

A photonic cluster state

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Imperial College
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[arXiv: 0810.2587](https://arxiv.org/abs/0810.2587)

Weizmann, March 2009

Proposal for conversion of single photon sources into devices which emit large strings of photonic cluster states in a controlled and pulsed-on-demand manner

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Terry R



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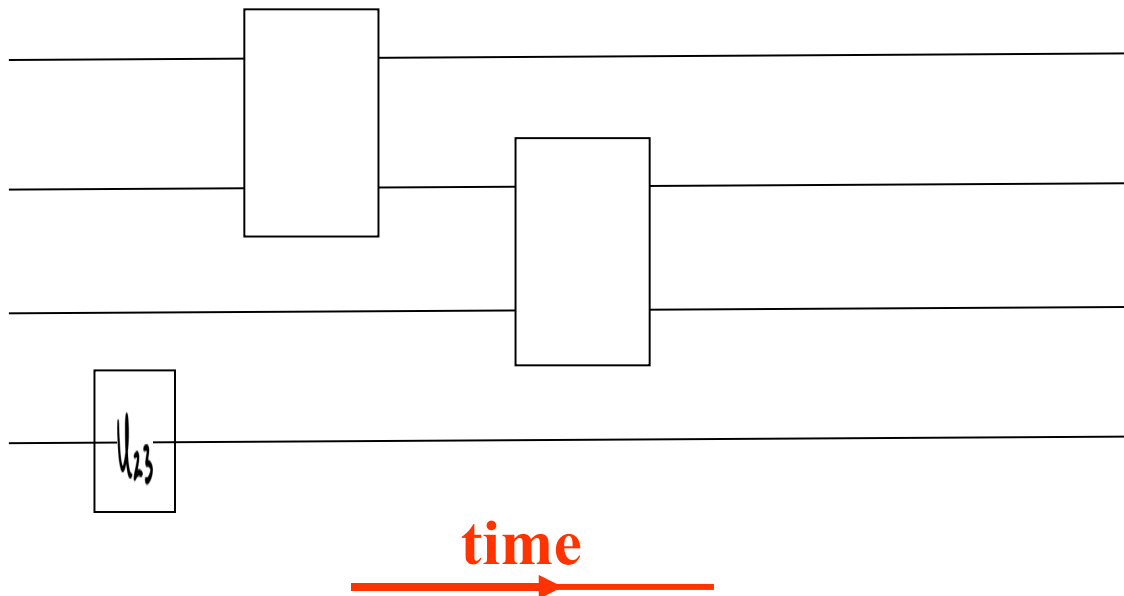
Weizmann, March 2009

Outline

- Cluster states and quantum computation
- Photonic cluster states
- Optical transitions in quantum dots
- The idealized “machine gun”
- Decoherence processes
- Outlook

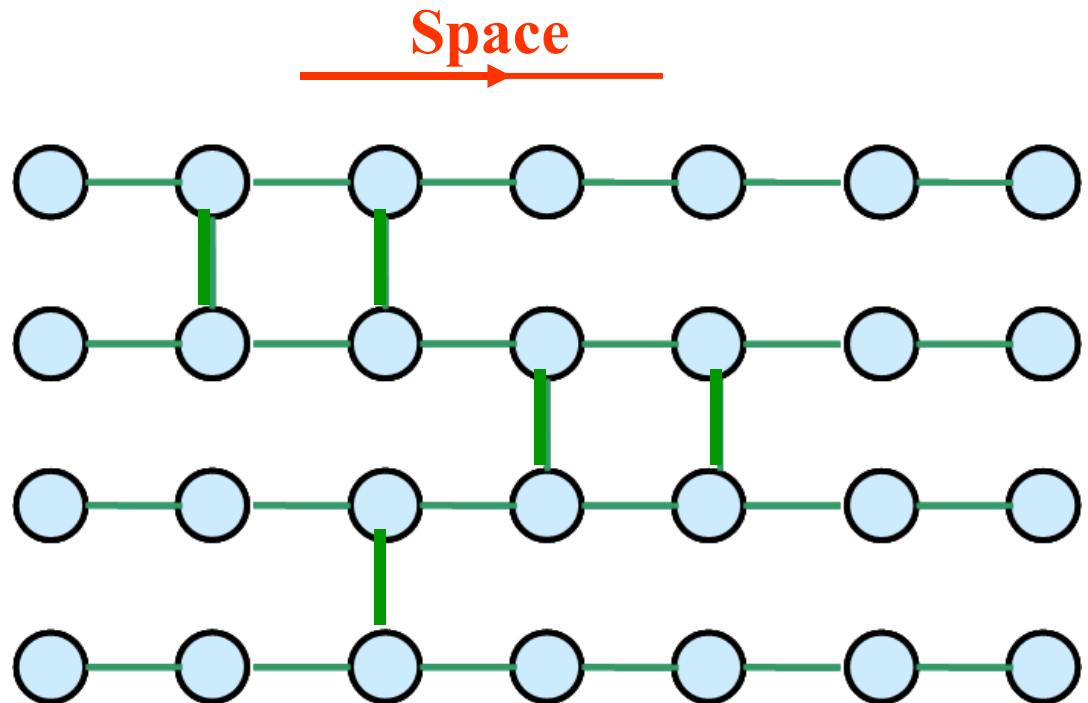
Quantum Computation

$$|\Psi_0\rangle \quad U(t) \quad \nearrow \text{Measurement}$$



A one way quantum computer

- Start from a many body entangled state
- Evolve by performing single particle measurements

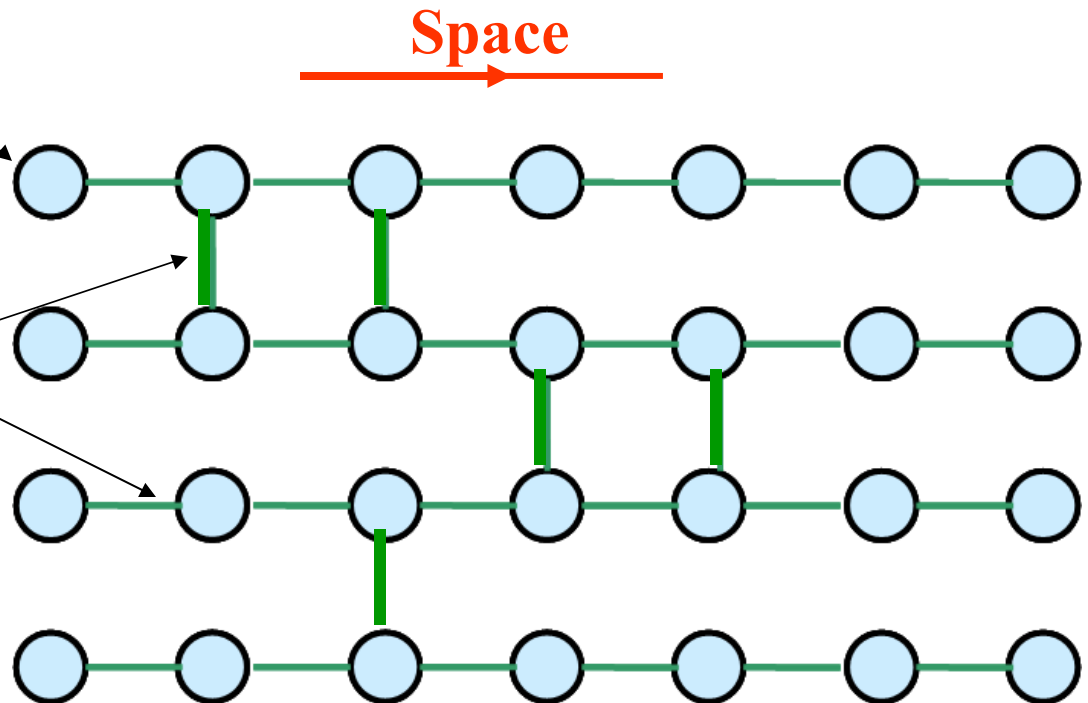


A one way quantum computer

- Start from a many body entangled state
- Evolve by performing single particle measurements

Initialize qubits
into $|0\rangle + |1\rangle$

Apply
Controlled-Z
(CPHASE)
unitary



So how do we build a photonic cluster state?

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So how do we build a photonic cluster state?

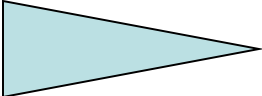
No easy way to get photons to interact

Starting from single photons

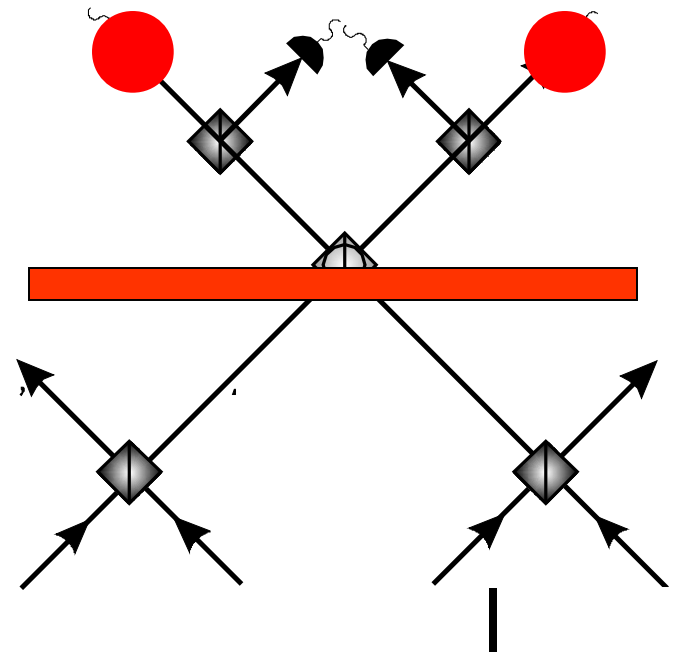
- Use interference and measurement to make effective interaction

(lesson from KLM)

4 single photons

 One Bell pair

Probability = $1/4$

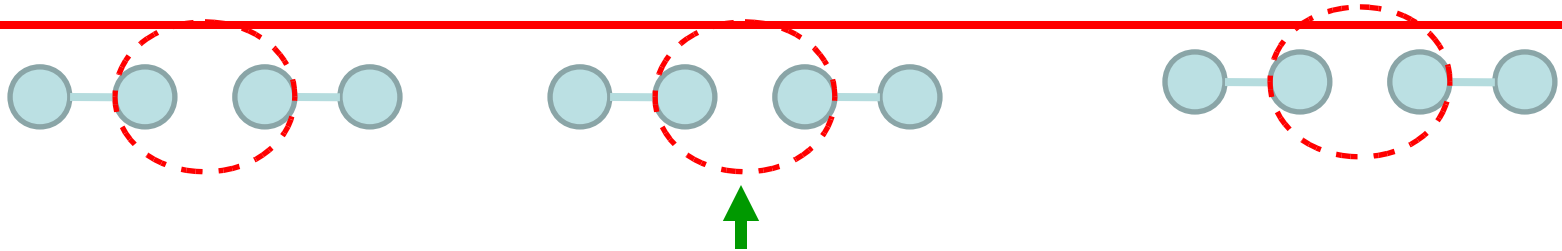


D. E. Browne and T. Rudolph PRL **95**, 010501 (2005).

Q. Zhang et. al., PRA **77**, 062316 (2008)

LOQC Starting from Bell pairs

LOQC Starting from Bell pairs

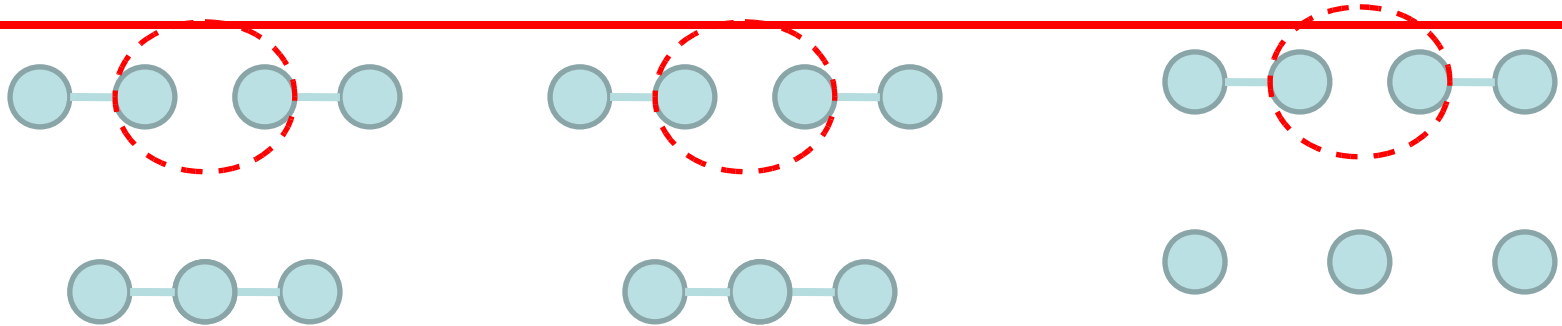


Interference and measurement of one photon

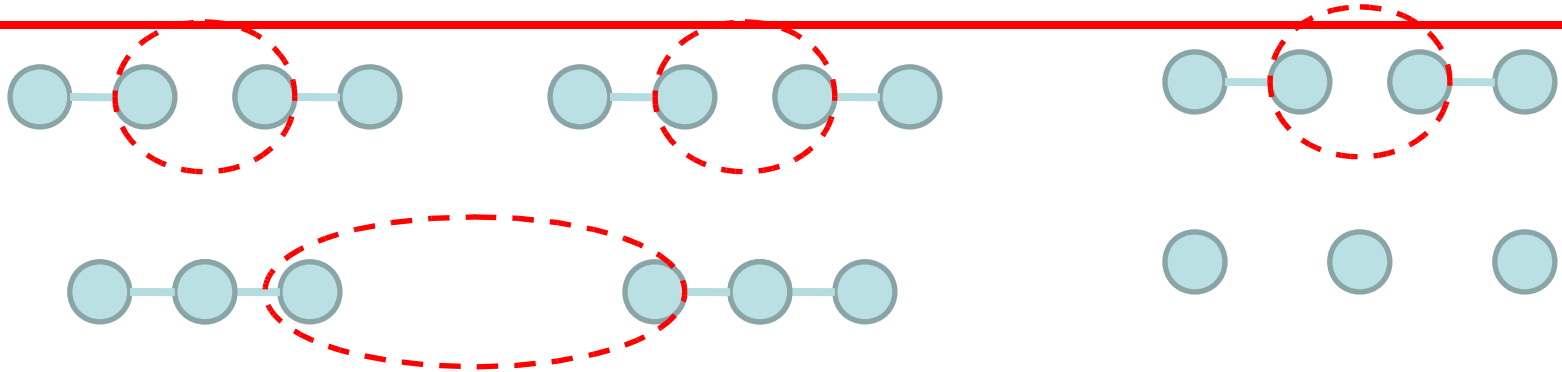
Probability = $1/2$

D. E. Browne and
T. Rudolph PRL 95,
010501 (2005).

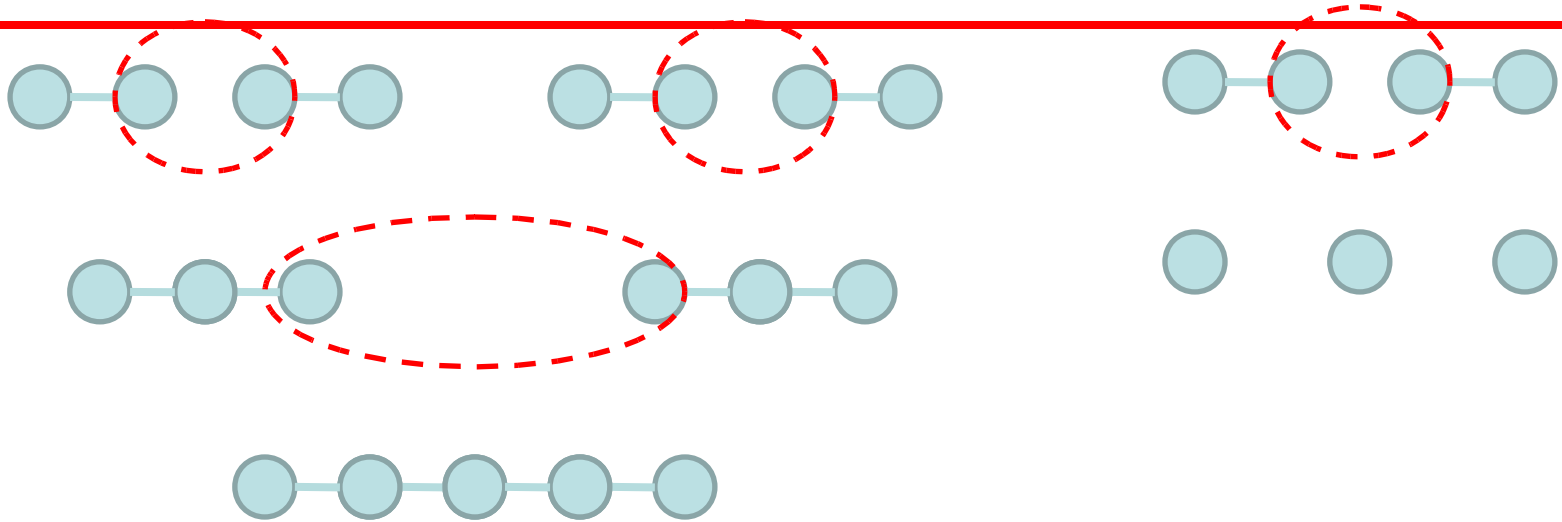
LOQC Starting from Bell pairs



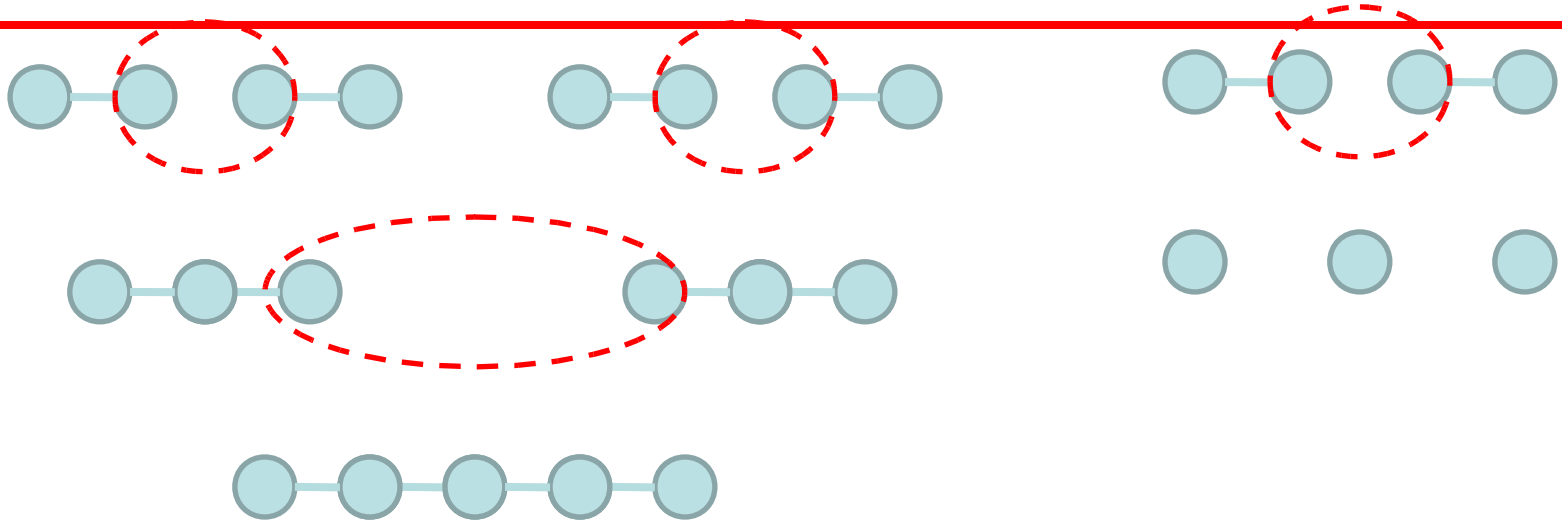
LOQC Starting from Bell pairs



LOQC Starting from Bell pairs



LOQC Starting from Bell pairs



Efficient but still resource intensive, requiring lots of quantum memory etc

LOQC Starting from Bell pairs

- Bell pairs produced directly from quantum dots via biexciton cascade?
- **Problems:**
 - (i) Exciton dephasing
 - (ii) Removes only the inefficiency of the first step

LOQC Starting from Bell pairs

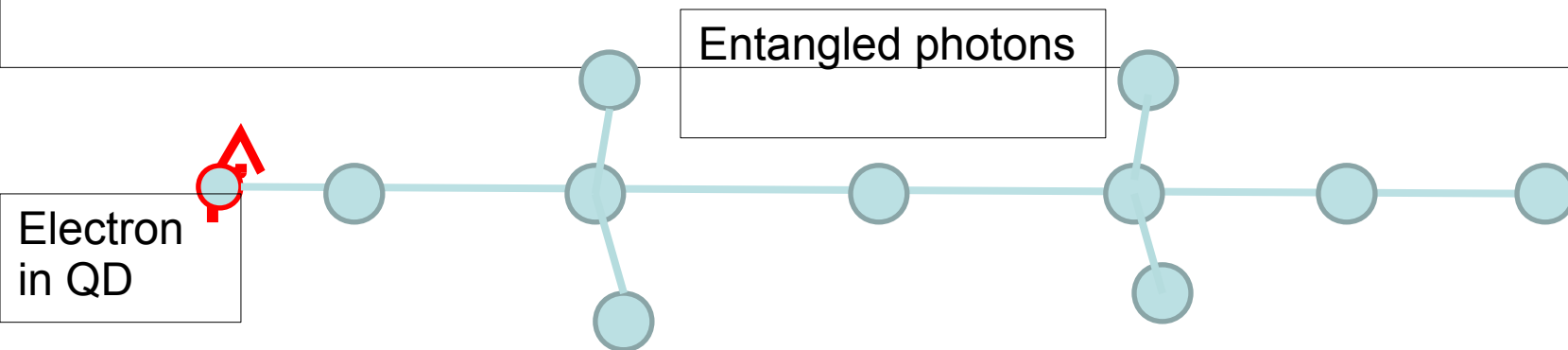
- Bell pairs produced directly from quantum dots via biexciton cascade?

- **Problems:**

- (i) Exciton dephasing

- (ii) Removes only the inefficiency of the first step

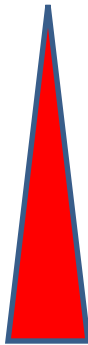
- **Solution:** Design a source which uses only single exciton recombination, but which can fire out optical cluster state:



Optical transitions in a quantum dot

$$J_z = +3/2$$

$$|\uparrow_h \uparrow_e \downarrow_e\rangle$$



$$|\uparrow_e\rangle$$

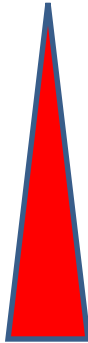
Optical transitions in a quantum dot

$$J_z = +3/2$$

$$|\uparrow_h \uparrow_e \downarrow_e\rangle$$



$$R$$



$$|\uparrow_e\rangle$$

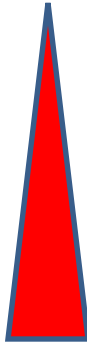
Optical transitions in a quantum dot

$$J_z = +3/2$$

$$|\uparrow_h \uparrow_e \downarrow_e\rangle$$



$$|R\rangle$$

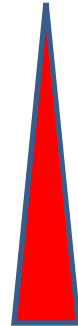


$$|\uparrow_e\rangle$$

$$|\square_h \uparrow_e \downarrow_e\rangle$$



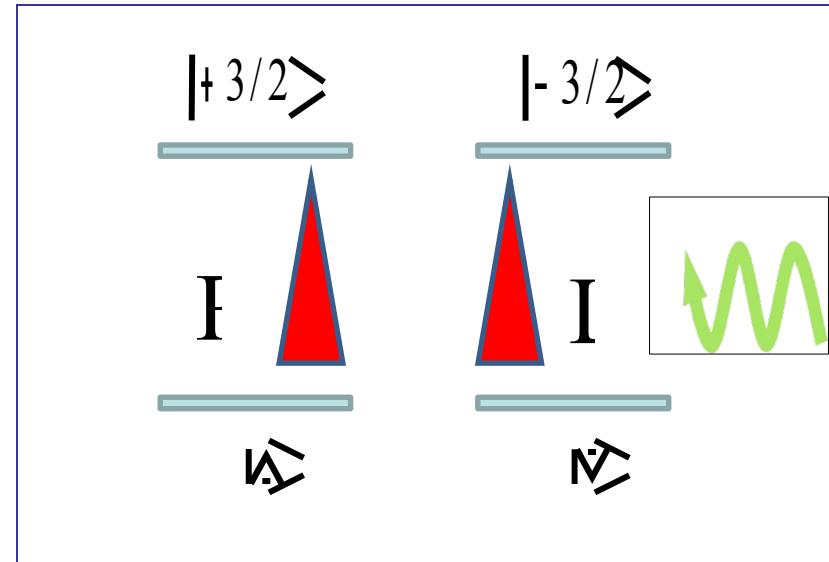
$$|L\rangle$$



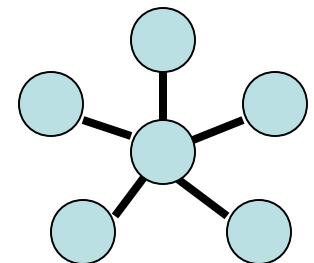
$$|\downarrow_e\rangle$$

GHZ state machine gun

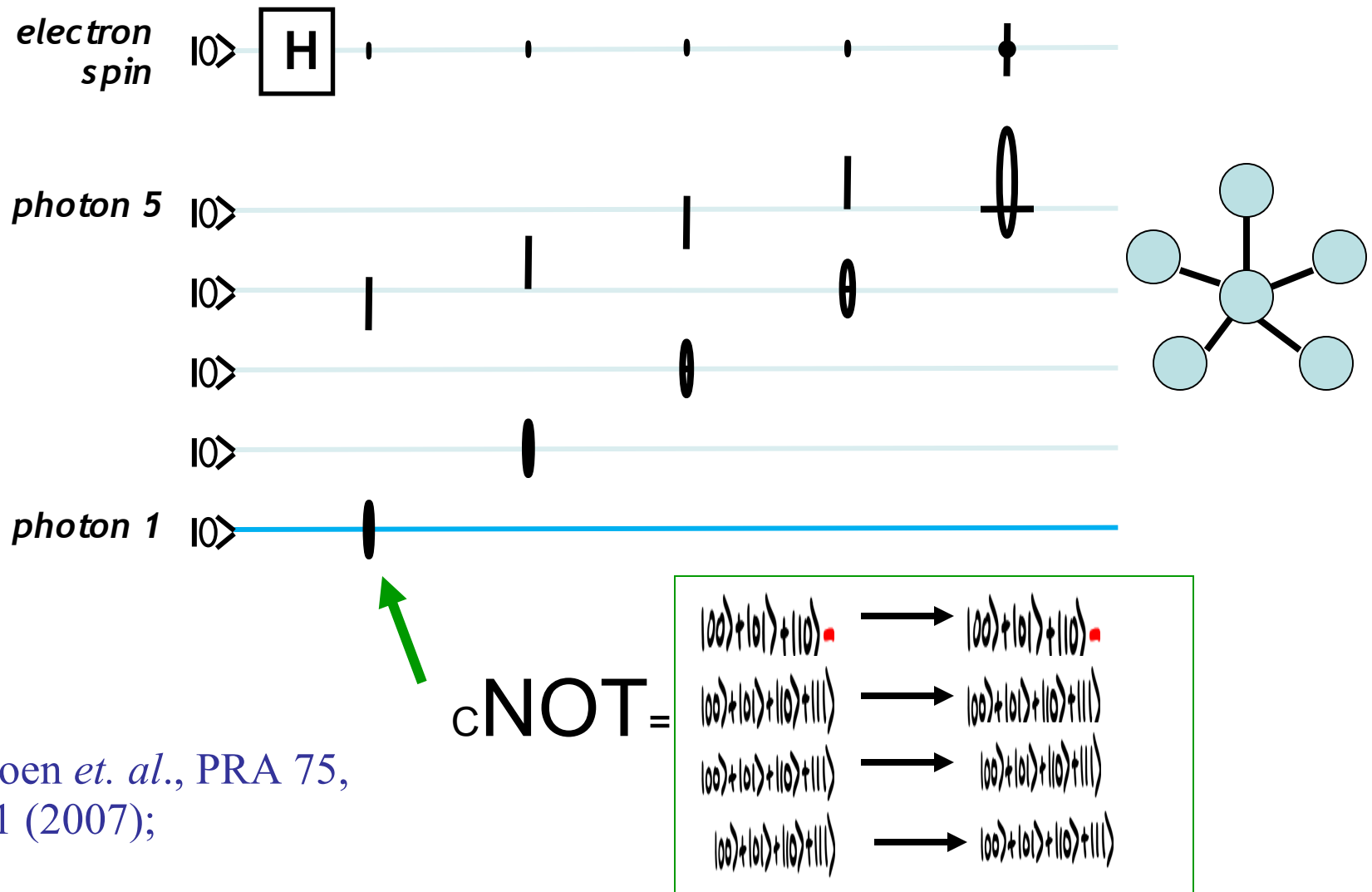
$$|\uparrow\rangle + |\downarrow\rangle \xrightarrow{\text{excitation}} |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$$



“Schrodinger’s cat” state:

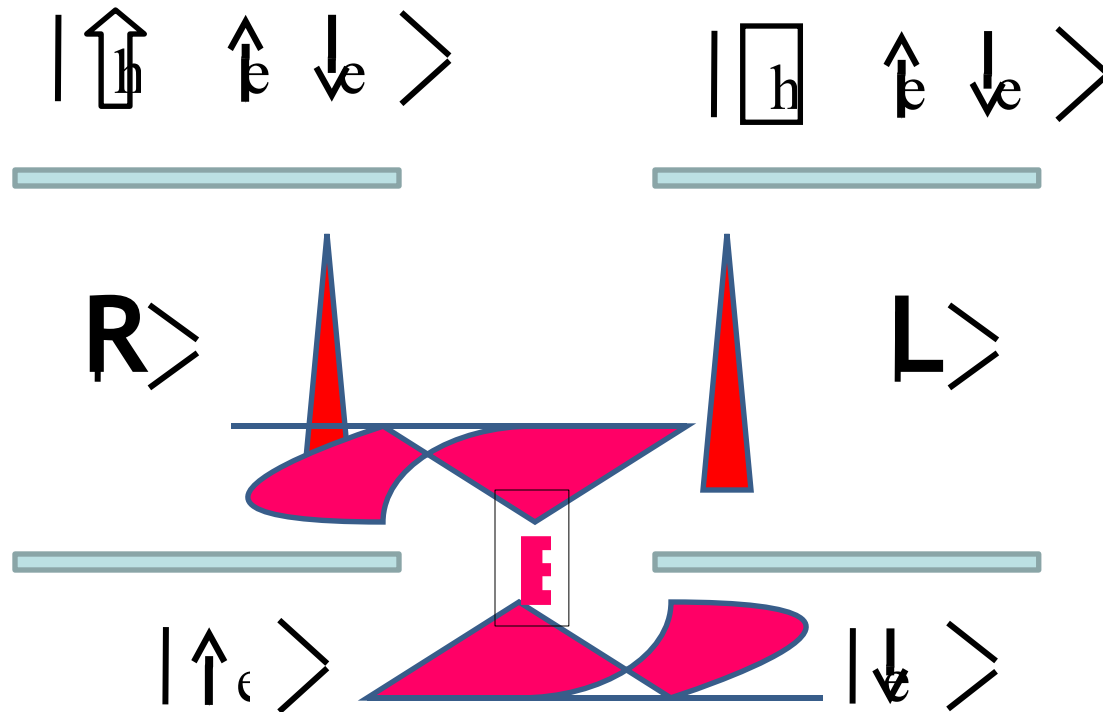


GHZ state machine gun



C. Schoen *et. al.*, PRA 75,
032311 (2007);

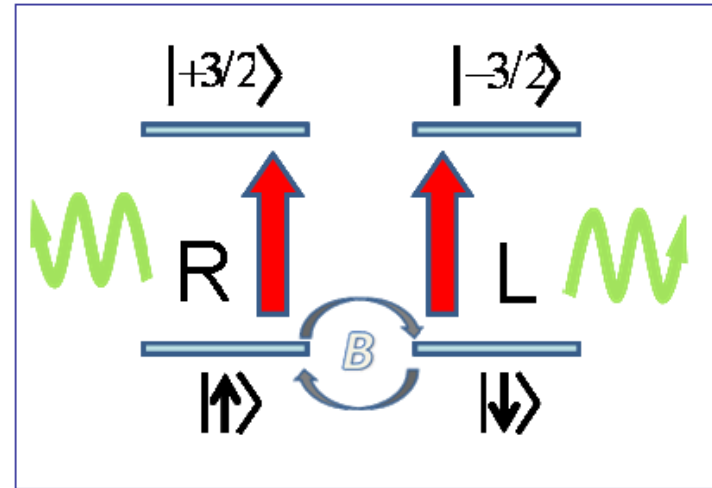
A cluster state machine gun



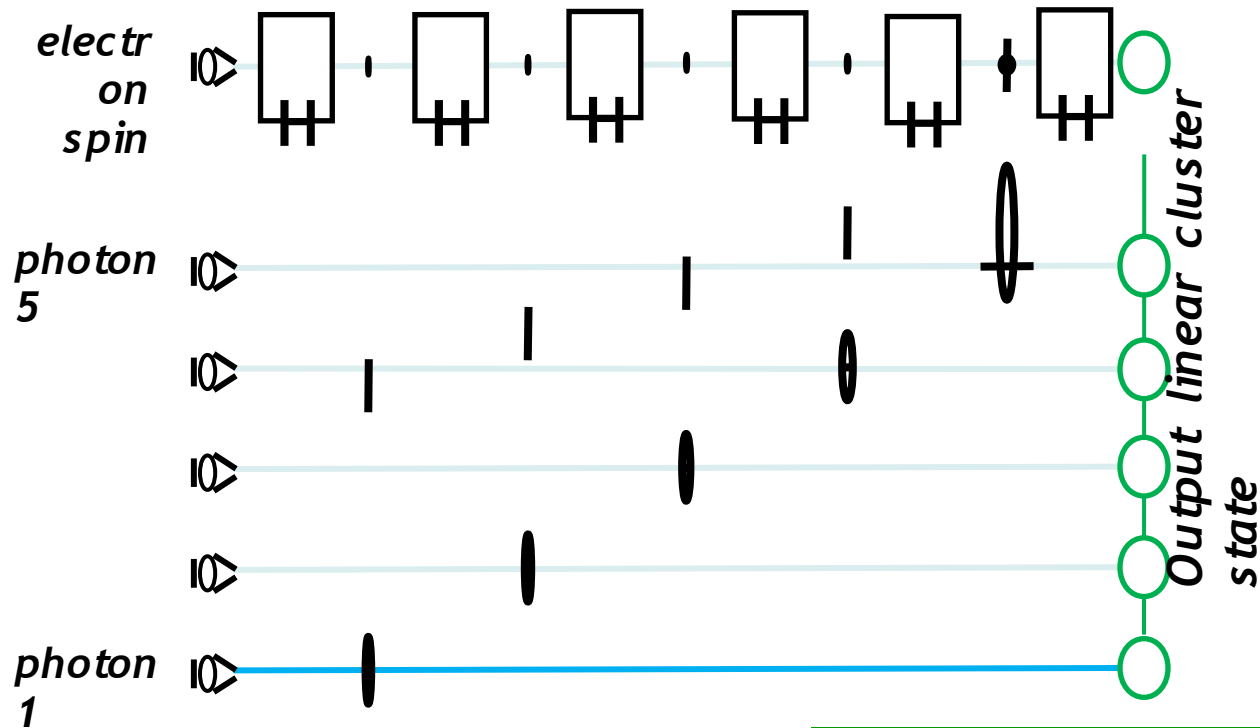
Add in-plane (xy) **static** Magnetic field

Ideal machine gun

precess.



Ideal machine gun



cNOT=

$$\begin{array}{lcl}
 |00\rangle + |01\rangle + |10\rangle & \longrightarrow & |00\rangle + |01\rangle + |10\rangle \\
 |00\rangle + |01\rangle + |10\rangle + |11\rangle & \longrightarrow & |00\rangle + |01\rangle + |10\rangle + |11\rangle \\
 |00\rangle + |01\rangle + |10\rangle + |11\rangle & \longrightarrow & |00\rangle + |01\rangle + |10\rangle + |11\rangle \\
 |00\rangle + |01\rangle + |10\rangle + |11\rangle & \longrightarrow & |00\rangle + |01\rangle + |10\rangle + |11\rangle
 \end{array}$$

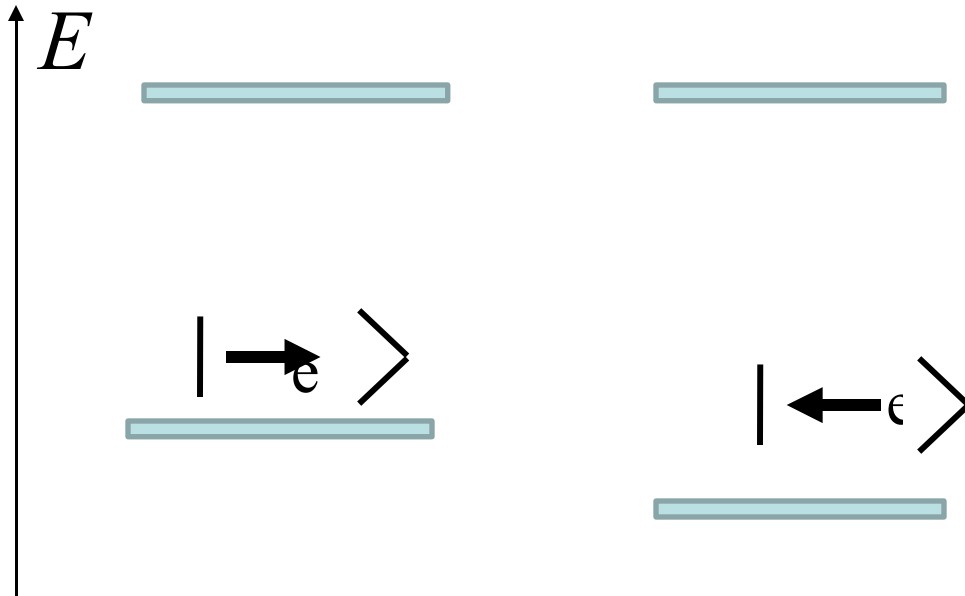
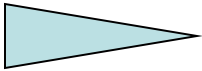
C. Schoen *et. al.*, PRA 75,
032311 (2007);

Decoherence (1)

- Finite lifetime of the excited state

Zeeman splitting

spectral “which path” information

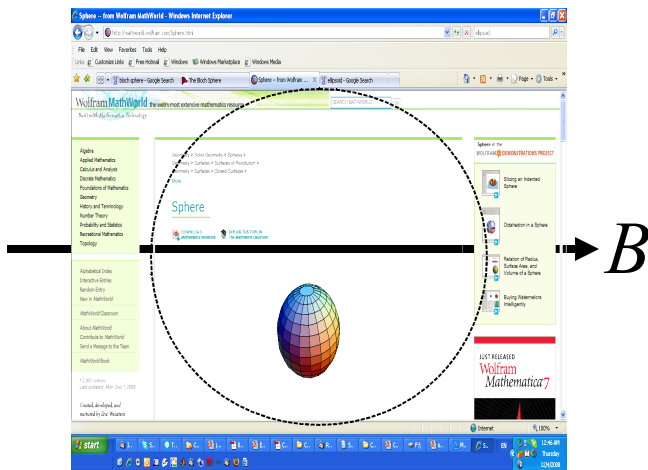


$$\Gamma_{\text{decay}} > g\mu B$$

$$\tau_{\text{decay}} < \hbar / g\mu B$$

Decoherence (2)

- Interaction of spin with its environment:
 - Dephasing ($T2$ time) (nuclear spins)
 - Spin relaxation ($T1$ time) (phonons)



$$\tau_1 \gg \tau_2 \gg \tau_{\text{decay}}$$

0.1sec 1μ sec 100psec

M. Kroutvar et. al., Nature 432, 81 (2004)

A. Greulich et al, Science 313, 341 (2006)



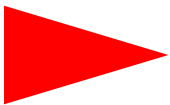
Limitation on the full process?

- Does the state degrade with number of photons (time)?
- Is the process limited by the $T2/1$ time?

NO

- Errors localize: constant, independent probability of error on each photon.

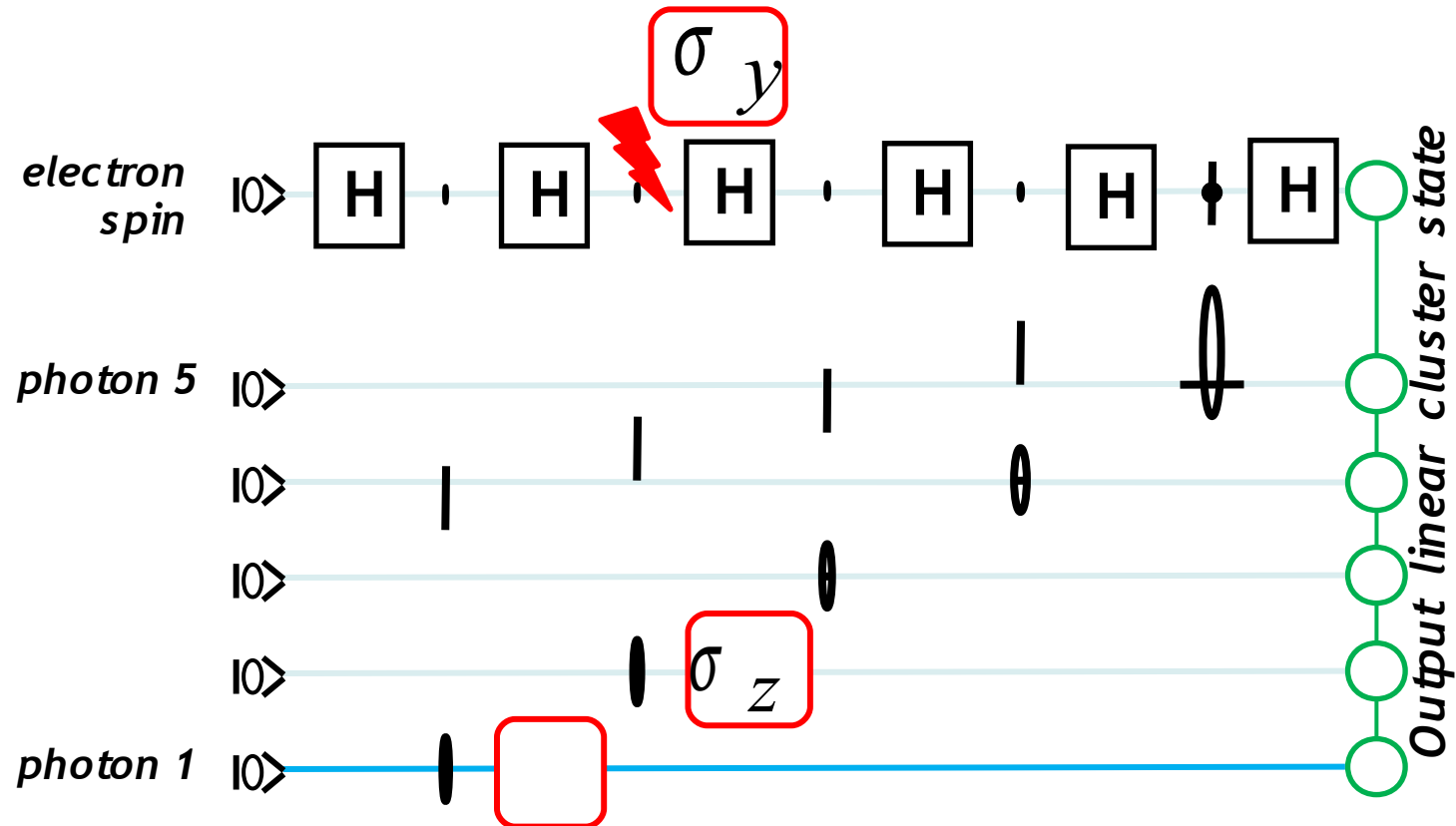
Process can continue irrespective of $T2/1$



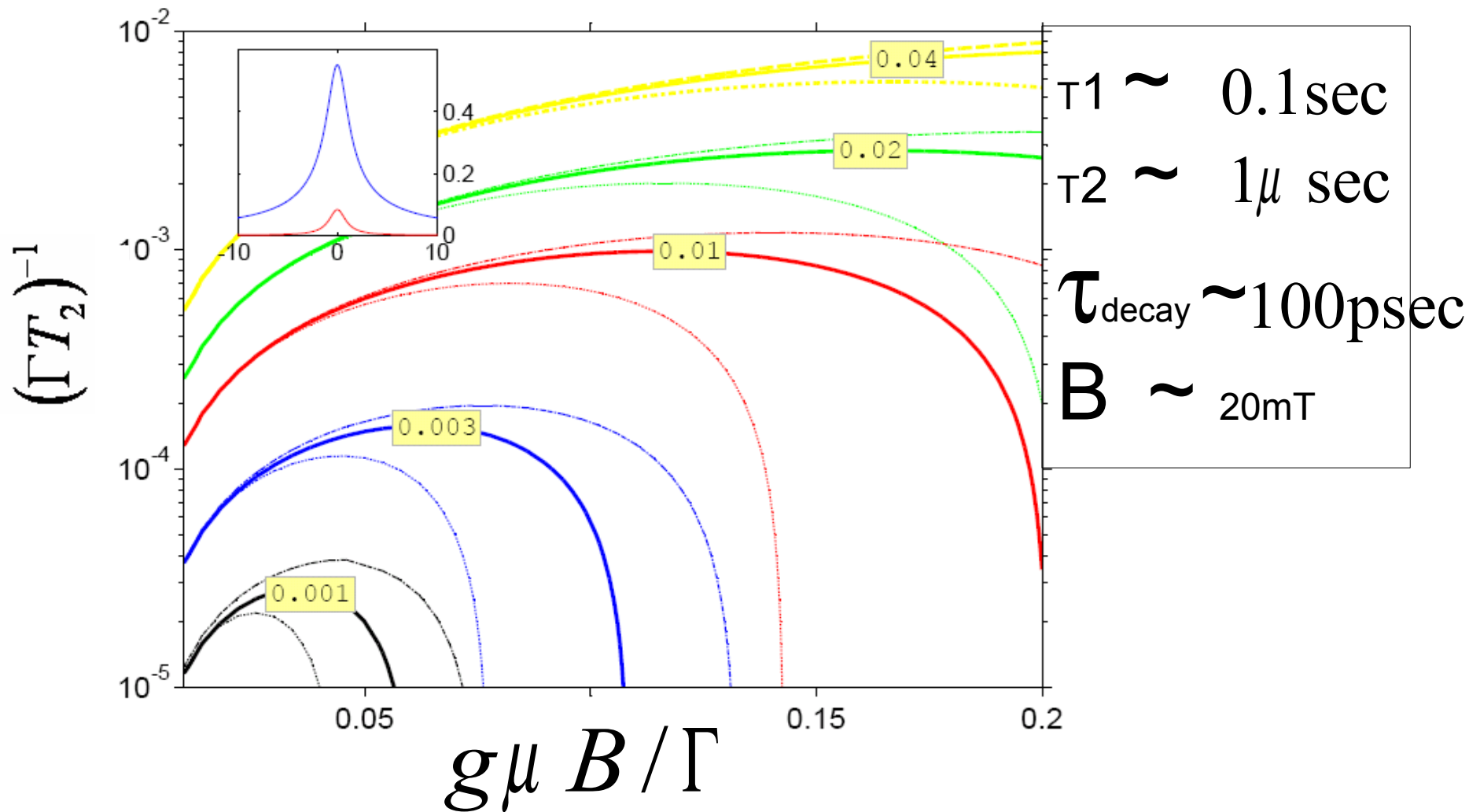
Localization of errors (2)

dephasing:

$$\rho_{n+1} = (1-p)\tilde{\rho}_{n+1} + p\sigma_Y^{\text{spin}}\tilde{\rho}_{n+1}\sigma_Y^{\text{spin}}$$

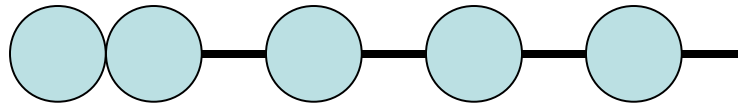


Error optimization



Constructing 2D cluster states

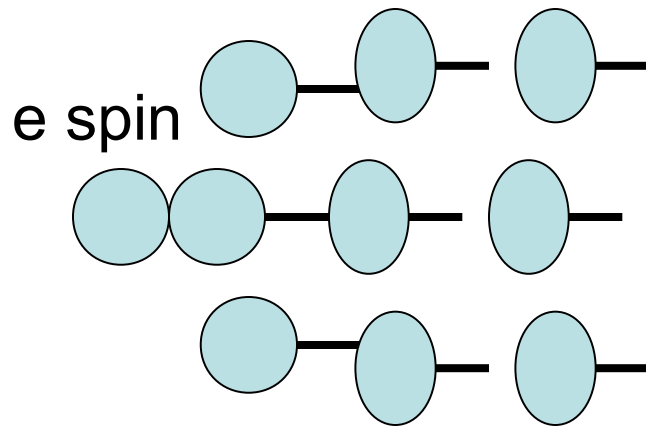
e spin



$$\pi / 2,$$

$$\pi / 2,$$

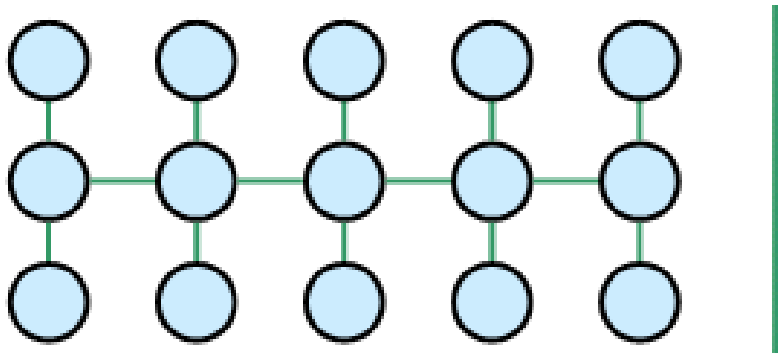
Constructing 2D cluster states



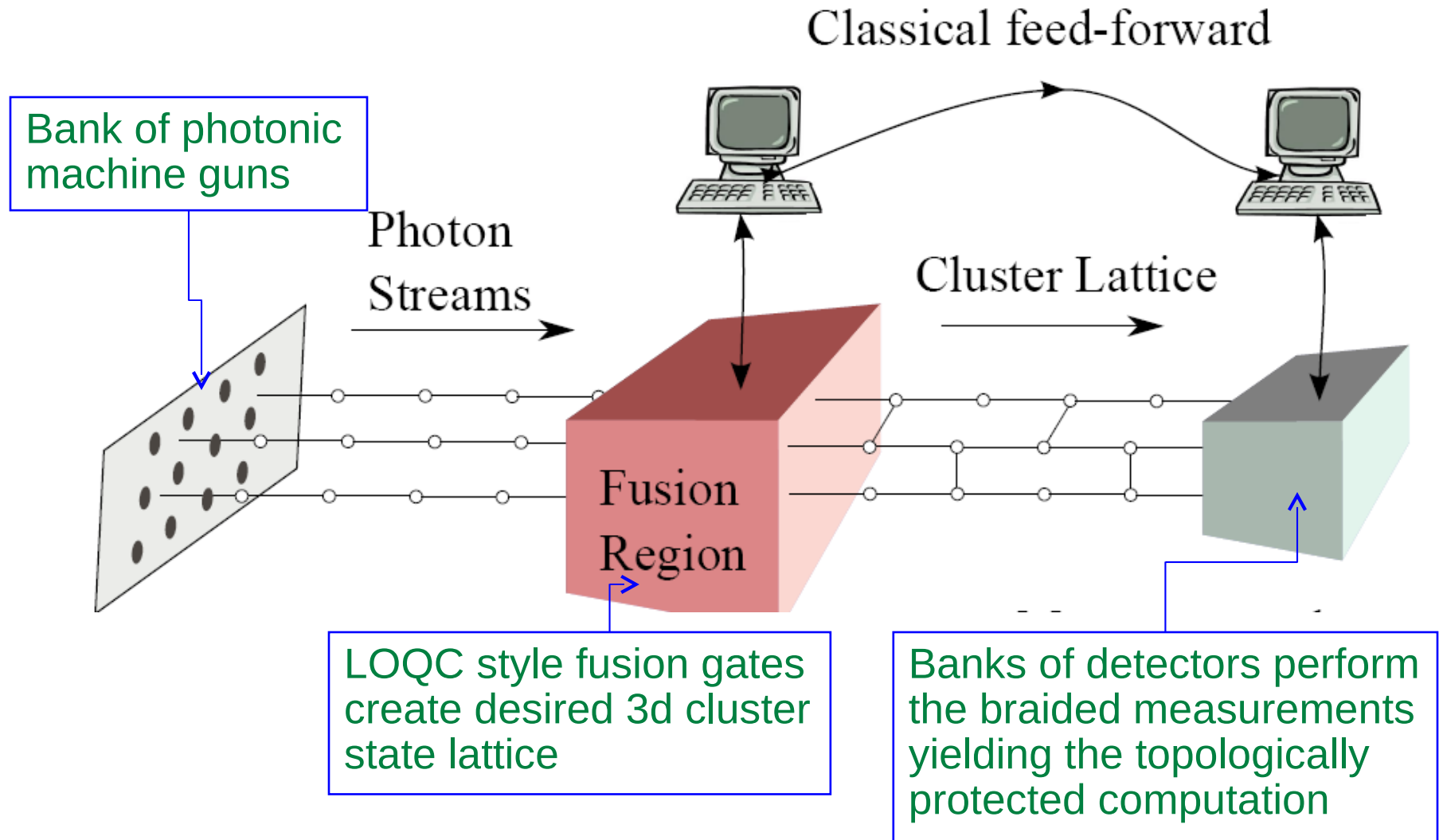
$$\pi / 2,$$

$$\pi / 2, \pi, \pi, \pi / 2, \pi, \pi,$$

Constructing 2D cluster states



Eventual architecture?



Summary and outlook

- Single quantum dots can emit large strings of 1D cluster states $\longrightarrow$ 2D clusters.
- Errors localize
- Process can proceed for times $\gg T_2, T_1$

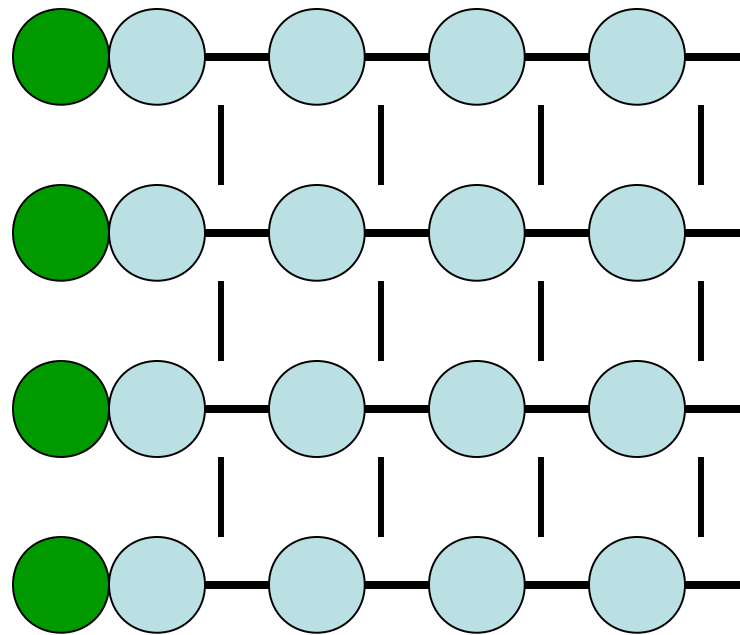
[arXiv: 0810.2587](https://arxiv.org/abs/0810.2587)

Summary and outlook

- Single quantum dots can emit large strings of 1D cluster states $\longrightarrow$ 2D clusters.
- Errors localize
- Process can proceed for times $\gg T_2, T_1$
- Outlook:
 - Coupling a number of quantum dots
 - Other single photon sources
 - Better bounds for non markovian environments.

[arXiv: 0810.2587](https://arxiv.org/abs/0810.2587)

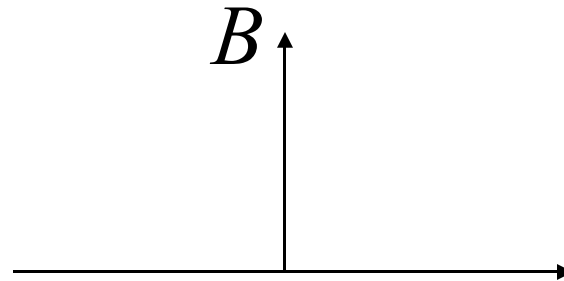
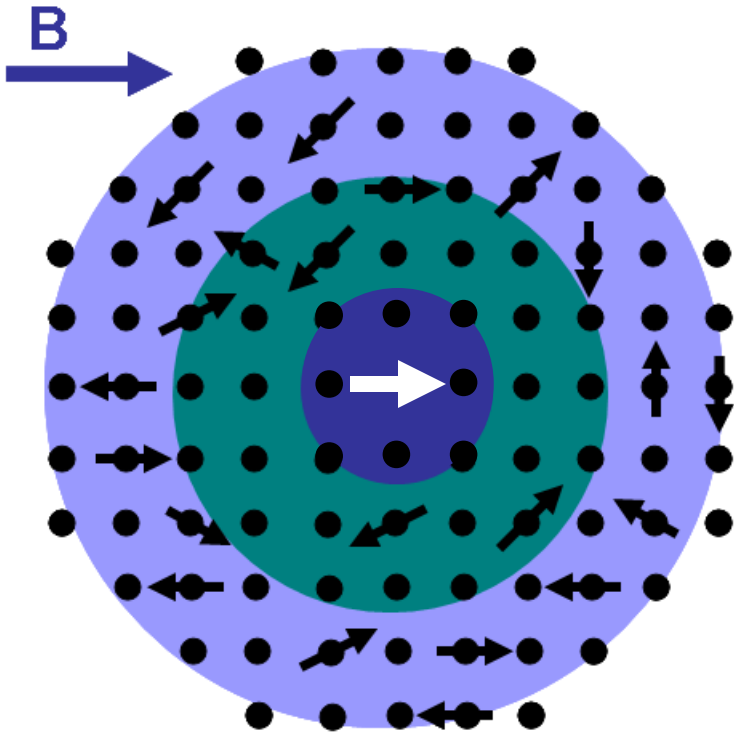
Parallel machine gun



Spin-bath Hamiltonian

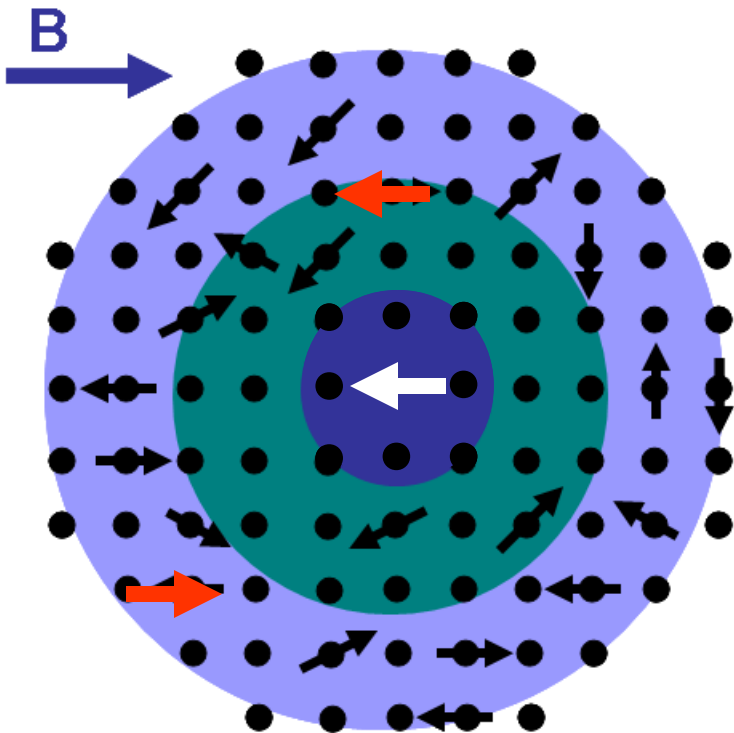
$$H = H_e + H_{eN} + H_N + H_{NN}$$

$$H_e = \mathbf{B} \cdot \mathbf{S}_e \quad \text{Zeeman}$$



Spin-bath Hamiltonian

$$H = H_e + H_N + H_{eN} + H_{NN}$$



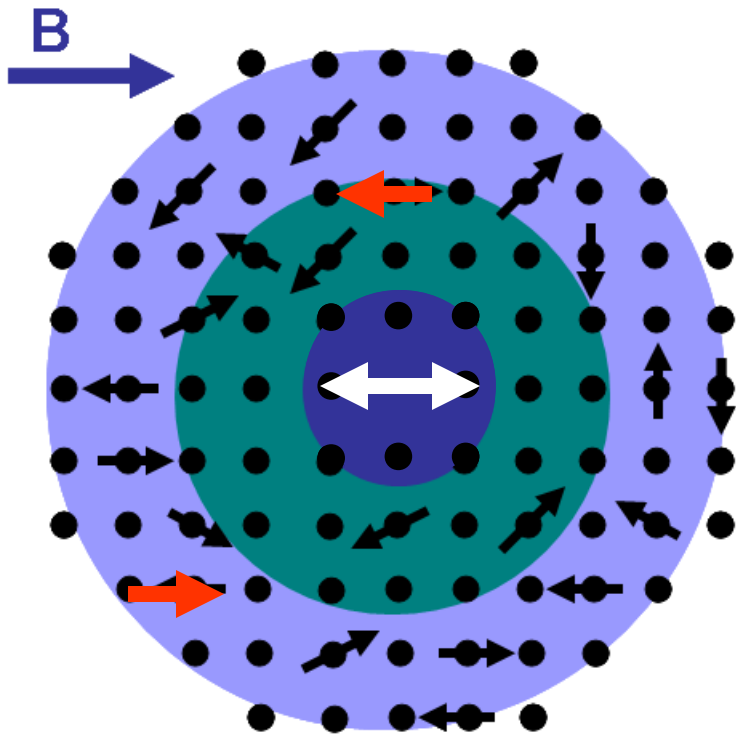
Energy scales

H_{eN}	$\alpha_n : 10^6 \text{ sec}^{-1}$
H_{NN}	$\beta_n : 10^2 \text{ sec}^{-1}$
H_e	$\mu_B : 10^{11} \text{ sec}^{-1} \text{ T}^{-1}$



Spin-bath Hamiltonian

$$H = H_e + H_N + H_{eN} + H_{NN}$$



Effective Interaction Hamiltonian



Spin wave-packet entanglement

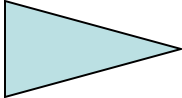
$$|\uparrow\rangle \rightarrow \sum_t A_t |\Omega(t)\rangle |t\rangle$$

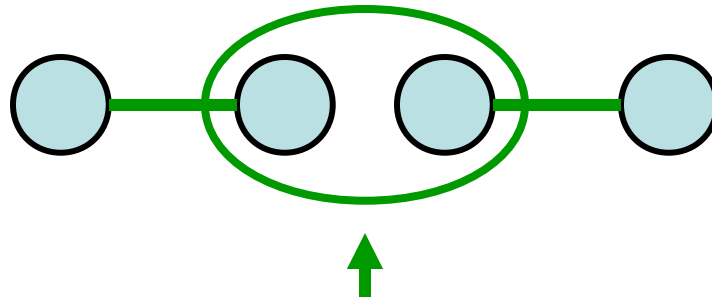
$$\rho(t_n + \tau) = U(\tau) \left(G^\dagger \rho(t_n) G + F^\dagger \rho(t_n) F \right) U^\dagger(\tau)$$

G= good mode function | F= bad mode function

$$g(k) = \frac{k - Z}{(k - Z)^2 - B^2}$$

Fuse larger clusters

- Two Bell pairs  3 photon C-state

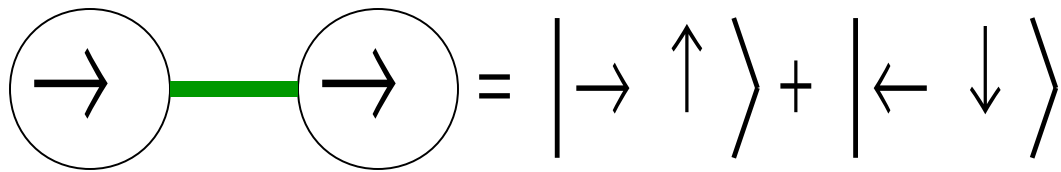


D. E. Browne and
T. Rudolph PRL 95,
010501 (2005).

Interference and measurement of one photon
Probability = $1/2$

- 3 photon C-state + Bell pair ...

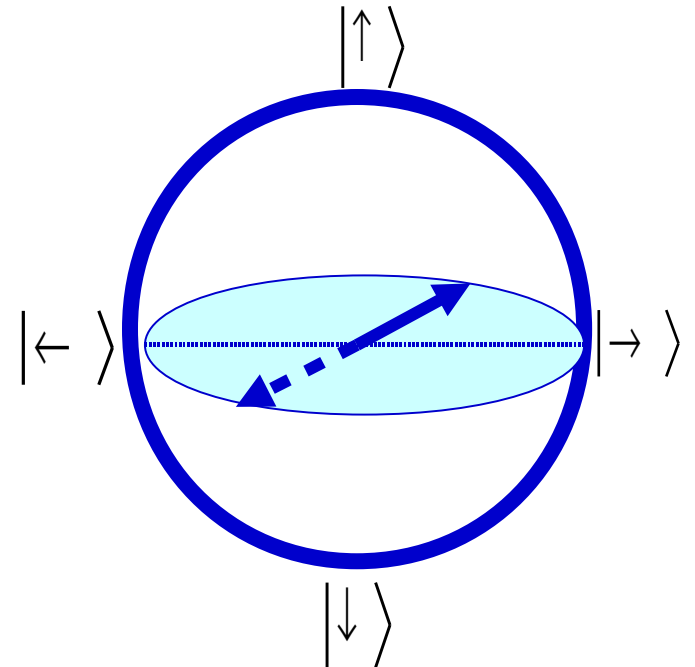
Two spins



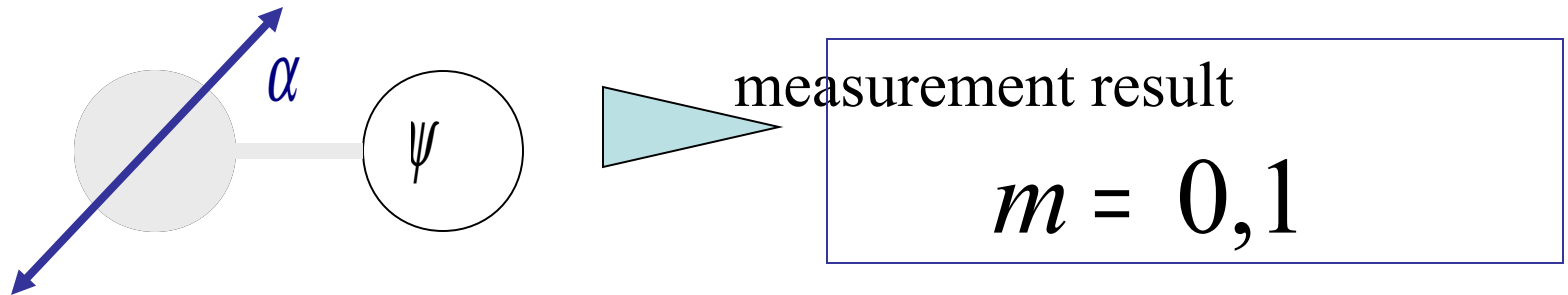
A diagram showing two white circles, each containing a right-pointing arrow. A thick green horizontal line connects the two circles. To the right of the second circle is an equals sign followed by the quantum state expression $|\rightarrow \uparrow\rangle + |\leftarrow \downarrow\rangle$.

$$\text{---} \rightarrow \text{---} = |\rightarrow \uparrow\rangle + |\leftarrow \downarrow\rangle$$

measure left spin
in xy plane basis



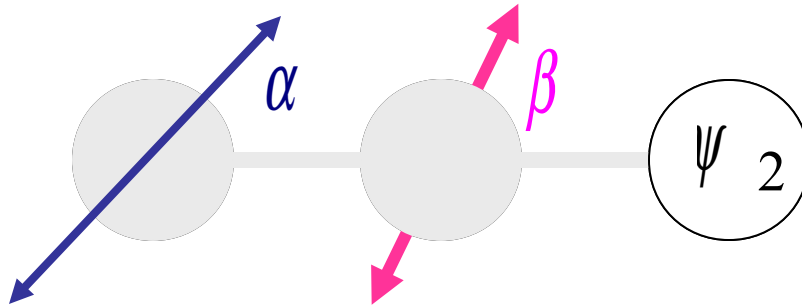
Two spins



$$\begin{aligned} |\psi\rangle &= (\sigma_x)^m \left(\cos(\alpha) |\uparrow\rangle + \sin(\alpha) |\downarrow\rangle \right) \\ &= (\sigma_x)^m H Z_\alpha |\rightarrow\rangle \end{aligned}$$

$$Z_\alpha = \exp(i\alpha \sigma_z) \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Three spins

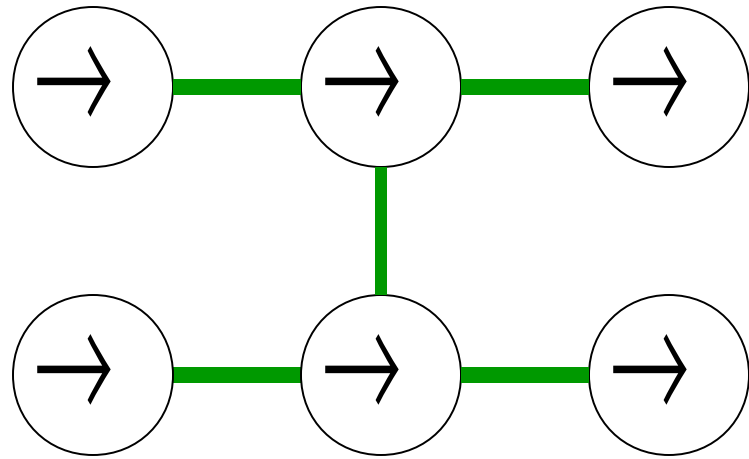


$$|\psi_2\rangle = \sigma \underbrace{HZ_\beta HZ_\alpha}_{X_\beta = \exp(i\beta \sigma_x)} |\rightarrow\rangle$$

Cluster states

Array of spins:

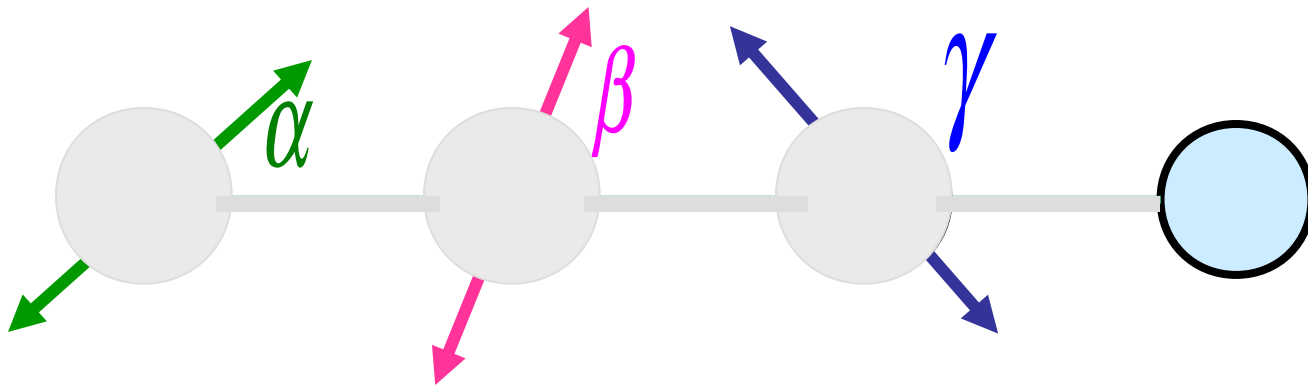
$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) =$$



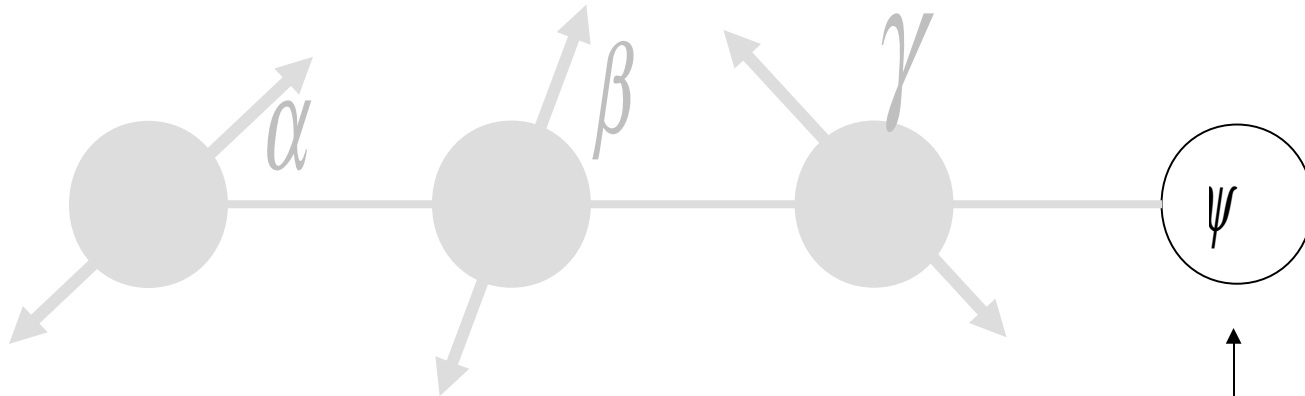
= control- σ_z

$$\left[\begin{array}{l} |\uparrow\uparrow\rangle \rightarrow |\uparrow\uparrow\rangle \\ |\uparrow\downarrow\rangle \rightarrow |\uparrow\downarrow\rangle \\ |\downarrow\uparrow\rangle \rightarrow |\downarrow\uparrow\rangle \\ |\downarrow\downarrow\rangle \rightarrow \text{---} |\downarrow\downarrow\rangle \end{array} \right.$$

1D chains



1D chains



Three measurements

Arbitrary rotation

$$|\psi\rangle = \sigma U(\alpha, \beta, \gamma) |\rightarrow\rangle$$

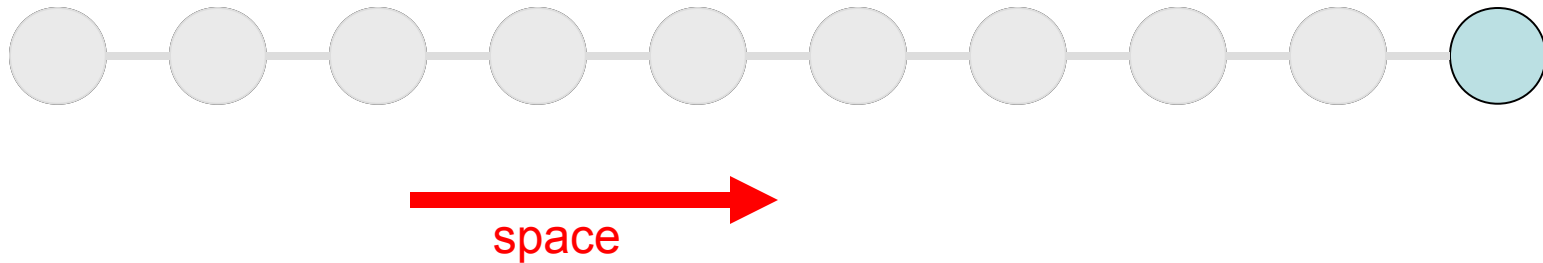
Note: Later angles depend on earlier measurement outcomes

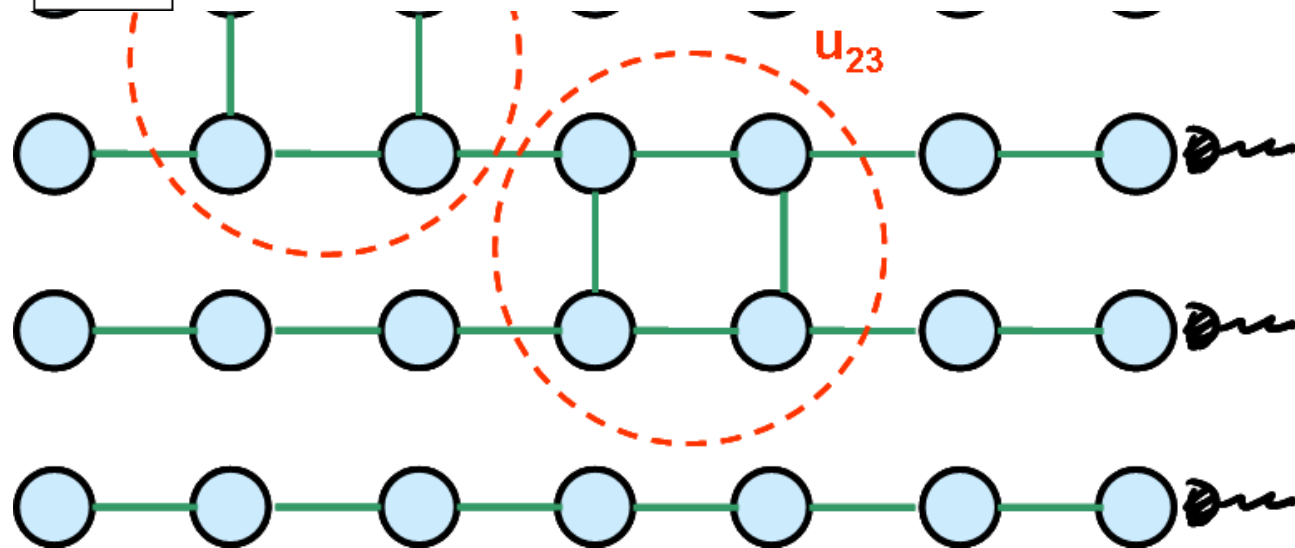
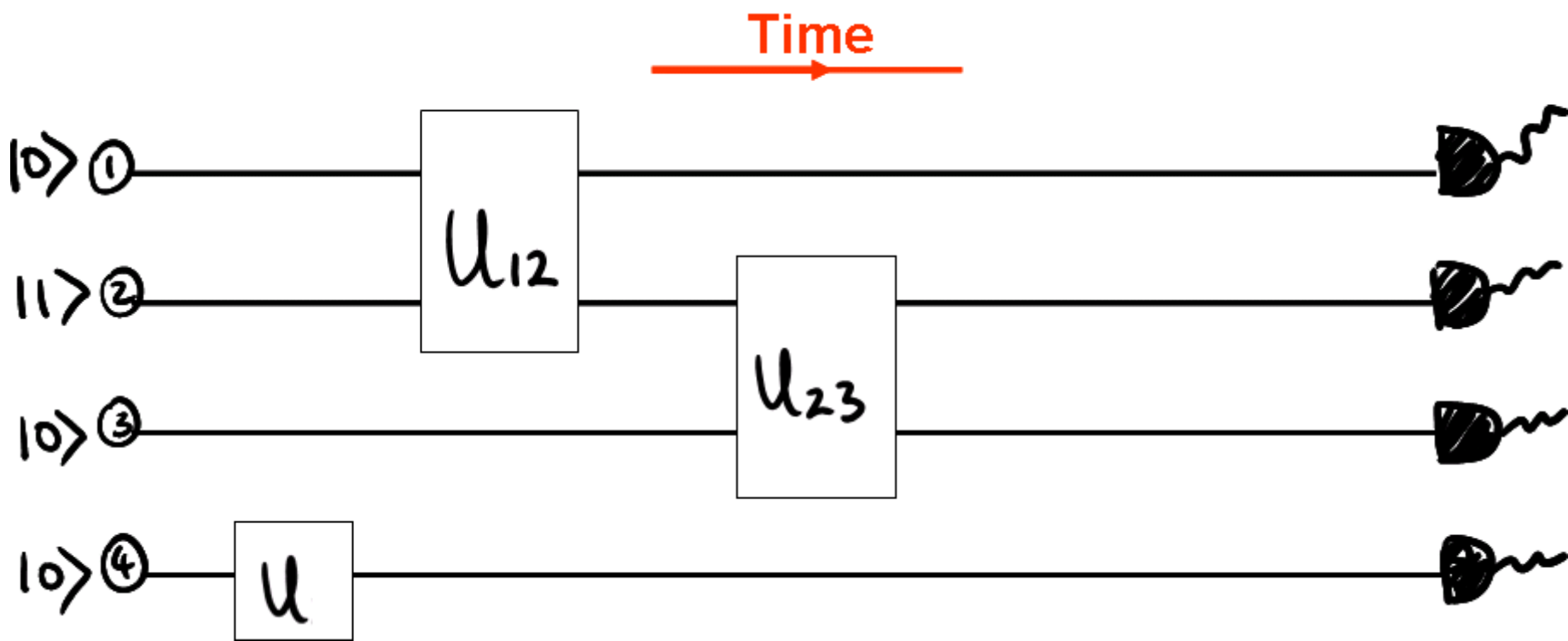
1D chains

Unitary time evolution

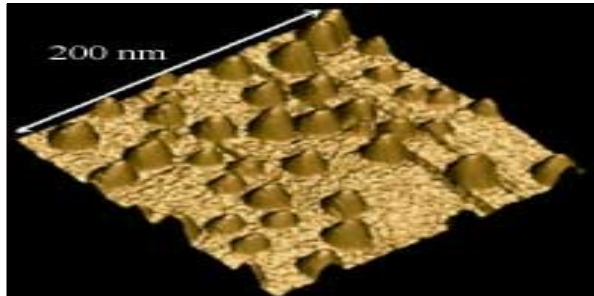


Evolution by measurements





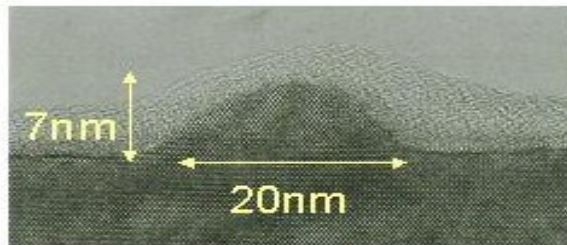
Strain induced self-assembled quantum dots



This scanning tunneling microscope image shows quantum dots made of indium arsenide and gallium arsenide. They are the type researchers used to observe electrons trapped inside.

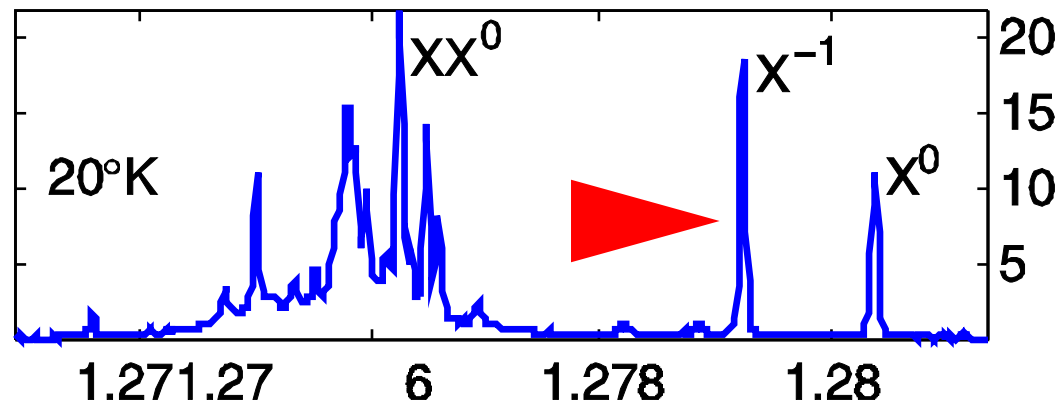
Source: University of Nottingham

STM (Nottingham)



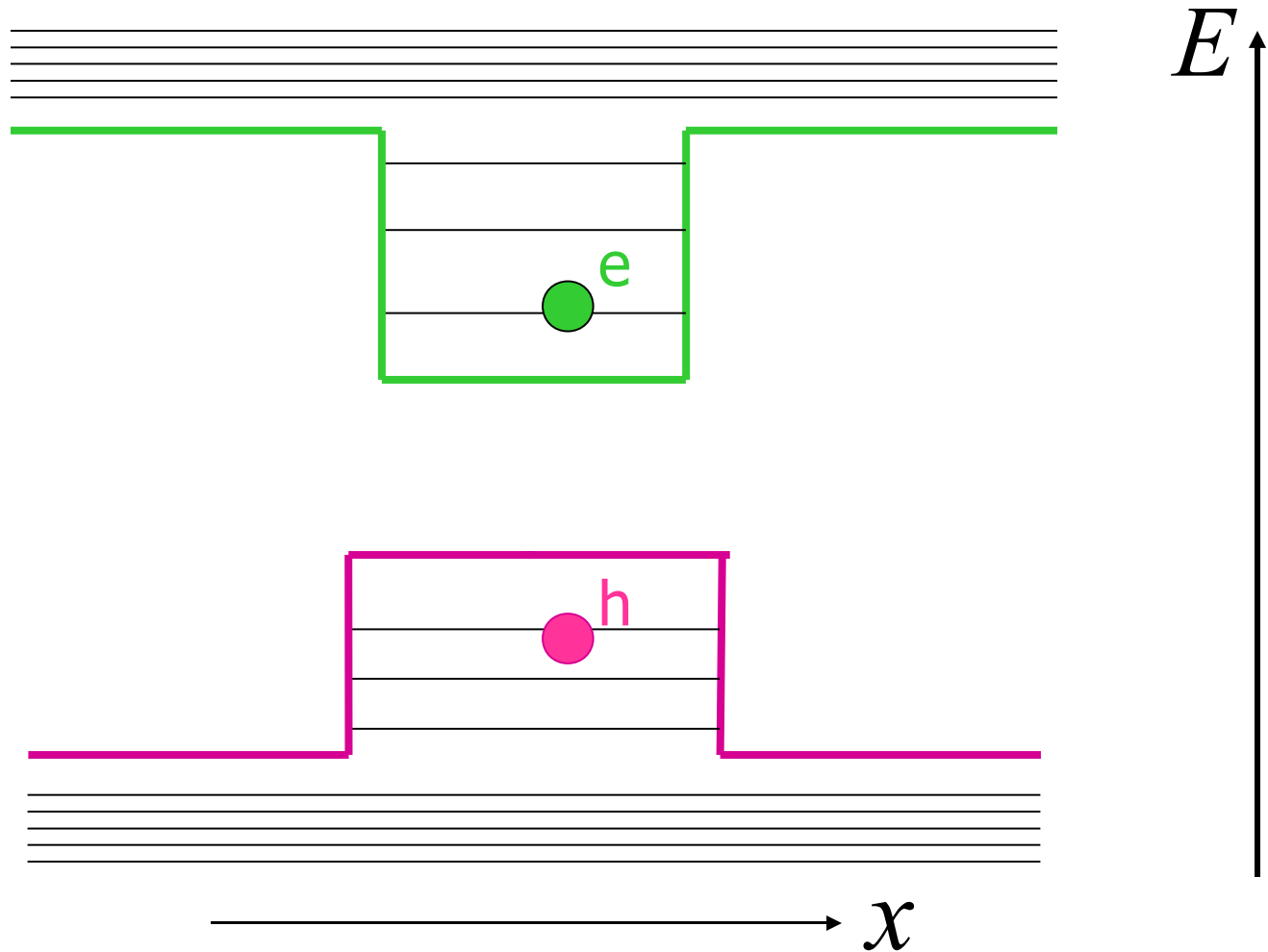
Cross-sectional TEM of an uncapped QD

PL spectra (Technion, UCSB)

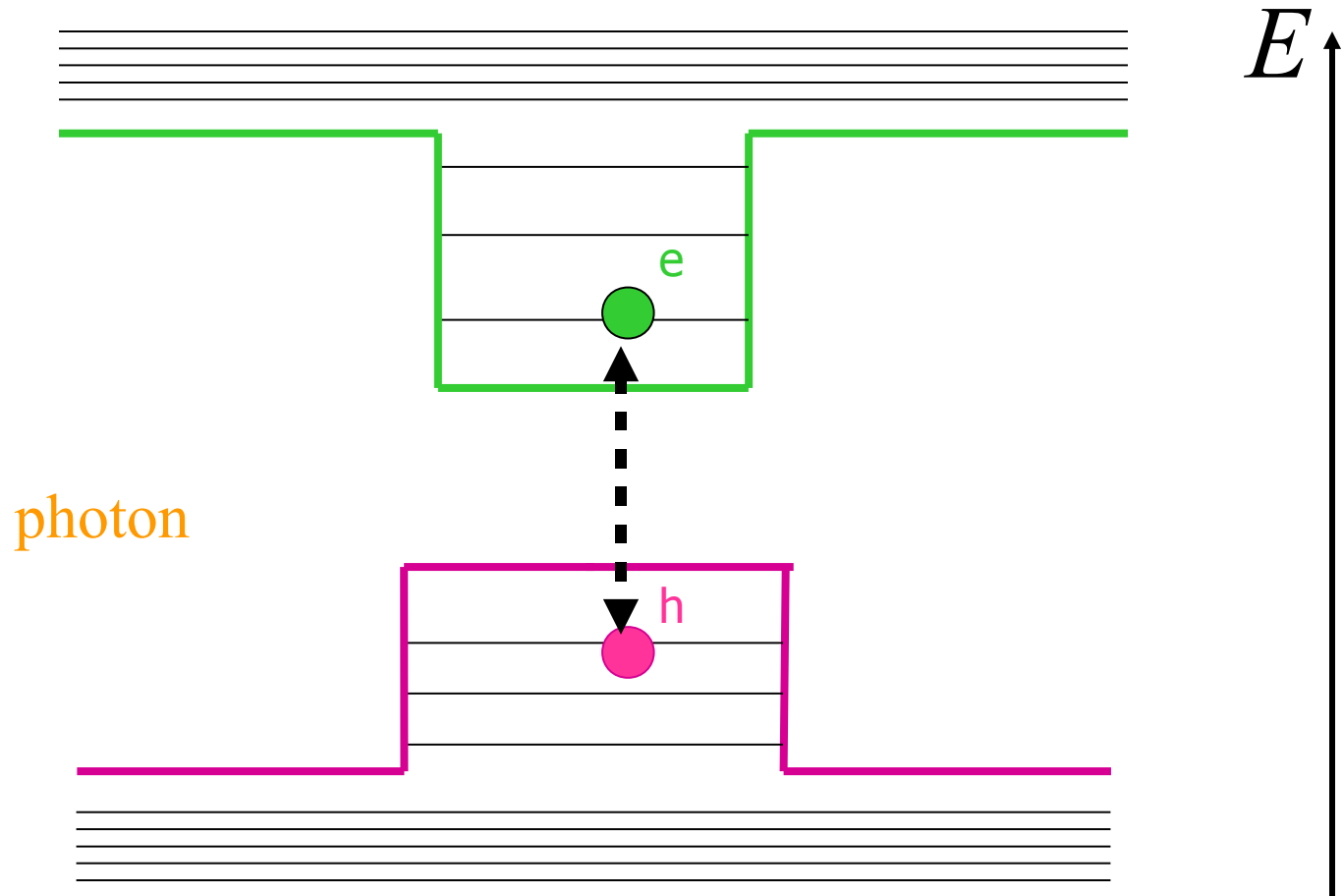


- N. Akopian *et. al.*, PRL **90**, 130501 (2006)
- Stevenson *et. al.*, Nature **439**, 179 (2006)
- M. Bayer *et. al.* PRB **65** 195315 (2002)
- Stefan Strauf *et. al.*, Nature Photonics **1**, 704 - 708 (2007)

Quantum dots



electron - hole recombination



Localization of errors (1)

ideally:

$$\tilde{\rho}_{n+1} = U^\dagger \rho_n U$$

$$\sum_k A_k^\dagger A_k = \mathbf{I}$$

decoherence:

dephasing:

$$\rho_{n+1} = (1 - p)\tilde{\rho}_{n+1} + p\sigma_Y^{\text{spin}}\tilde{\rho}_{n+1}\sigma_Y^{\text{spin}}$$

