

Going beyond Gaussian limits on continuous variable processing and measurement

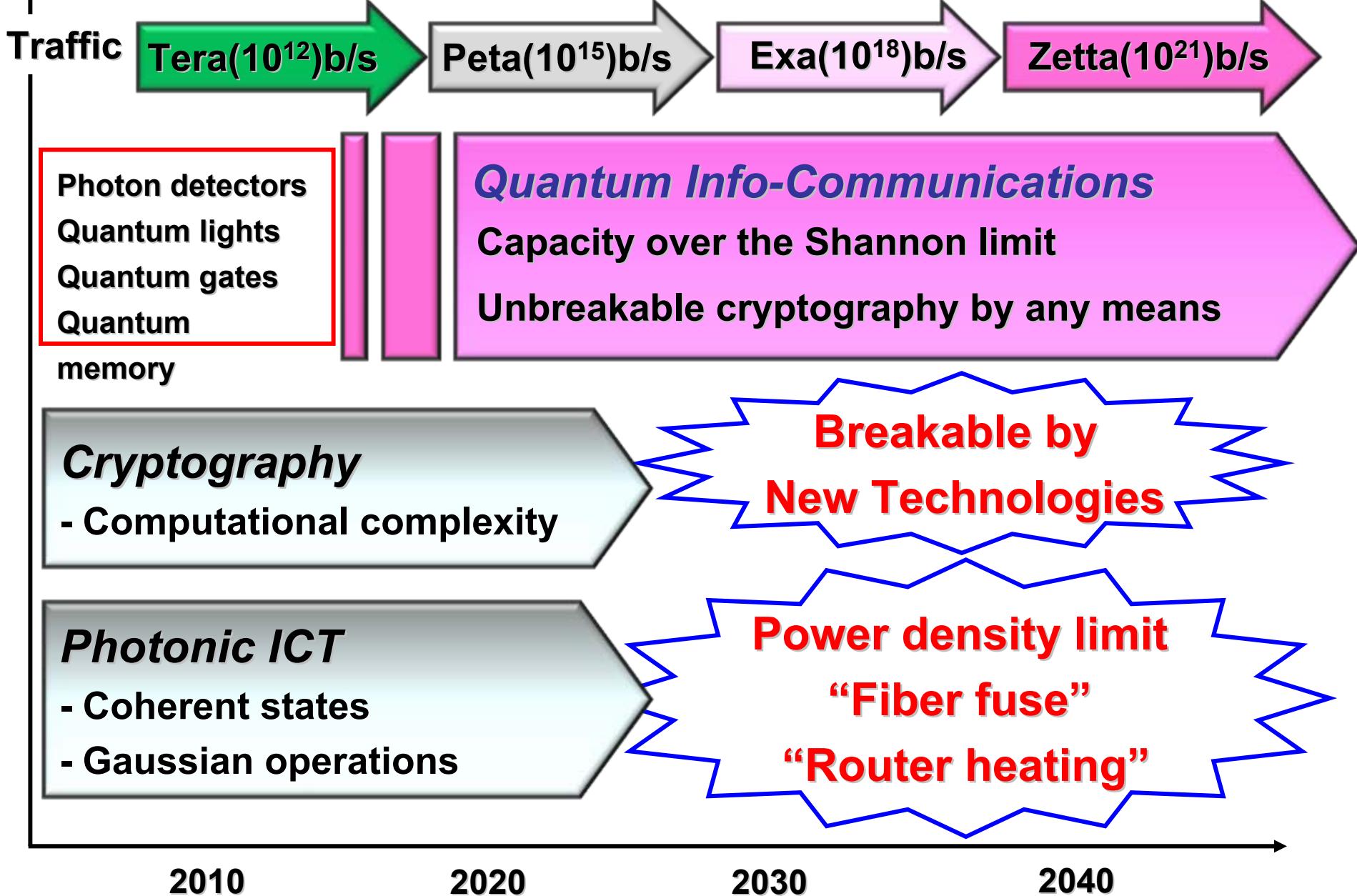
H. Takahashi, J. S. Neergaard-Nielsen,
M. Takeuchi, K. Wakui, M. Takeoka,
K. Hayasaka, M. Fujiwara, A. Waseda,
H. Tanaka, and M. Sasaki



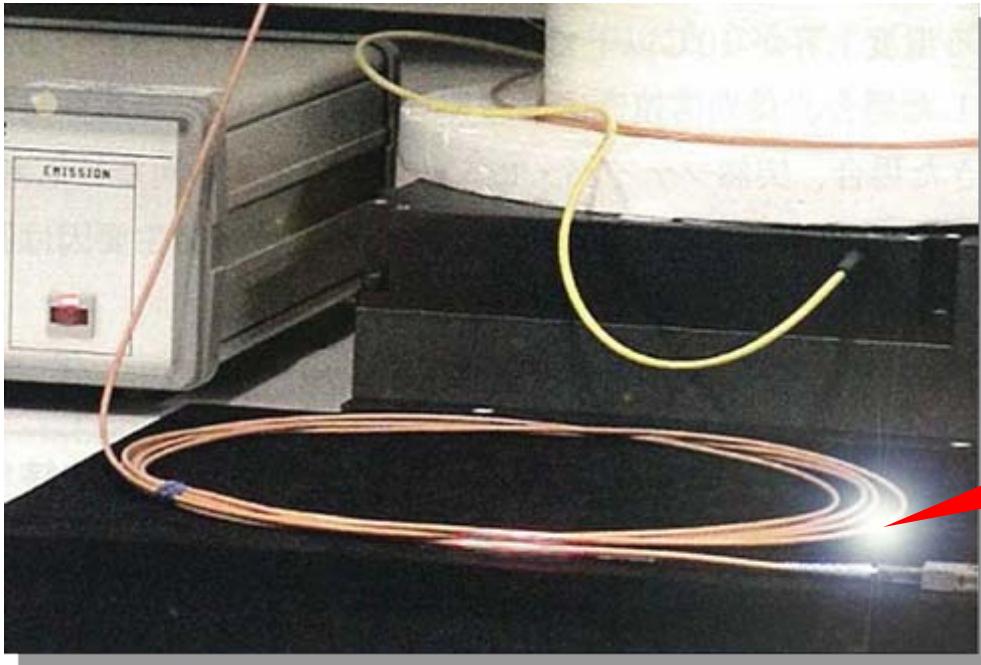
- Quantum enhanced capacities of optical fiber
- Non-Gaussian paradigm of CVs

State control and Measurement

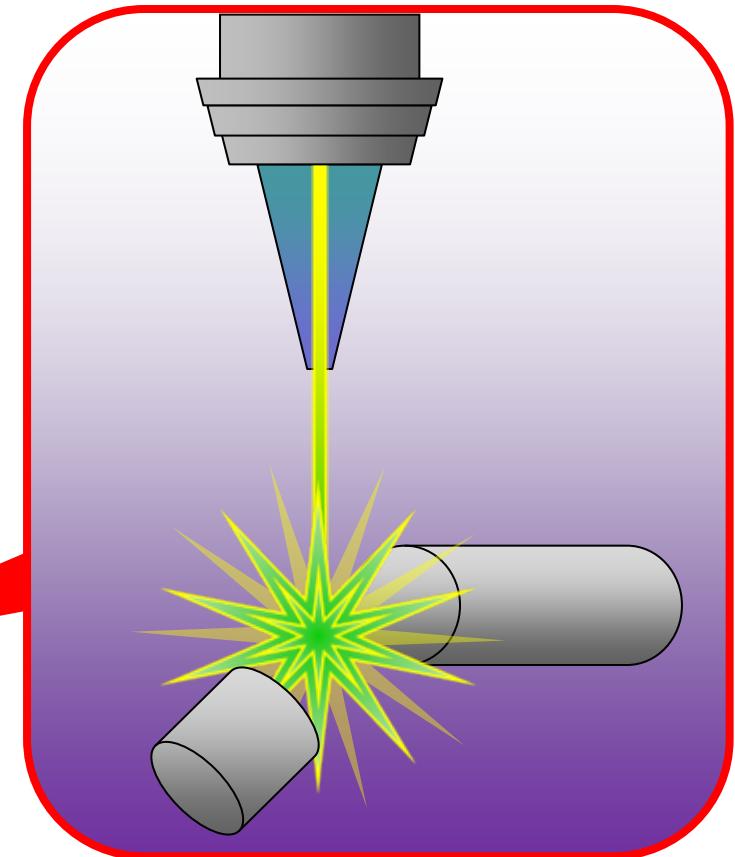
Roadmap of Info-Communications Technology



**The power density in backbone fibers
is now the same as that of a laser welder.**



Laser welder



**No more signals can be multiplexed
into a single fiber line.**

Roadmap of Info-Communications Technology

Traffic

Tera(10^{12})b/s

Peta(10^{15})b/s

Exa(10^{18})b/s

Zetta(10^{21})b/s

Photon detectors
Quantum lights
Quantum gates
Quantum
memory

Quantum Info-Communications

Capacity over the Shannon limit

Unbreakable cryptography by any means

Cryptography

- Computational complexity

Optimal way to transmit
the maximum information
with the minimum power

Photonic ICT

- Coherent states

- Gaussian operations

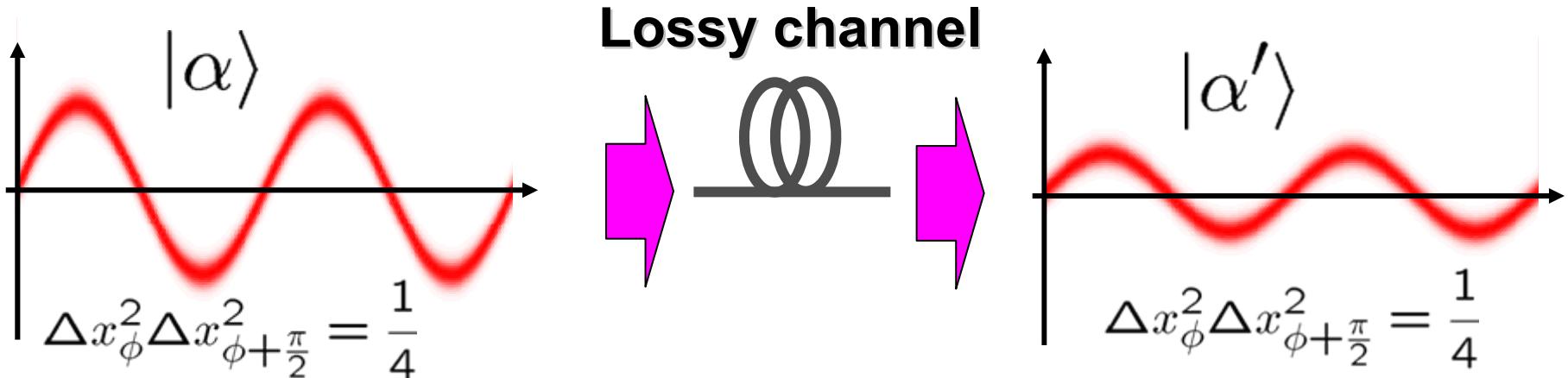
2010

2020

2030

2040

Coherent state



- The purity & the minimum uncertainty relation are never lost even after transmission through a lossy channel.
- No other states satisfy this property.



The coherent state is the best signal carrier for communications in any future.

Our approach

Quantum control of continuous variables (CVs)

Photonic ICT

- Dense coherent codec
 $|\alpha_k\rangle$
- WDM
- Gaussian operations
(Amp, routing, ...)

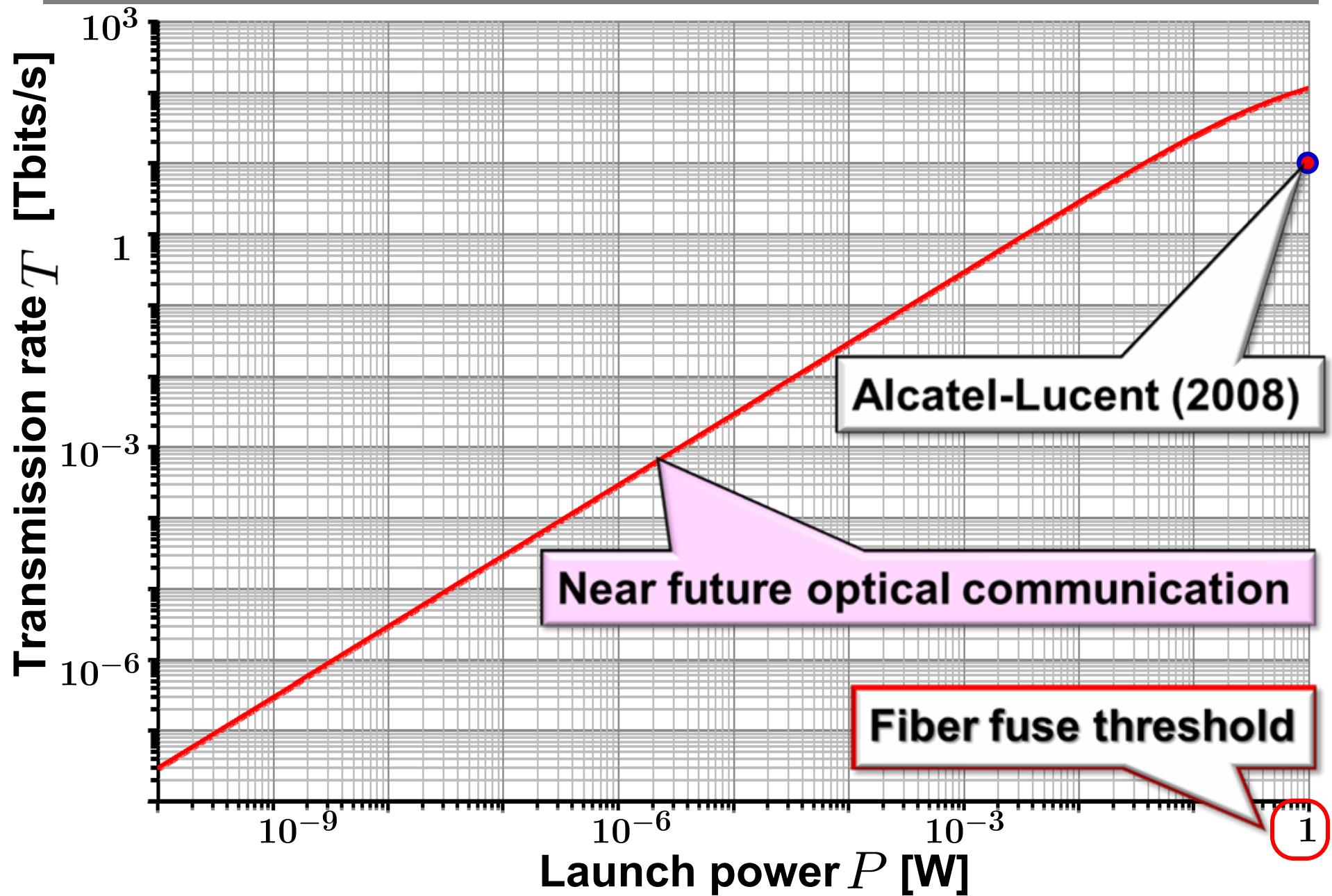
Quantum ICT

- Q Cryptography
- DV QKD
- CV QKD
- Q. noise random cipher
- Q Communications
- Holevo capacity

Secure and Power Minimum Photonic Network

Quantum enhanced capacities of optical fiber

Transmission rate vs Launch power

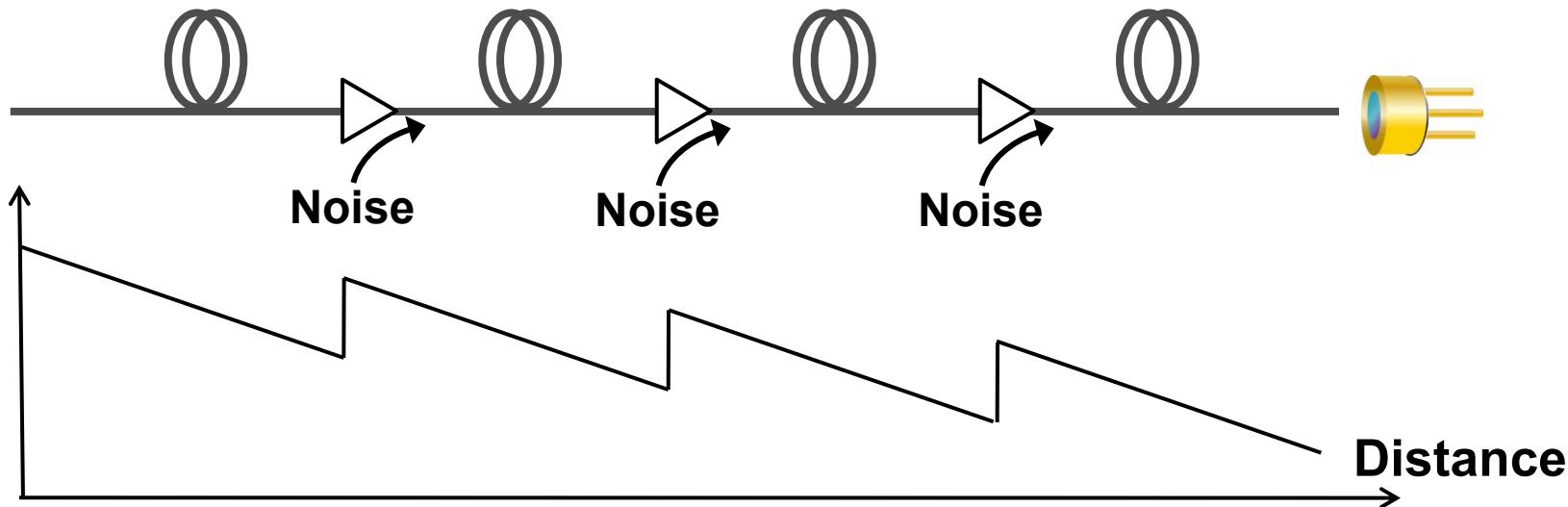
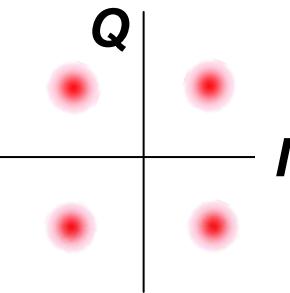


Photonic communication

Alcatel-Lucent (2008)

16.4 Tbps through a single fiber over 2,550 km (in the lab)

- 4 state signals (2bit/pulse)
- 164 wavelength division multiplexing modulated at 100 Gbps
- 50 amplifiers in every 51 km



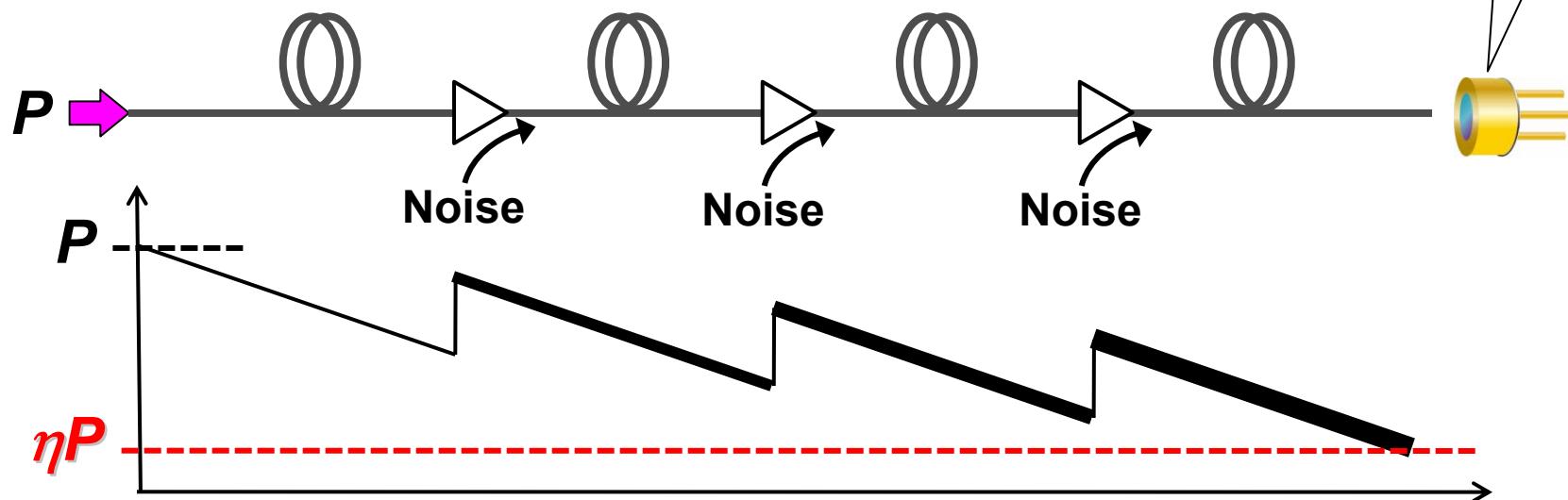
To forecast near future optical communication

Rough modeling by $C = W \log_2 \left(1 + \frac{\eta P}{N} \right)$

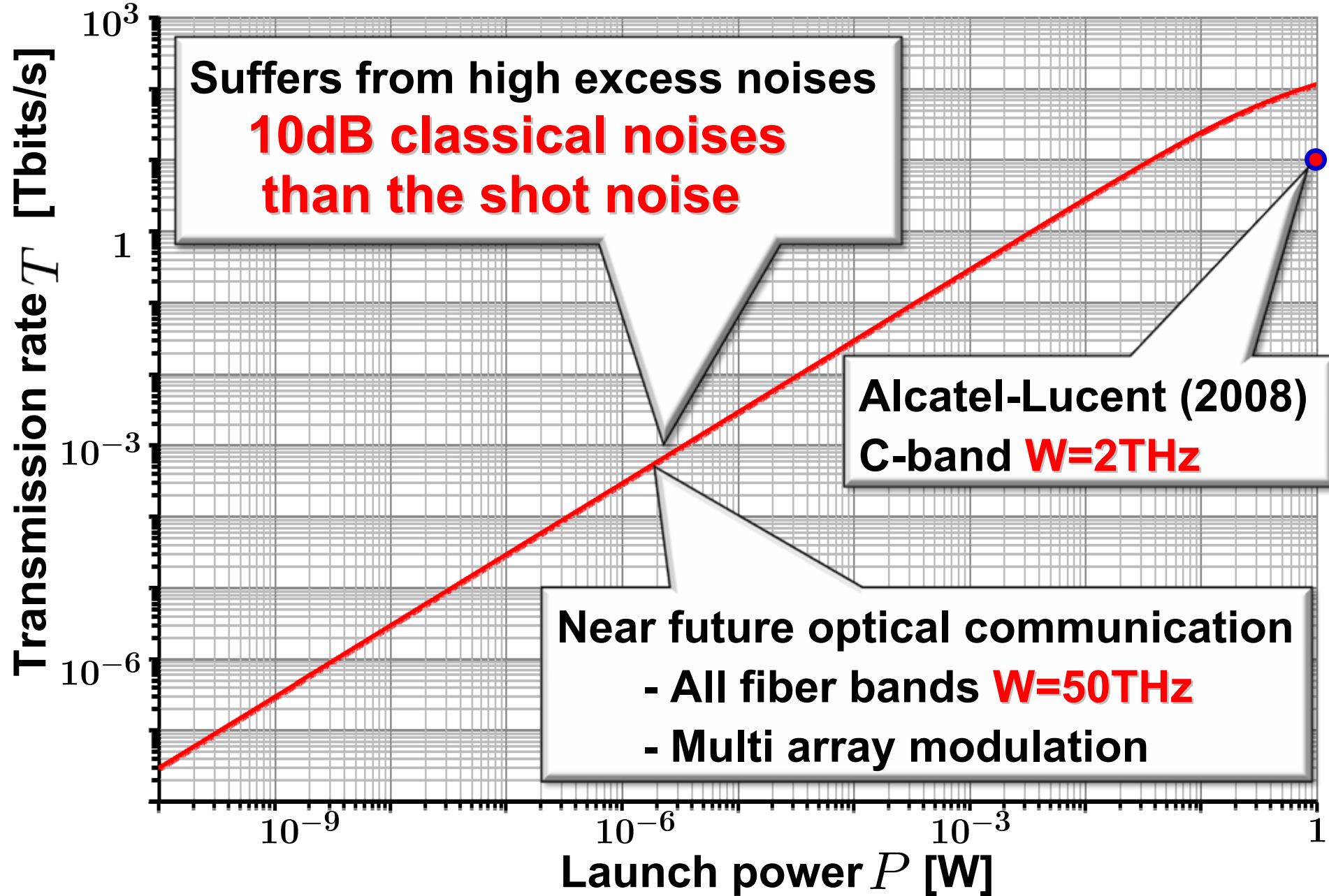
- (1) Transmittance $\eta \sim 10^{-2}$
(2) Noise power $N \sim 100\text{uW}$

- Gain & number of amplifiers
- Noises (amplifier, detector, background)

SNR~20dB
at receiver

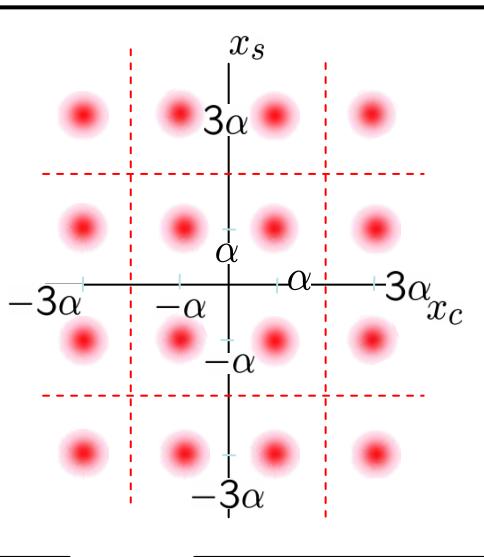


Max performance of current technology



Quantum-limited coherent system

Only the shot noise. No classical excess noises.



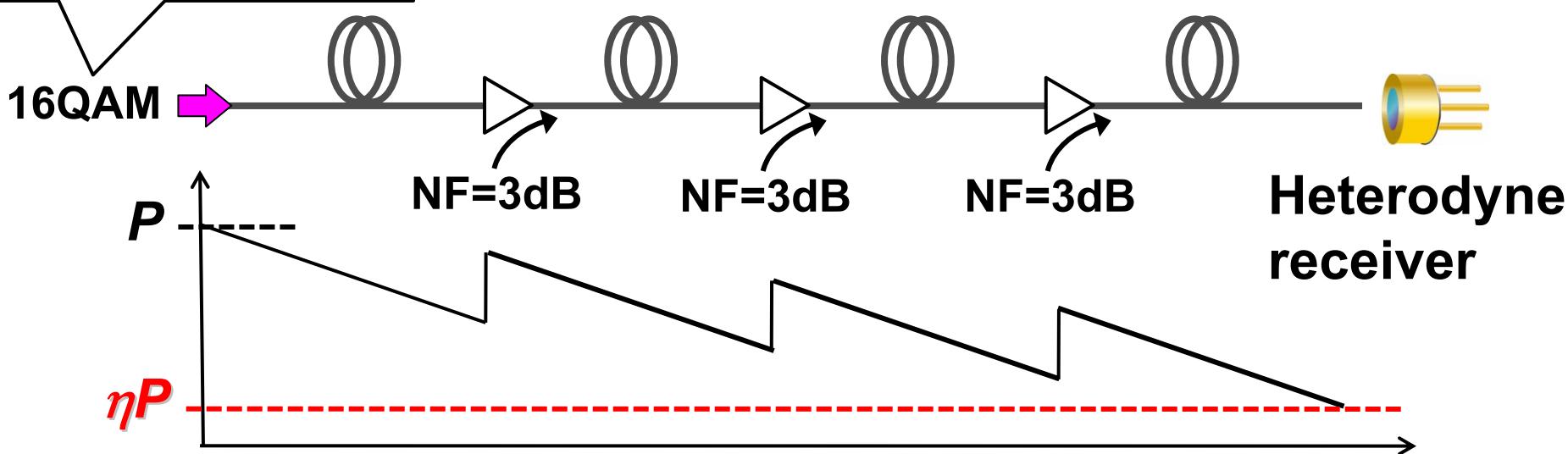
$$C = W \log_2 \left(1 + \frac{\eta P}{N} \right)$$

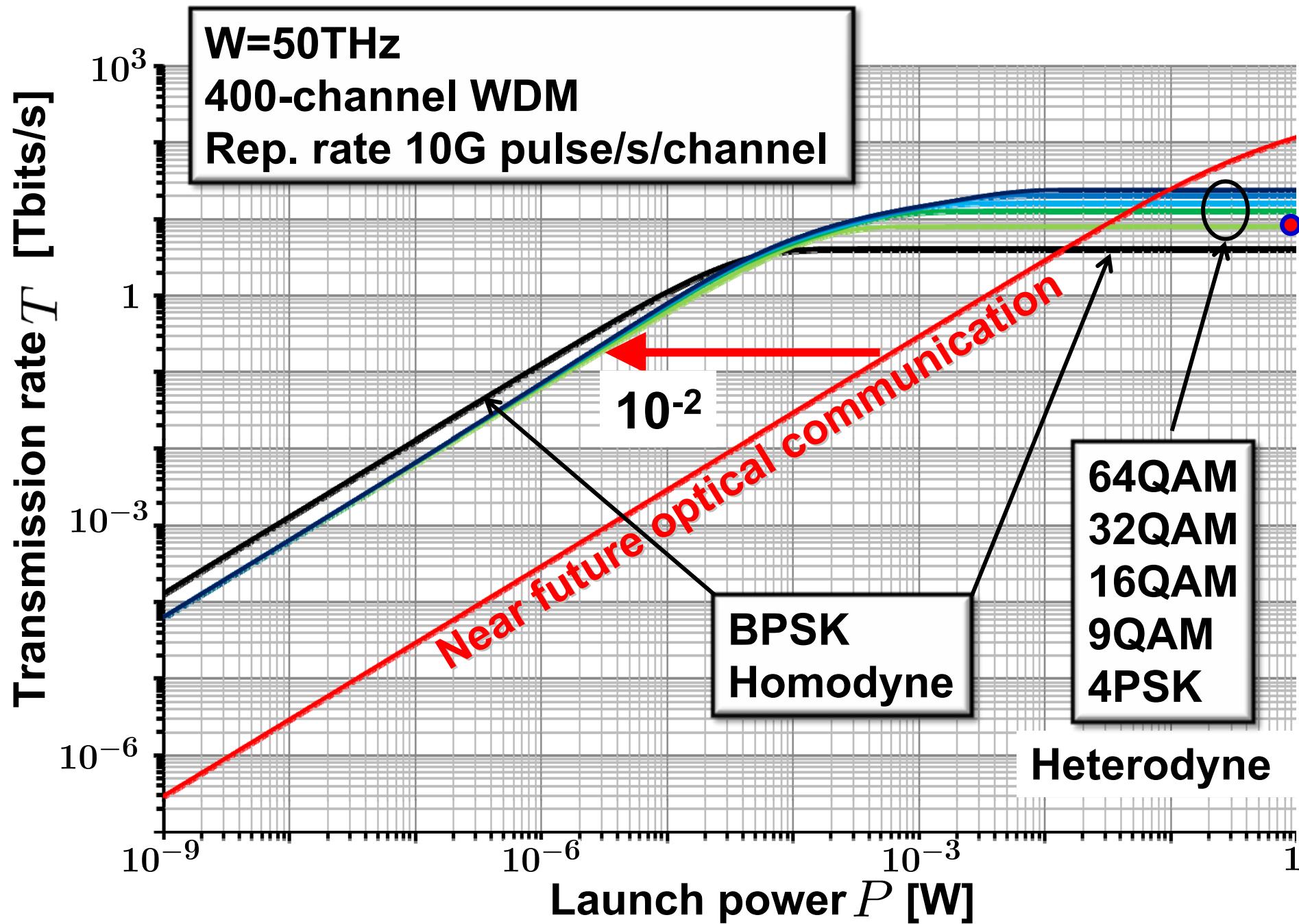
$$\eta = 10^{-2}$$

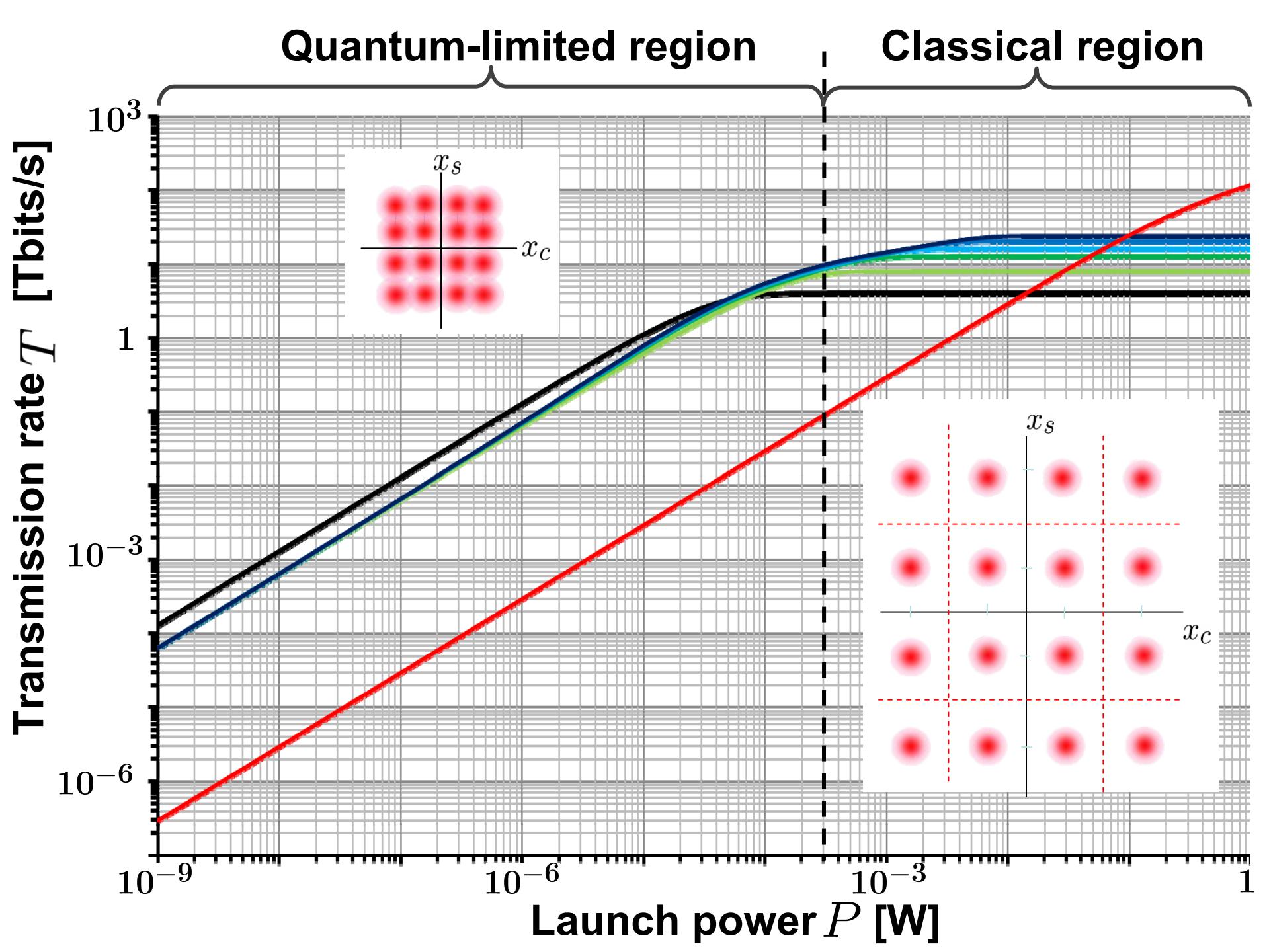
$$P = \bar{n} W h f_c$$

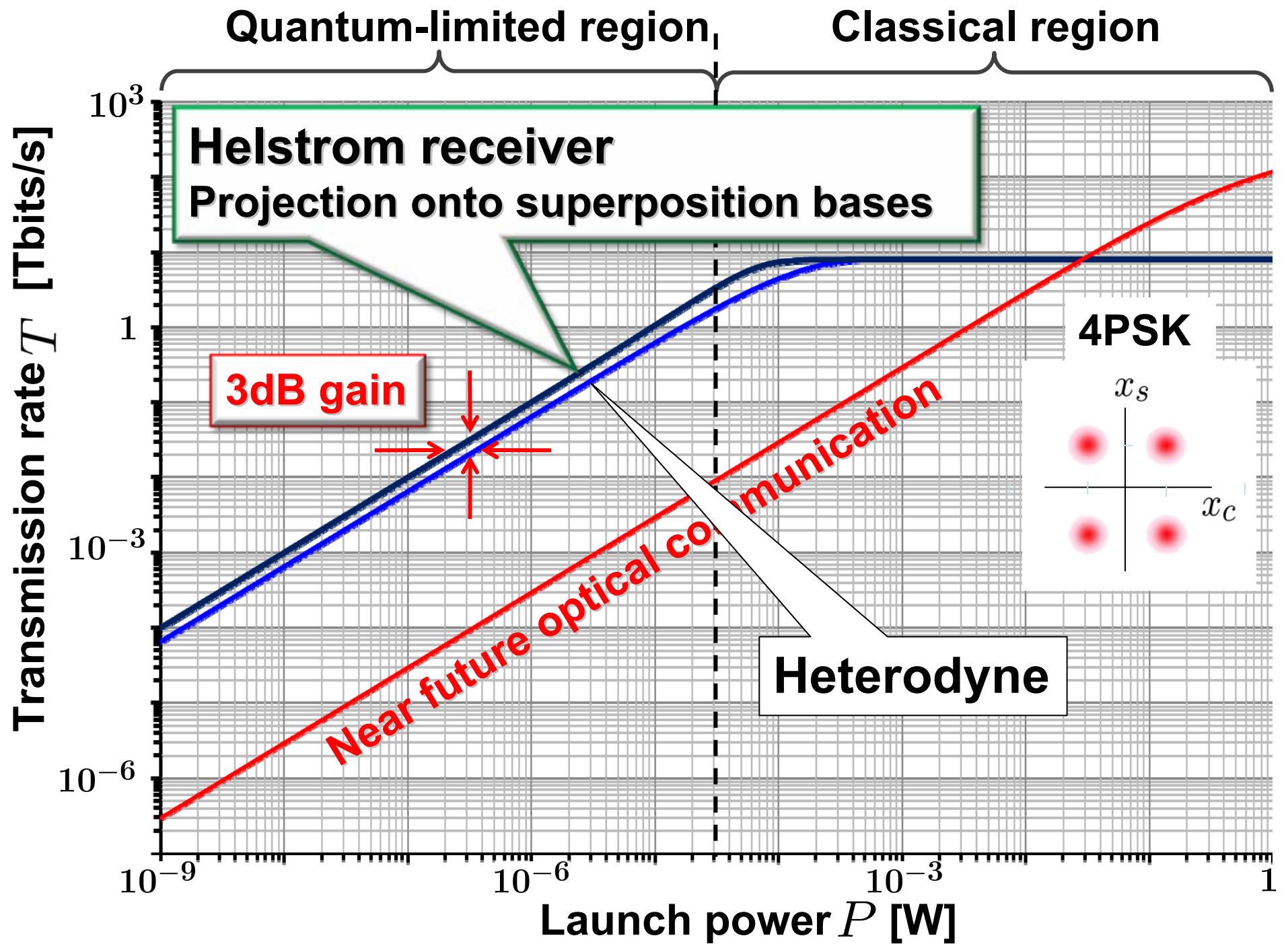
$$N = W h f_c$$

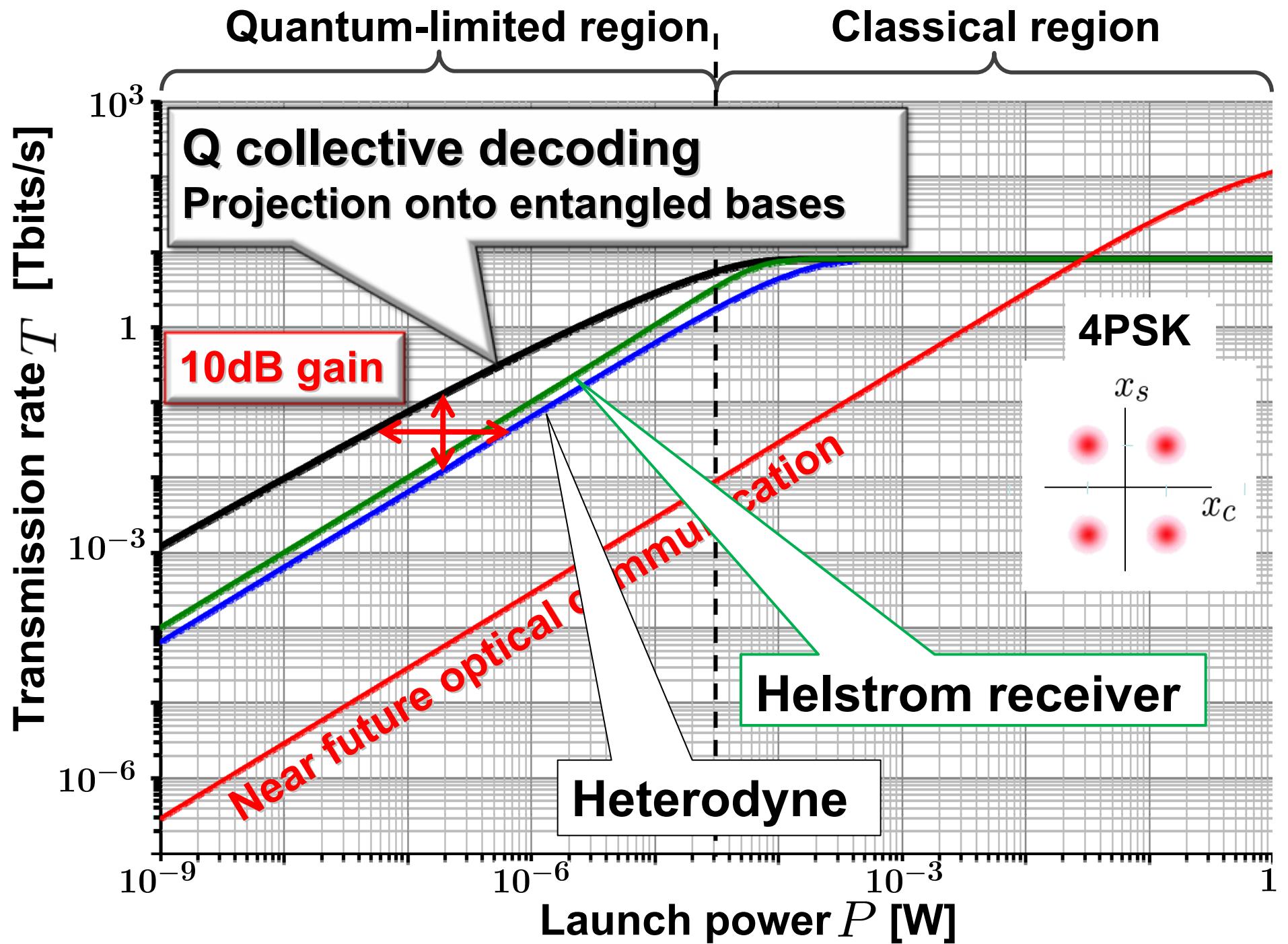
**Q. Amplifiers
Ideal Dyne Detectors**









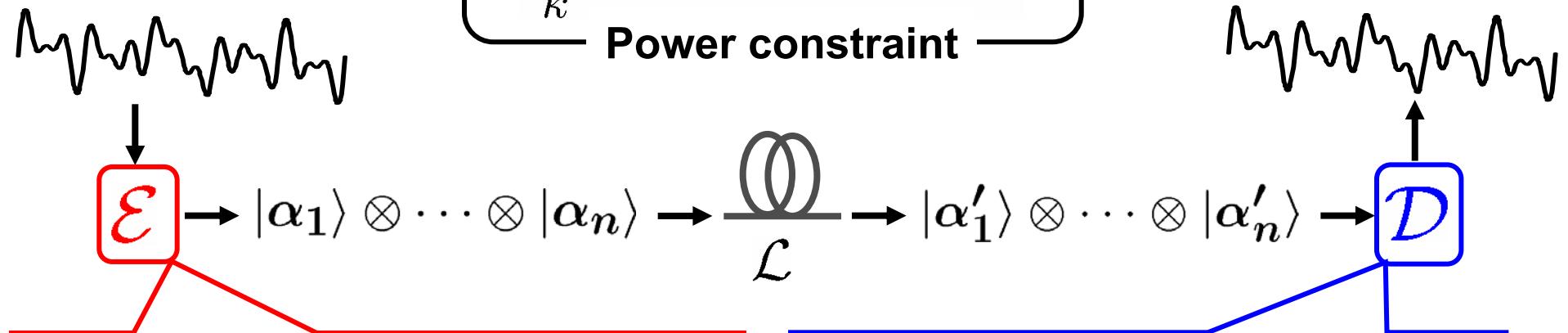


Ultimate capacity of lossy bosonic channel

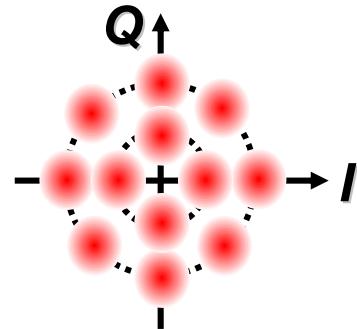
Giovannetti, et al., Phys. Rev. Lett. 92, 027902 (2004).

$$\sum_k \hbar \omega_k \langle \hat{a}_k^\dagger \hat{a}_k \rangle = \mathcal{E}.$$

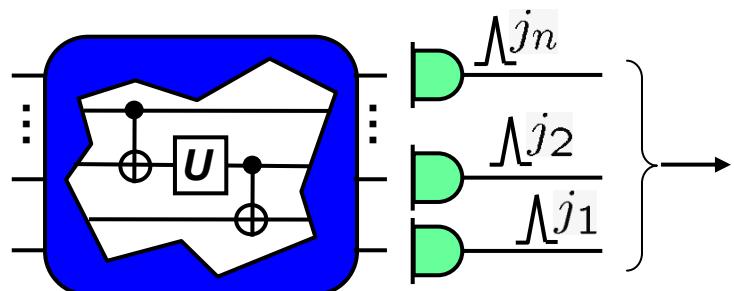
Power constraint



Dense coherent modulation



Q. collective decoding



Quantum theoretical limit
Single-fiber capacity can
never exceed Peta bps

Transmission rate T [Tbits/s]

10^3

1

10^{-3}

10^{-6}

10^{-9}

Launch power P [W]

Near future
optical
communication

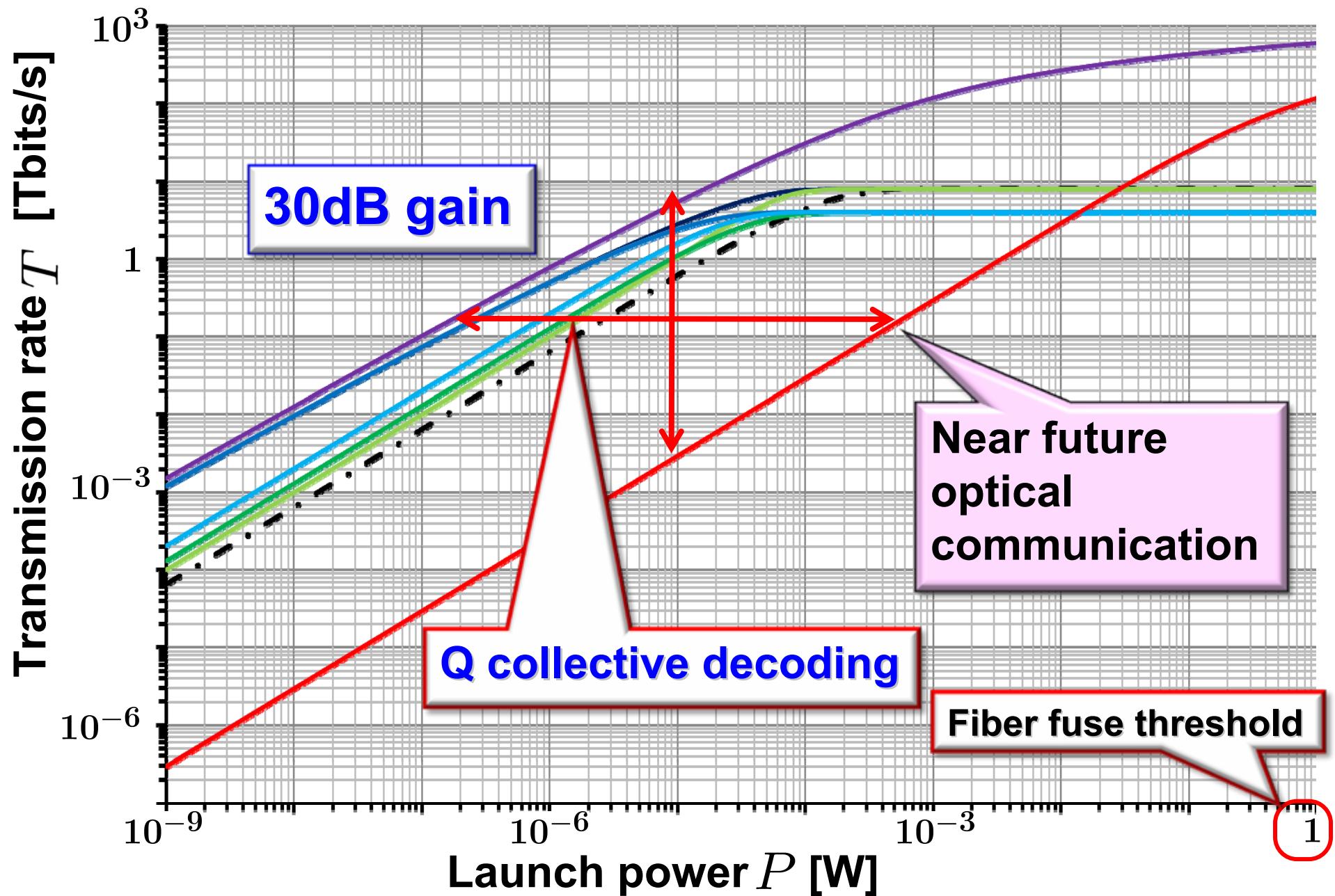
- Reduce excess noises
- Ideal coherent systems

Q collective decoding

Fiber fuse threshold

1

To realize power minimum communication



To realize power minimum communications

Peta bps with 1W launch power to a single fiber

(1) Low loss WDM technology

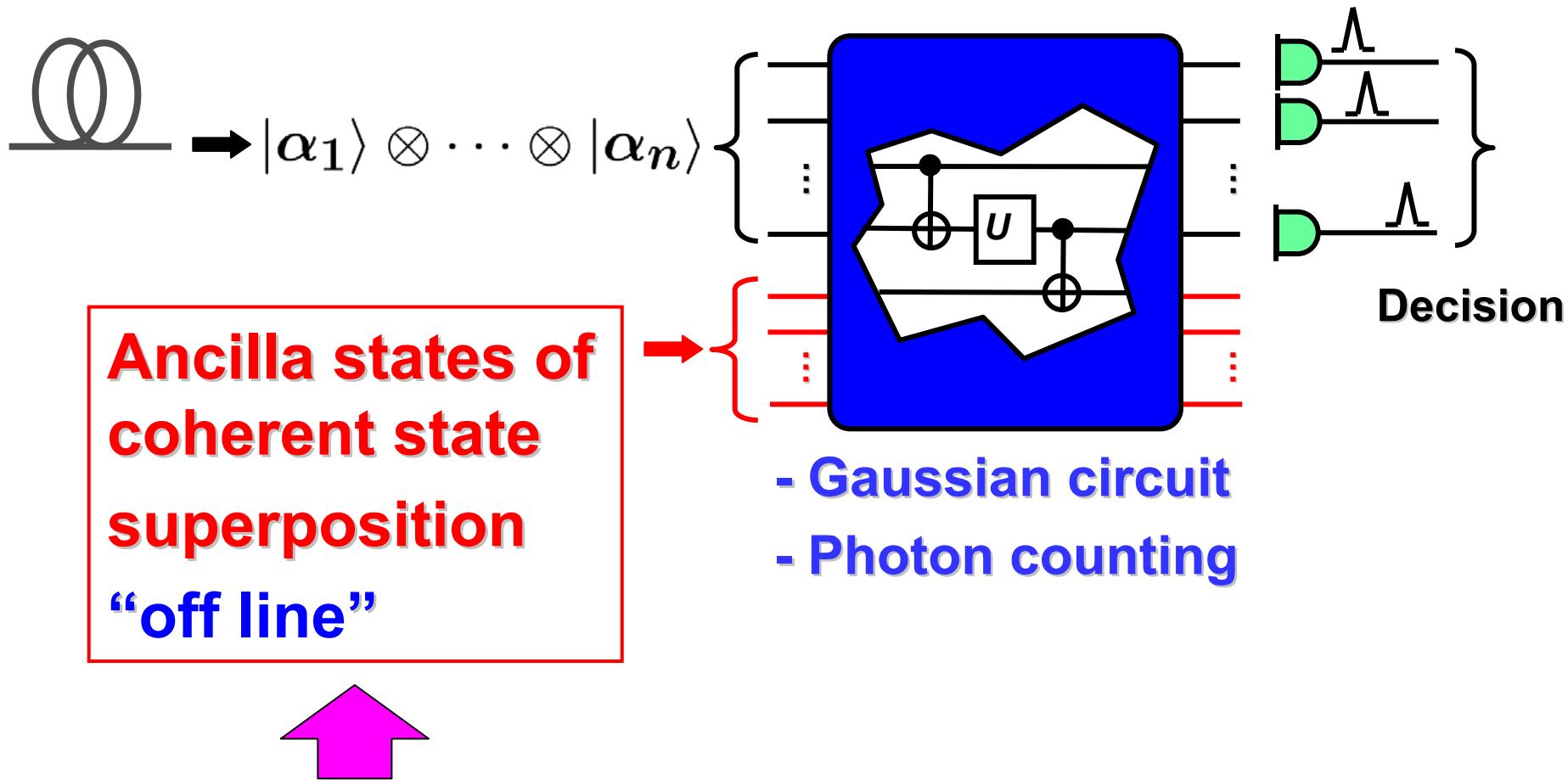
(2) Narrow band (<10GHz) quantum channel

- Dense coherent modulation

- Quantum-limited amplifier ($NF=2$)

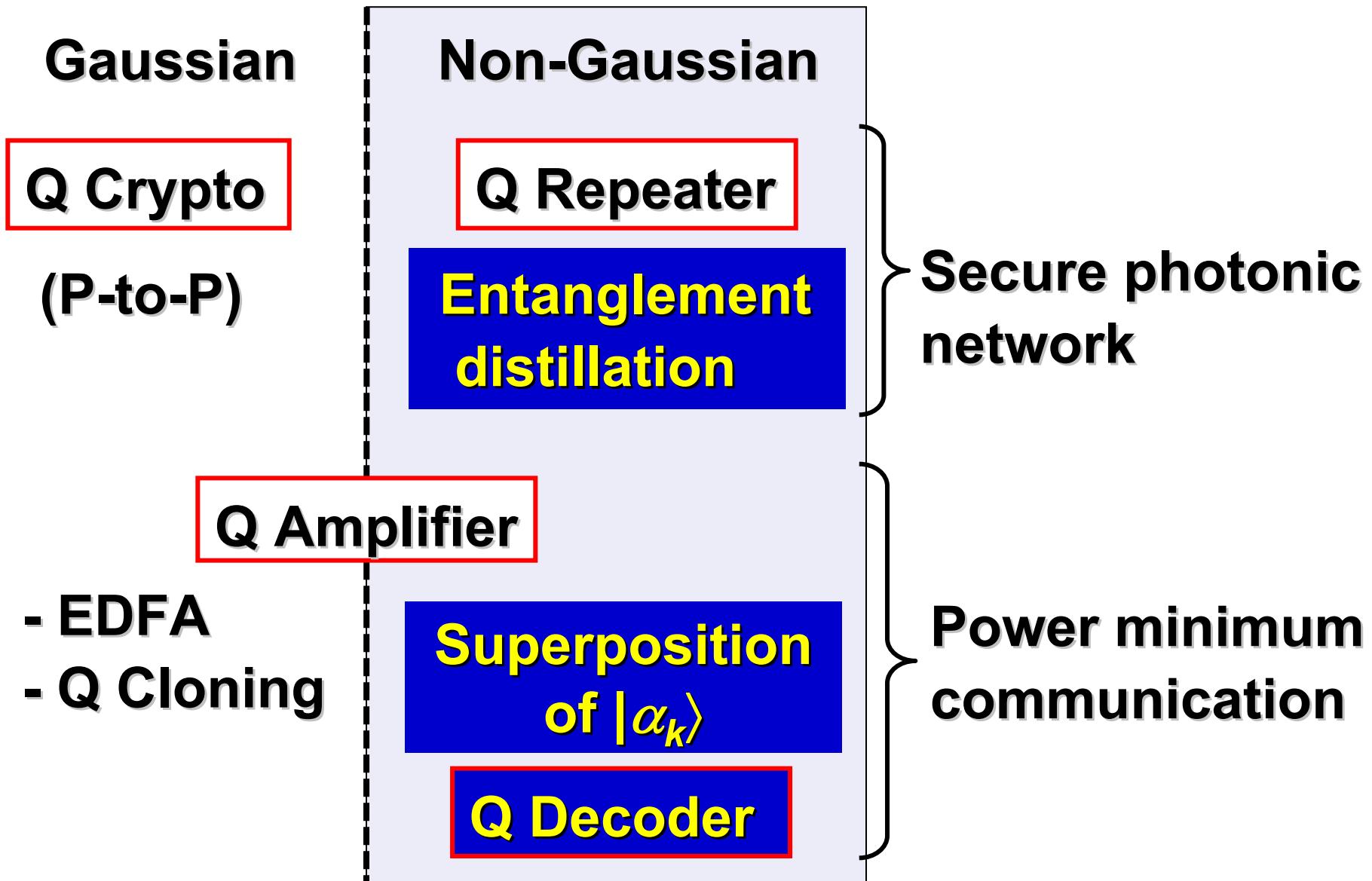
- Quantum collective decoder

Quantum collective decoding



Highly non-linear process with very low loss
beyond Gaussian operations

Quantum control of CVs



Low loss non-Gaussian operation

A currently feasible scheme

“Photon subtraction from a squeezed state”

Yurke, Schleich, & Walls, PRA42,1703(1990).

Dakna, et al., PRA 55, 3184 (1997).

$$\hat{a}^{\hat{S}} |0\rangle \sim |\alpha\rangle - |-\alpha\rangle$$

$$\hat{a}^2 \hat{S} |0\rangle \sim |\alpha\rangle + |-\alpha\rangle$$

Recent progress on non-Gaussian operations

Single photon subtraction

- Ourjoumtsev, et al. Science 312, 83 (2006). ([CNRS](#))
- Neergaard-Nielsen, et al. RRL, 97, 083604 (2006). ([NBI](#))
- Wakui, et al., Opt. Express 15, 3568 (2007). ([NICT](#))

Photon addition and subtraction → the commutation rule

- Parigi, et al., Science 317, 1890 (2007).

Cat state generation from $|n\rangle$

- Ourjoumtsev, et al. Nature, 448, 784 (2007).

Breeding a cat state

- Takahashi, et al. PRL101, 233605 (2008).

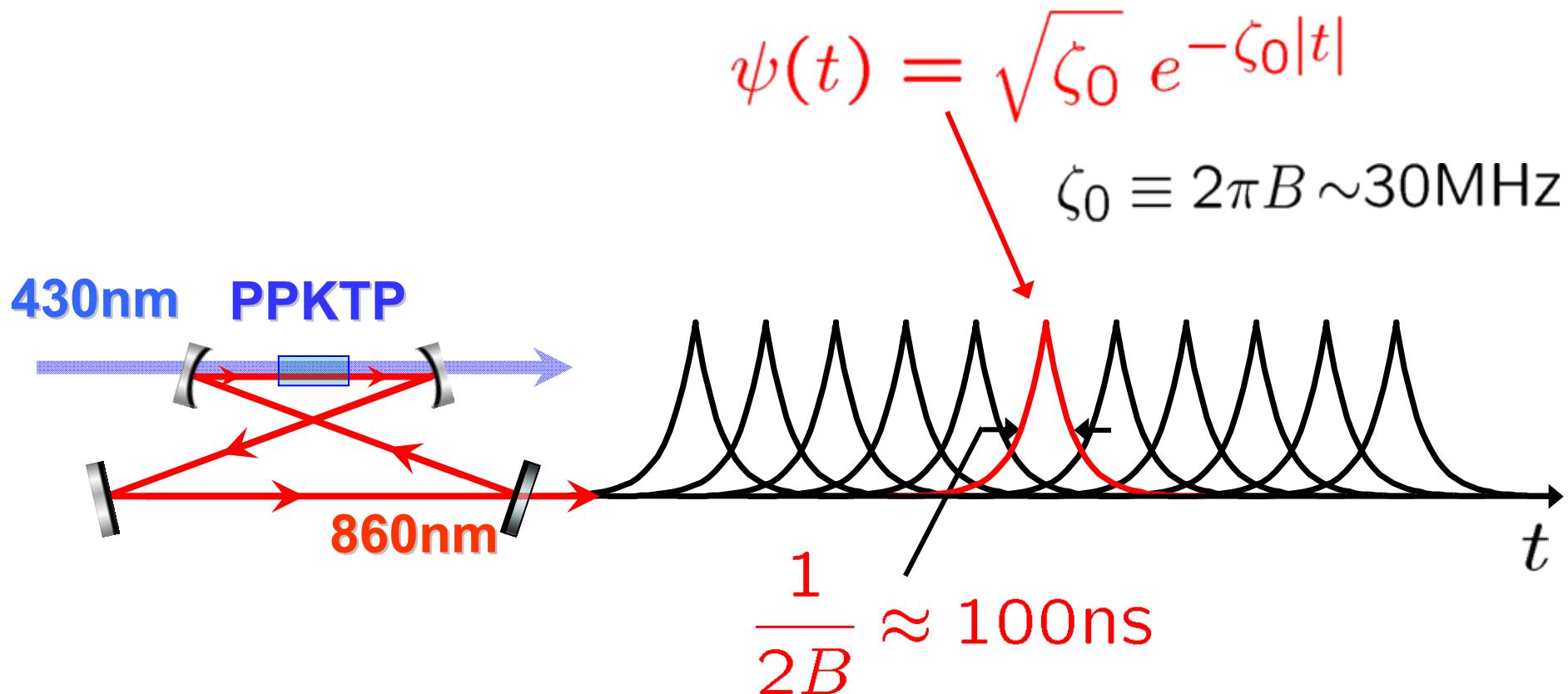
Noise-free generation of an entangled cat state

- Ourjoumtsev et al., Nature Physics (2009).

Three-photon subtraction by TES (NIST)

CW squeezed state generated by OPO

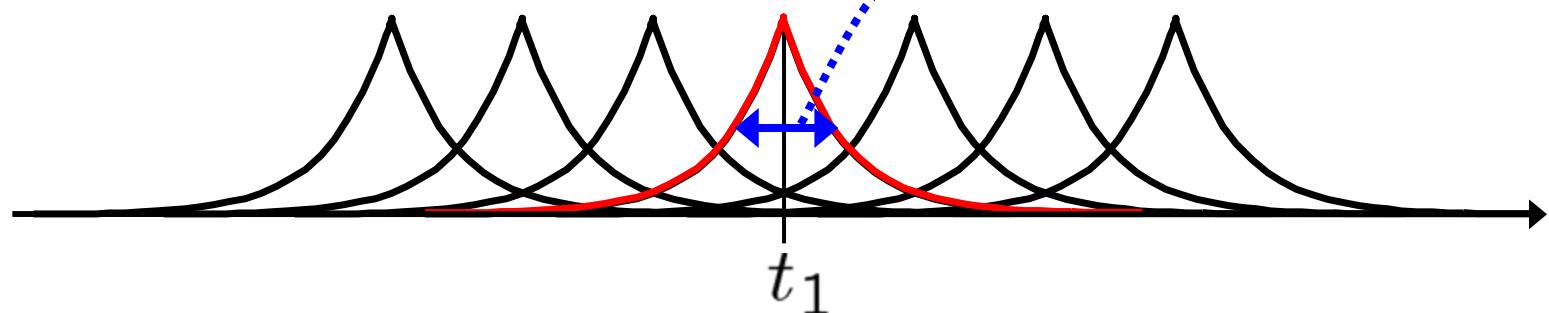
Highly pure squeezed state



Trigger signals at Si APD

Time scale of squeezed state

$$\frac{1}{2B} \sim 100\text{ns}$$

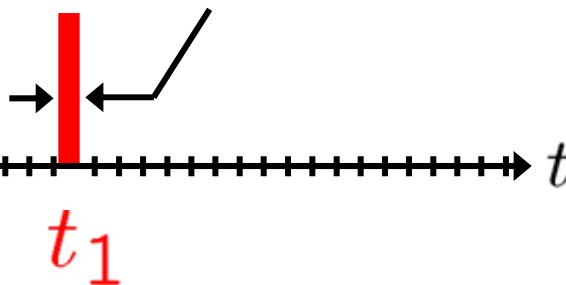


$\hat{S}|0_A\rangle$

$\hat{a}(t_1)\hat{S}|0_A\rangle$



Time resolution $T \sim 0.5\text{ ns}$

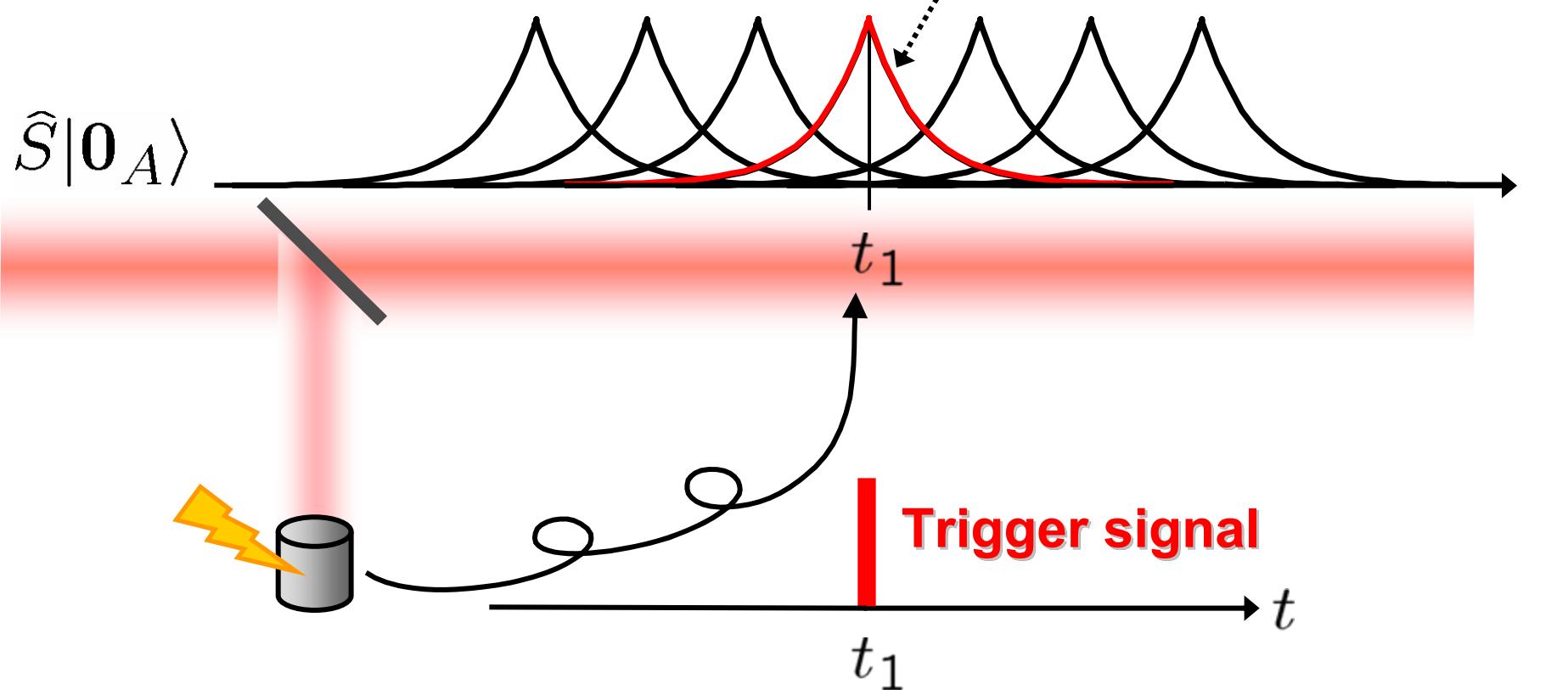


Single-photon subtracted state

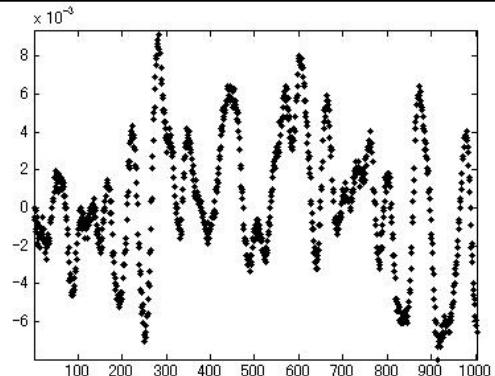
“Squeezed single-photon state”

$$\hat{a}(t_1)\hat{S}|0_A\rangle = \hat{S} \int_{-\infty}^{\infty} dt \hat{a}^\dagger(t) \nu(t - t_1) |0_A\rangle$$

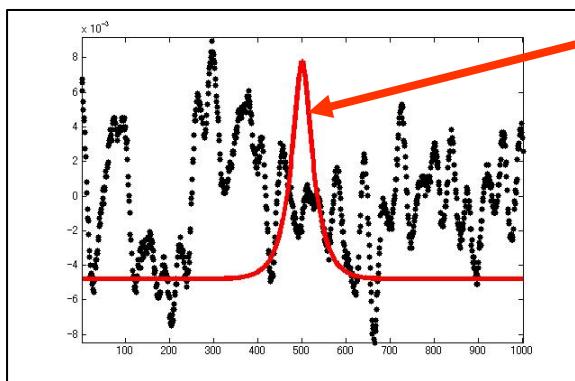
Takeoka, et.al. PRA 77, 062315 (2008).



Homodyne distribution



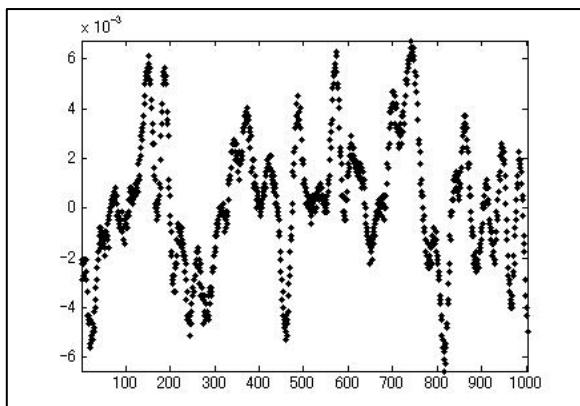
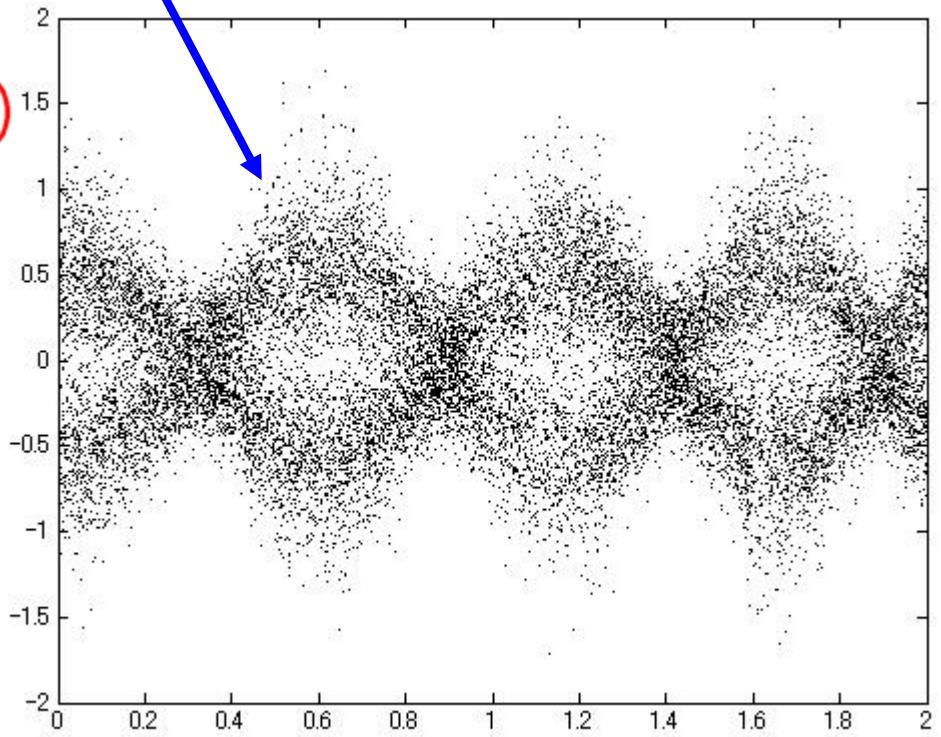
$$\hat{X}_\phi(t)$$



$$\psi(t - t_1)$$



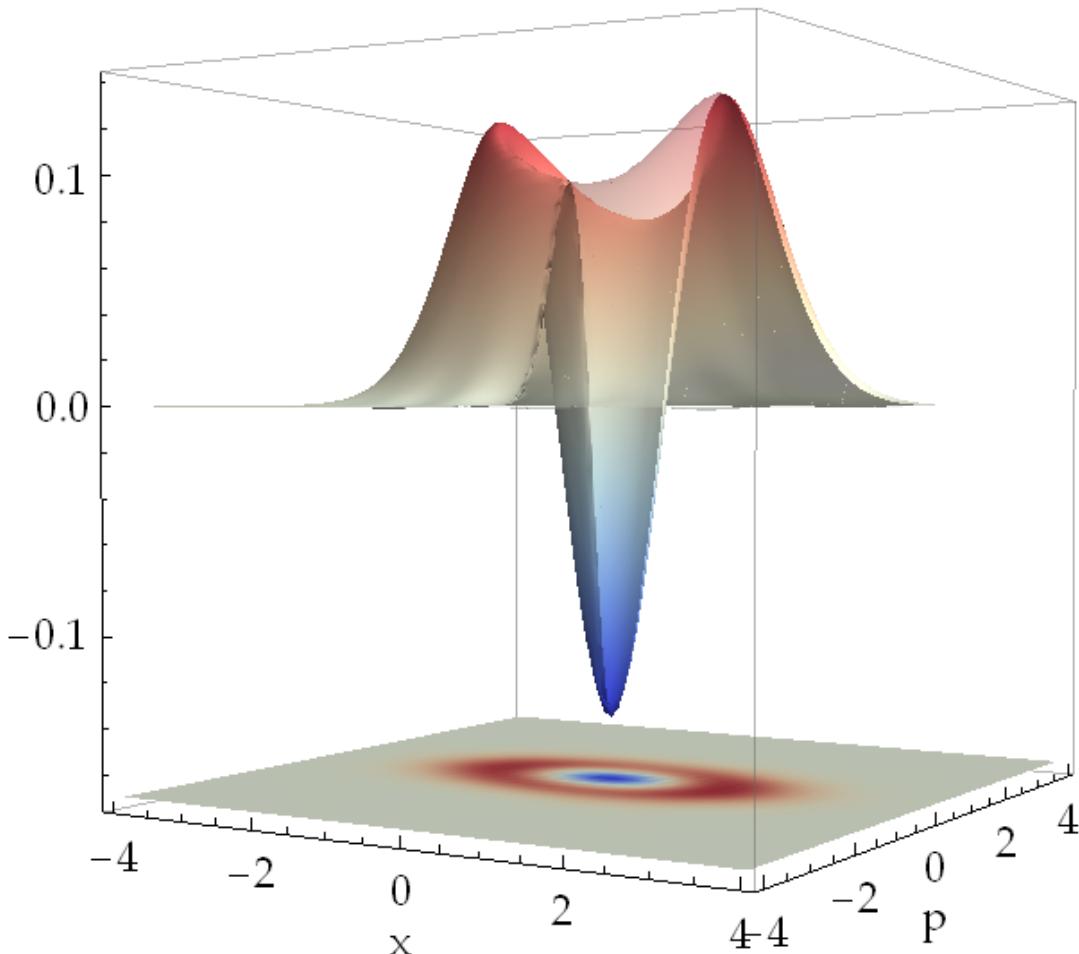
$$\hat{X}_\phi \equiv \int dt \psi(t - t_1) \hat{X}_\phi(t)$$



Mølmer, PRA73, 063804 (2006).
Sasaki & Suzuki, PRA73, 043807 (2006).
Nielsen & Mølmer PRA 75, 023806 (2007).

The most non-classical optical state ever observed directly without any correction of detection inefficiency

$W(0,0) = -0.142$



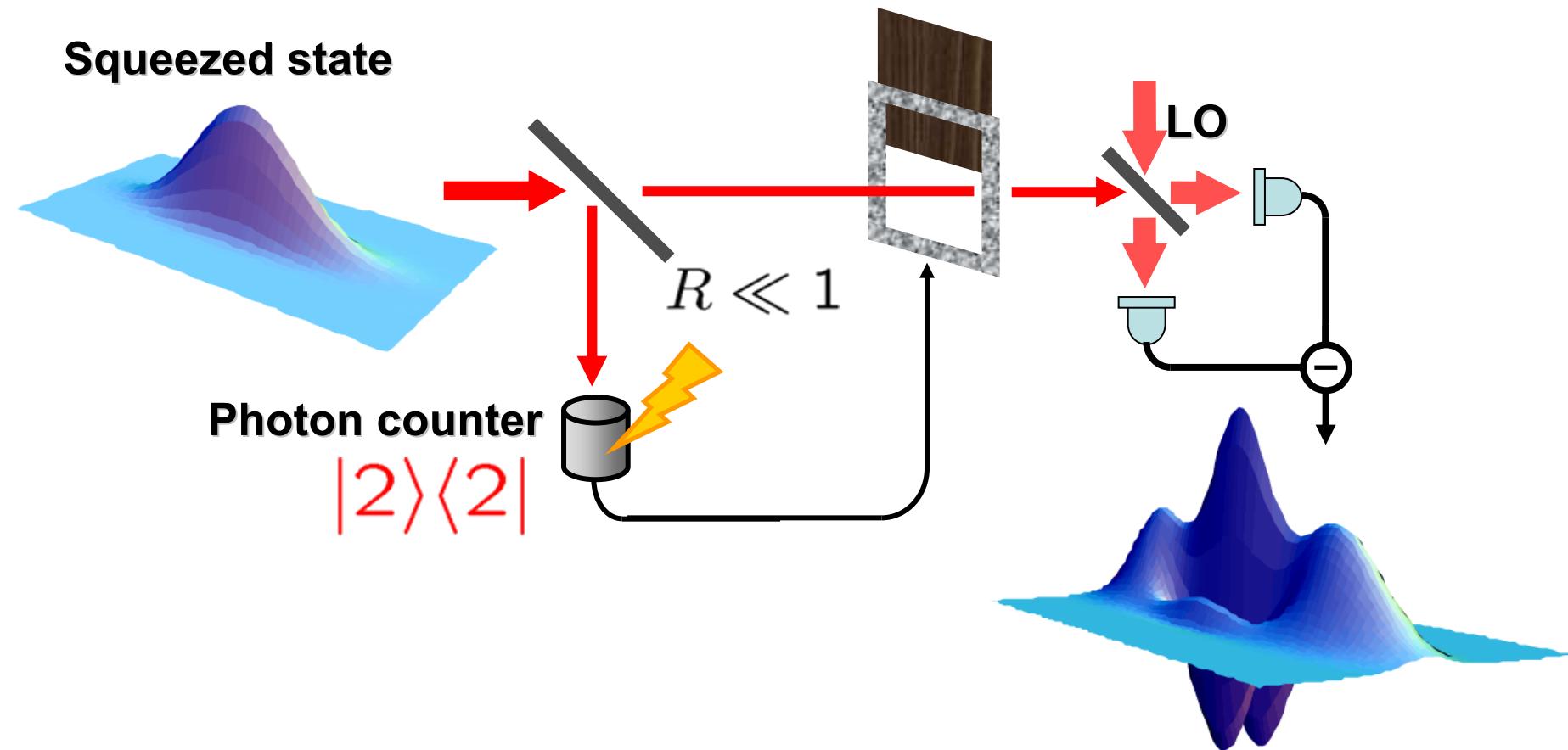
After optimizing

- electrical filters
- a way of coupling between the detectors and the sampling oscilloscope,

Two photon subtraction

$$\hat{a}^2 \hat{S} |0\rangle \sim |\alpha\rangle + |-\alpha\rangle$$

Squeezed state



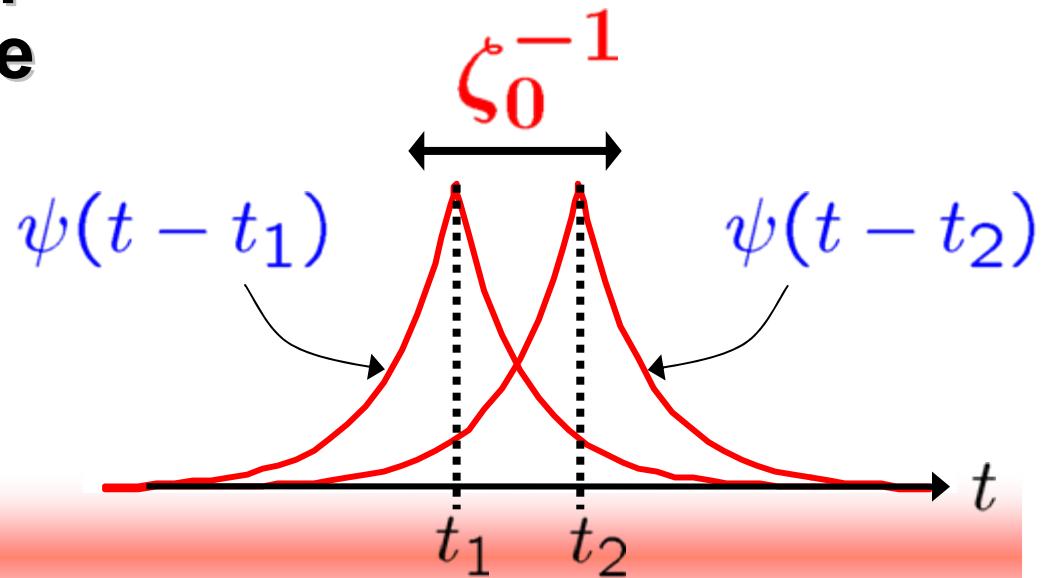
Even-number Schrödinger kitten state

Two-photon subtraction in the CW scheme

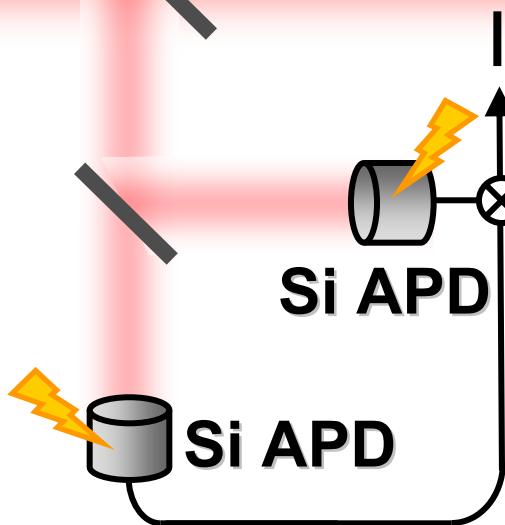
Time-separated photon subtraction **within** a coherence time



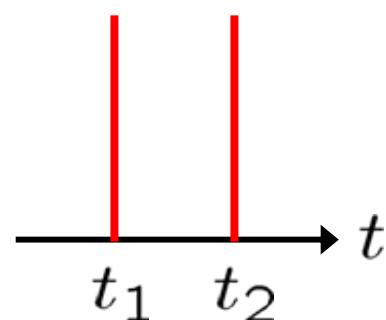
Two modes



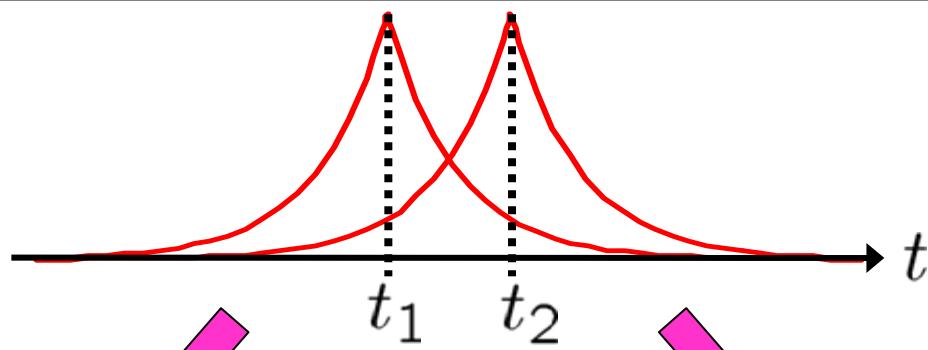
$\hat{S}|0_A\rangle$



Photon clicks

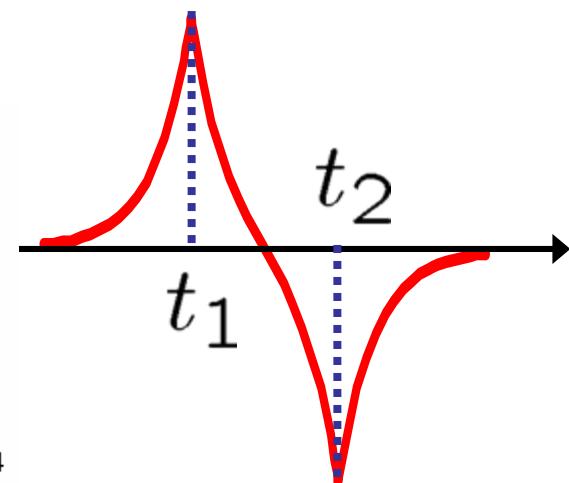
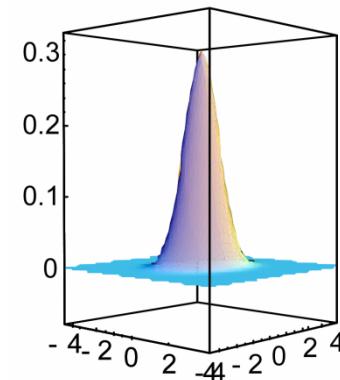
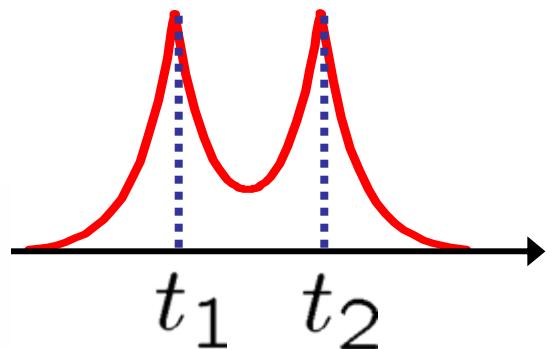
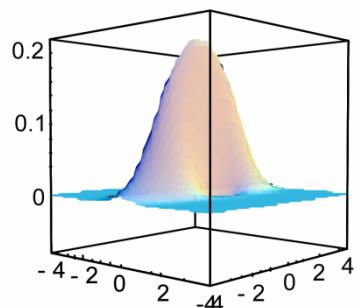


Two orthonormal modes in the main beam



$$\hat{A}_+ \equiv \int dt \Psi_+(t) \hat{a}(t)$$

$$\hat{A}_- \equiv \int dt \Psi_-(t) \hat{a}(t)$$

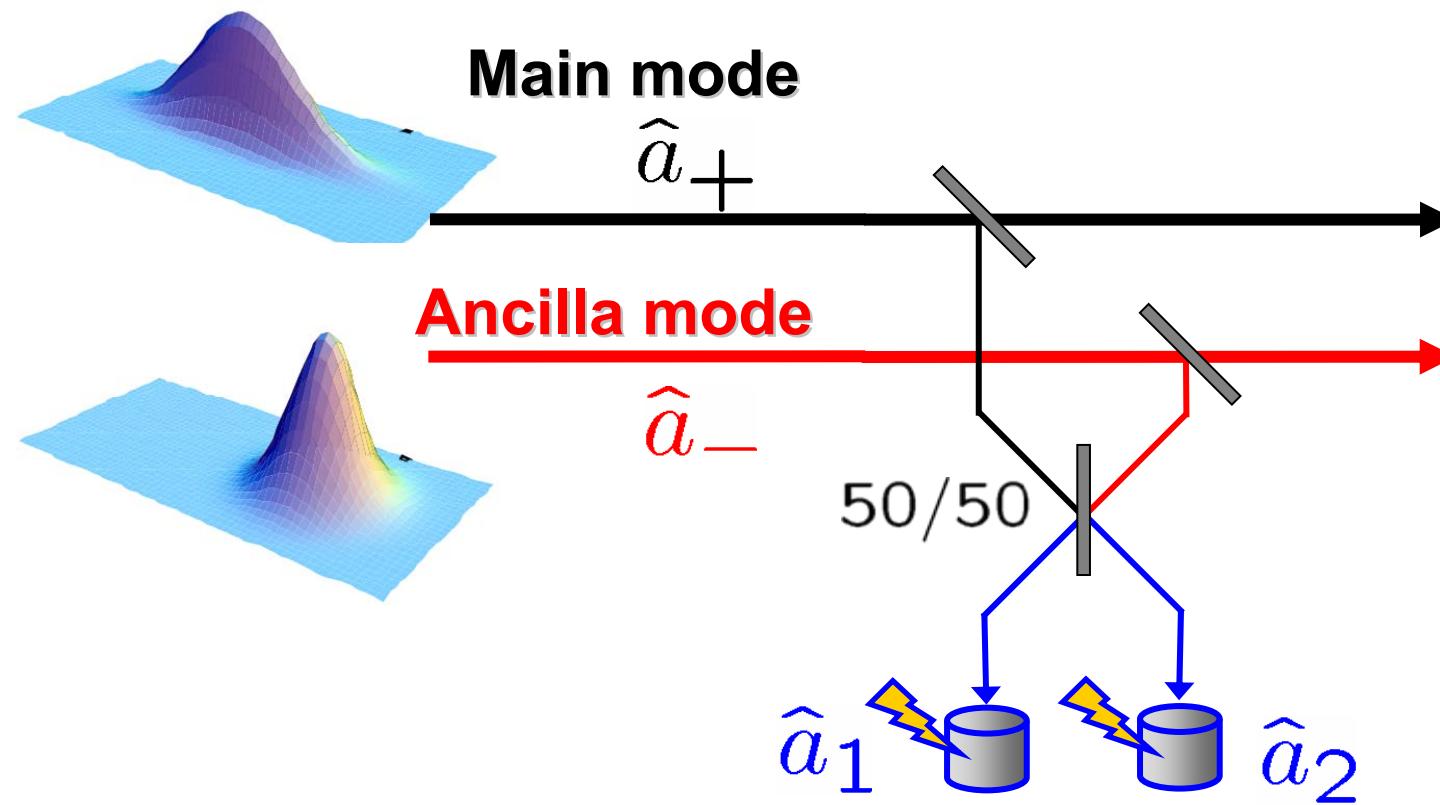


Contains most of photons.

~ the vacuum state.

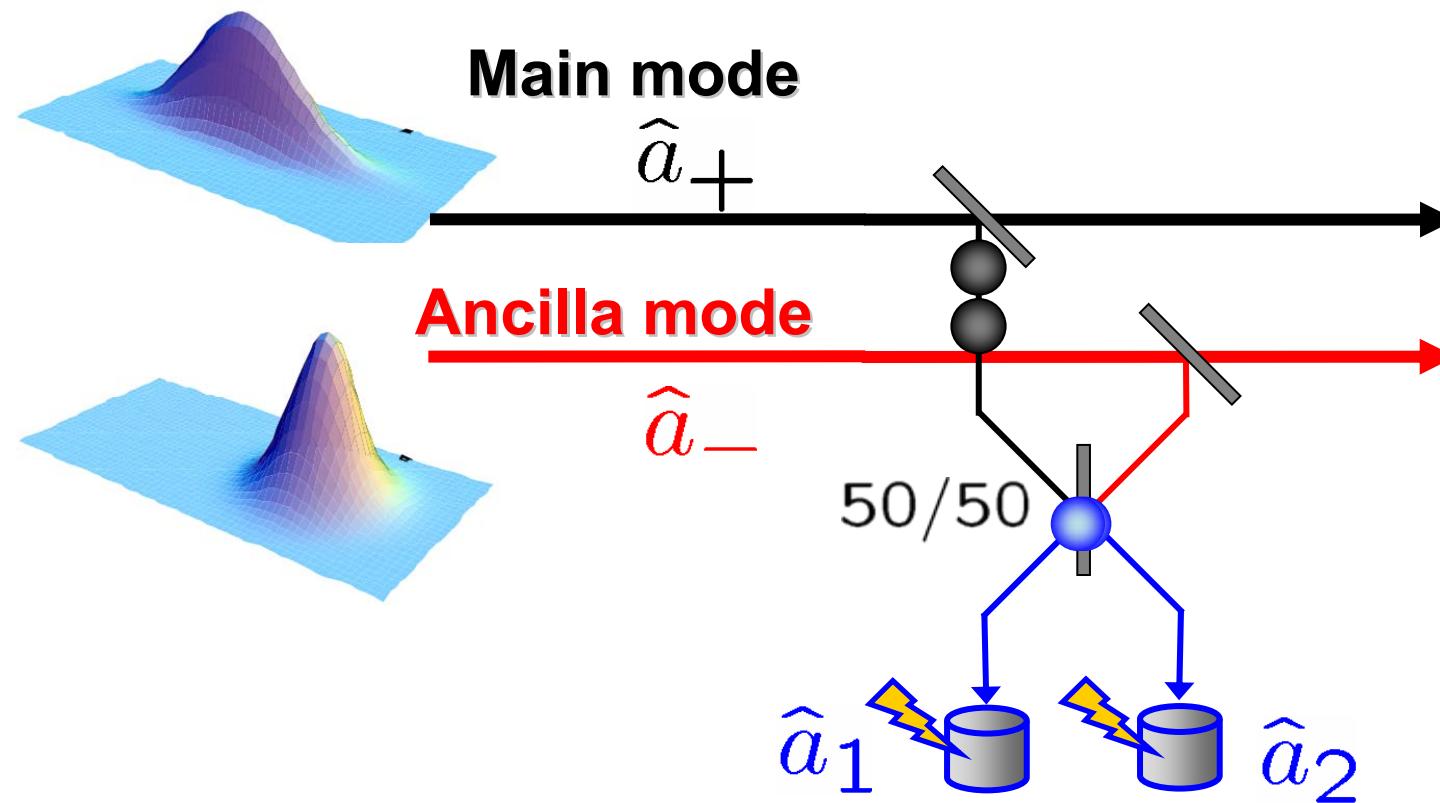
Indistinguishable photon subtraction from the two modes in the extended space

$$\hat{a}_2 \hat{a}_1 |0_{\text{sq+}}\rangle |\alpha_-\rangle$$



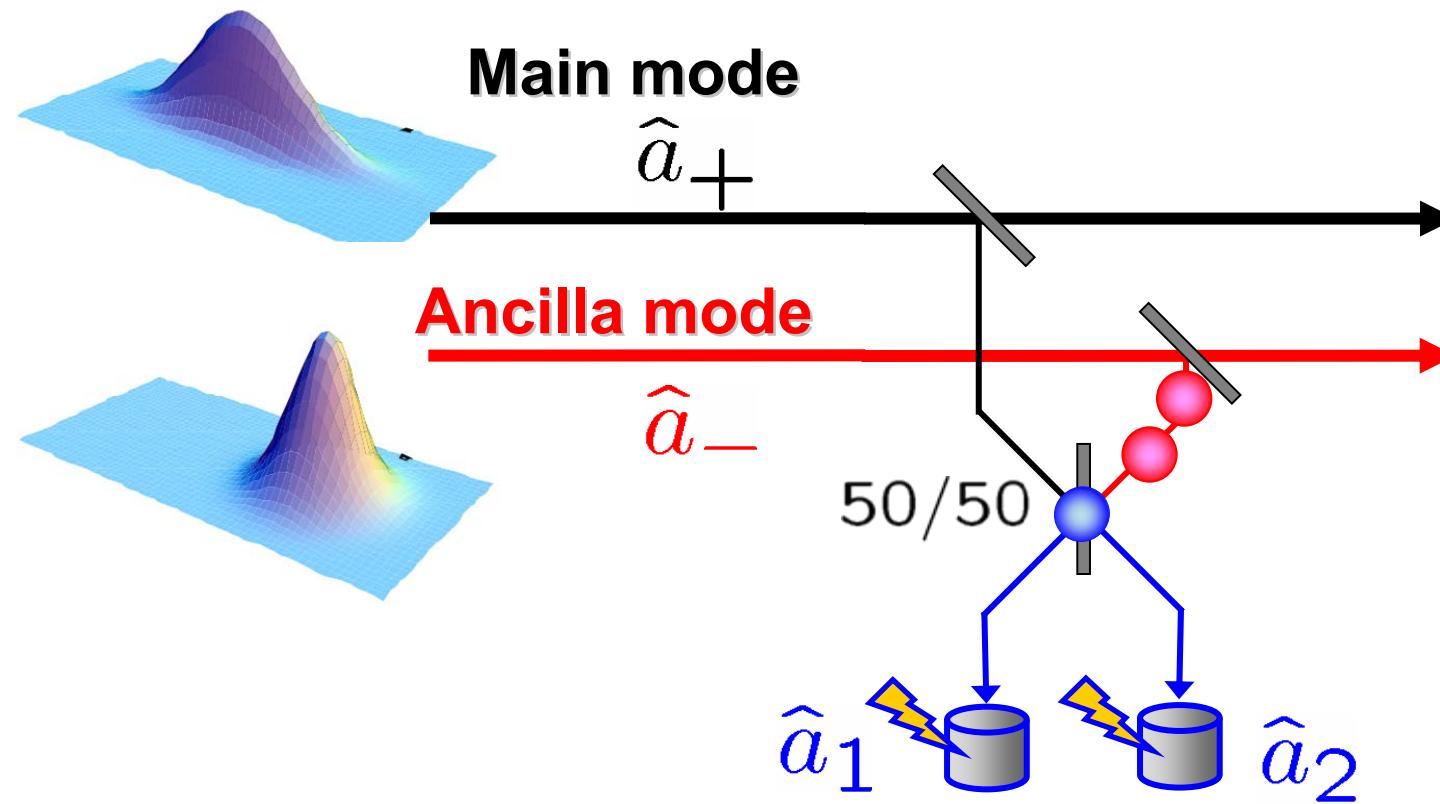
Indistinguishable photon subtraction from the two modes in the extended space

$$\hat{a}_2 \hat{a}_1 |0_{\text{sq+}}\rangle |\alpha_-\rangle$$



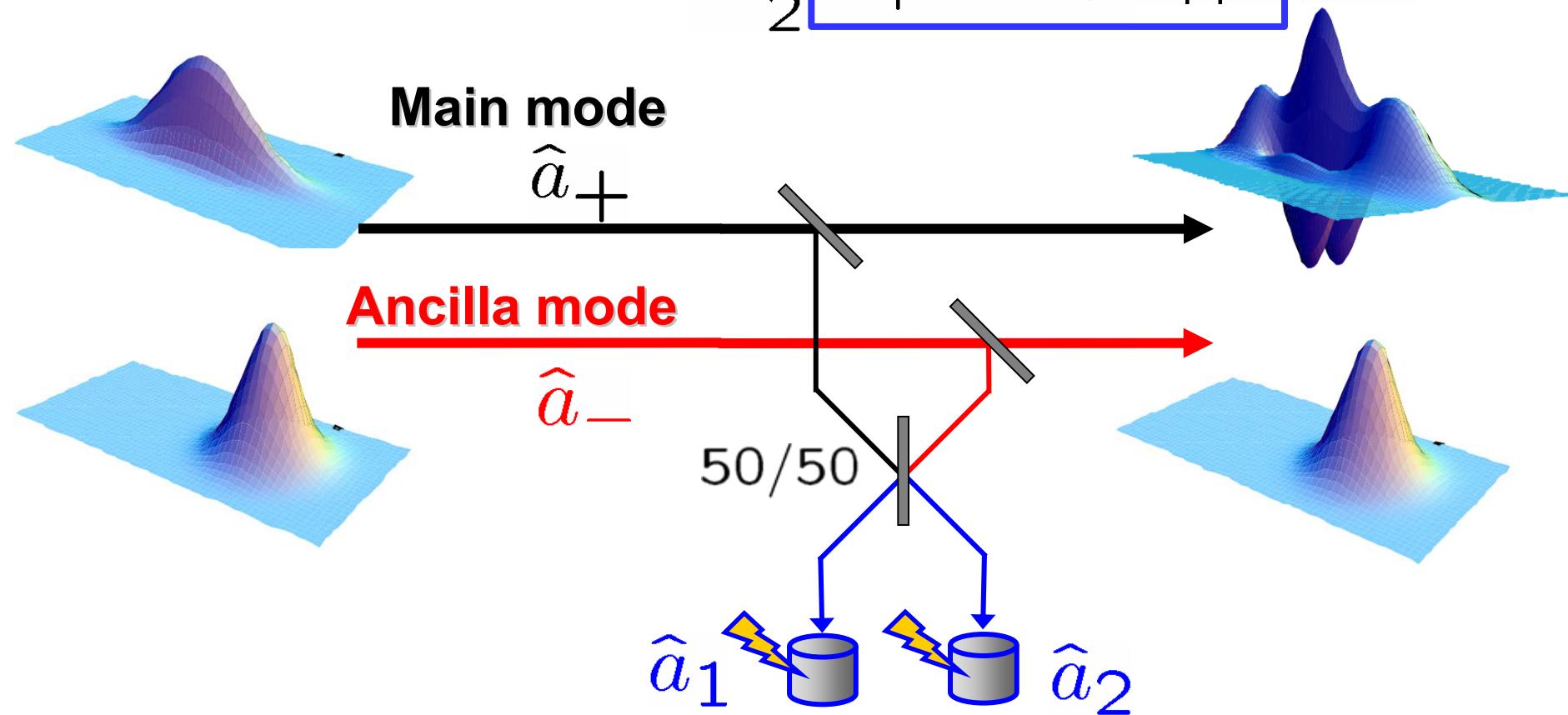
Indistinguishable photon subtraction from the two modes in the extended space

$$\hat{a}_2 \hat{a}_1 |0_{\text{sq}+}\rangle |\alpha_-\rangle$$



Indistinguishable photon subtraction from the two modes in the extended space

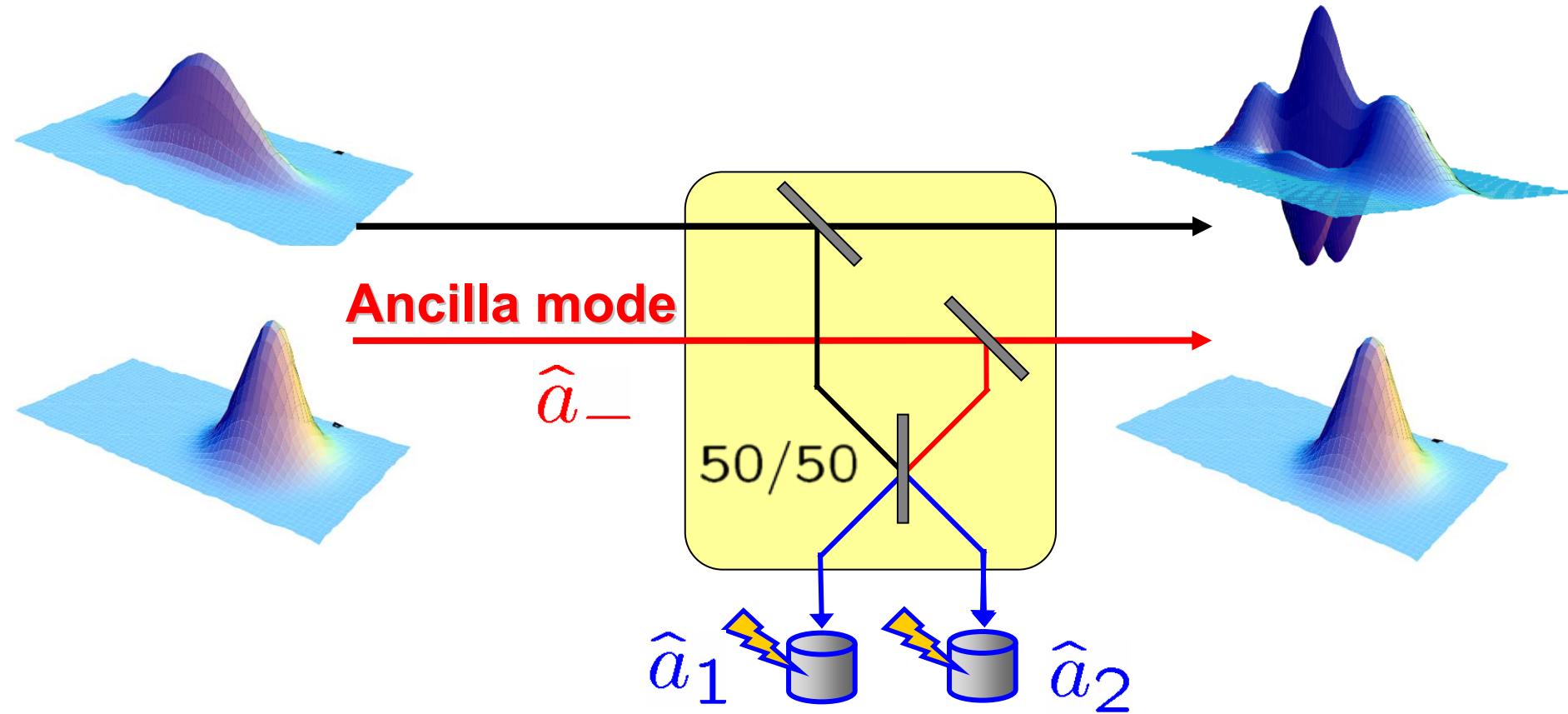
$$\begin{aligned}\hat{a}_2 \hat{a}_1 |0_{\text{sq}+}\rangle |\alpha_-\rangle &= \frac{1}{2} (\hat{a}_+^2 - \hat{a}_-^2) |0_{\text{sq}+}\rangle |\alpha_-\rangle \\ &= \frac{1}{2} (\hat{a}_+^2 - \alpha_-^2) |0_{\text{sq}+}\rangle |\alpha_-\rangle\end{aligned}$$



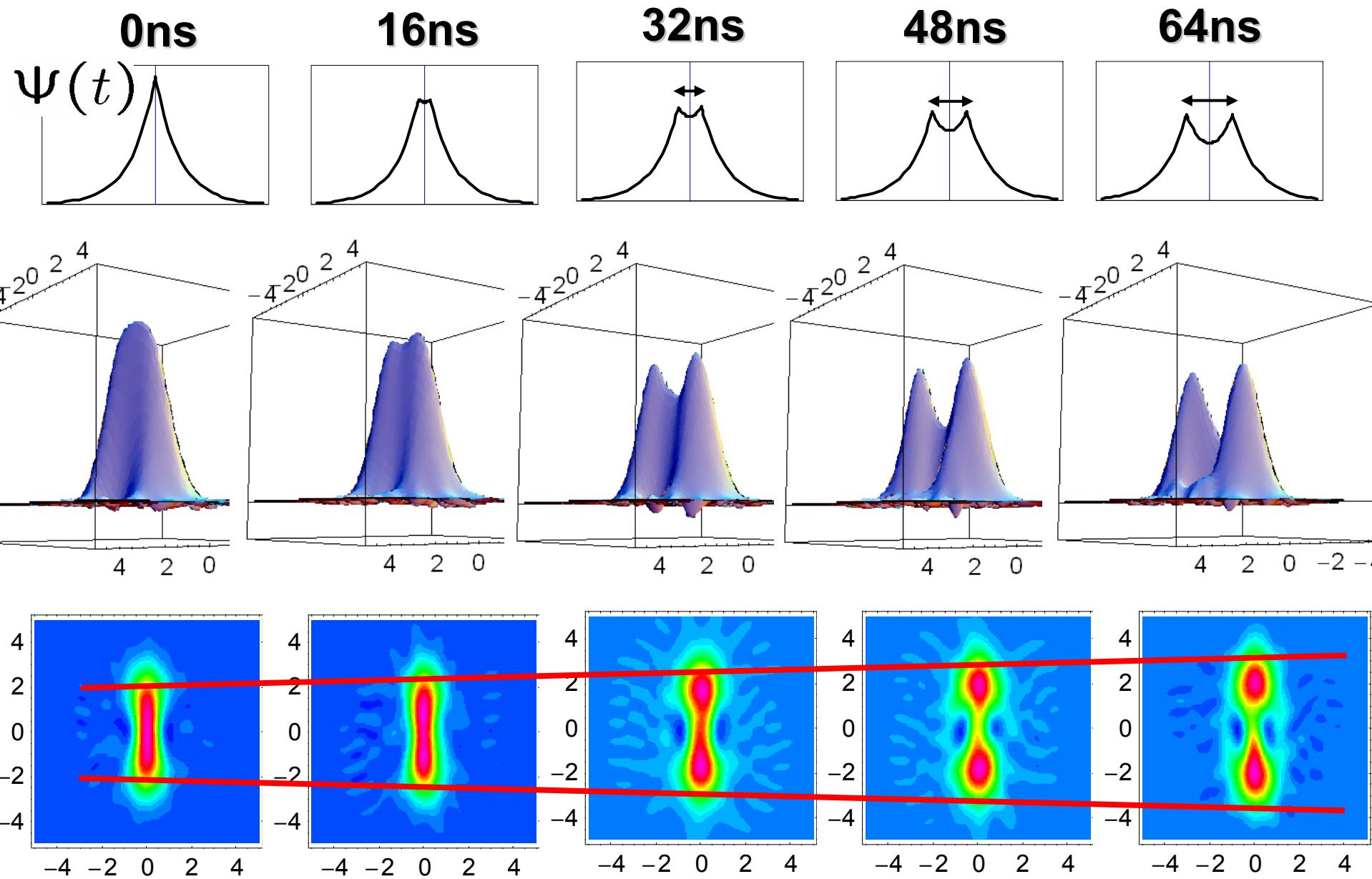
Ancilla-assisted control of cat states

Takeoka, et.al. PRA 77, 062315 (2008).

Sasaki, et.al. PRA 77, 063840 (2008).



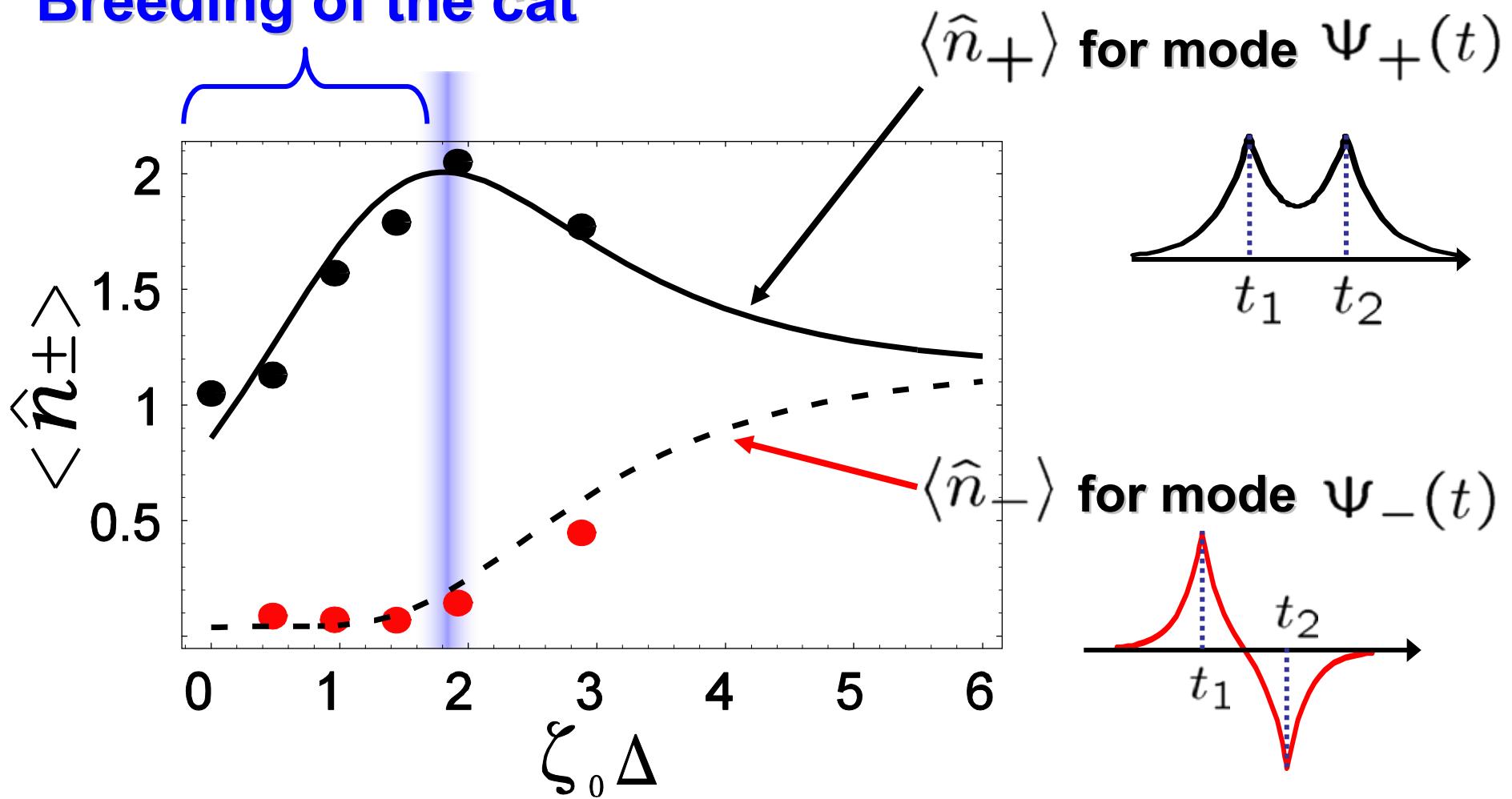
Cat breeding : Takahashi, et al. PRL101, 233605 (2008).

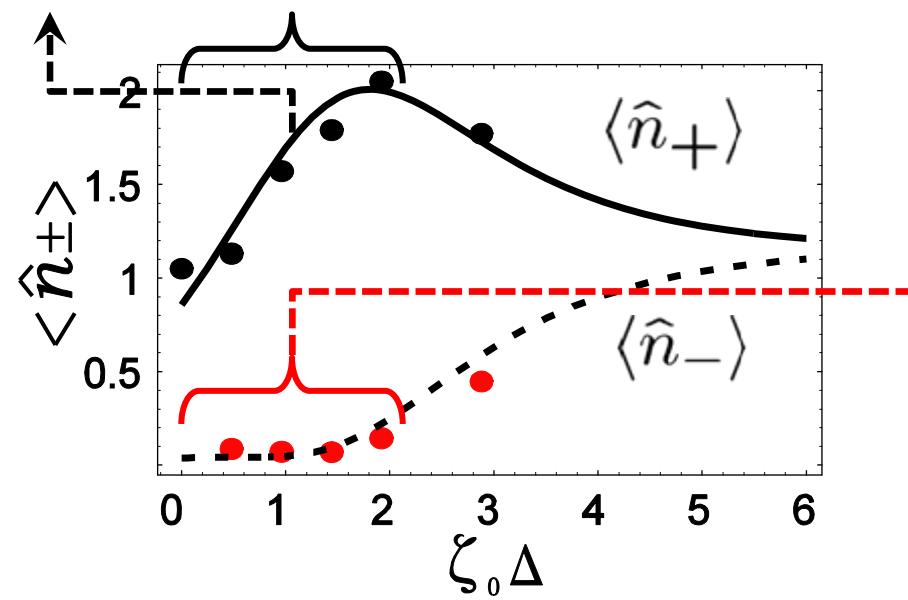
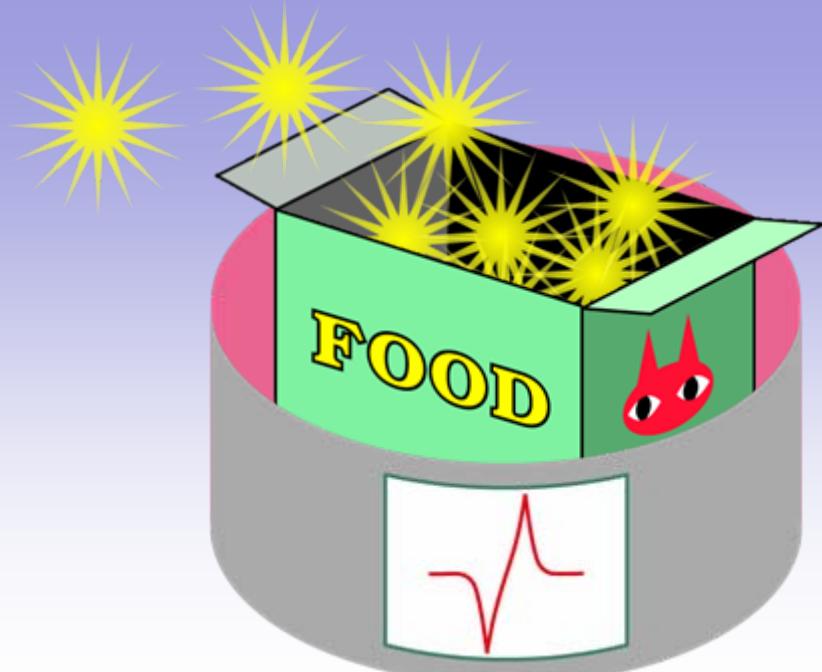
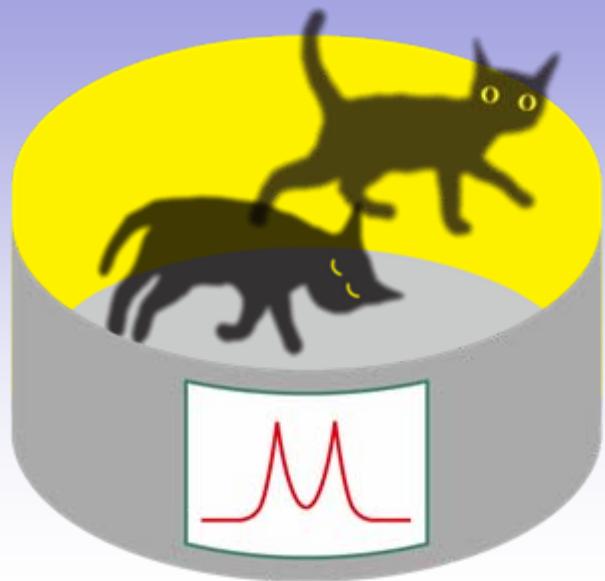


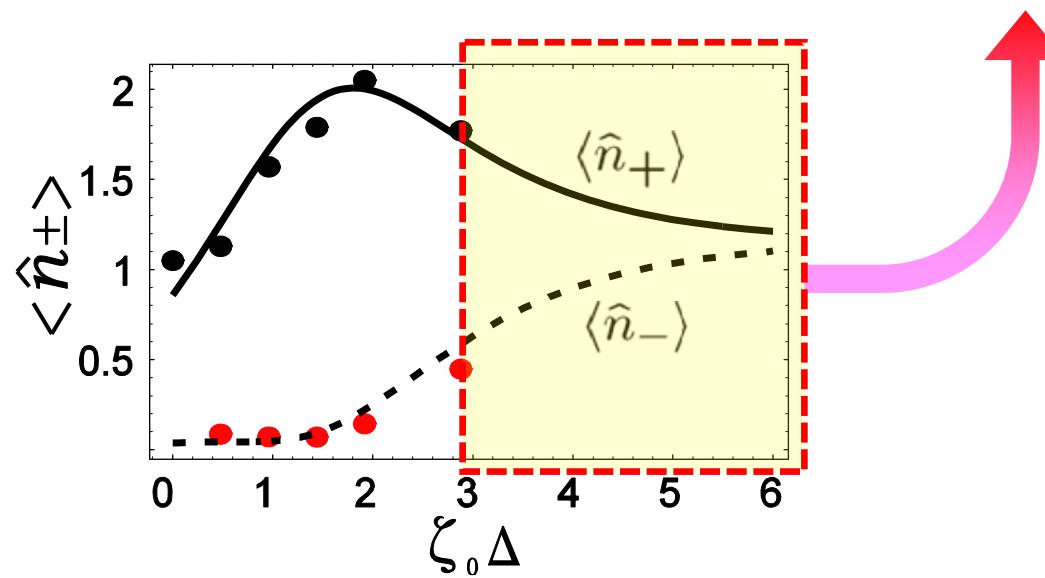
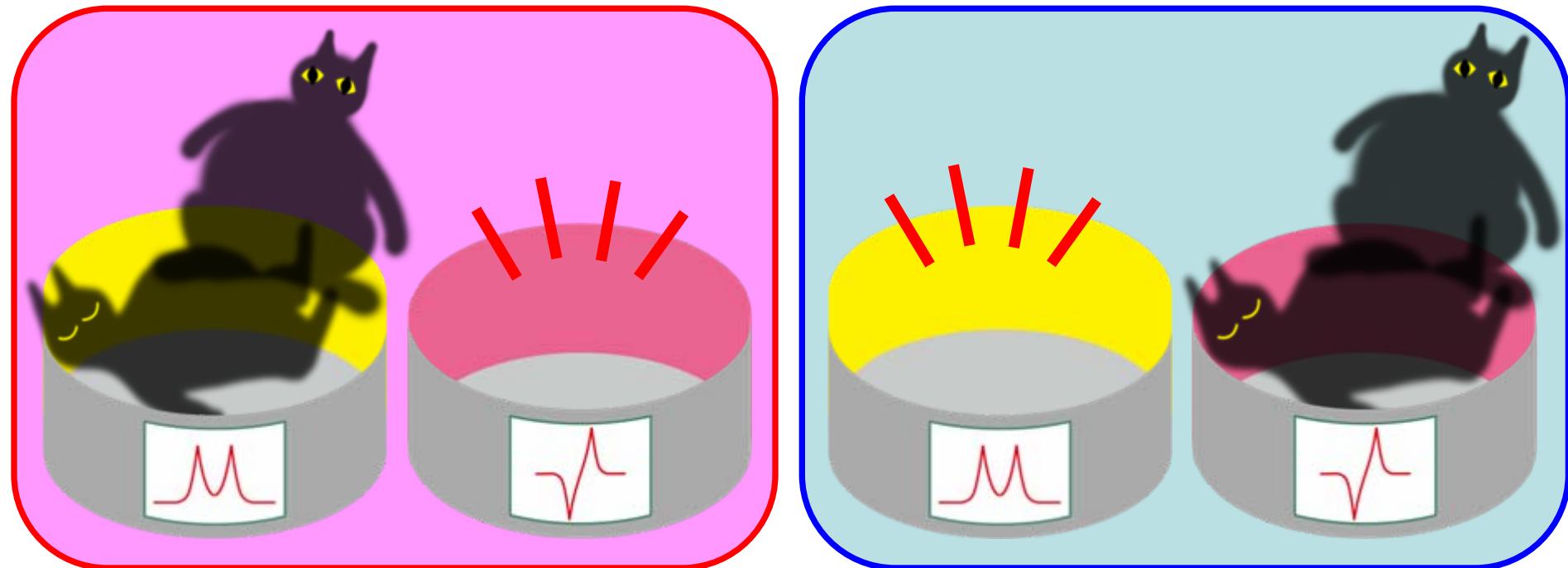
Average photon numbers

Sasaki, et.al.
PRA 77, 063840 (2008).

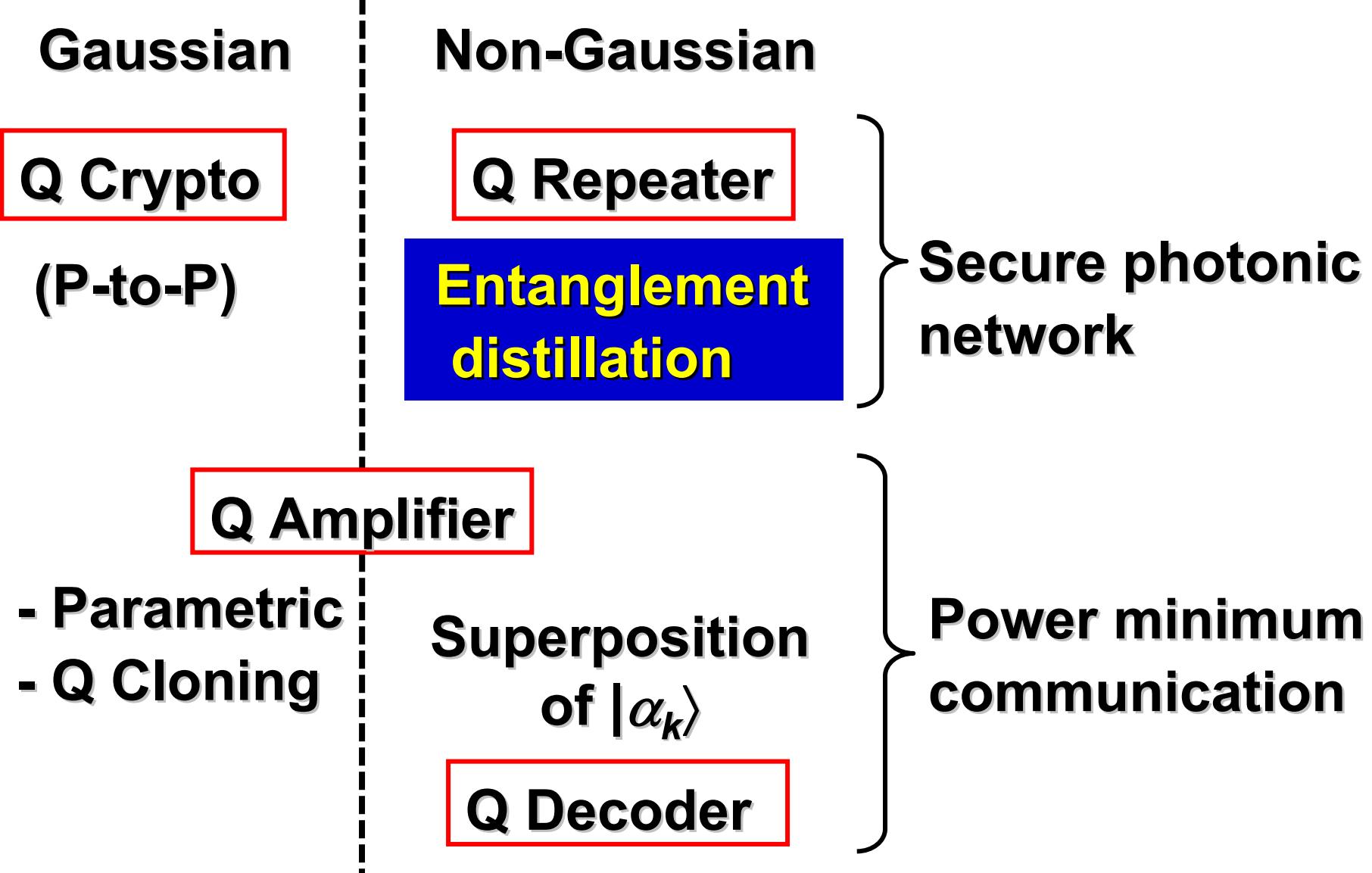
Breeding of the cat





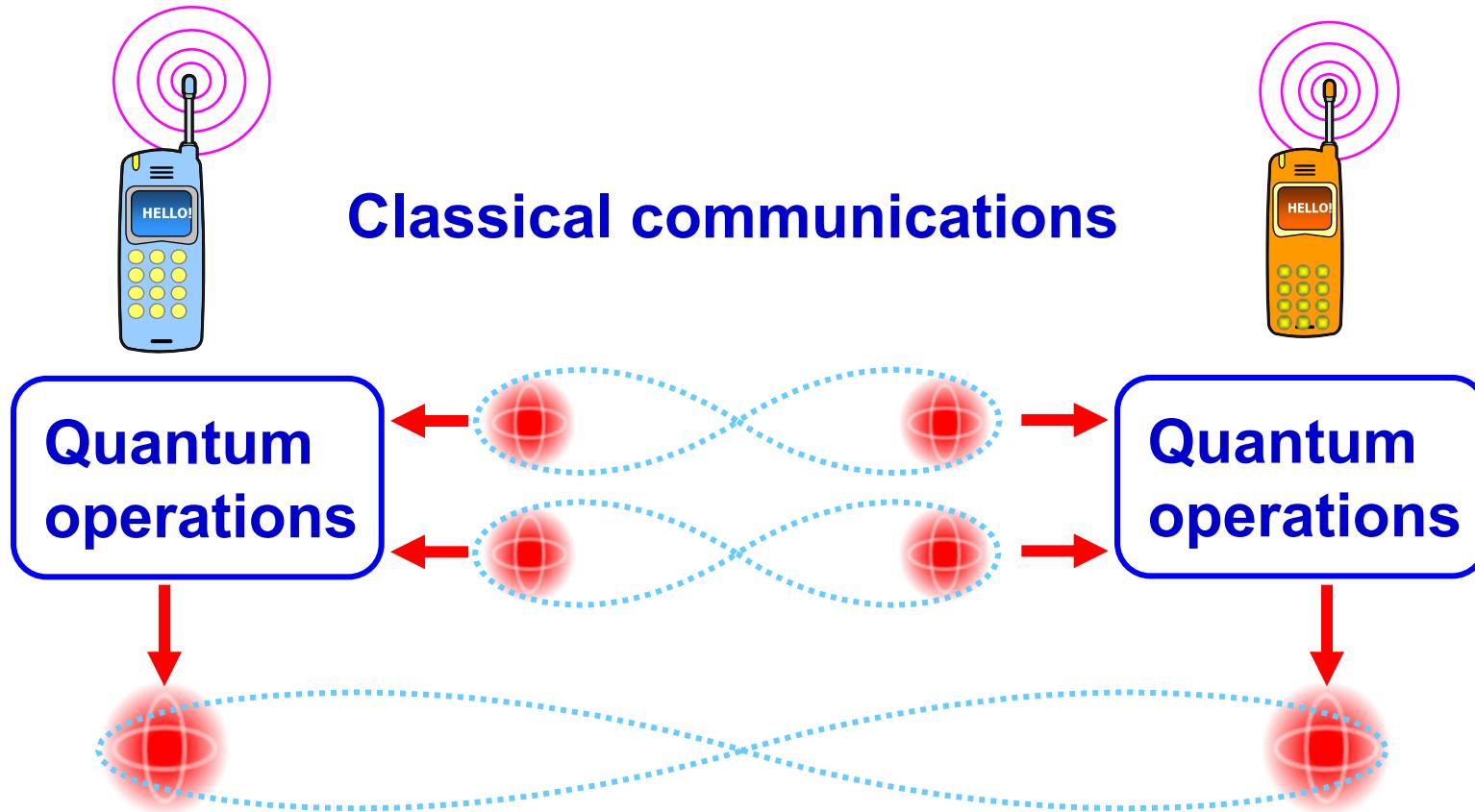
$|\text{even cat}\rangle_+ |\text{0}\rangle_- + |\text{0}\rangle_+ |\text{even cat}\rangle_-$ 

Quantum control of CVs



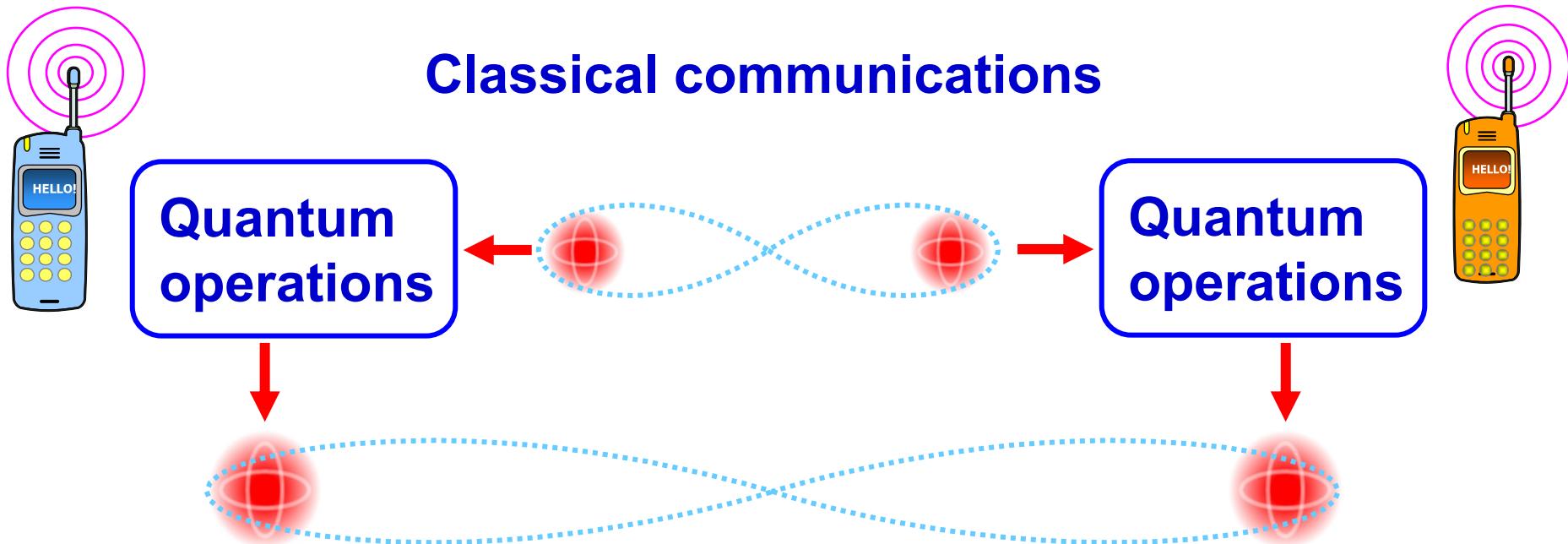
Entanglement distillation

- Two parties are separated in space,
- Q. operations are allowed only at each local node, while two parties can communicate classically.



Given a single pair of entangled state

- The entanglement can never be increased deterministically using only LOCC.
- One can, however, increase the entanglement probabilistically by a non-unitary filtering.



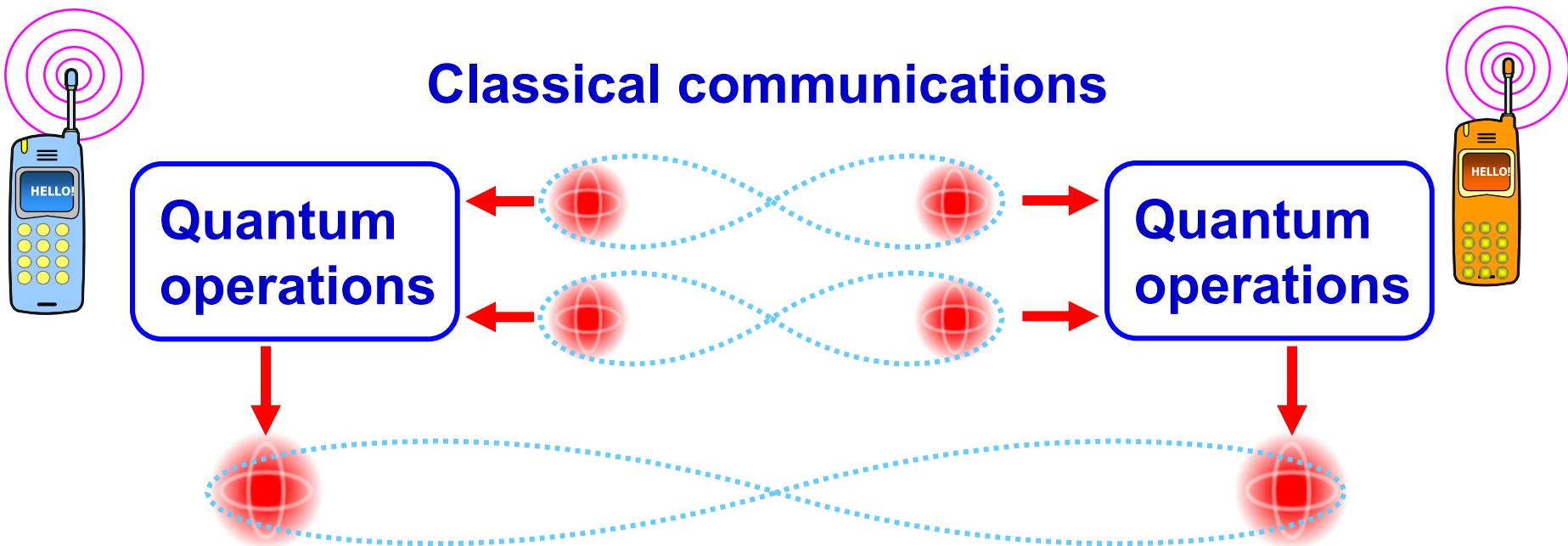
Qubit system : Kwiat et. al. Nature, 409, 1014 (2001).

Entanglement distillation from multiple pairs

Demonstrated with qubit systems,

- Pan et. al. Nature, 423, 417 (2003)
- Yamamoto et. al. Nature 421, 343 (2003)

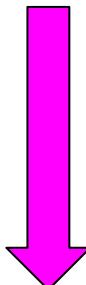
→ A single highly entangled qubit pair was extracted from two less-entangled qubit pairs.



Entanglement distillation for CV states

No-go theorem

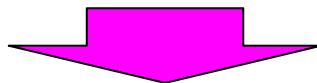
Gaussian LOCC alone can never distill the entanglement from Gaussian inputs even probabilistically.



Eisert, Scheel, & Plenio, PRL 89, 137903 (2002).
Fiurasek, PRL, 89, 137904 (2002)
Giedke and Cirac, PRA, 66, 032316 (2002)

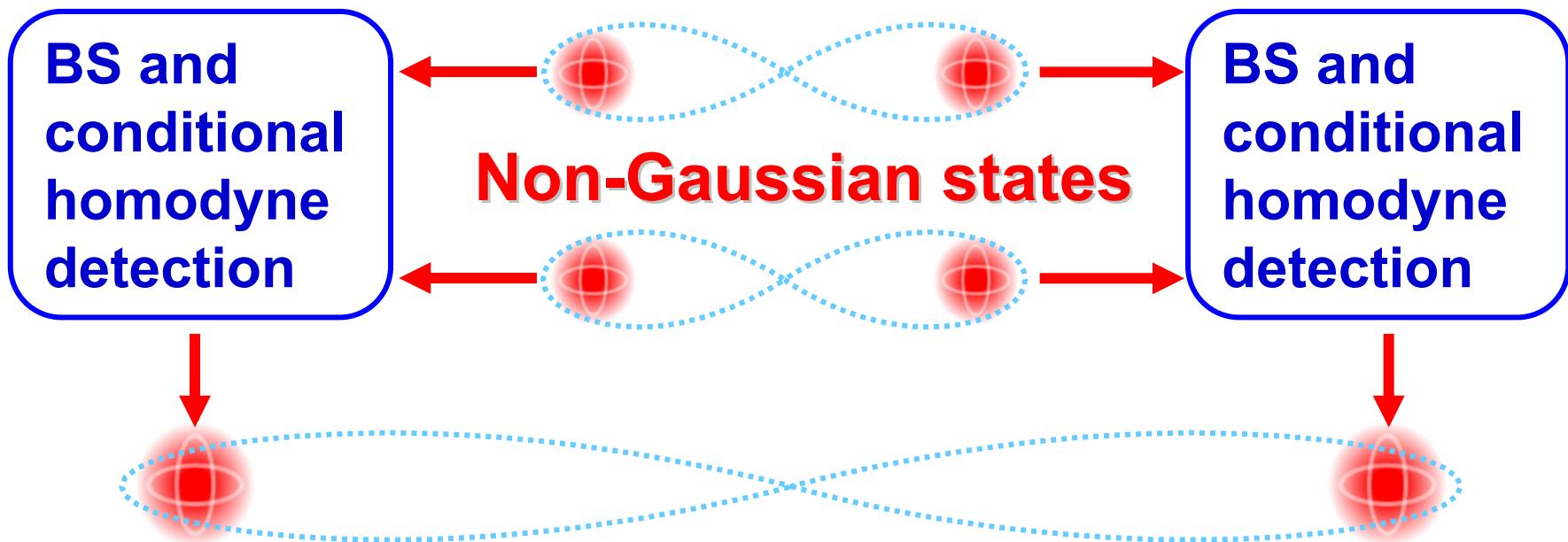
Non-Gaussian operation

If input entangled states are **non-Gaussian states** (phase-diffusion or time-varying attenuation noises), then it is not the case of the no-go theorem.



Well established Gaussian operations could distill the decohered entanglement.

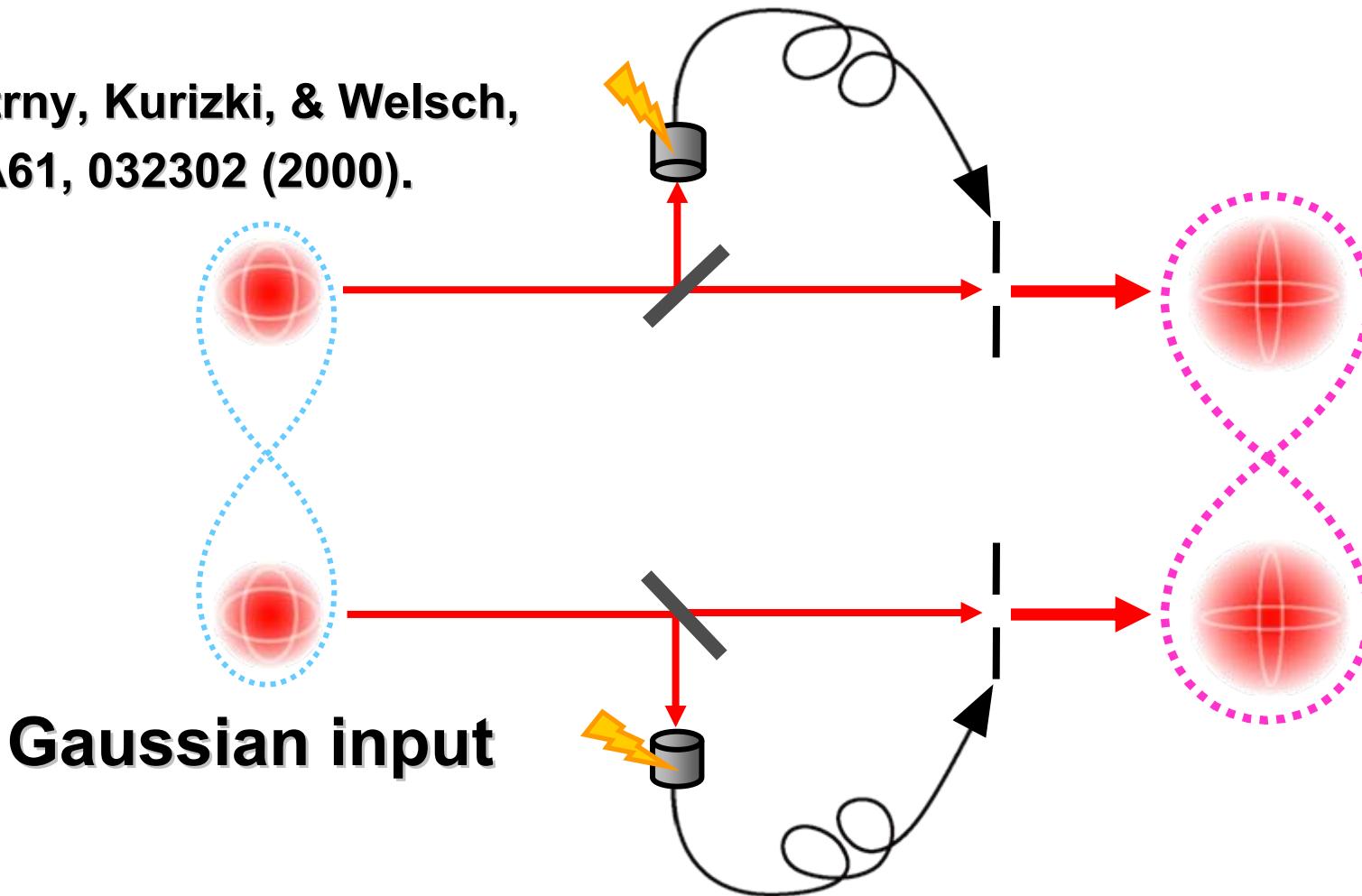
Dong et al., Nature Physics 4, 919 (2008).
Hage et al., Nature Physics 4, 915 (2008).



Entanglement distillation for a Gaussian input to overcome the no-go bound of Gaussian operation

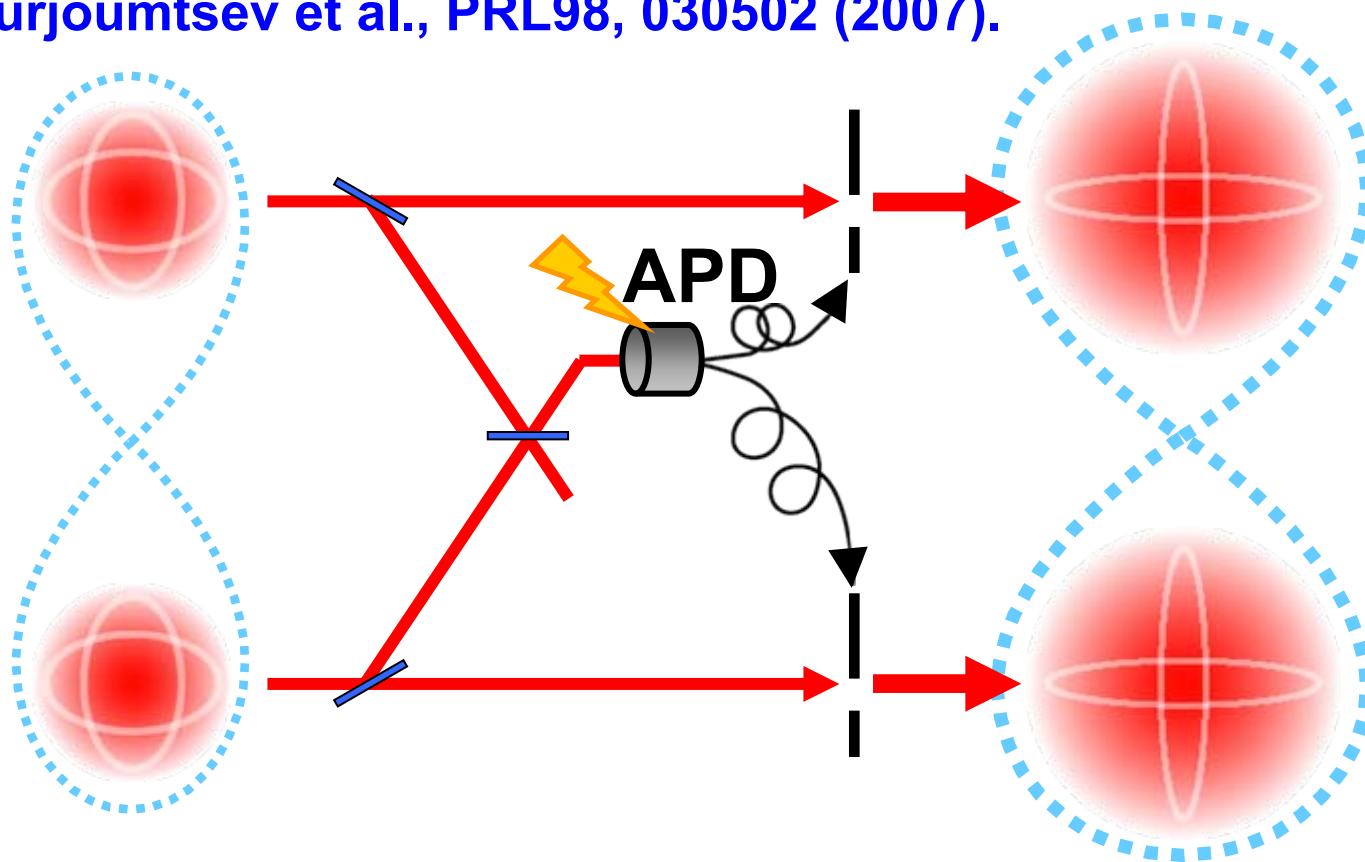
Probabilistic distillation with a single pair

Opatrny, Kurizki, & Welsch,
PRA61, 032302 (2000).



Entanglement enhancement by non-local operation (coherent single-photon subtraction)

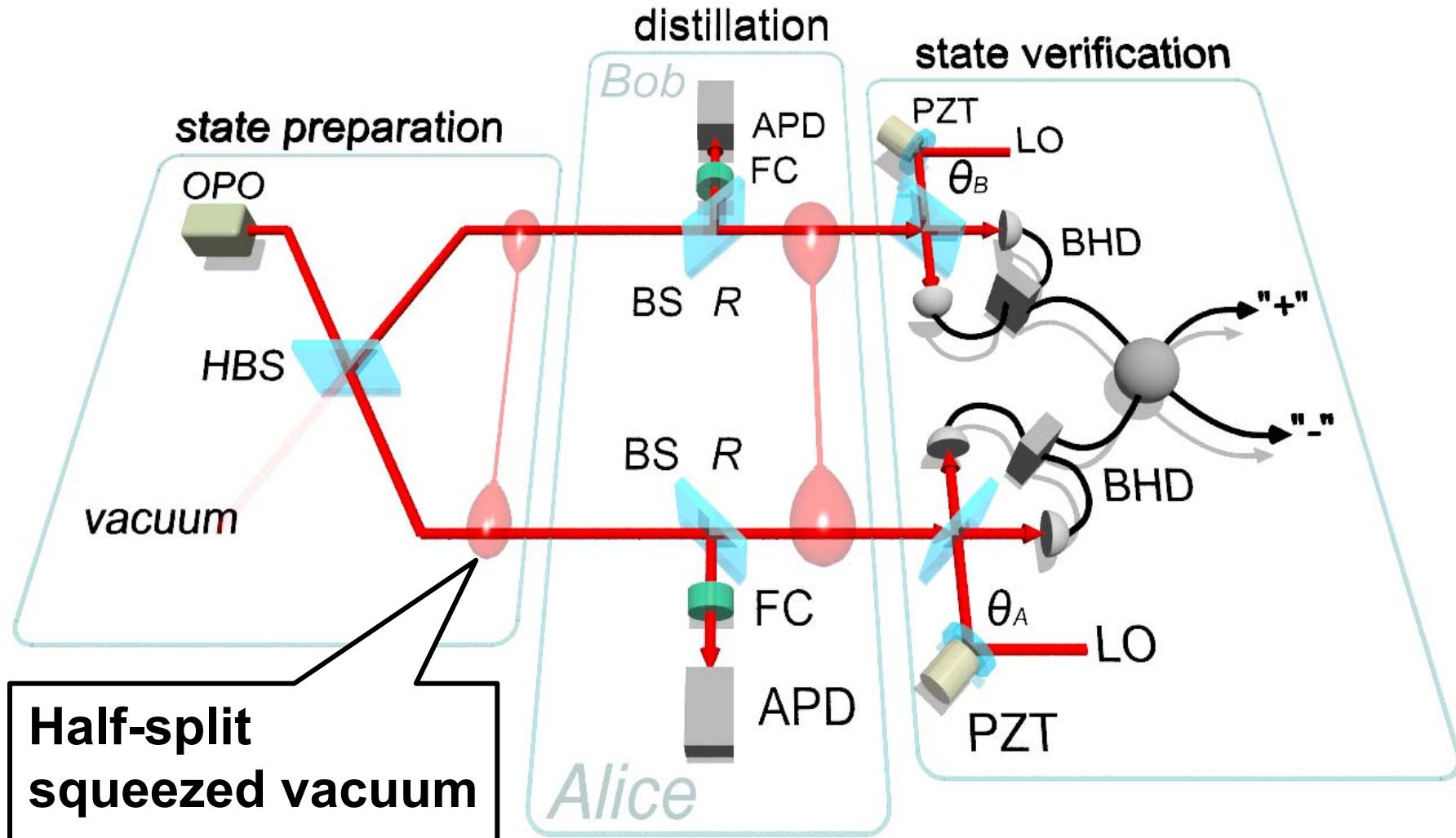
Ourjoumtsev et al., PRL98, 030502 (2007).



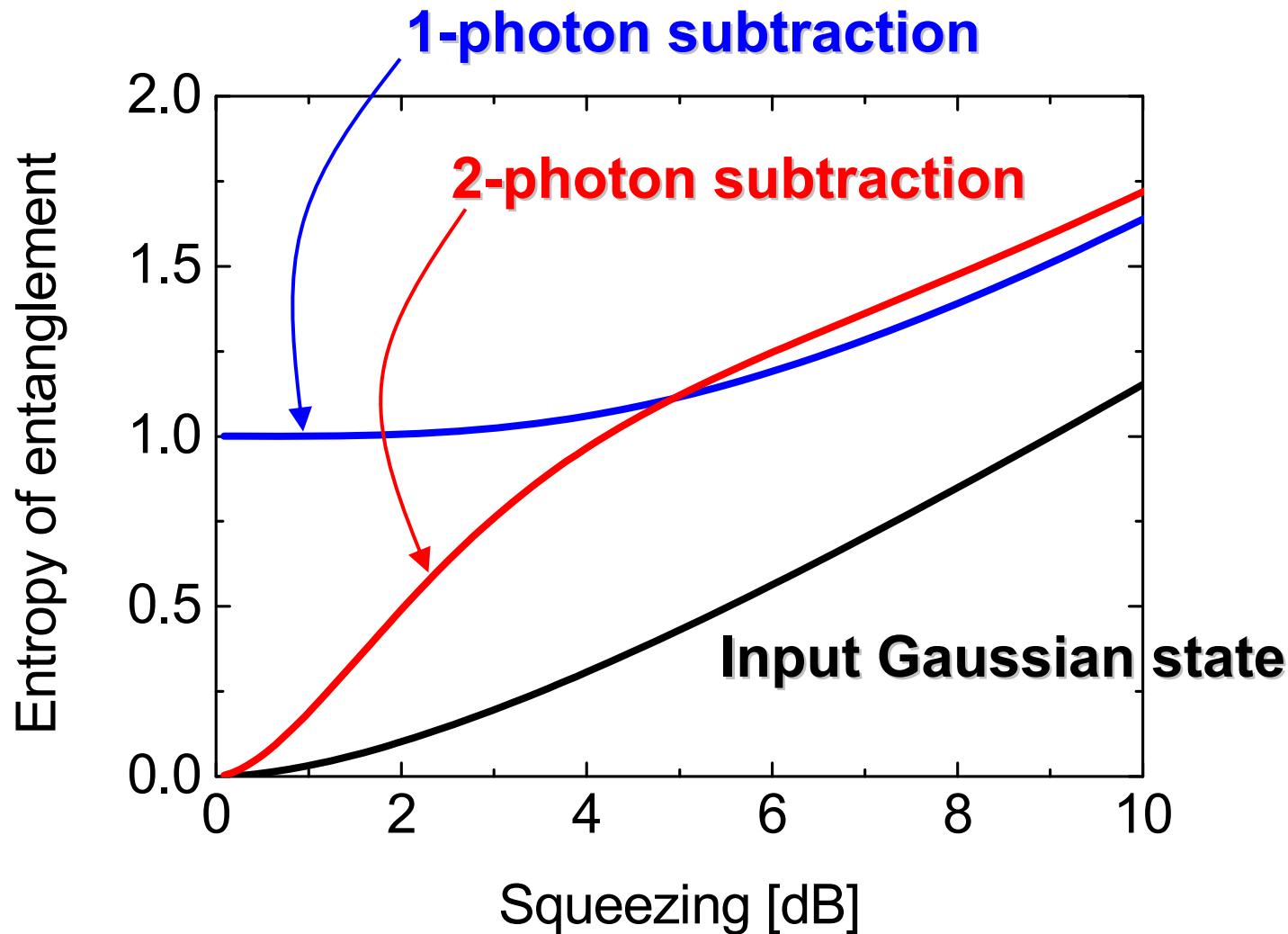
→ Applied to noise-free generation of an entangled cat state from two separable squeezed vacua.
Ourjoumtsev et al., Nature Physics (2009).

Our experiment

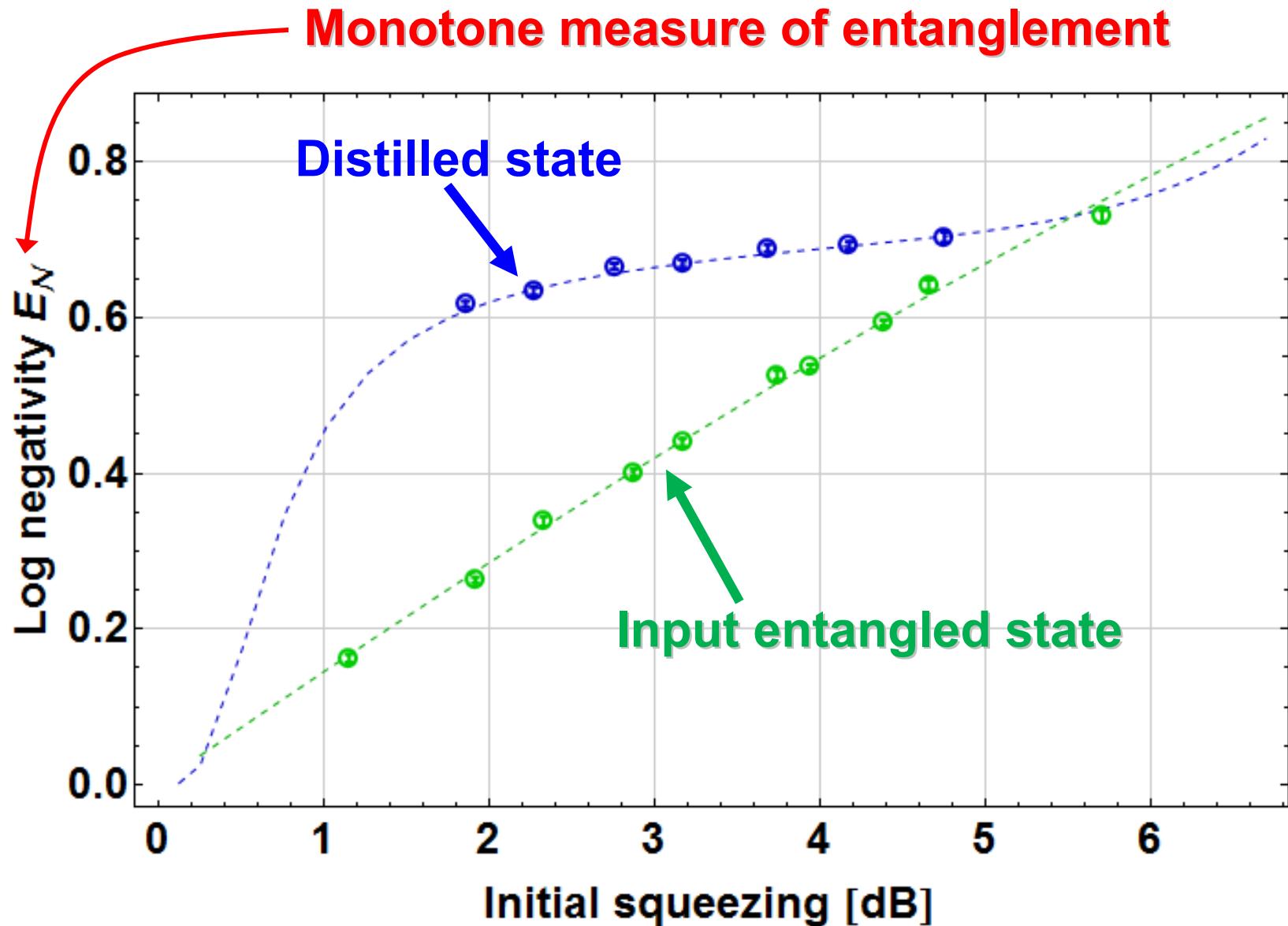
Photon are subtracted at each separated node
“original spirit of entanglement distillation”



Ideal case of half-split squeezed vacuum



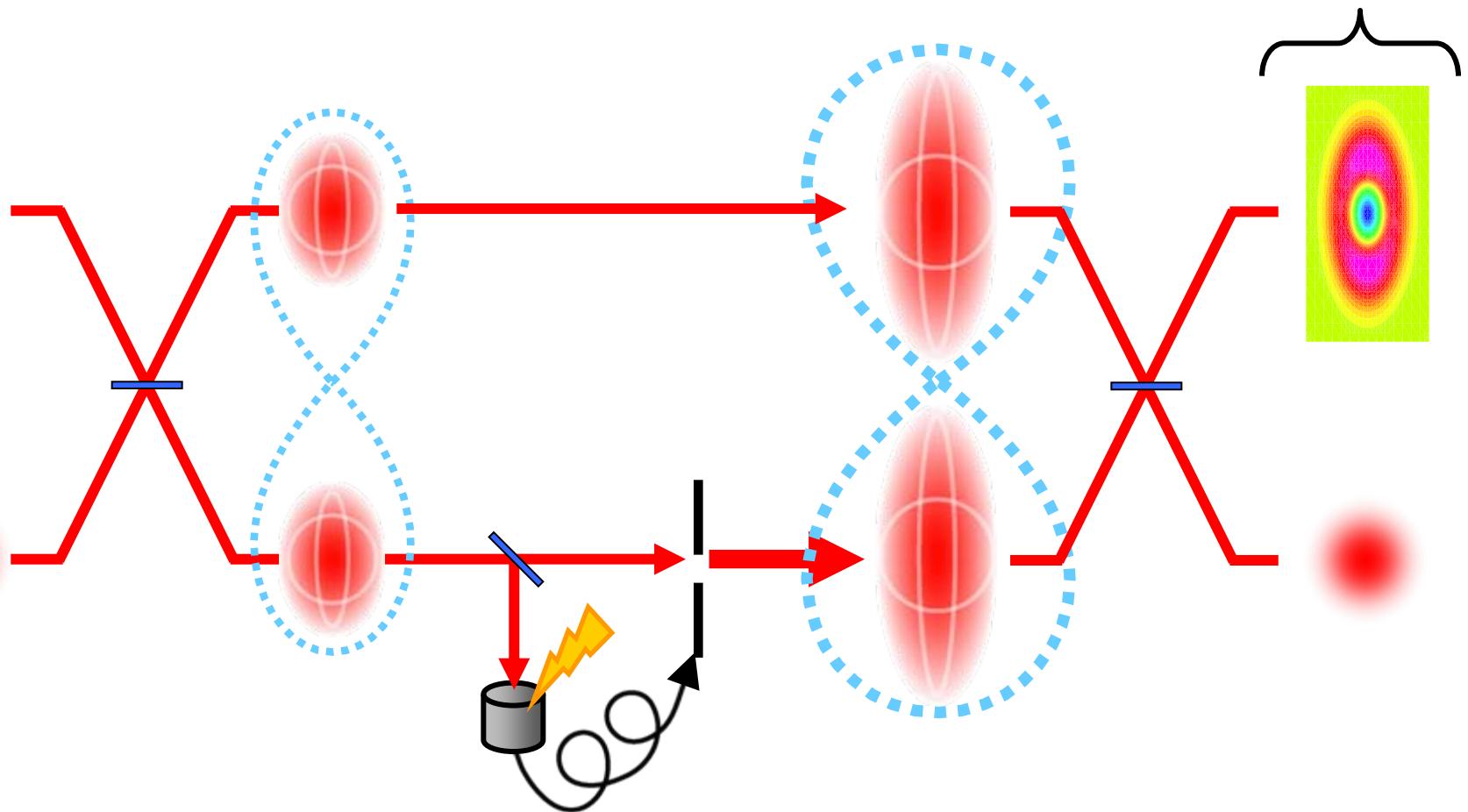
Distillation by single-photon subtraction

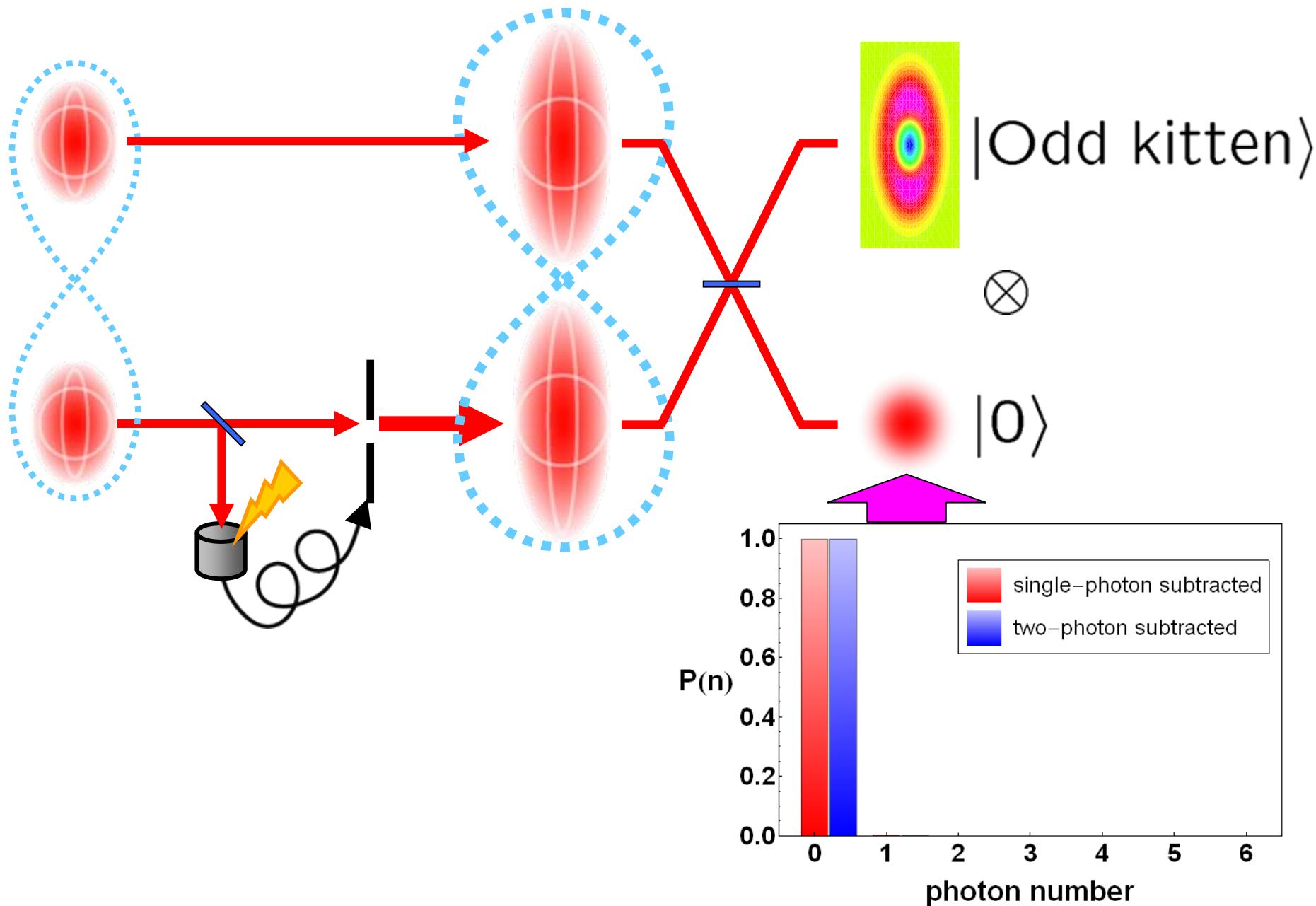


To see the data quality more explicitly

We beam-split the distilled entangled state

$$\rightarrow | \text{Odd kitten} \rangle \otimes | 0 \rangle$$

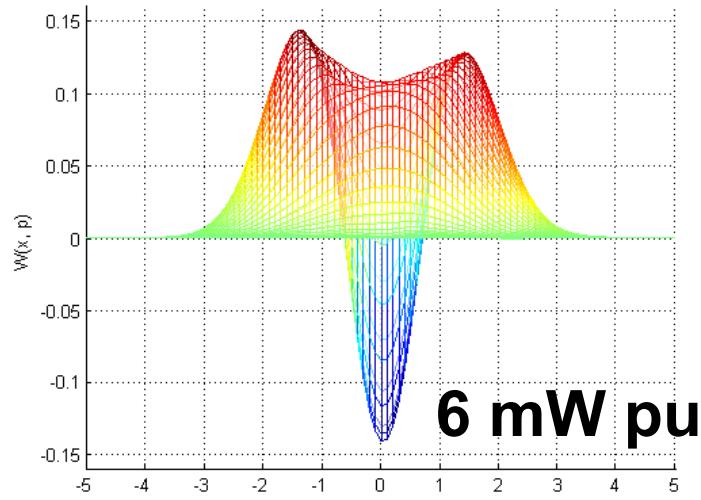




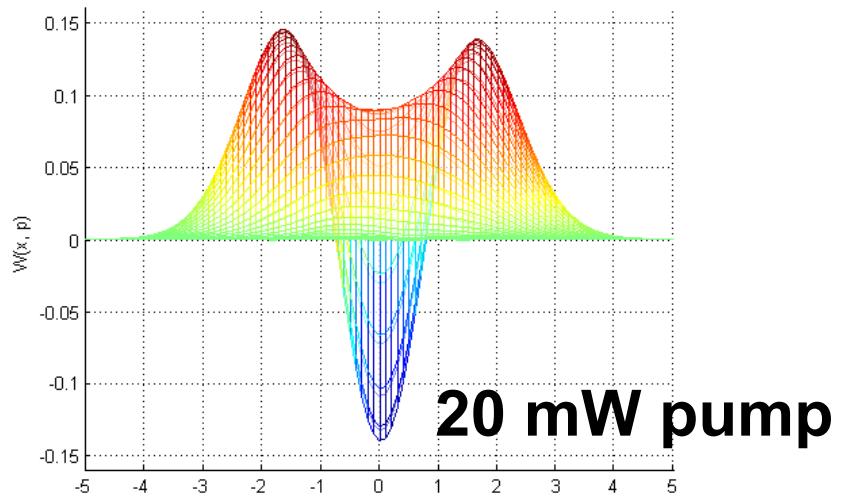
The 2nd port was really in the vacuum state.

Disentangled kitten states

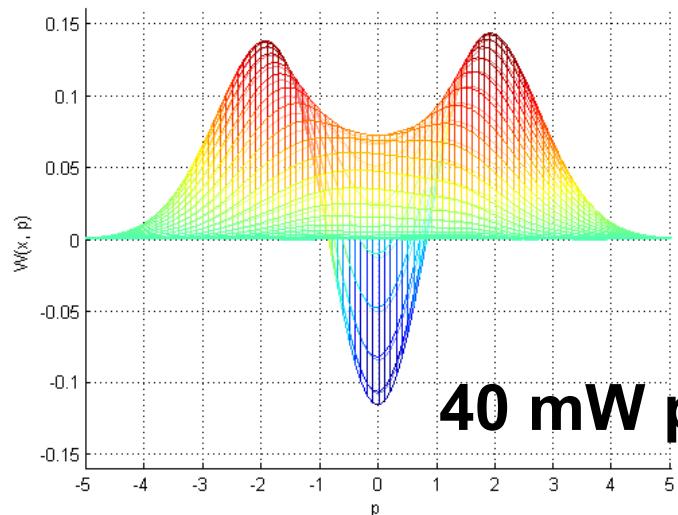
Deeply negative Wigner function (no correction)



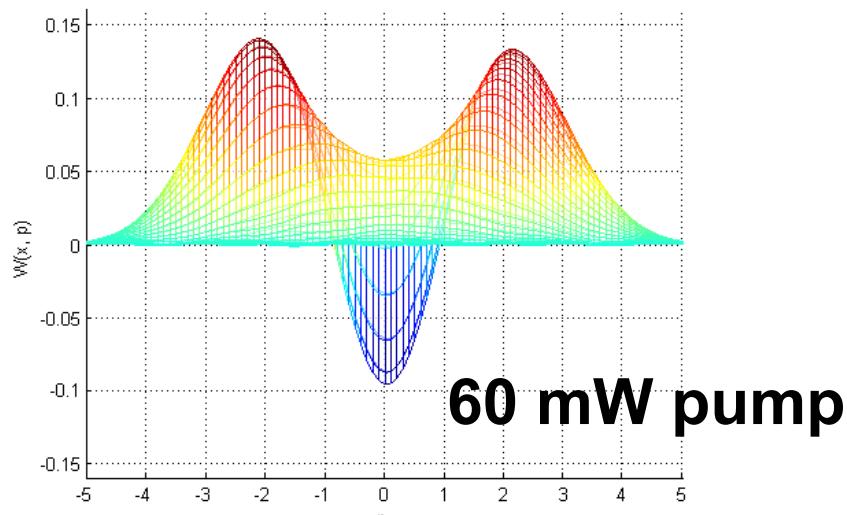
6 mW pump



20 mW pump

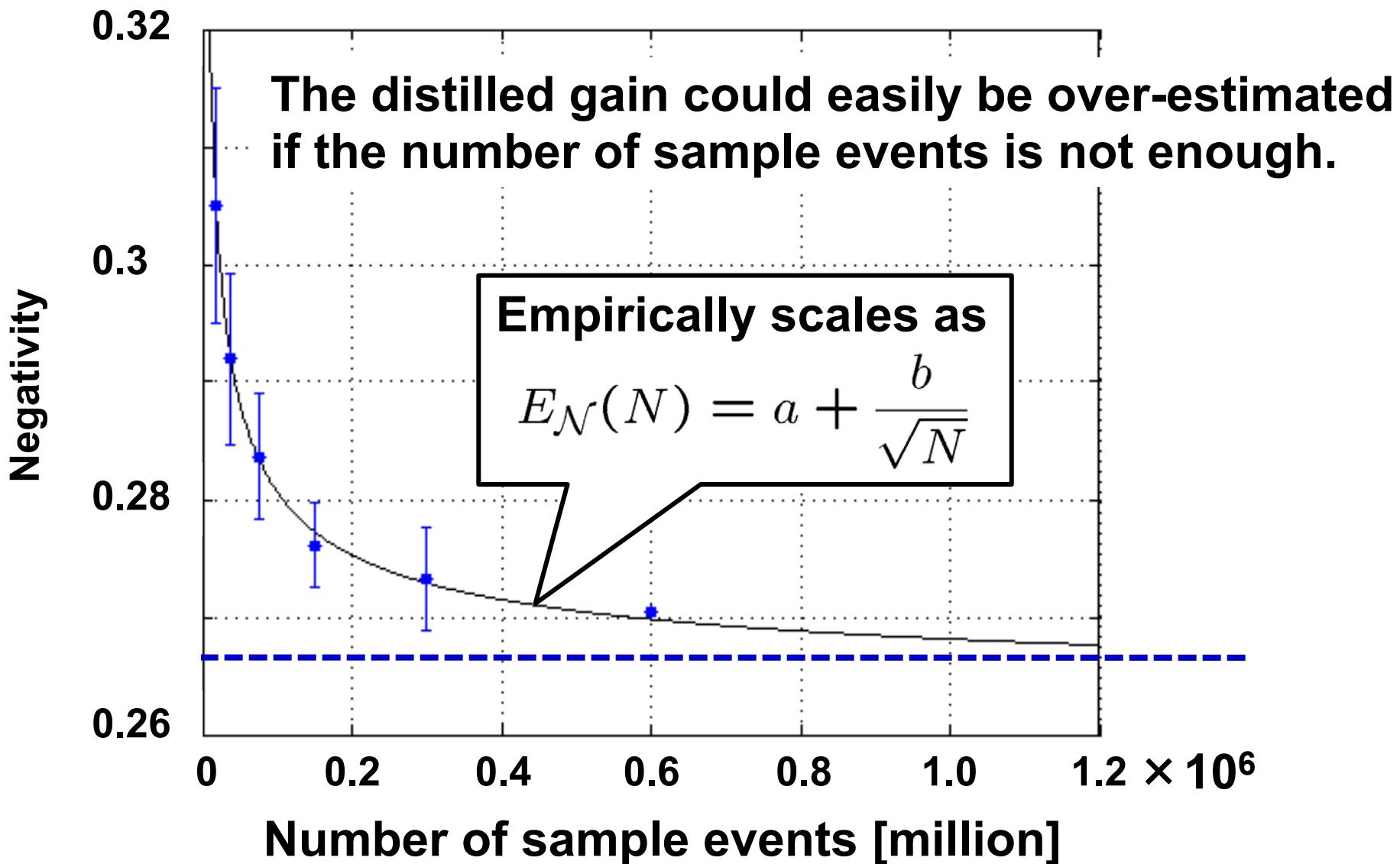


40 mW pump



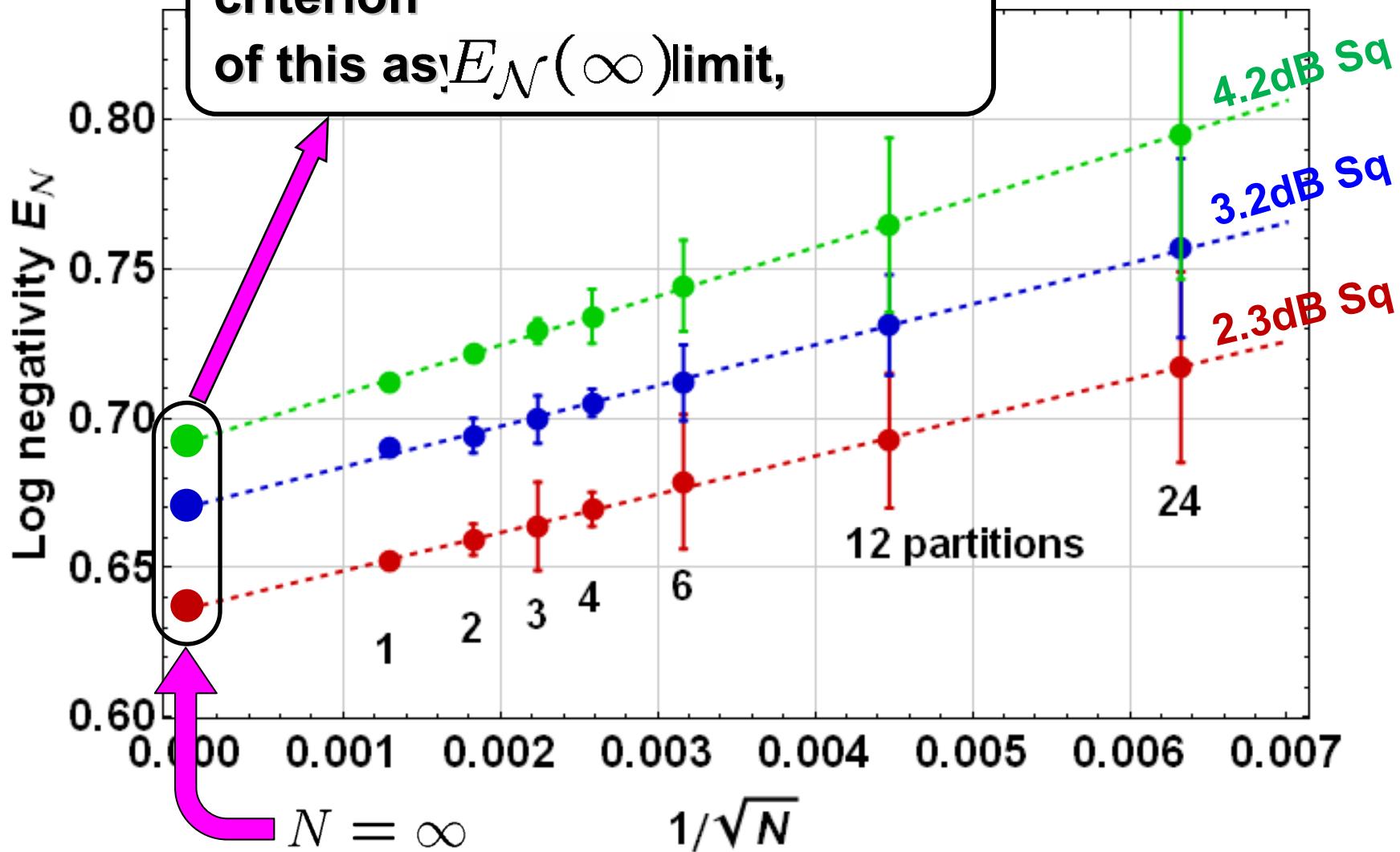
60 mW pump

Evaluation of entanglement is extremely sensitive to statistical noise.



Extrapolate the log negativity to $N \rightarrow \infty$

We used the stringent criterion of this as $E_N(\infty)$ limit,



Distillation by two-photon subtraction

1-photon subtraction

$$|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B$$

Event rate
~ 2000c/s

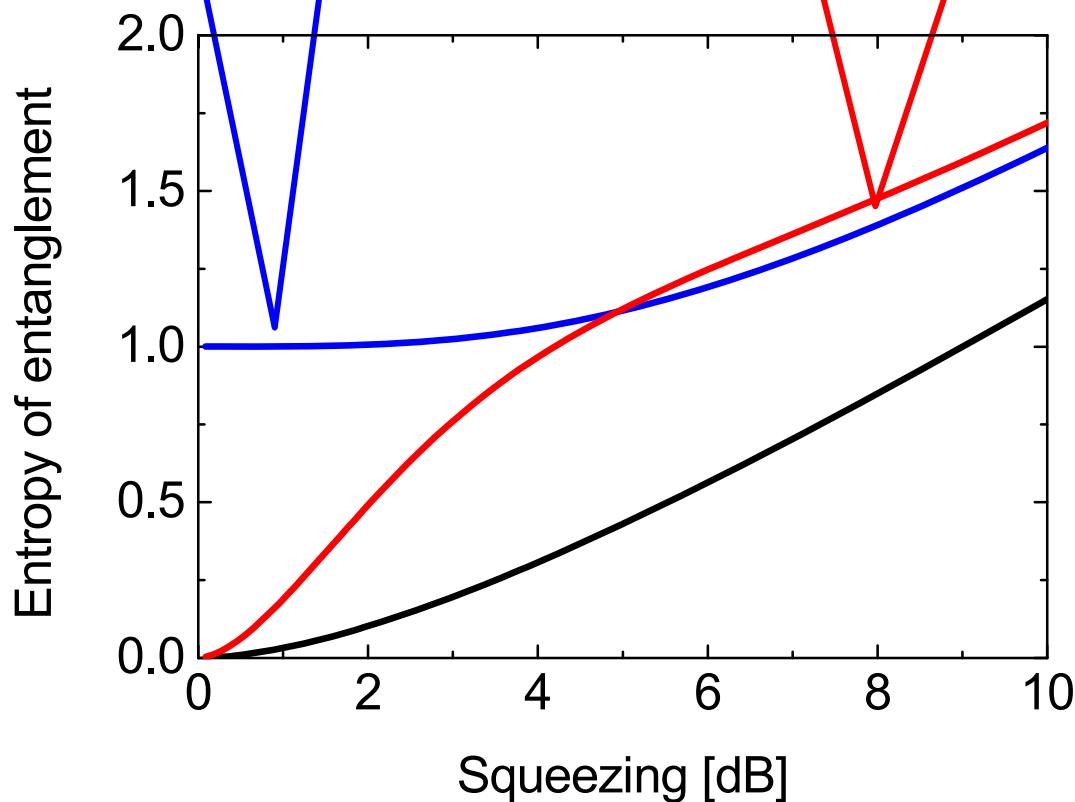
Non-Gaussian

2-photon subtraction

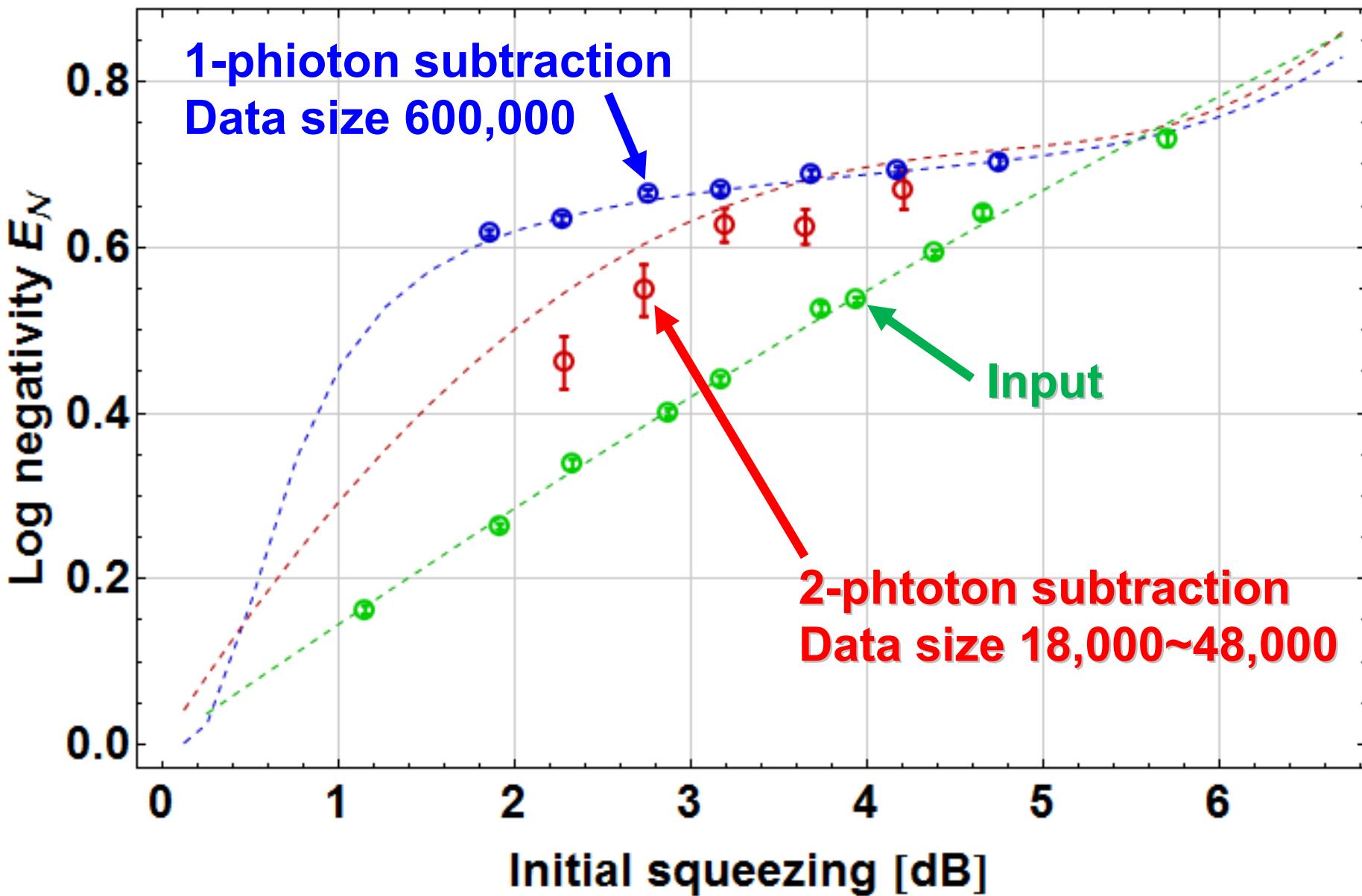
The gain is not so remarkable.

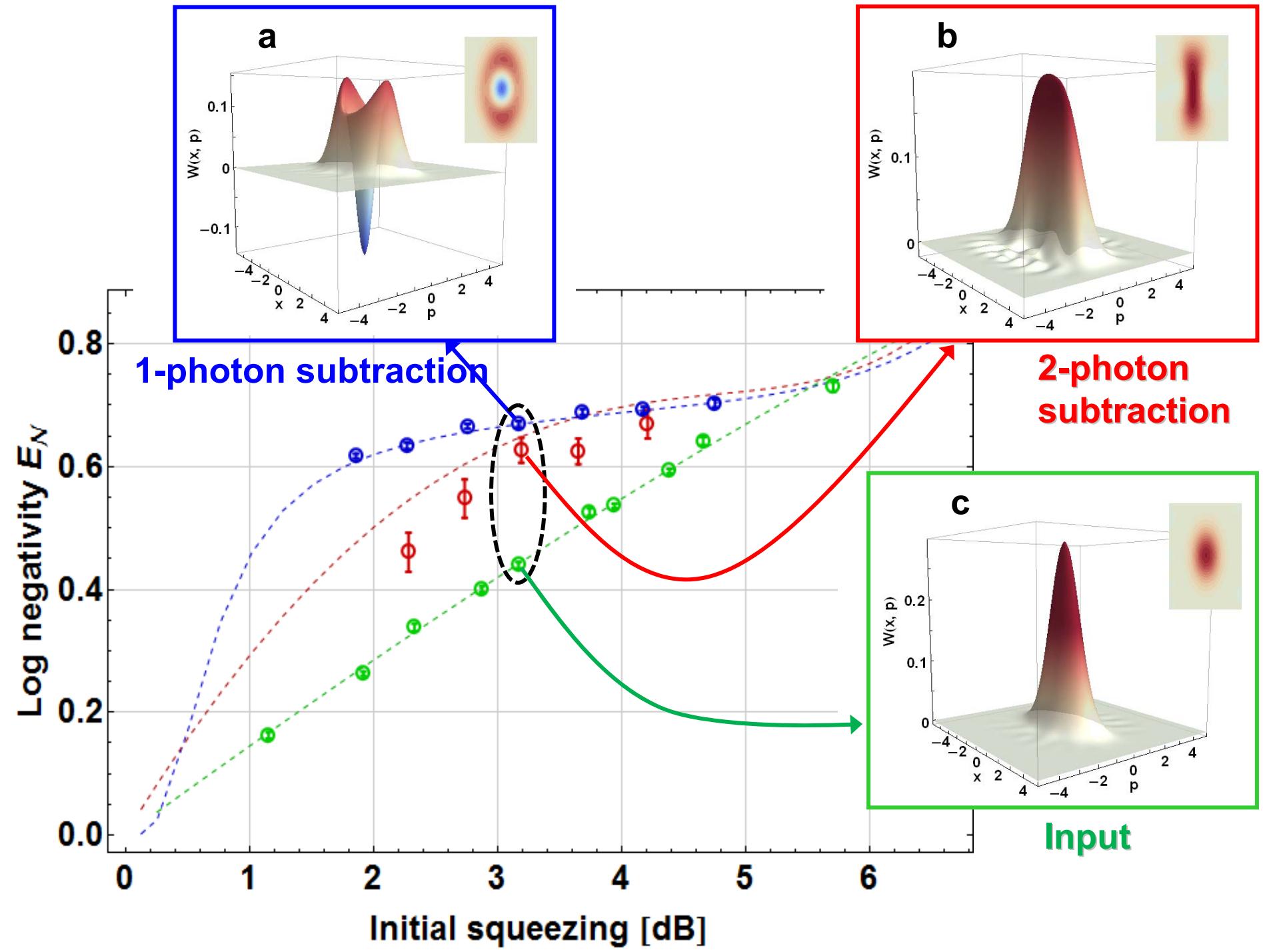
Gaussian like entanglement

“more useful
for CV
processing”



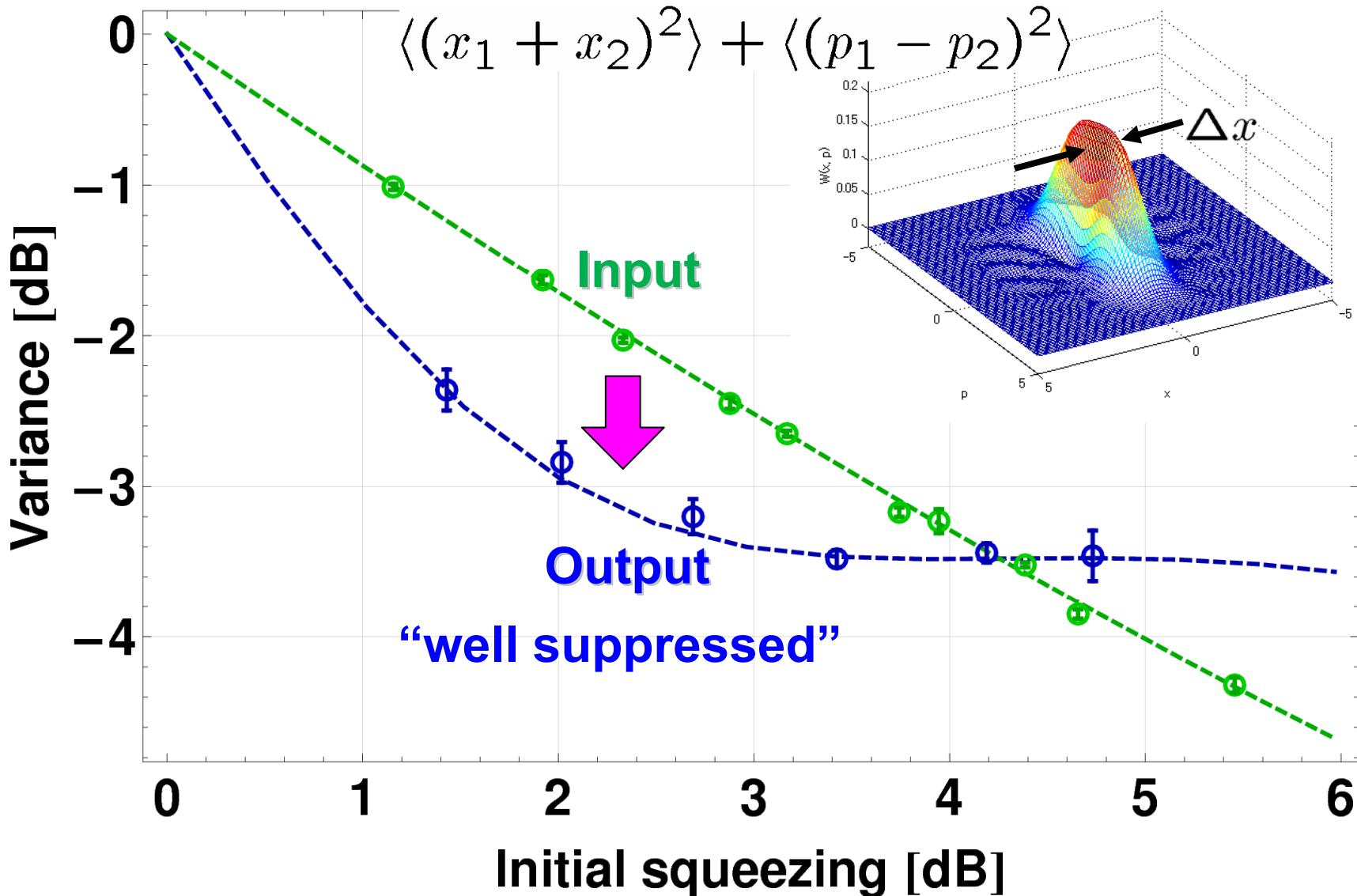
Event rate
~ a few c/s





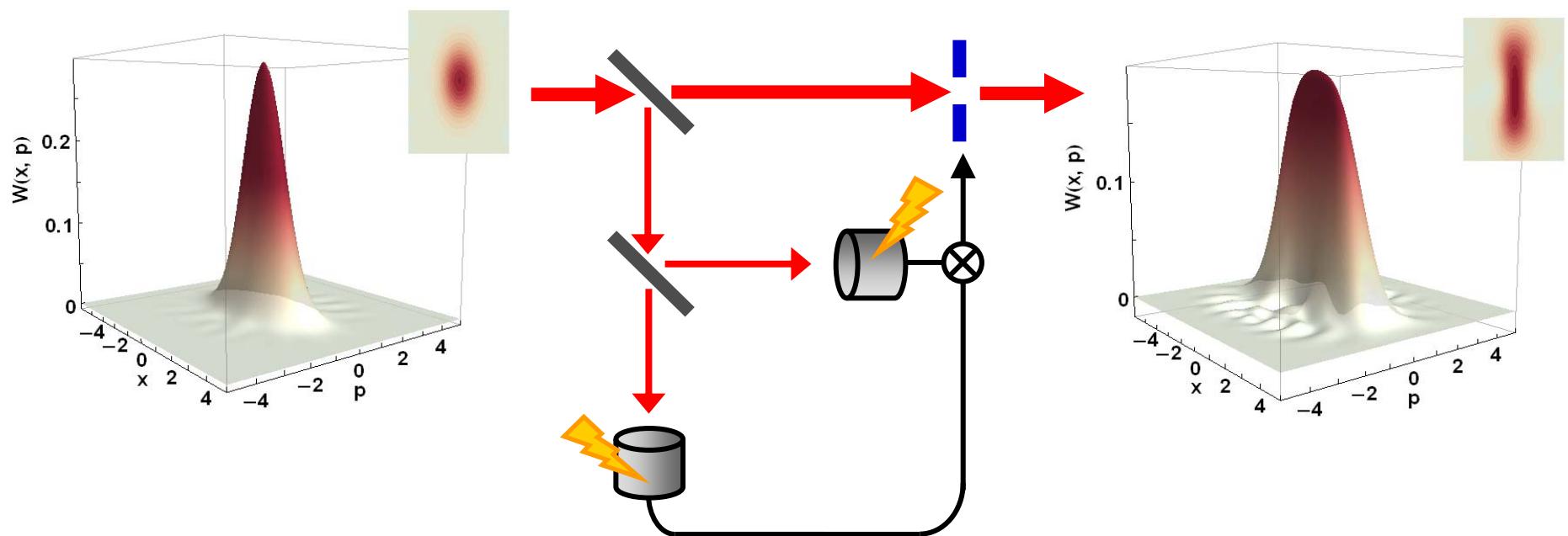
Gaussian-like entanglement gain

We may use the correlated variance



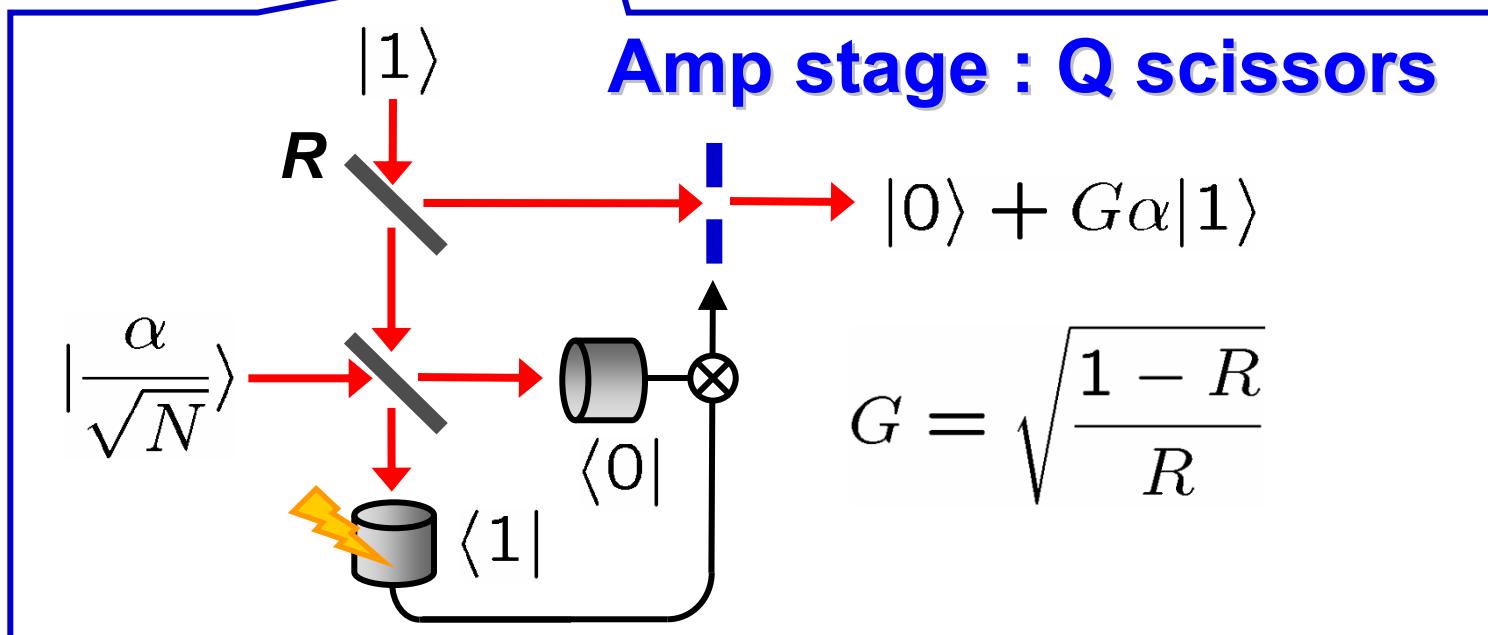
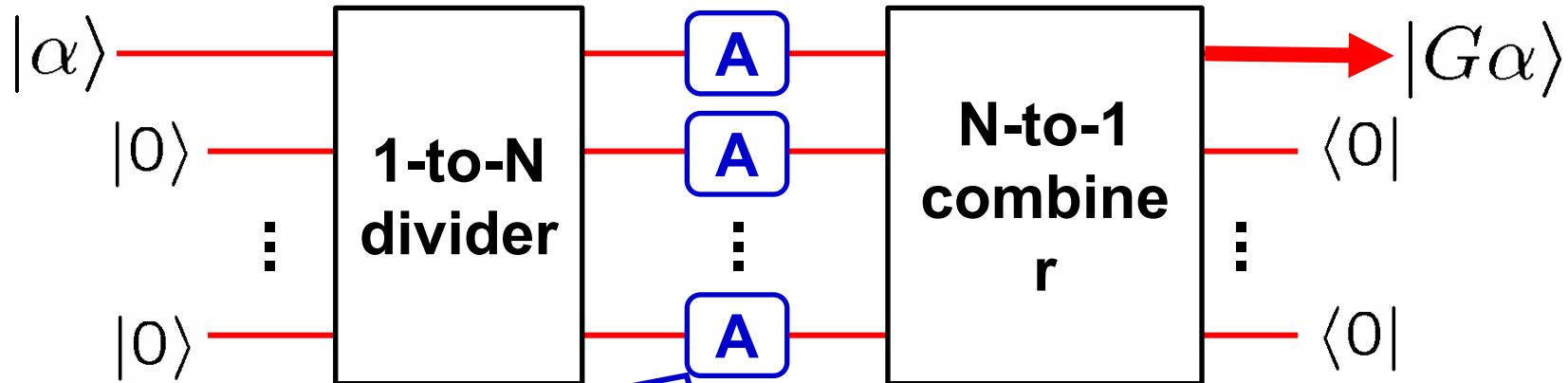
Two photon subtraction as Quantum amplifier for squeezed state

It is not deterministic but it can be noiseless
in principle.



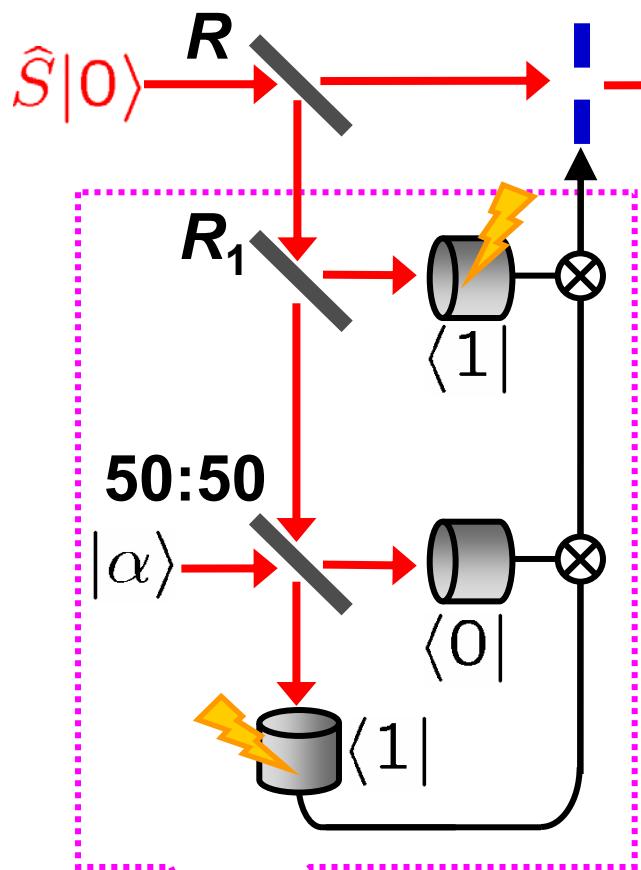
Noiseless coherent state amplifier

Xiang, Ralph, Lund, Walk, & Pryde, quant-ph/0907.3638



Extension of our scheme

Takeoka & Sasaki, PRA75, 064302 (2007).

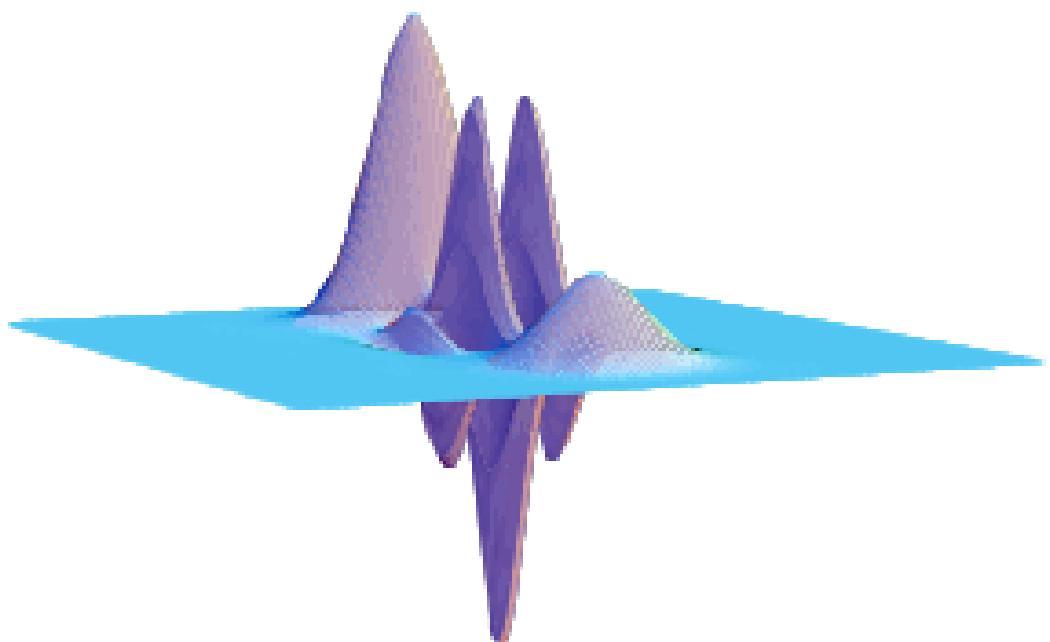


Even cat

Odd cat

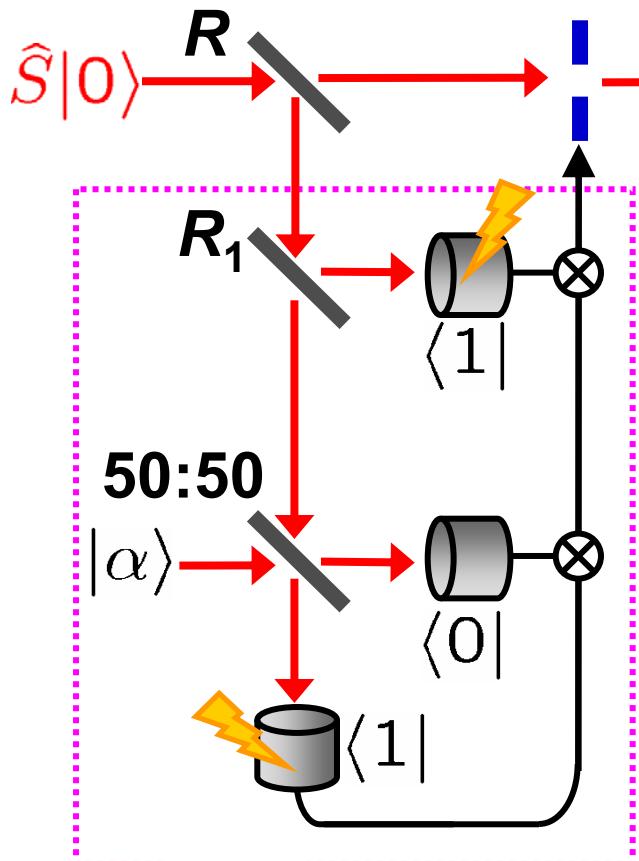
Arbitrary superposition

Projection onto
 $\beta^* \langle 1| + \langle 2|$



Extension of our scheme

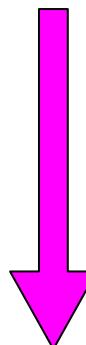
Takeoka & Sasaki, PRA75, 064302 (2007).



Projection onto
 $\beta^* \langle 1 | + \langle 2 |$

Even cat

Odd cat



Small squeezing

$$|0\rangle + \sqrt{\frac{1-R}{(1-R_1)R}}\alpha|1\rangle$$

Q scissors for noiseless coherent state amplifier

Quantum receiver with superconducting photon detectors

Superconducting photon detectors

- Very low noise
- Veeeery broadband

Transition Edge Sensor (TES)

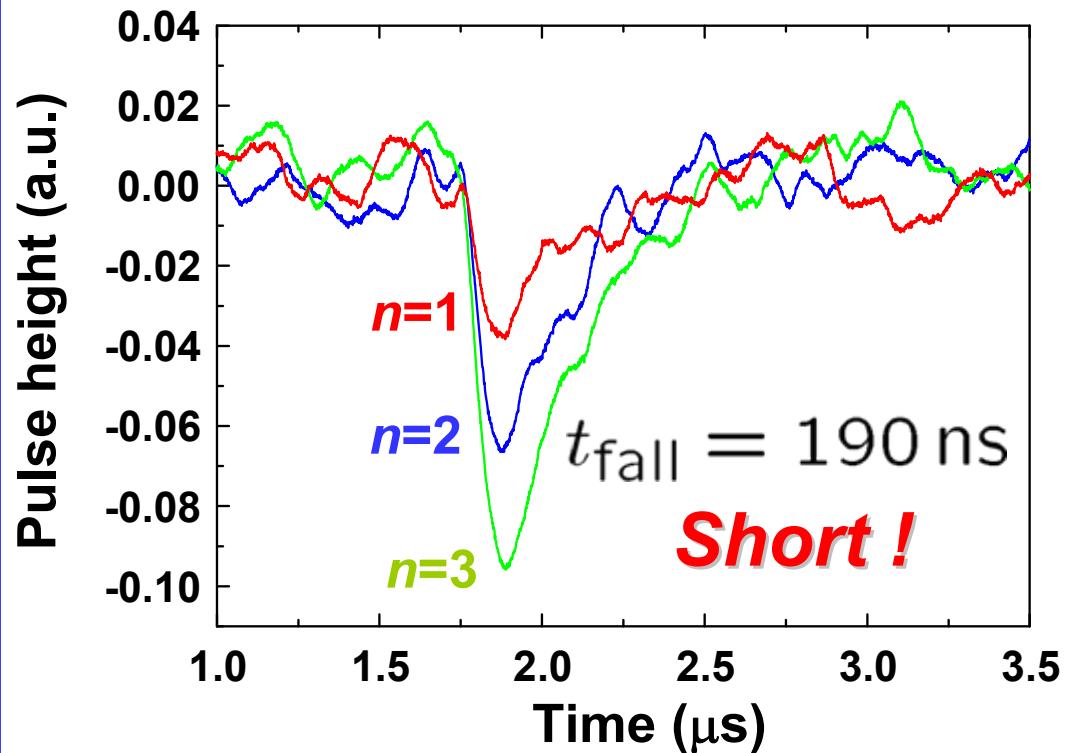
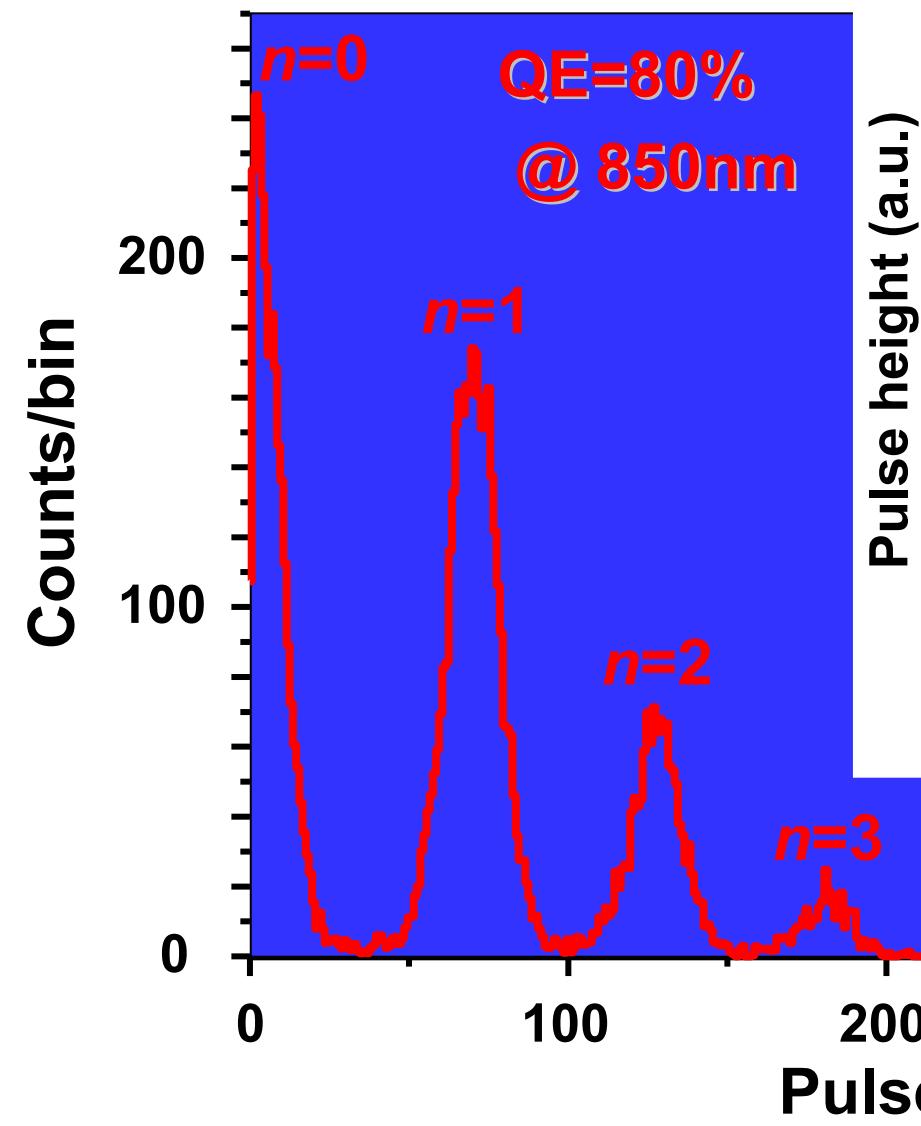
“Remarkable photon number resolution”

Lita, Miller, & Nam, Opt. Exp. 16, 3032 (2008).

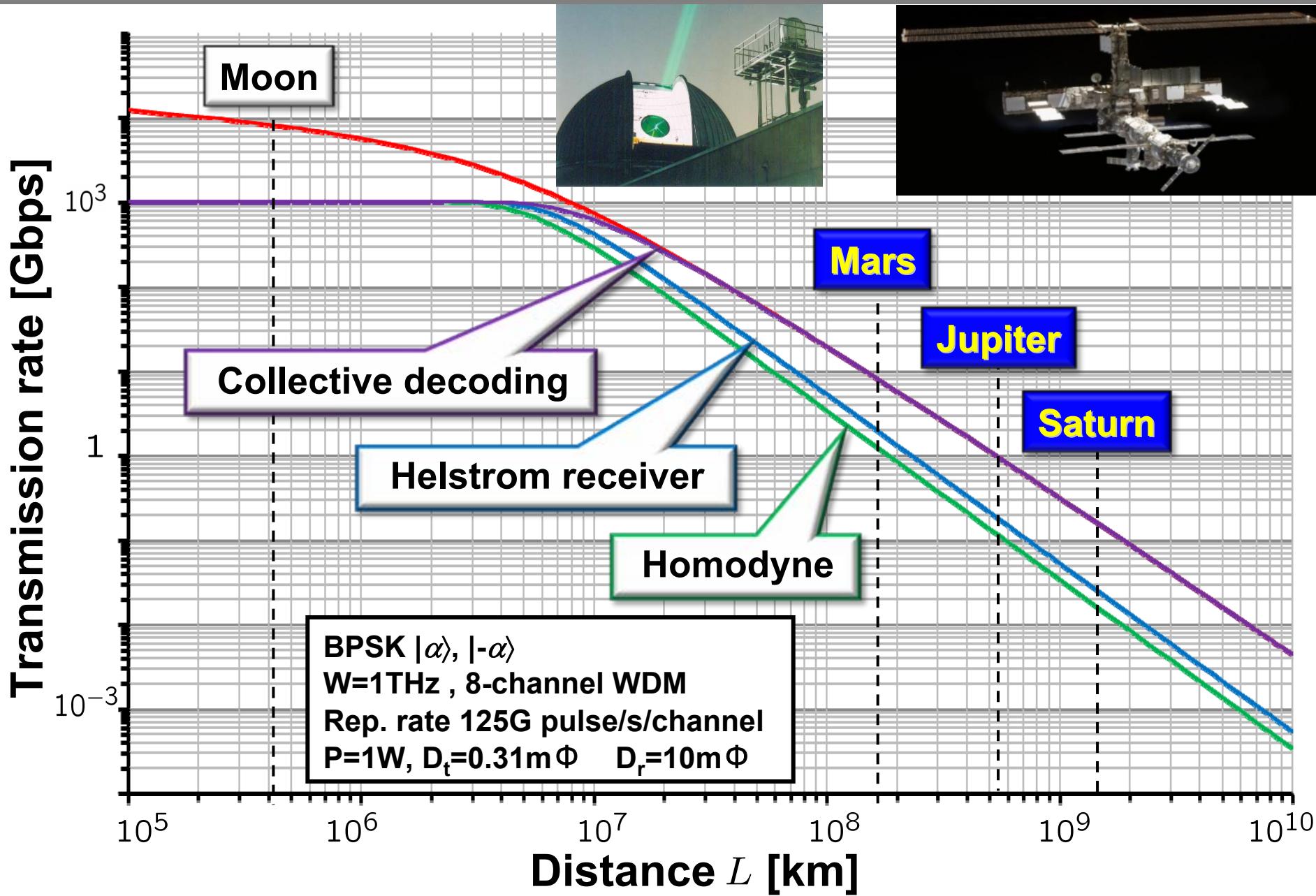
- Tungsten
- Detection efficiency ~ 95% at 1556nm.
- Recovery time ~ 800ns

Photon number resolving detector with Ti-TES

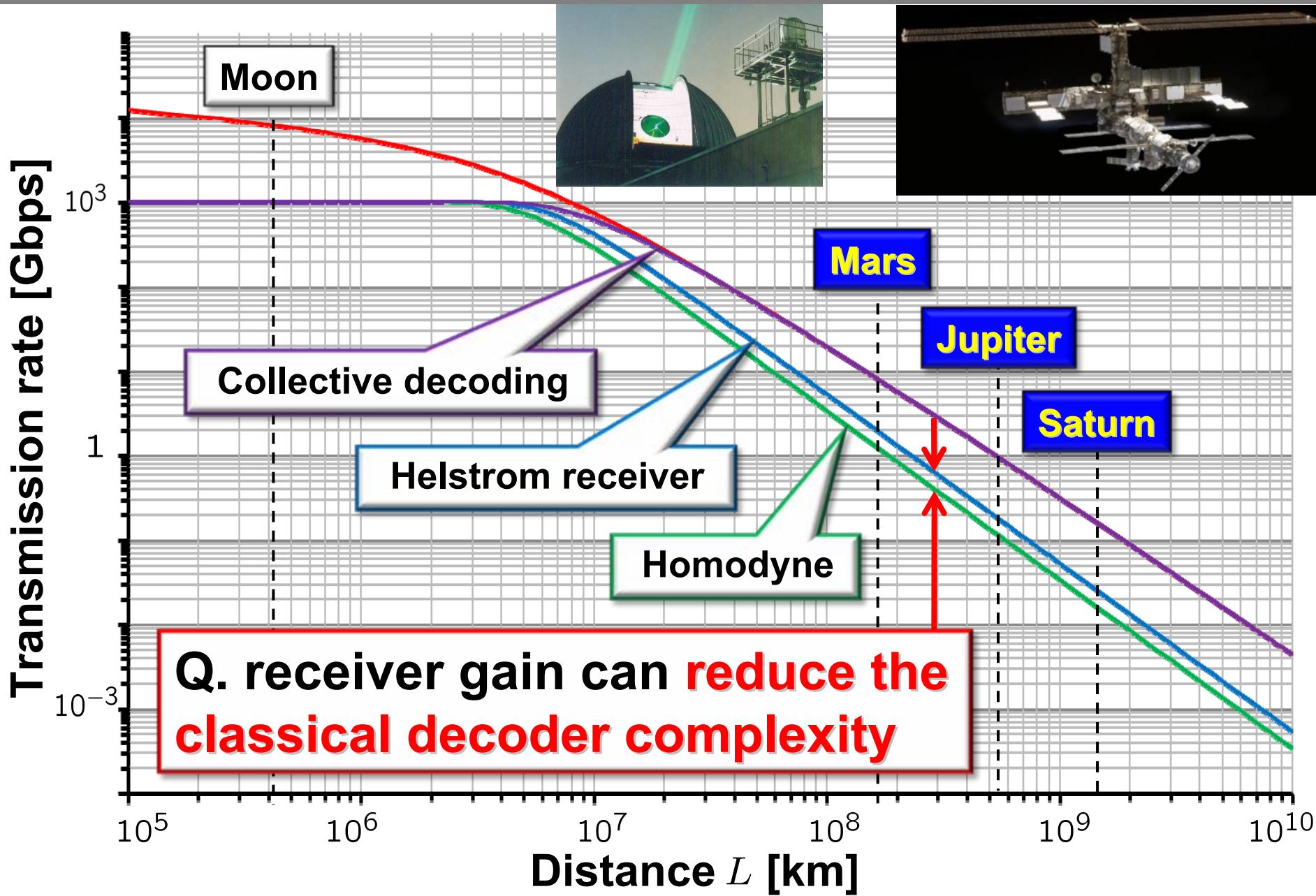
D. Fukuda et al. (AIST)



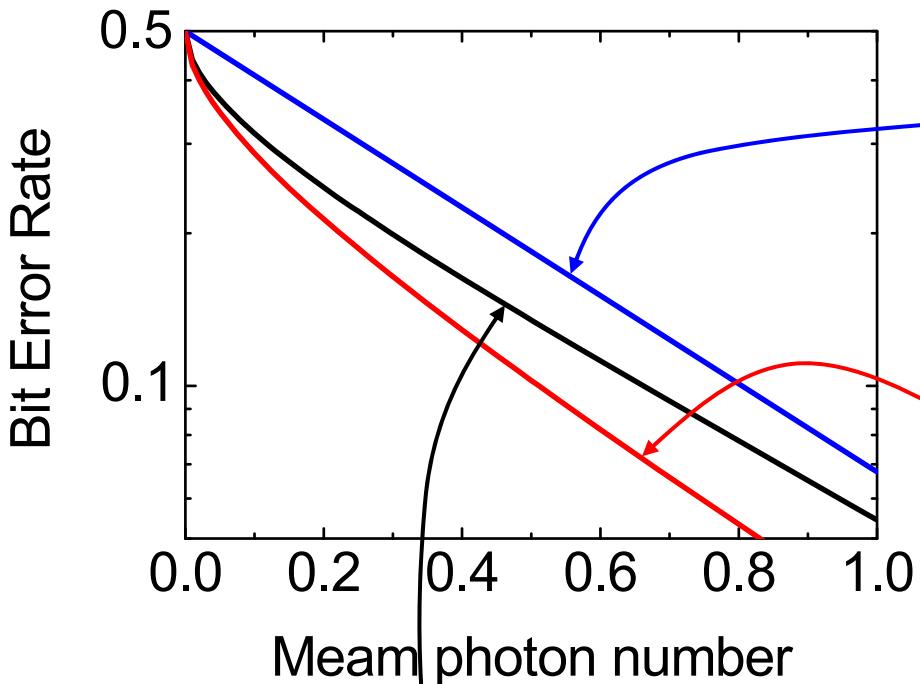
TES with high QE could be applied to Q. receiver



Large-scale error-correction code is employed



Discrimination of on/off signals



Direct detection

$$\begin{aligned} |0\rangle &\rightarrow |0\rangle\langle 0| \\ |\alpha\rangle &\rightarrow \sum_{n \geq 1} |n\rangle\langle n| \end{aligned}$$

Helstrom receiver

$$\begin{aligned} |0\rangle &\rightarrow c_0 |0\rangle - c_1 |\alpha\rangle \\ |\alpha\rangle &\rightarrow c_1 |0\rangle + c_0 |\alpha\rangle \end{aligned}$$

Displacement receiver

$$\begin{aligned} |0\rangle &\rightarrow \hat{D}(\beta)|0\rangle \xrightarrow{\text{dashed}} |0\rangle\langle 0| \\ |\alpha\rangle &\rightarrow \hat{D}(\beta)|\alpha\rangle \xrightarrow{\text{solid}} \sum_{n \geq 1} |n\rangle\langle n| \end{aligned}$$

Discrimination of on/off signals

Directly observed gain

Bit error rate

0.5
0.4
0.3
0.2
0.1

Mean photon numbers of
optical coherent signals
 $|\alpha|^2/2$

Helstrom bound

0.1 0.2 0.4 0.6 0.8 1.0

Displacement receiver

$|\alpha\rangle$
 $|0\rangle$

$\hat{D}(\beta)$

TES

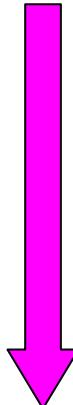
System DE $\sim 70\%$

Theoretical limit
by direct detection

Summary

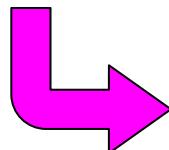
Elementary operations of non-Gaussian control

- Generation and control of cat states
- Entanglement distillation for CVs
- Non-Gaussian Q. receivers with TES



- Noiseless generation of nonlocal entangled cats
- Noiseless Q amplifier

Basic tool box of non-Gaussian control of CVs



Quantum enhanced photonic ICT

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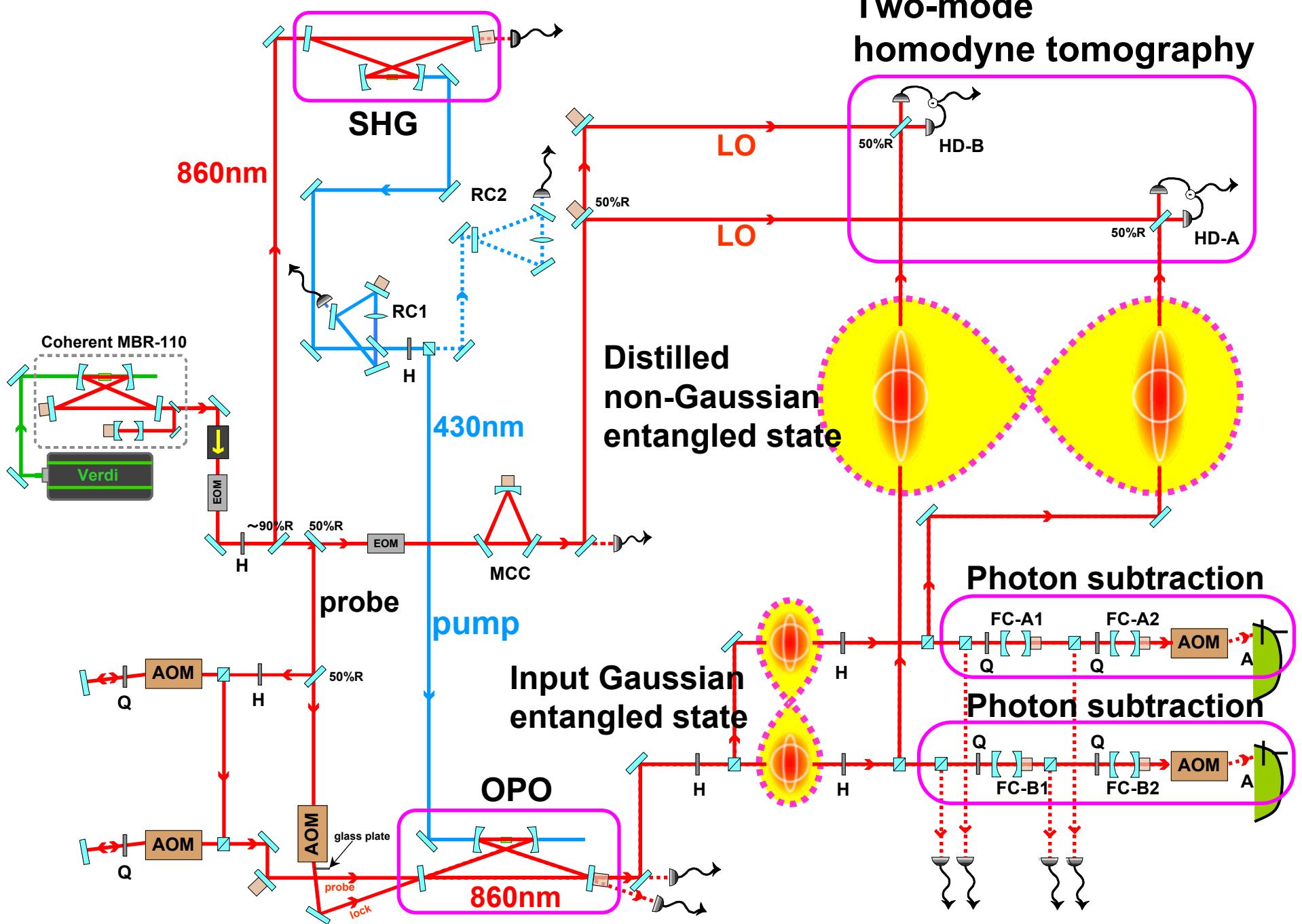
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JST/SORST



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AIST

A. Waseda
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Two-mode homodyne tomography



References

Single-photon subtraction

Wakui, et al, Opt. Express 15, 3568 (2007).

Sasaki & Suzuki, PRA 73, 043807 (2006).

Two-photon subtraction

Takahashi, et.al. PRL 101, 233605 (2008).

Takeoka, et.al. PRA 77, 062315 (2008).

Sasaki, et.al. PRA 77, 063840 (2008).