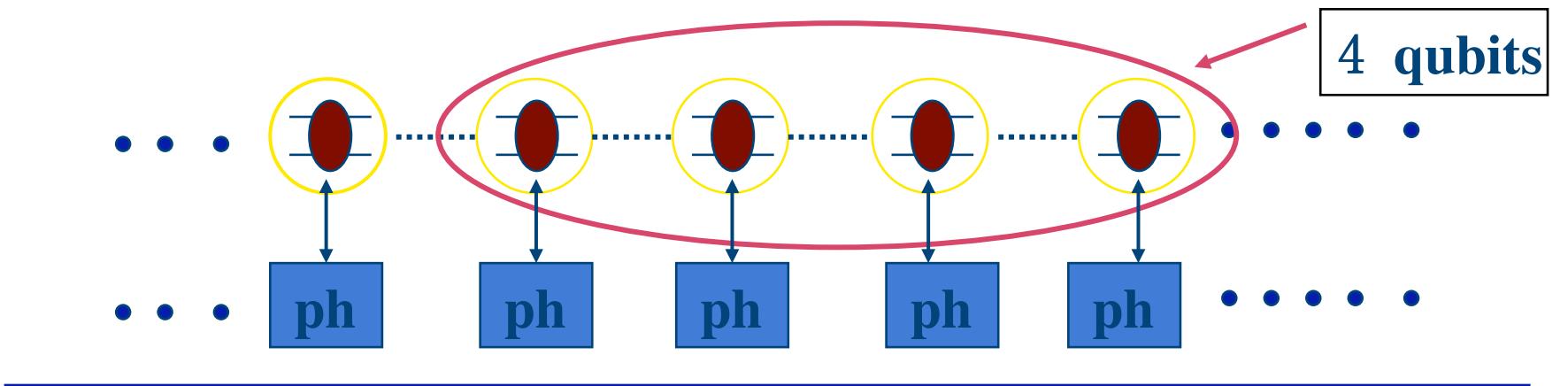
# **Universal Disentanglement: N-Qubit Case**

**Markov phase noises**

# N-qubit system: Dephasing noises

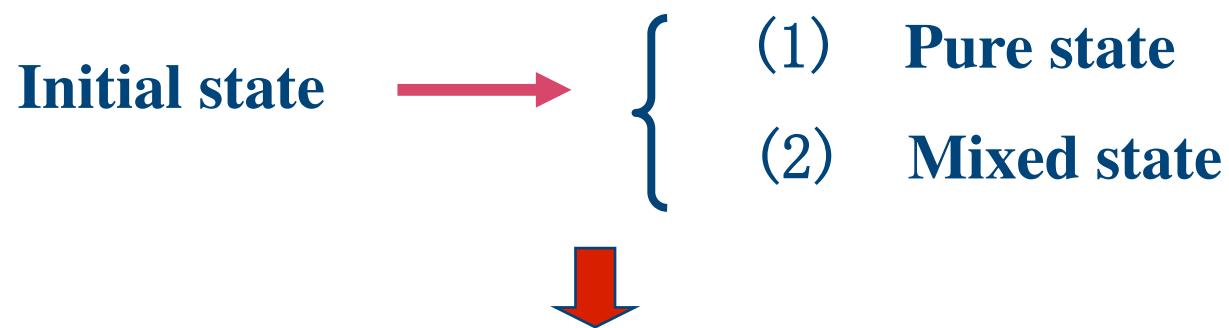
No cross-talk between two participating qubits





# Disentanglement dynamics

How does a pre-arranged entangled state decay under the influence of local thermal noises?



**Any pre-arranged entanglement will be completely destroyed in a finite time if the following condition is met:**

## Condition for ESD

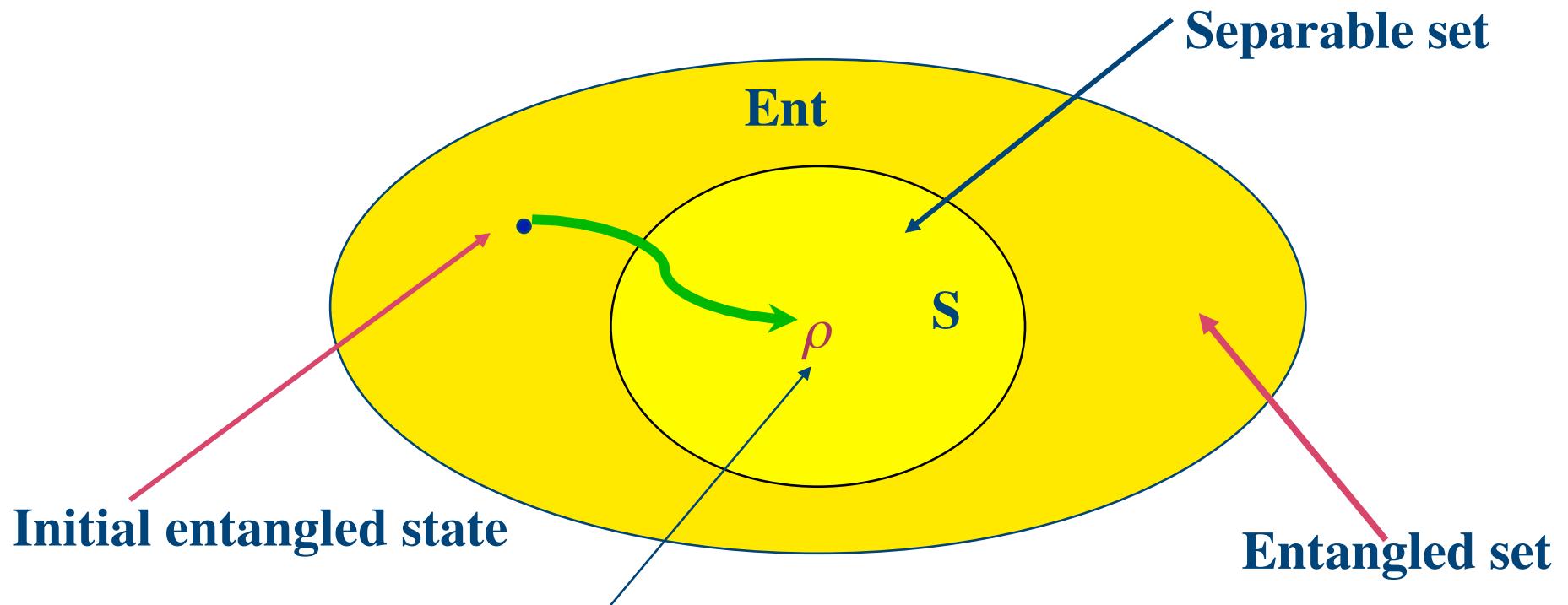
$$\prod \rho_{ii} = \rho_{11}\rho_{22} \dots \rho_{NN} \neq 0$$

Steady state

$$\rho(t \rightarrow \infty) = diag(\rho_{11}, \dots, \rho_{NN})$$

# Disentanglement under local phase noises

Space of all density matrices for N qubits

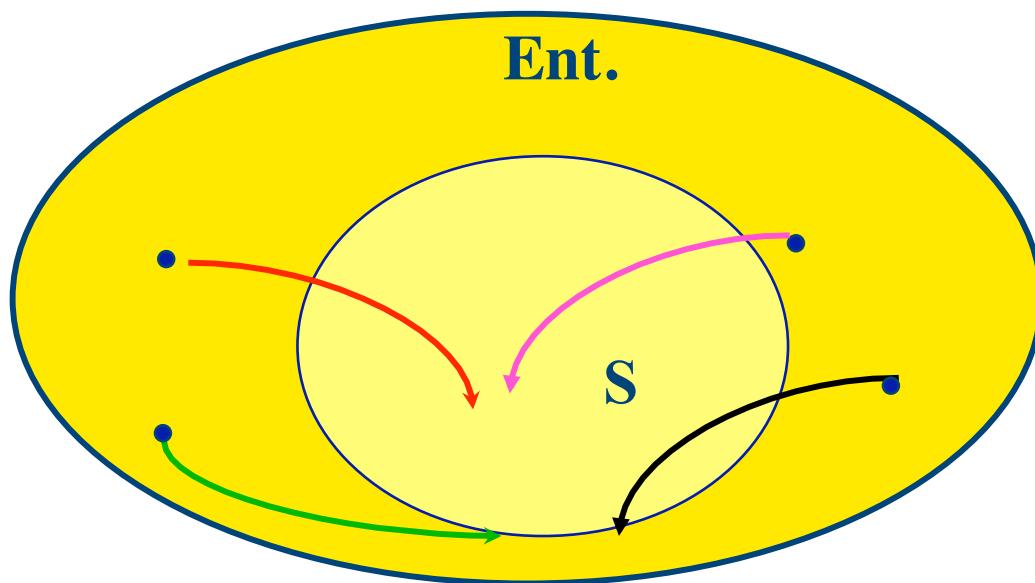


$$\rho(t = \infty) = \text{diag}(N_1, N_2, \dots)$$

is an interior point

# Why sudden death happens?

Space of all density matrices for N qubits



Topological explanation

Physical reason?

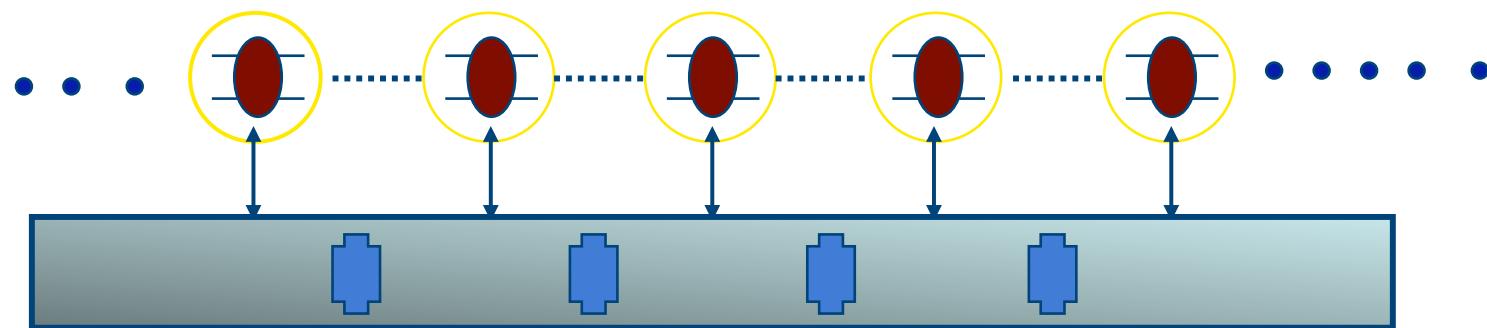
Quantum control

- {
- Direct coherence control
- Feedback control
- Error correction ....

# **Future Prospect**

# Decoherence suppression of N-Qubit Systems

- Identify noise sources
- Learn to live with noises
- Minimize the effect of noises



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Environment engineering

# Conclusion

- All real quantum systems are open
- Entanglement is a dynamic variable
- Disentanglement dynamics is different from decoherence
- Quantum processes: non-Markovian noise
- A future direction: Disentanglement control

# Quantum Information Science @ Stevens



# **Team at Stevens**

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## **Graduate Students (PhD):**

Brittany Corn  
Samuel Hedemann  
Wenchong Shu  
Jie Xu  
Tao Yang

One Postdoc is going to join us soon

# Acknowledgement

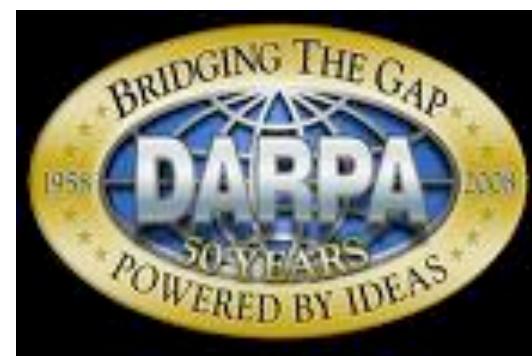
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Thank You