

# Merging photonic crystal cavities and single quantum dots: A practical source of entangled photon pairs

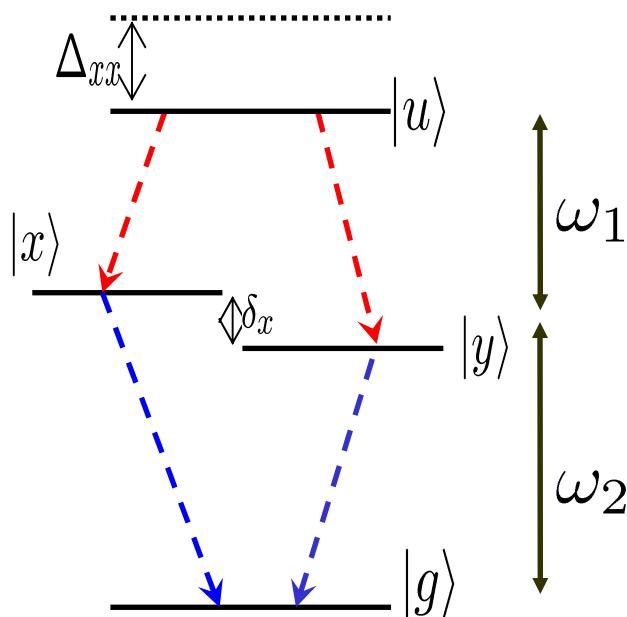
P. K. Pathak and S. Hughes



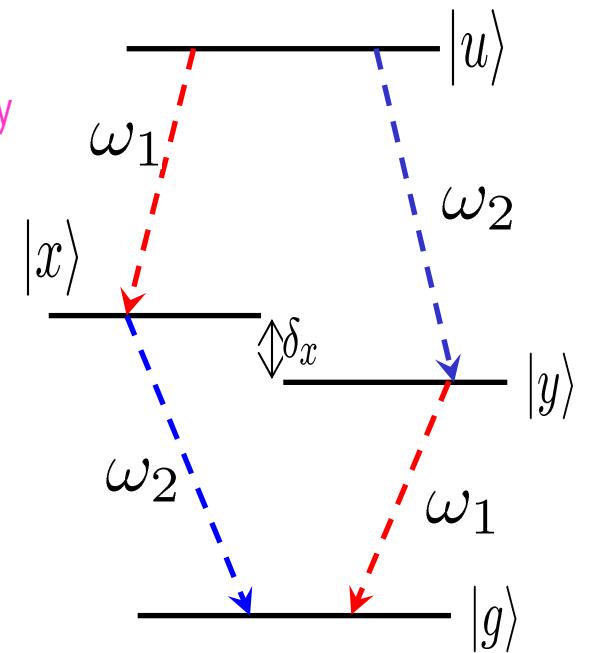
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# Entangled photons from a single quantum dot

“Within” generation



“Across” generation



Removing binding energy

M. E. Reimer *et al.*,  
arXiv:0706.1075 (07)

Problem : Anisotropic energy gap

Removing anisotropic gap

- \* Spectral filtering
- \* External fields
- \* Cavity QED

Problem: Time order

Reordering time  
Time delay

# Quantum state of the emitted photon pair

For within generation

$$|\psi\rangle = \sum_{k,l} c_{kl} |\mathbf{1}_k\rangle_x |\mathbf{1}_l\rangle_x + d_{kl} |\mathbf{1}_k\rangle_y |\mathbf{1}_l\rangle_y$$

For across generation

$$|\psi\rangle = \sum_{k,l} c_{kl} |\mathbf{1}_k\rangle_x |\mathbf{1}_l\rangle_x + d_{lk} |\mathbf{1}_l\rangle_y |\mathbf{1}_k\rangle_y$$

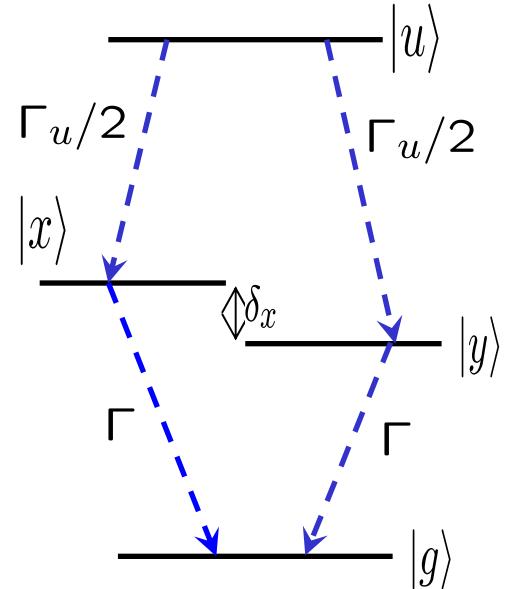
$$c_{kl}(d_{kl}) = \frac{\sqrt{\Gamma_u \Gamma / 2\pi^2}}{(\omega_k + \omega_l - \omega_u + i\Gamma_u)(\omega_l - \omega_{x(y)} + i\Gamma)}$$

Using the polarization states as basis

Off-diagonal element of two-photon density matrix:

$$\gamma = \rho_{xxyy} = \int \int c_{kl}^* d_{kl} d\omega_k d\omega_l$$

$$\text{Concurrence } C = 2|\gamma|$$



# Requirements for generating entangled photons

within generation

$$\gamma = \frac{\Gamma_u \Gamma}{2\pi^2} \int \int \frac{d\omega_k d\omega_l}{|\omega_k + \omega_l - \omega_u + i\Gamma_u|^2 (\omega_l - \omega_y + i\Gamma)(\omega_l - \omega_x - i\Gamma)}$$

Concurrence  $C = 2|\gamma| = |\frac{\Gamma_u/2}{\Gamma + i\delta_x}|$  Only significant  $\delta_x \rightarrow 0$

→ Degeneracy in excitons is required

across generation

$$\gamma = \frac{\Gamma_u \Gamma}{2\pi^2} \int \int \frac{d\omega_k d\omega_l}{|\omega_k + \omega_l - \omega_u + i\Gamma_u|^2 (\omega_k - \omega_y + i\Gamma)(\omega_l - \omega_x - i\Gamma)}$$

Vanishes → no entanglement! ??

$$|\psi\rangle = \sum_{k,l} c_{kl} |1_k\rangle_x |1_l\rangle_x + d_{lk} |1_l\rangle_y |1_k\rangle_y$$

Distinguishable paths in time

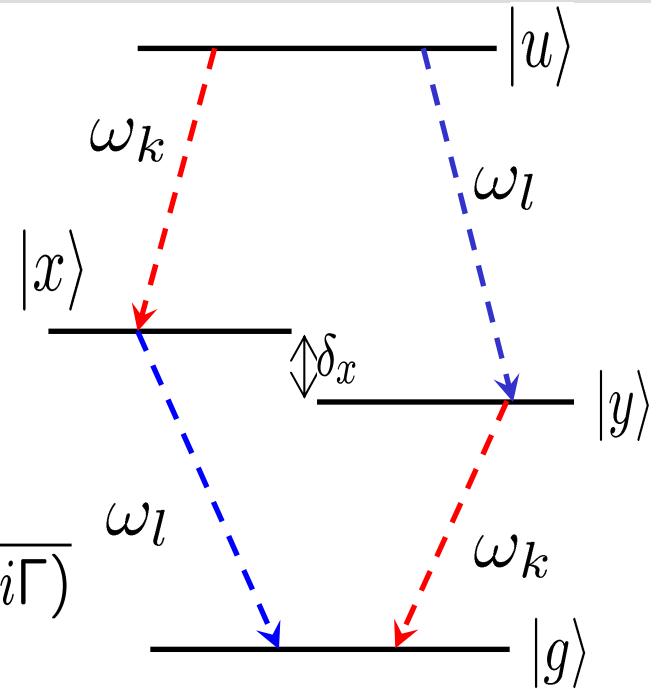
→ Time reordering is required

# Time reordering

For time delay of photons:

$$W_{\text{opt}} = e^{i(\omega_k - \omega_l)t_0}$$

$$\gamma = \frac{\Gamma_u \Gamma}{2\pi^2} \int \int \frac{W_{\text{opt}}(\omega_k, \omega_l) d\omega_k d\omega_l}{|\omega_k + \omega_l - \omega_u + i\Gamma_u|^2 (\omega_k - \omega_y + i\Gamma)(\omega_l - \omega_x - i\Gamma)}$$



For time delayed photons

$$\gamma = \frac{2\Gamma e^{-2\Gamma t_0}}{\Gamma_u} (1 - e^{-\Gamma_u t_0})$$

Avron et al., PRL 100, 120501 (2008)

Pathak and Hughes, PRL 103, 048901 (2009)

# Results for time delayed photons

For quantum dots

$$\Gamma_u/\Gamma = 2$$

$$\gamma = 0.25$$

concurrence = 0.5

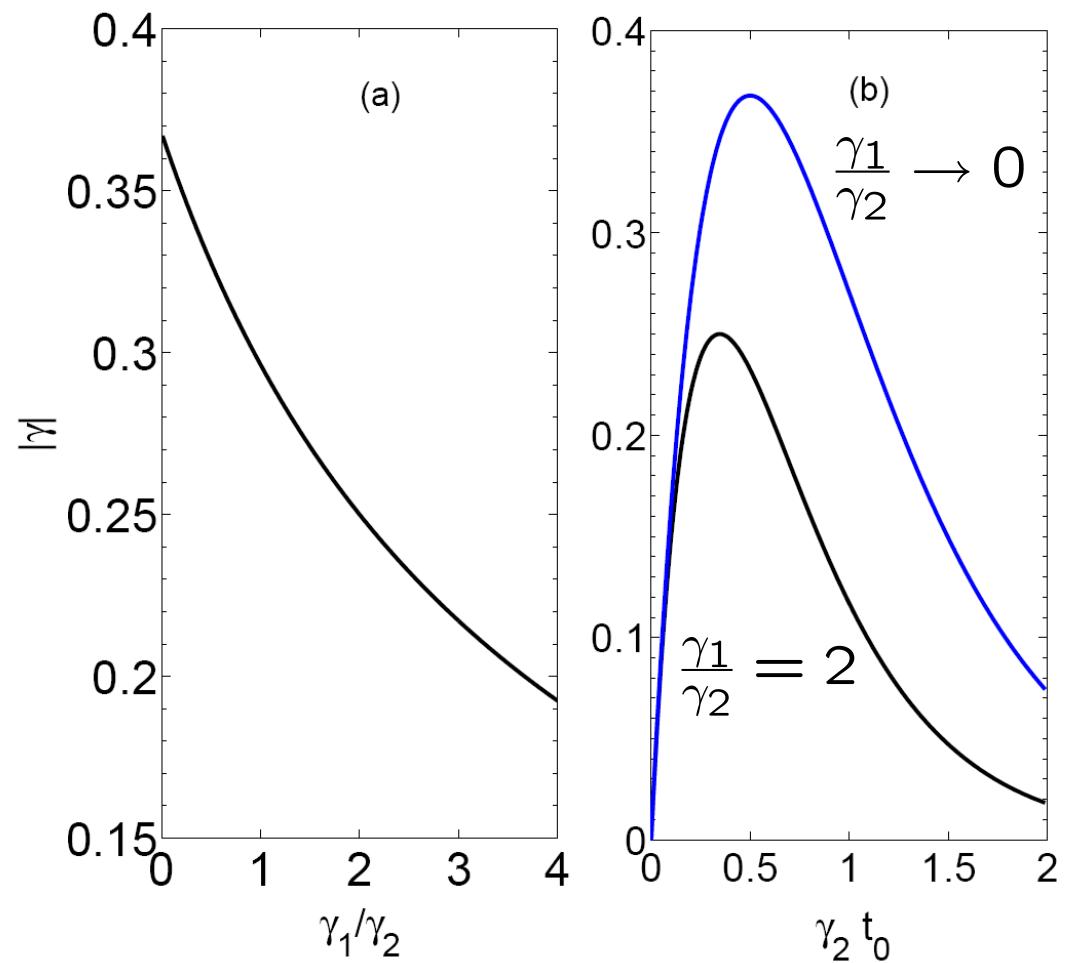
If somehow manipulate decay rates of biexciton and excitons:

$$\Gamma_u/\Gamma \rightarrow 0$$

$$\gamma = 1/e$$

concurrence = 0.73

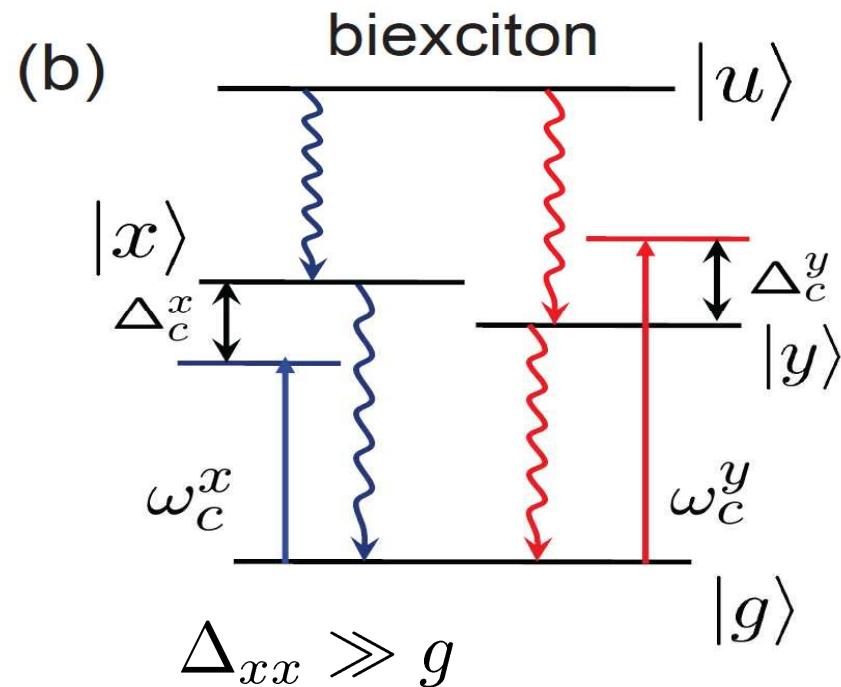
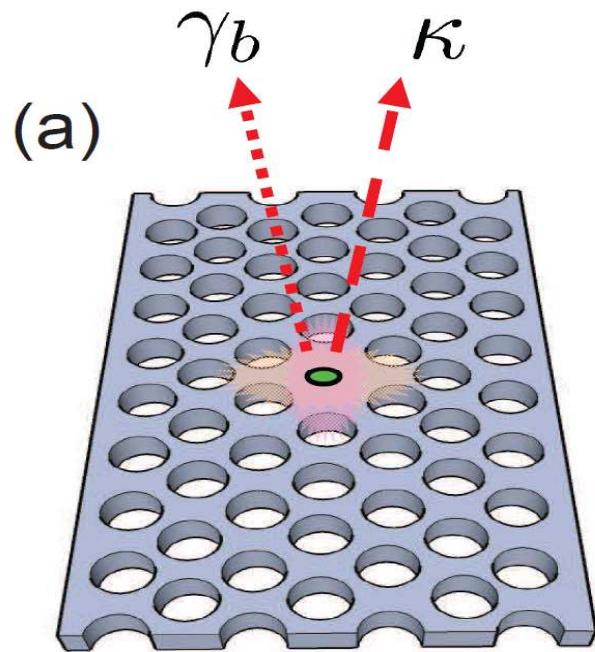
Pathak & Hughes, PRL 103, 048901 (2009)



→ Manipulation of decay rates is necessary

# Cavity-assisted generation of the entangled photons

Johne et al., PRL 100, 240404 (2008)  
Pathak & Hughes, PRB 79, 205416 (2009)



1. Spontaneous decay
2. Leak from cavity modes

\*biexciton decays spontaneously  
\*excitons coupled with cavity

Related cavity-tuning via AFM-oxidation:  
see K. Hennessy et al., APL 89, 041118 (2006)  
Jelena's talk for temp tuning.

## Two-photon probabilities

Spontaneously (via radiation modes) emitted pair:

$$S_r(\omega_k, \omega_l) = \frac{\Gamma_u \Gamma_r}{\pi^2} \frac{|\omega_l - \omega_c^\alpha + i\kappa|^2}{[(\omega_k + \omega_l - \omega_u)^2 + \gamma_1^2] |\omega_l - \omega_\alpha + ig_+^\alpha|^2 |\omega_l - \omega_\alpha + ig_-^\alpha|^2}$$

Cavity-induced splitting of spectra

$$g_\pm^\alpha = \frac{1}{2} \left( \kappa + \gamma_2 - i\Delta_c^\alpha \pm i\sqrt{4g^2 - (\kappa - \gamma_2 - i\Delta_c^\alpha)^2} \right)$$

$\gamma_i$  → Radiative + nonradiative broadening

One photon emitted spontaneously from biexciton  
and the other emitted via the cavity from exciton

$$S_c(\omega_k, \omega_l) = \frac{\Gamma_u \kappa}{\pi^2} \frac{g^2}{[(\omega_k + \omega_l - \omega_u)^2 + \gamma_1^2] |\omega_l - \omega_\alpha + ig_+^\alpha|^2 |\omega_l - \omega_\alpha + ig_-^\alpha|^2}$$

## Emitted spectrum from the polaritons

$$S_c(\omega_l) = \int_{-\infty}^{\infty} S_c(\omega_k, \omega_l) d\omega_k = \frac{g^2 \kappa}{|\omega_l - \omega_\alpha + ig_+^\alpha|^2 |\omega_l - \omega_\alpha + ig_-^\alpha|^2}$$

Exciton spectrum has two peaks corresponding to

$$\omega_l \approx \omega_{x,y}^\pm$$

**Polariton state:**  $\omega_{x,y}^\pm = \frac{1}{2} \left( \omega_c^{x,y} + \omega_{x,y} \pm \sqrt{(\Delta_c^{x,y})^2 + 4g^2} \right)$

Biexciton spectrum of photons has peaks at

$$\omega_k \approx \omega_u - \omega_{x,y}^\pm$$

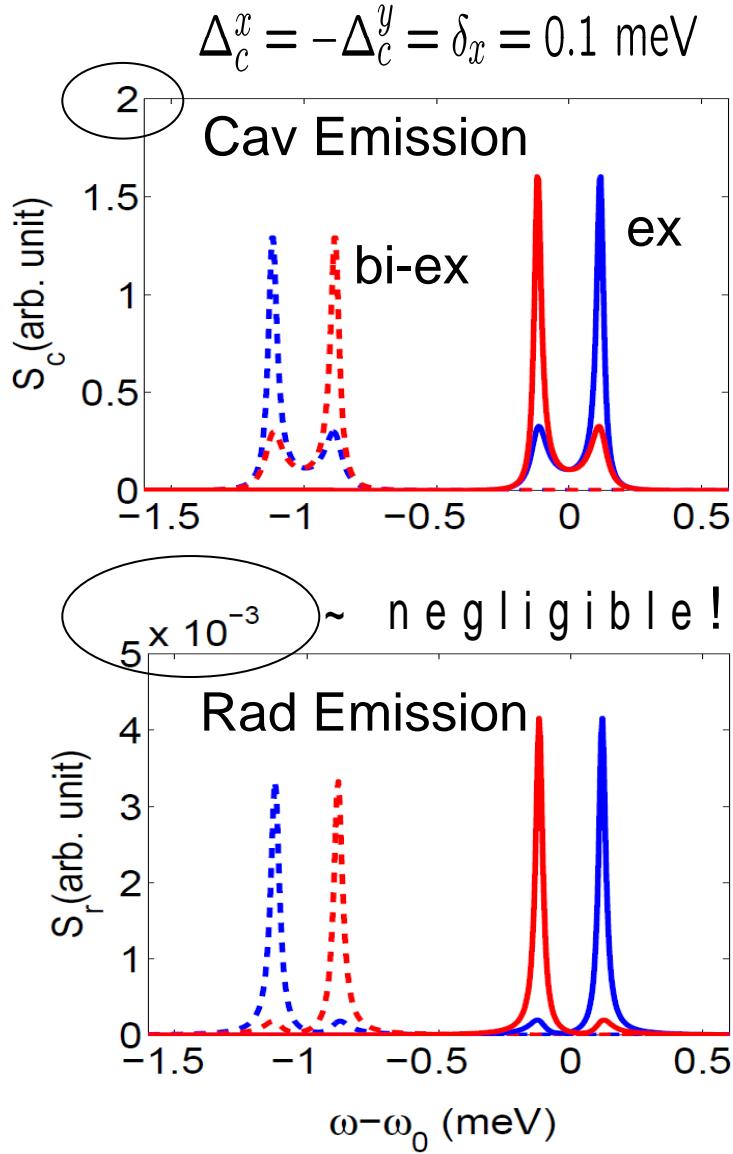
Recover one-photon results of:

S. Hughes and P. Yao, Opt. Express **17**, 3322 (2009)

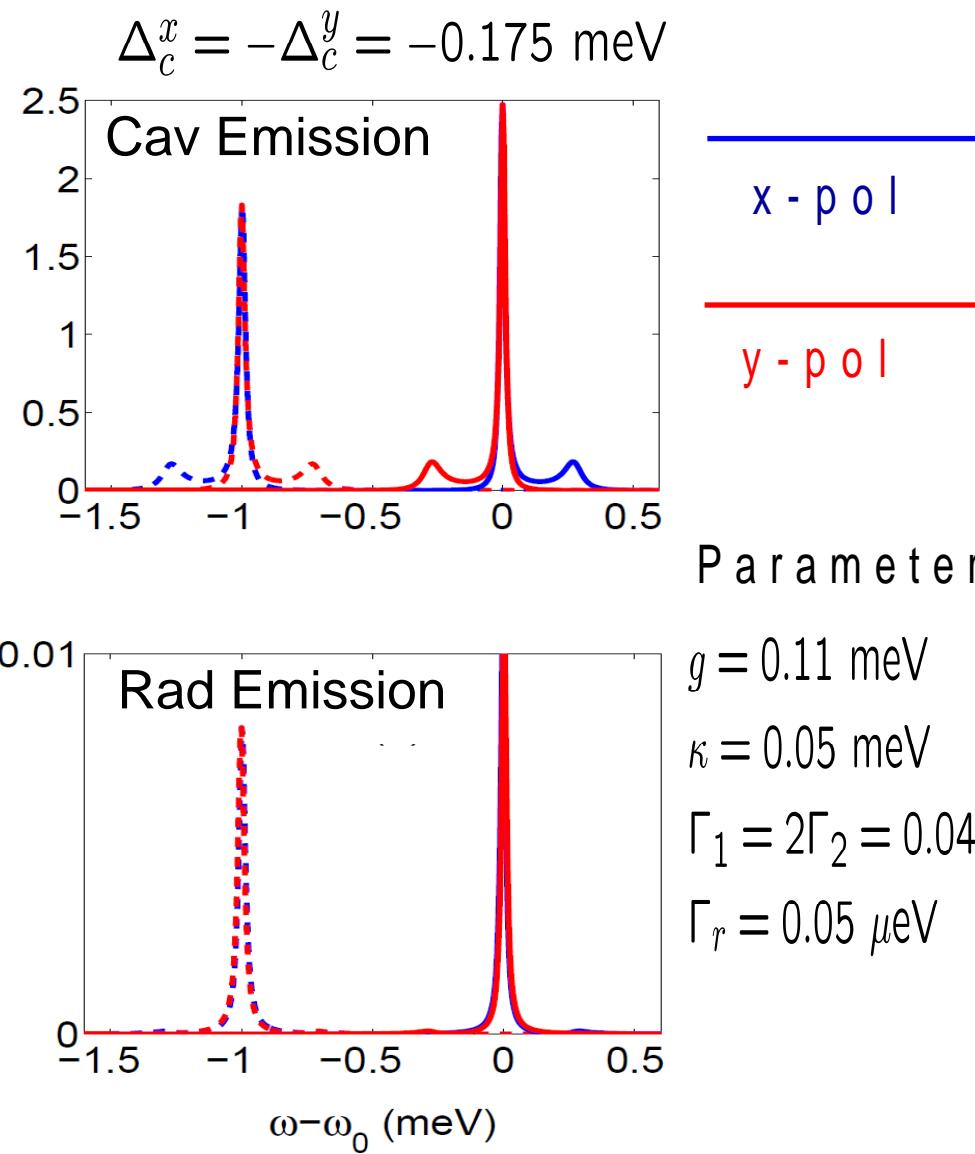
G. Cui and M. G. Raymer, PRA **73**, 53807 (2006)

# Two photon spectra

Scheme 1



Scheme 2



Parameters:

$$g = 0.11 \text{ meV}$$

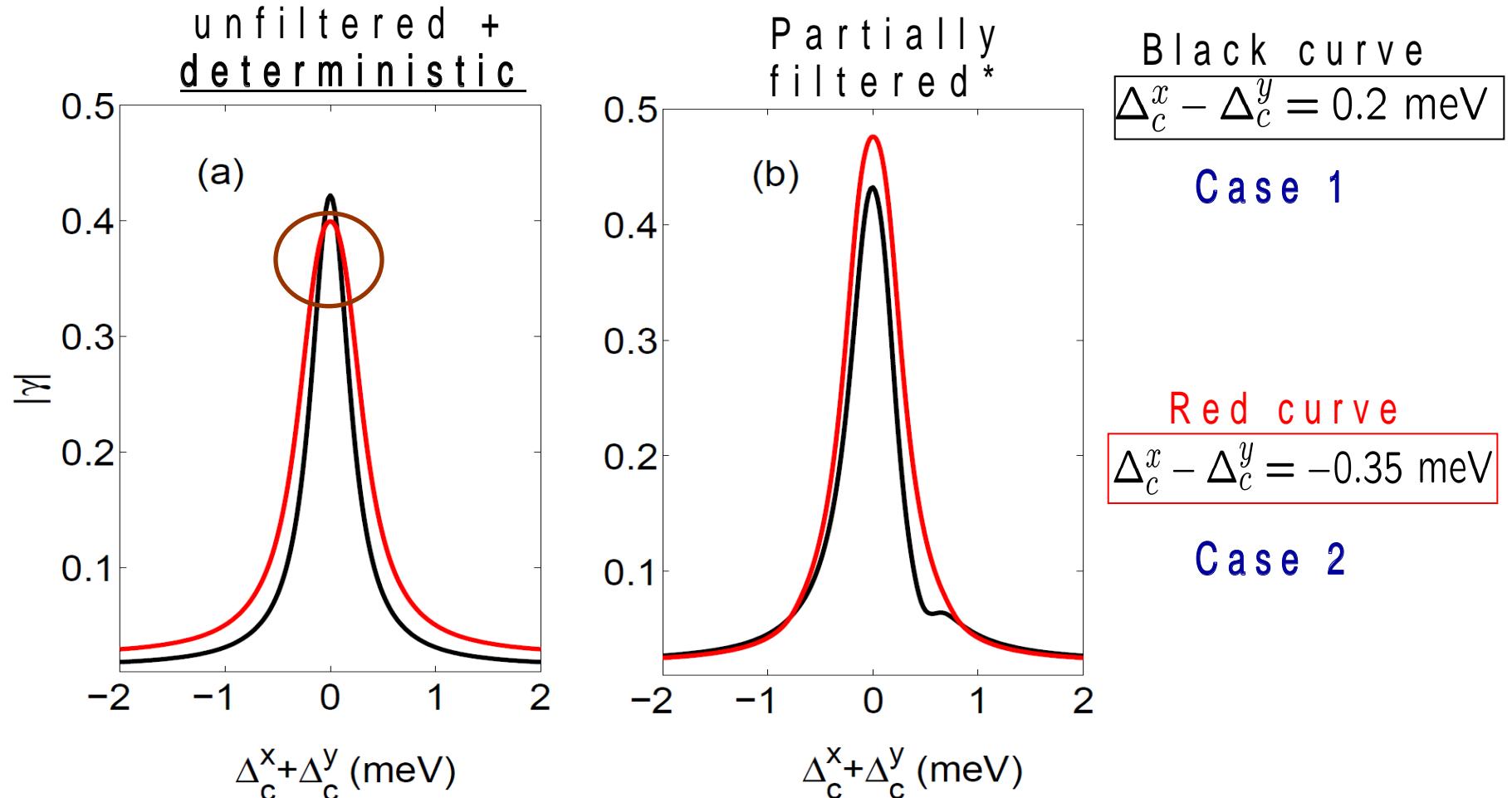
$$\kappa = 0.05 \text{ meV}$$

$$\Gamma_1 = 2\Gamma_2 = 0.04 \text{ meV}$$

$$\Gamma_r = 0.05 \mu\text{eV}$$

# Two-photon entanglement (Concurrence C=2|γ|)

Pathak and Hughes, PRB 79, 205416 (2009)



\* The filter function corresponds to two spectral windows of width  $w=0.2$  meV centered at  $\omega_x^-$  and  $\omega_u - \omega_x^-$

## Remarks

- ◆ Pros
  - We have developed a general theory for a single QD biexciton-exciton cascade coupled with a PC-cavity.
  - We predict deterministic Concurrence values larger than 0.8 using experimental parameters.
  - Improvements can be made with a simple spectral filter, but with a reduced probability.
- ◆ Cons
  - The biexciton state is not coupled with cavity modes and has very long life time.
  - The photons emitted in spontaneous decay of biexciton have poor collection efficiency.

# Coupling biexciton to cavity modes\*

Lateral gating can tune the biexciton in resonance with the same cavity modes: Reimer et al, PRB 78, 195301 (2008).

$$H_I(t) = \hbar g_1^x |x\rangle\langle g| \hat{a}_c^x e^{i\Delta_c^x t} + \hbar g_2^x |u\rangle\langle x| \hat{a}_c^x e^{i(\omega_{ux}-\omega_c^x)t} \\ + \hbar g_1^y |y\rangle\langle g| \hat{a}_c^y e^{i\Delta_c^y t} + \hbar g_2^y |u\rangle\langle y| \hat{a}_c^y e^{i(\omega_{uy}-\omega_c^y)t} + \text{H.C.}$$

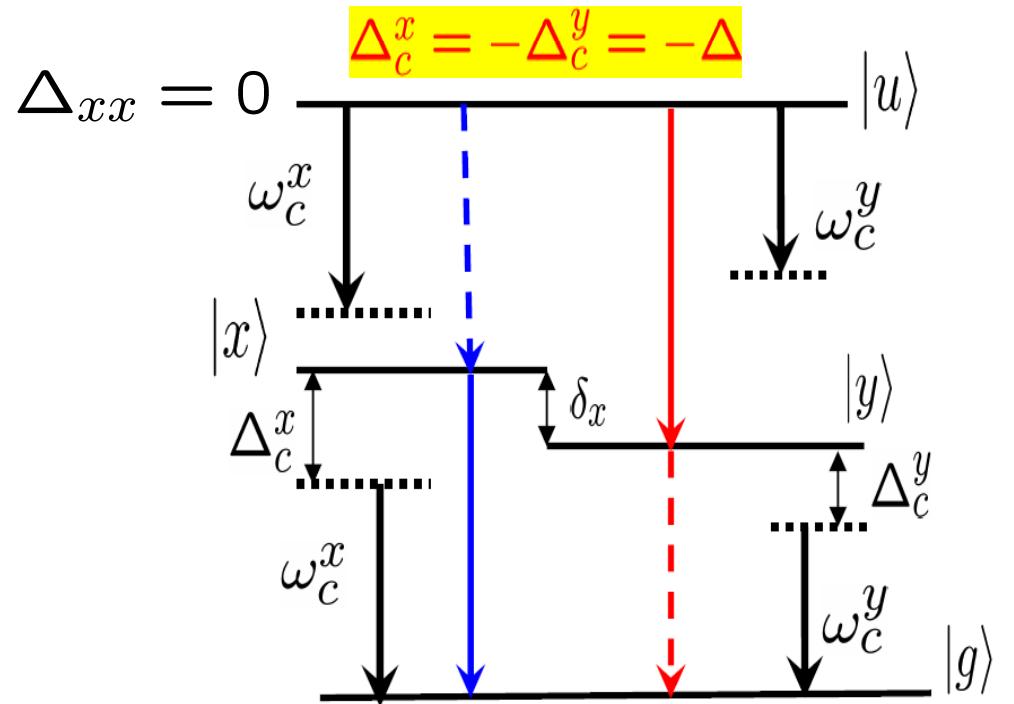
Biexciton decay rates

$$\Gamma_1^x = \frac{(g_2^x)^2 \kappa}{[\kappa^2 + (\Delta_c^x - \delta_x)^2]}$$

$$\Gamma_1^y = \frac{(g_2^y)^2 \kappa}{[\kappa^2 + (\Delta_c^y + \delta_x)^2]}$$

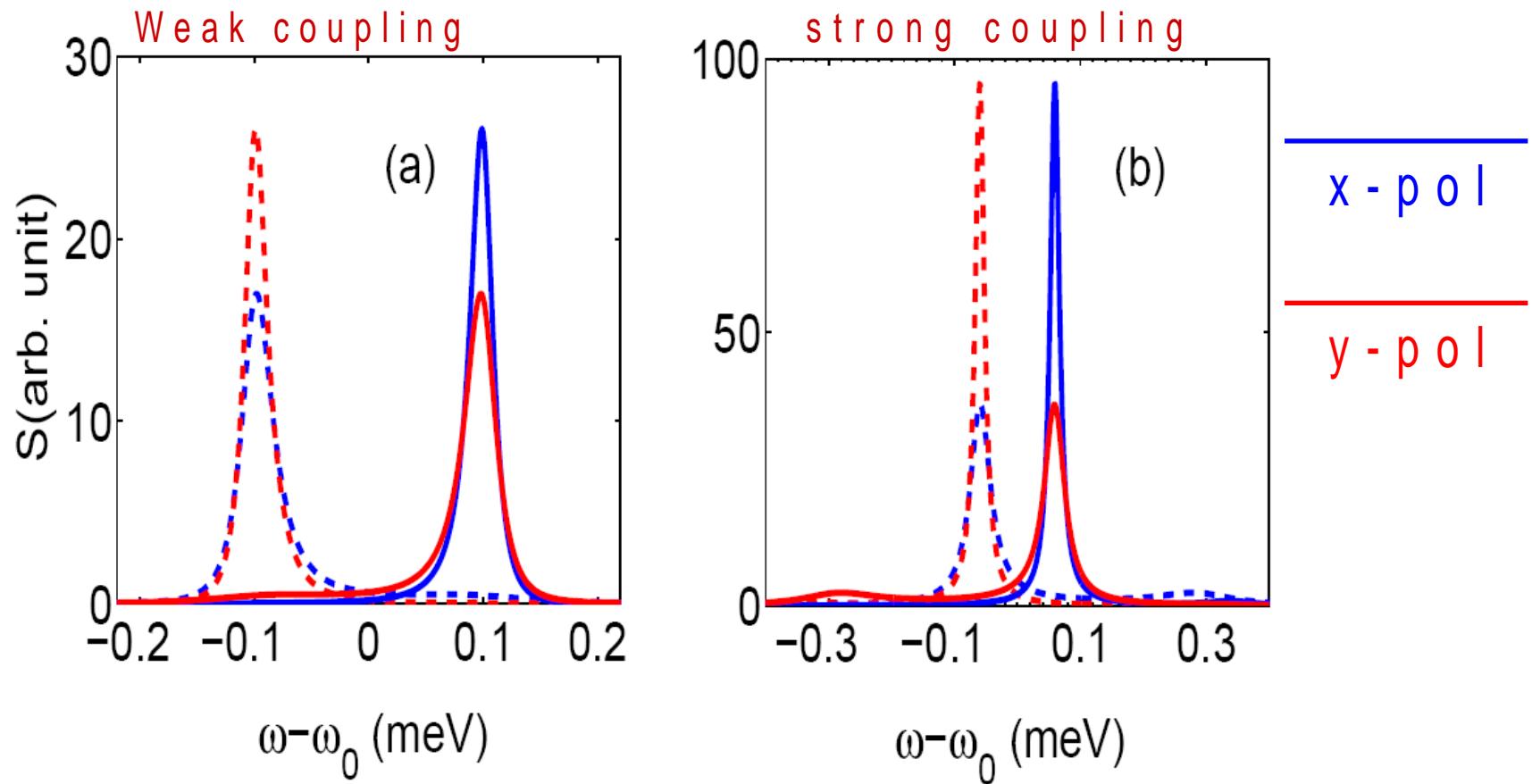
Excitons decay rate

$$\Gamma_2^i = \frac{(g_1^i)^2 \kappa}{(\kappa^2 + \Delta_c^i)^2}$$



\* see, Pathak & Hughes, arXiv:0906.3035

# Spectrum of photons emitted from cavity



$S(\omega)$  for  $\delta_x = 0.2$  meV,  $\gamma_1 = 2\gamma_2 = 0.004$  meV,  $\kappa = 0.05$  meV, for (a)  $g_1^x = g_2^x = g_1^y = g_2^y = 0.02$  meV, and  $\Delta_c^x = \Delta_c^y = 0$  meV, and for (b)  $g_1^x = g_2^x = g_1^y = g_2^y = 0.1$  meV, and  $\Delta_c^x = -\Delta_c^y = -0.2$  meV.

# Time reordering and entanglement

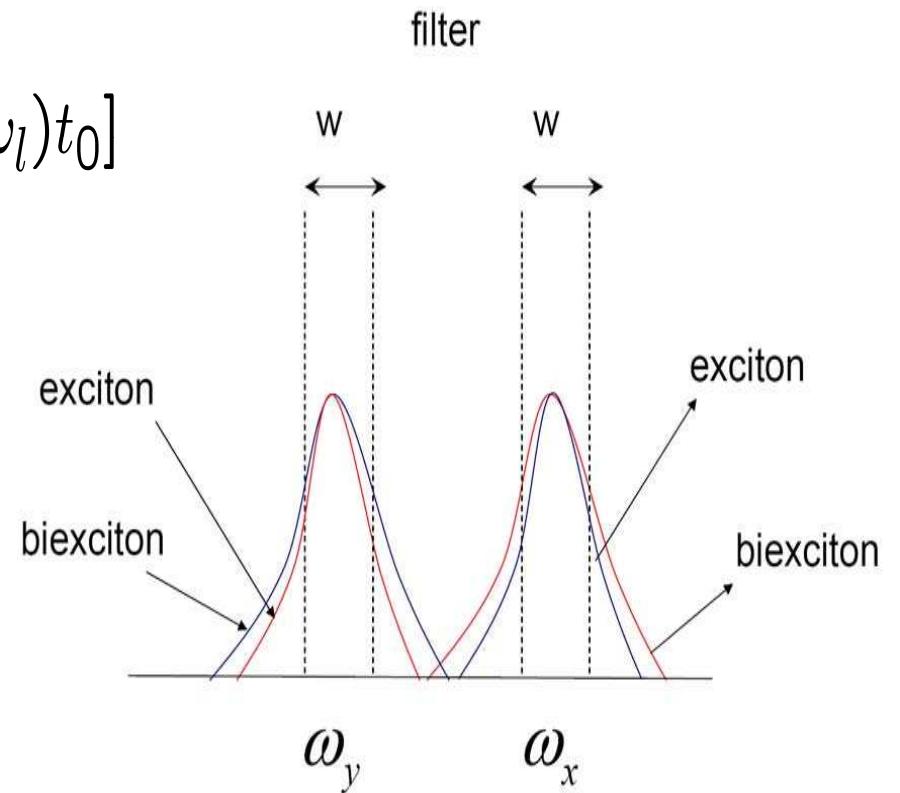
Off-diagonal element of density matrix:

$$\gamma = \frac{\int \int c_{kl}^{x*} c_{lk}^y d\omega_k d\omega_l}{\int \int |c_{kl}^x|^2 d\omega_k d\omega_l + |c_{lk}^y|^2 d\omega_k d\omega_l} = 0$$

Time delay:  $W_{\text{opt}} = \exp[-i(\omega_k - \omega_l)t_0]$

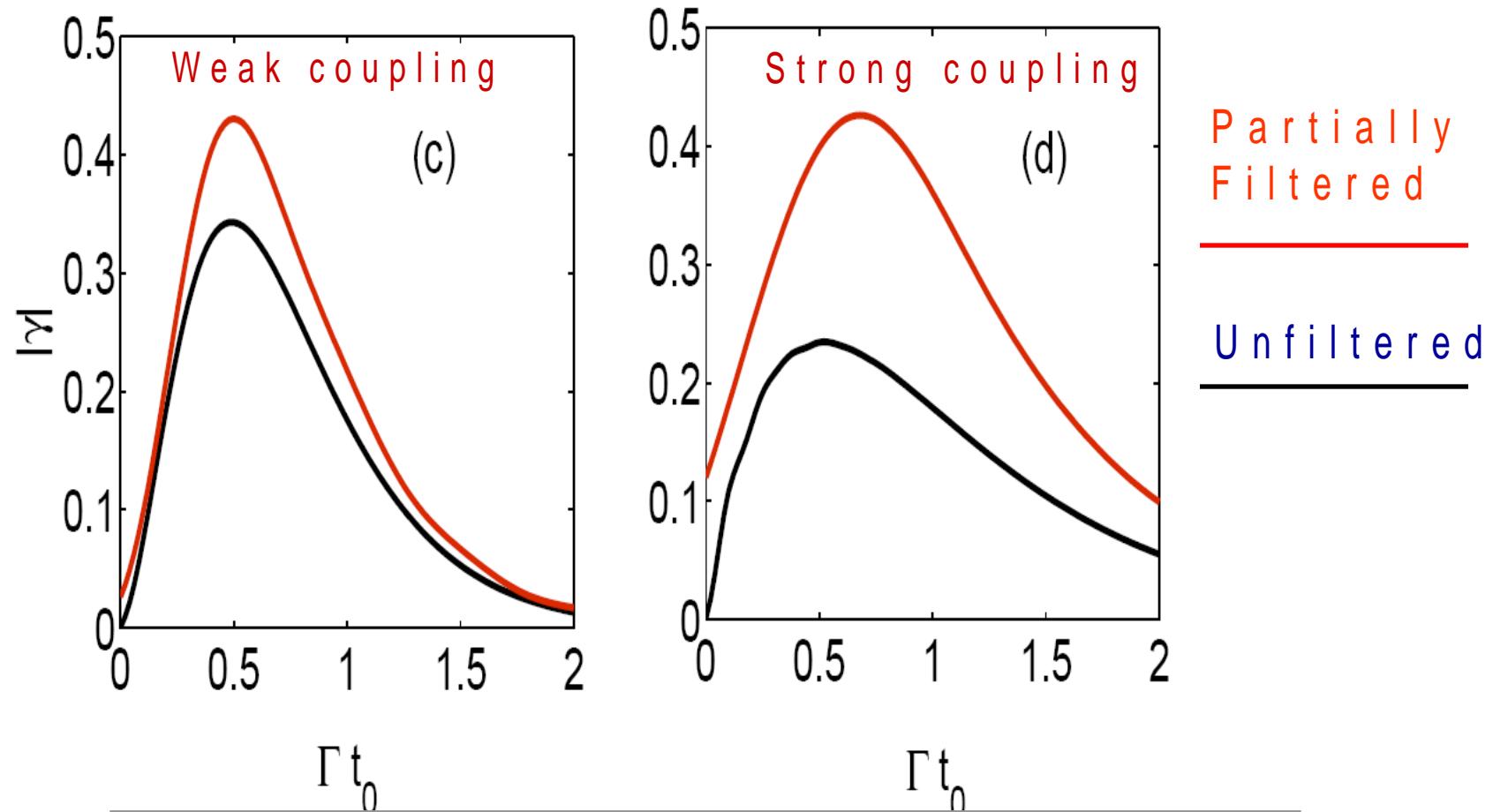
Distillation of entanglement

$$F(\omega_k, \omega_l) = \begin{cases} 1, & \text{for } |\omega_k - \omega_1| < w, \\ 1, & \text{for } |\omega_l - \omega_2| < w, \\ 0, & \text{otherwise.} \end{cases}$$



See N. Akopian et al, PRL 96, 130501 (2006).

## Across-generated entangled photon pair

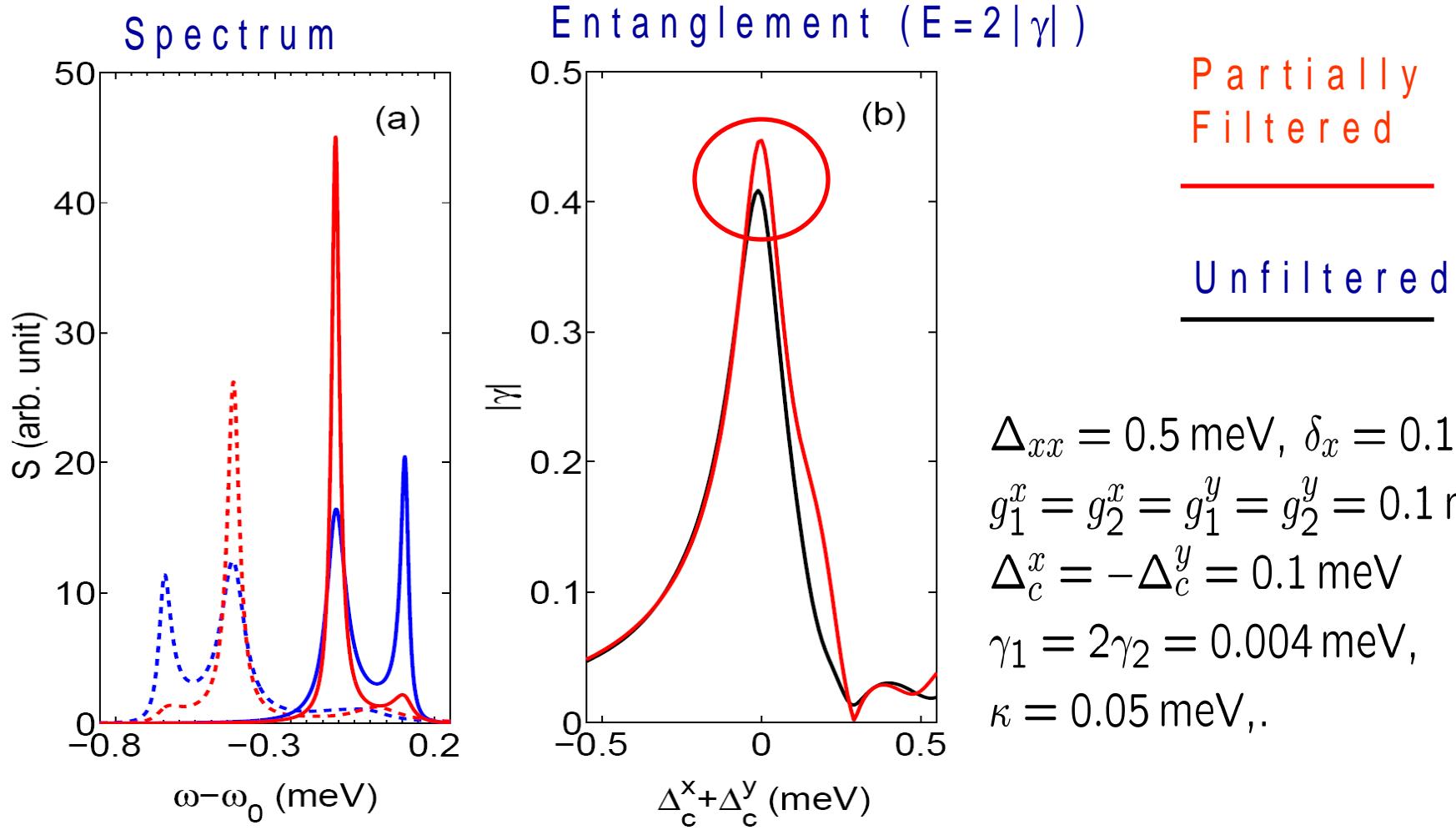


For (c) two spectral windows of width  $w = 0.05$  meV, centered at  $\omega_x$  and  $\omega_y$ , and for (d) the two spectral windows of width  $w = 0.03$  meV, centered at  $\omega_x^-$  and  $\omega_y^+$ .

## Within generation of entangled photon pair

State of photons:

$$|\psi\rangle = \sum_{k,l} [c_{kl}^x |1_k\rangle_x |1_l\rangle_x + c_{kl}^y |1_k\rangle_y |1_l\rangle_y]$$



# Summary

- ◆ Developed a general theory to describe QD coupling to the quasi-degenerate photonic crystal cavity modes, including coupled excitons and biexcitons.
- ◆ Maximum possible concurrence in “across-generation” by manipulating the QD decay rates is 0.73, and higher values of entanglement are possible using spectral filtering followed by a time delay.
- ◆ Efficient “within generation” of entangled photons is also possible, with and without a reduced biexciton binding energy.
- ◆ Planar photonic crystals show great promise as a platform technology for chip-based quantum light sources, and the field is still very young.

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Thank You.