Synthesizing Arbitrary Photon States in a superconducting resonator

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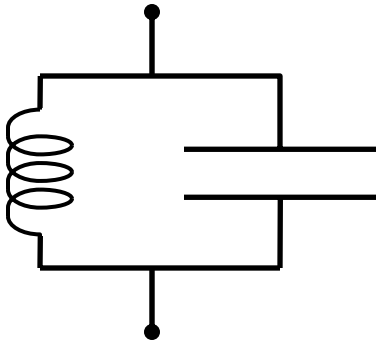
- Quantum Integrated Circuits
  - Quantum currents & voltages
  - Microfabricated “atoms”
- Digital to Analog Converter

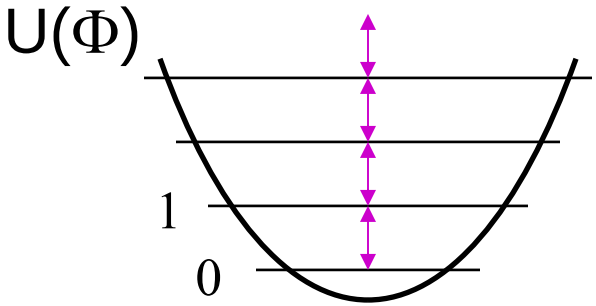
Classical: V = a[0] 1 + a[1] 2 + a[2] 4 + …a[9] 512

Quantum: $\Psi = \alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle + \ldots \alpha_9 |9\rangle$

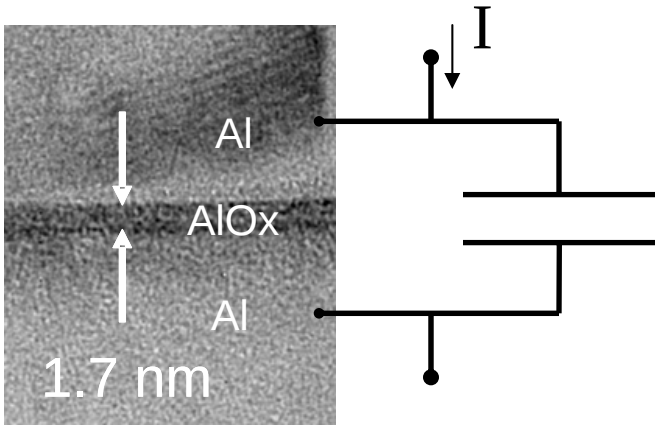
Josephson Qubits & Nonlinearity

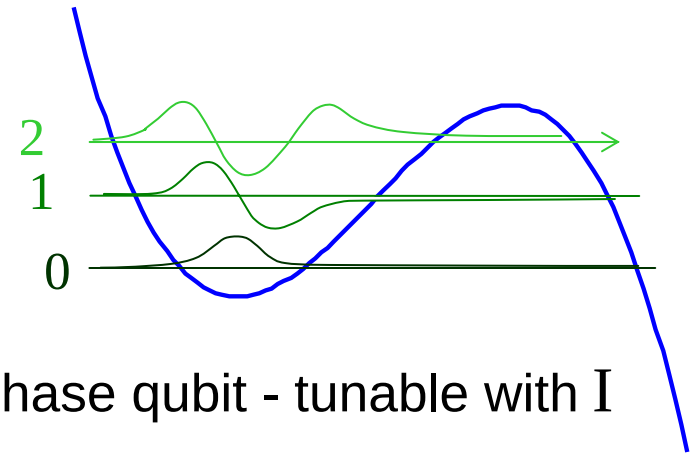
- LC oscillator (linear): no qubit possible





- Josephson junction: <u>non-linear</u> inductance with <u>1 photon</u> (low loss)





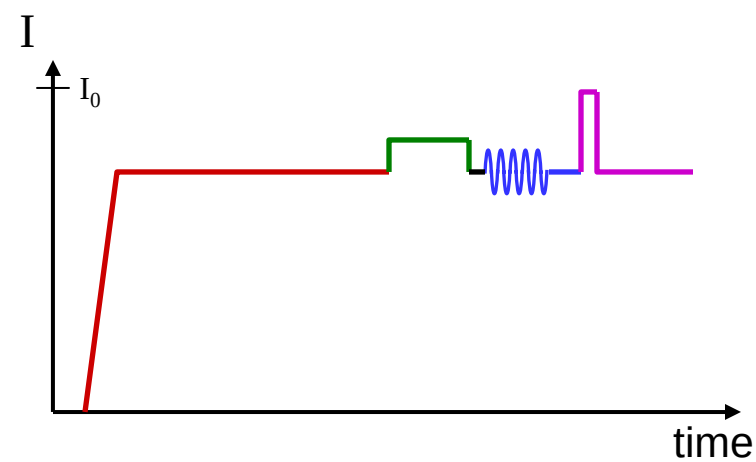
Phase qubit - tunable with I

Josephson Phase Qubit

(1) State Preparation

Wait for decay to |g⟩

$(20\,\mathrm{mK}\,;\,kT << \hbar\omega_{10}\,;\,6\,\mathrm{GHz})$



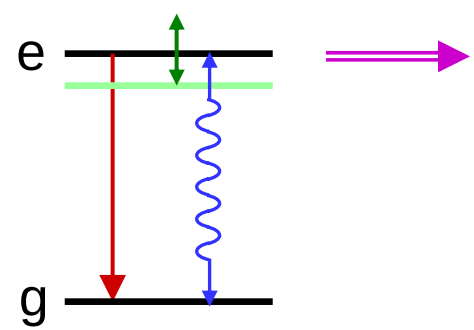
(2) Qubit logic with current bias I

DC current shifts energy; σ_z

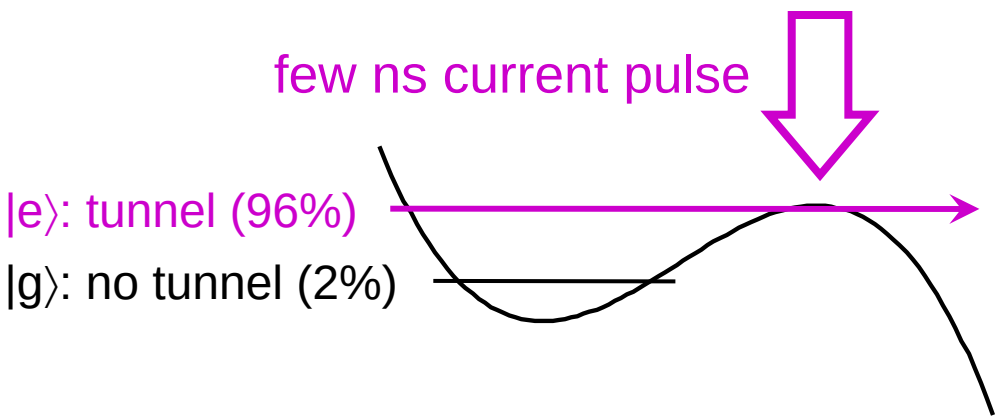
Microwaves for transition; σ_x, σ_y

(3) State Measurement: pulse I

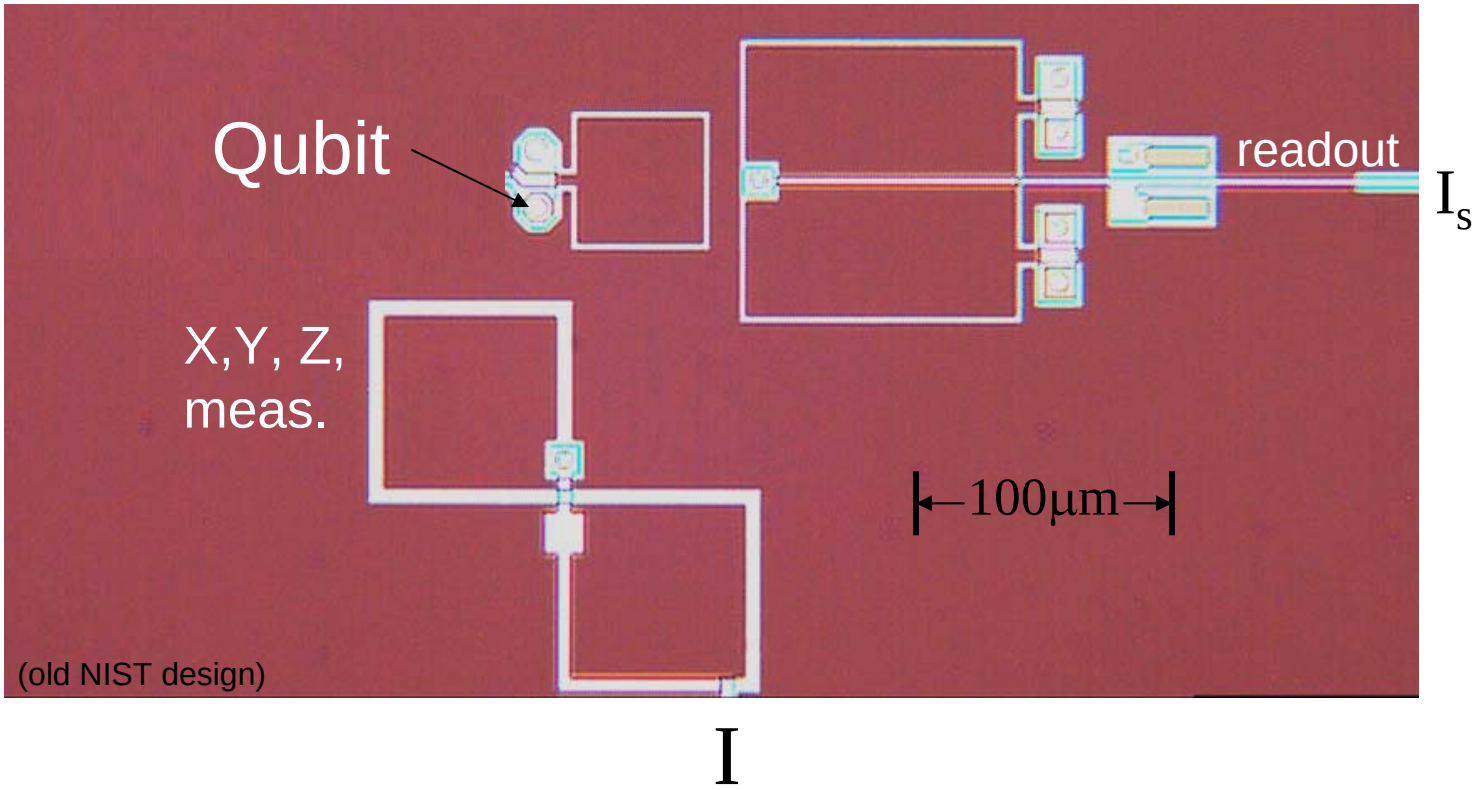
Fast single shot – high fidelity



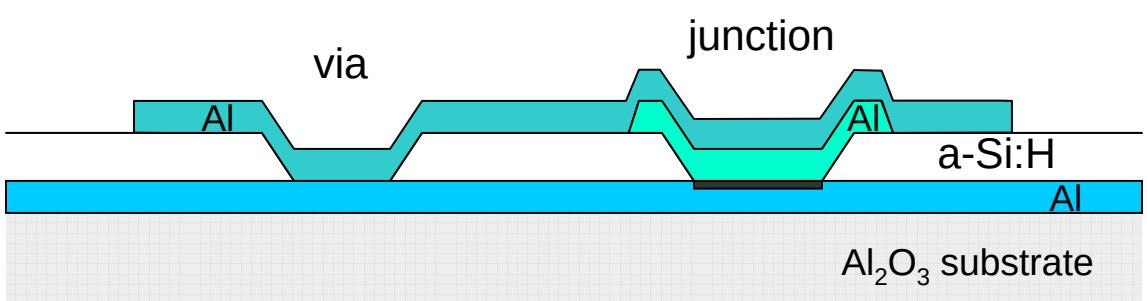
few ns current pulse



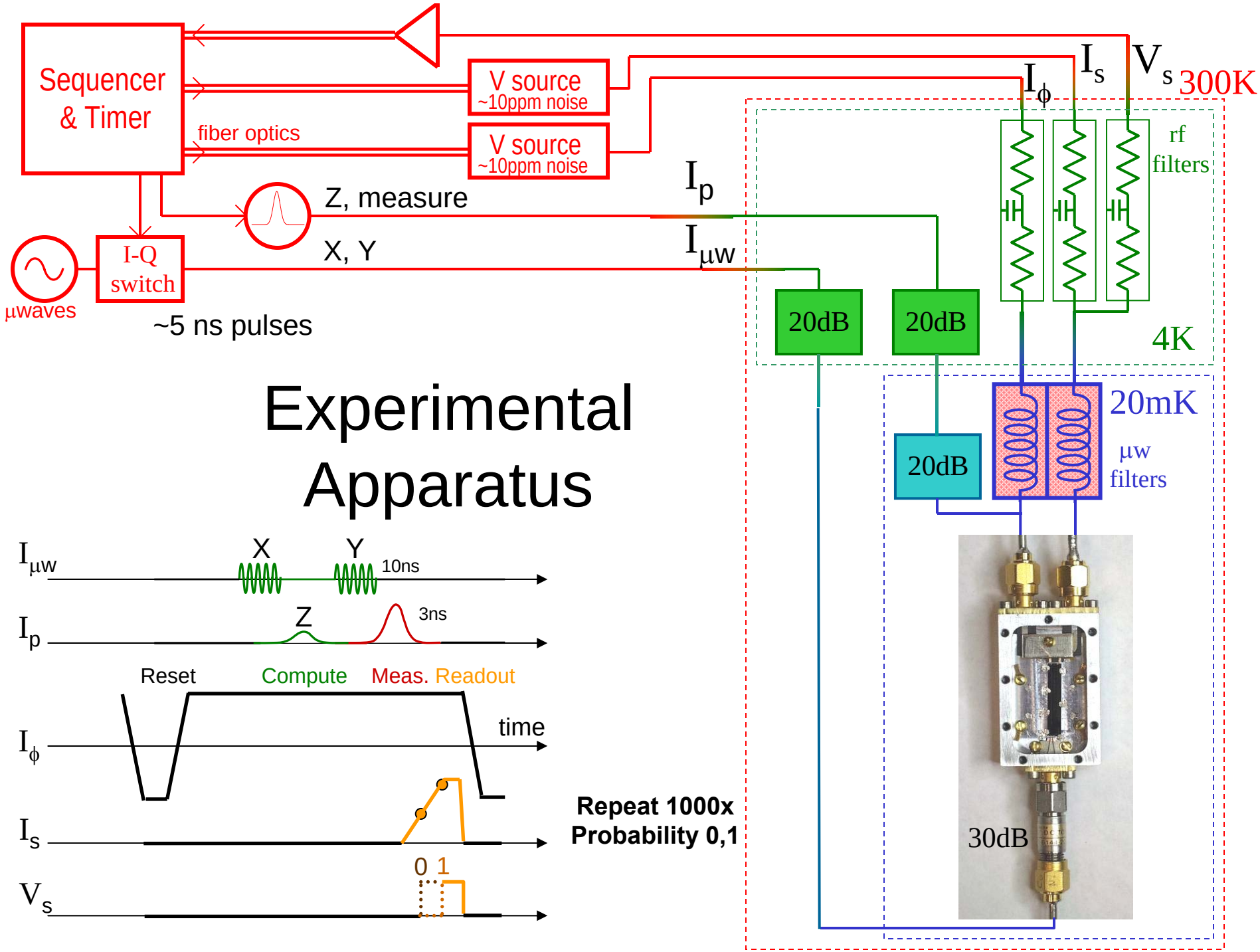
IC Fabrication



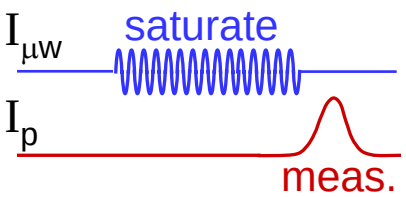
Al junction process
& optical lithography

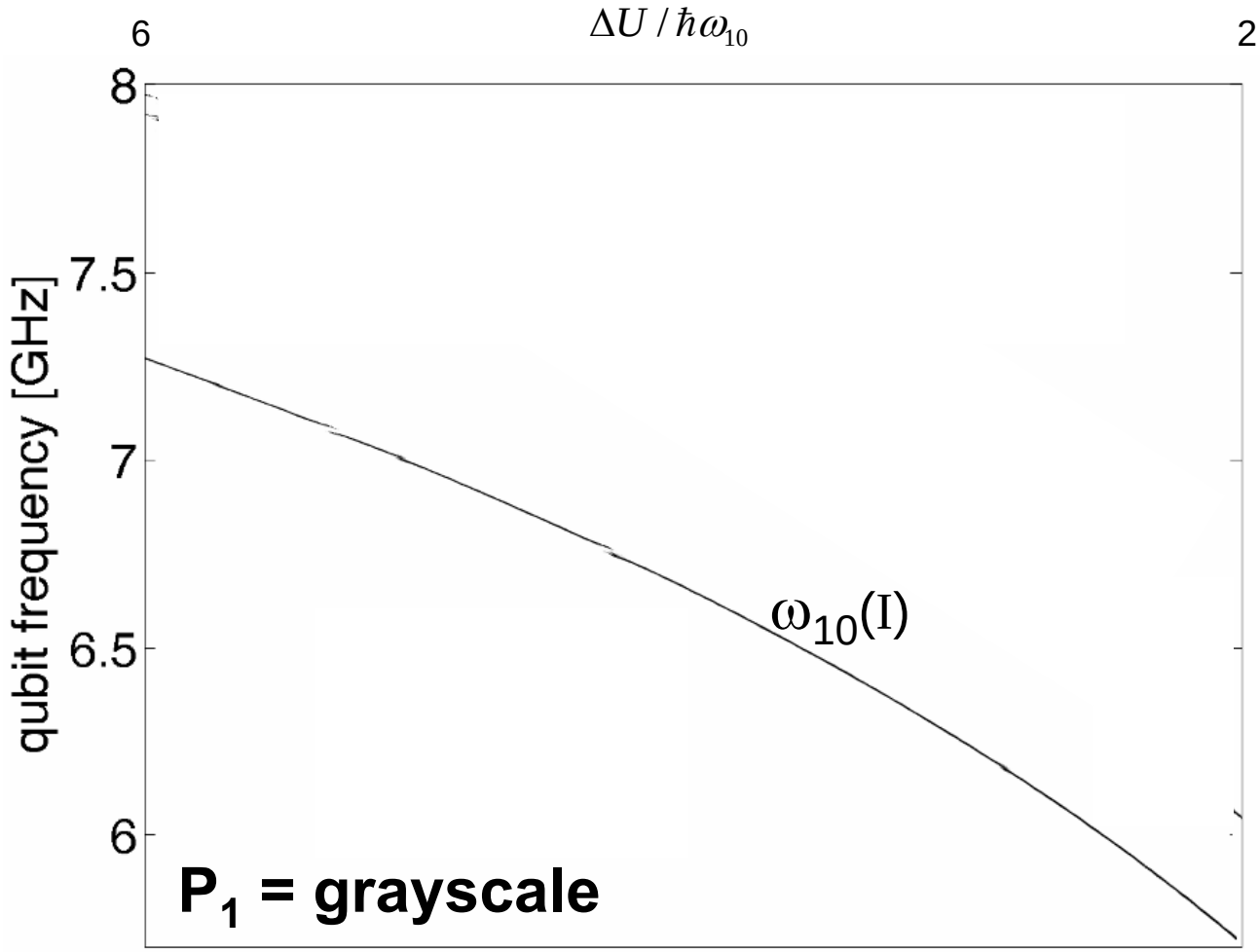


Experimental Apparatus

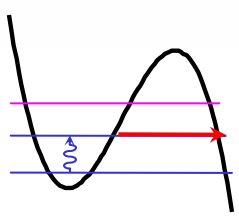


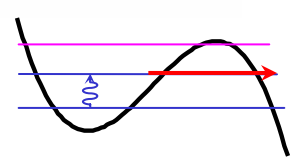
Spectroscopy





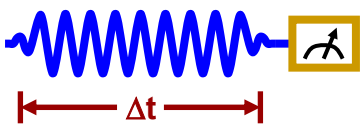
Bias current I (au)



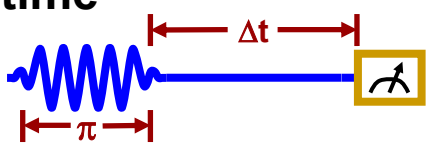


Qubit Characterization

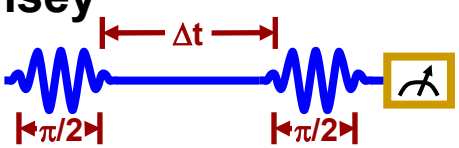
- **Rabi**

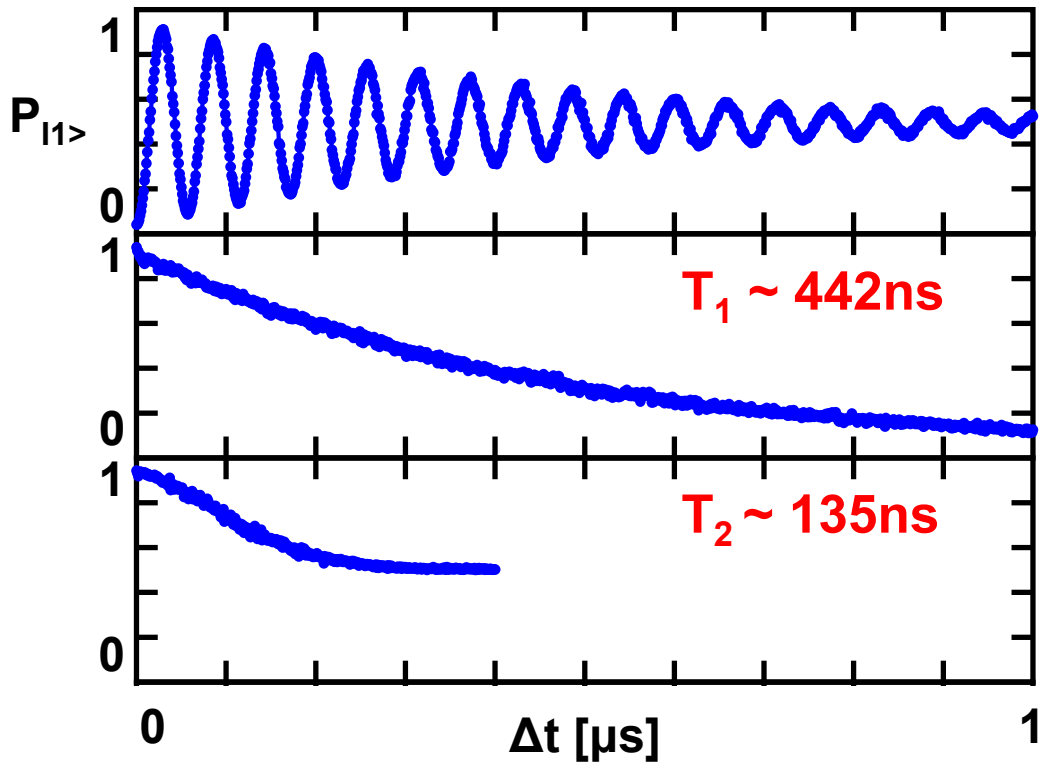


- **Lifetime**



- **Ramsey**

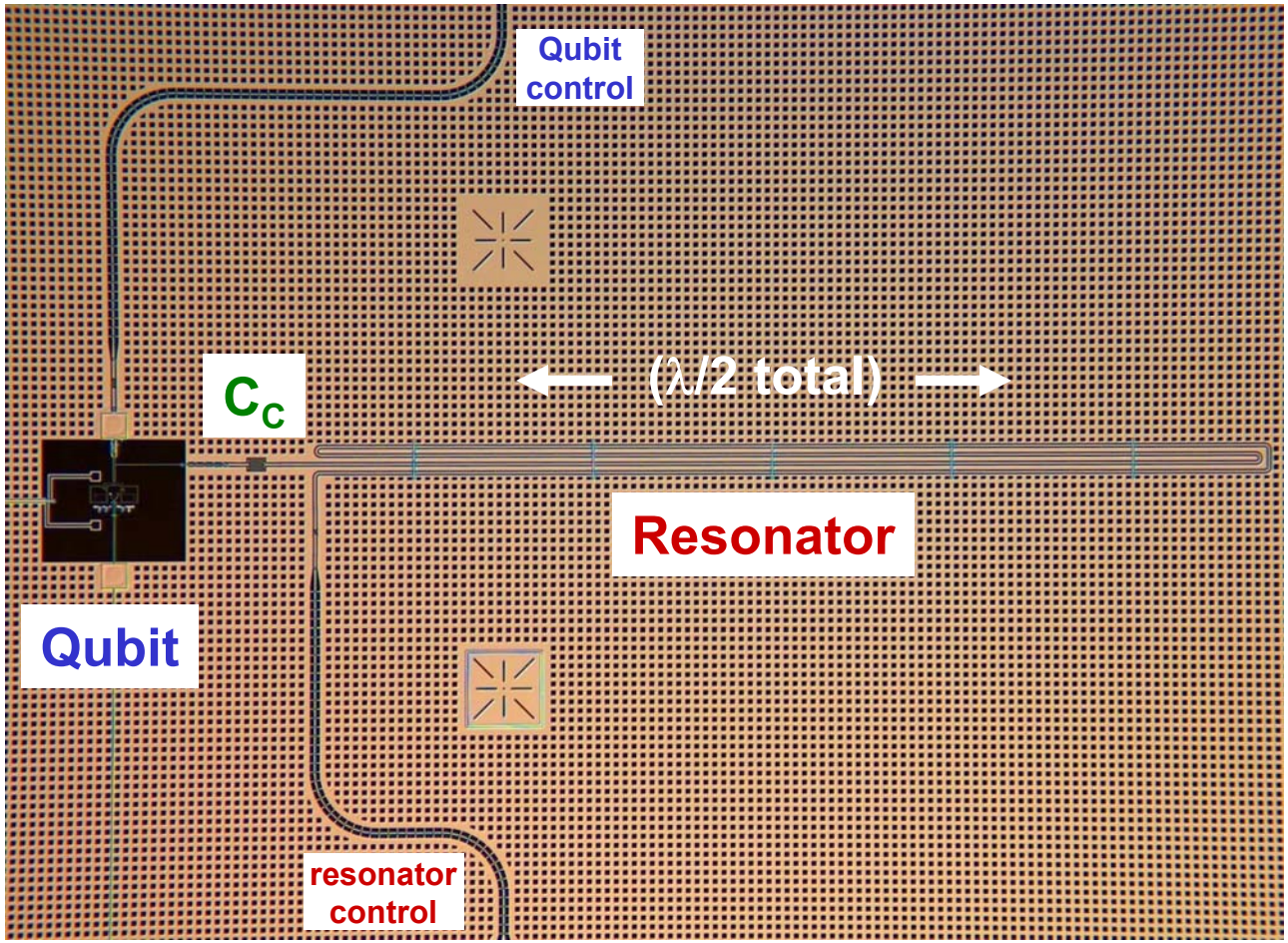


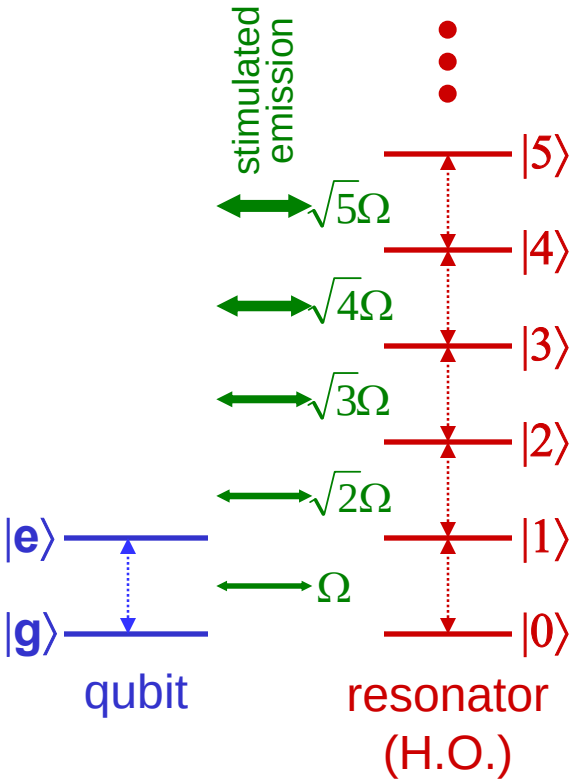


τ_{op} ~ 8ns

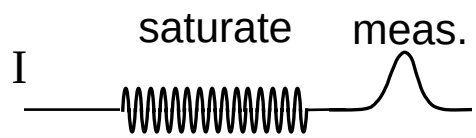
Qubit Coupled to Photons (Harmonic Oscillator)

Qubit-resonator first demonstrated in Saclay (1987),
GHz coupling easy in phase qubits
Design follows cQED (Yale)
Readout through qubit (Saclay87, NIST), high fidelity
Similar to ion traps (NIST), Rydberg (ENS)

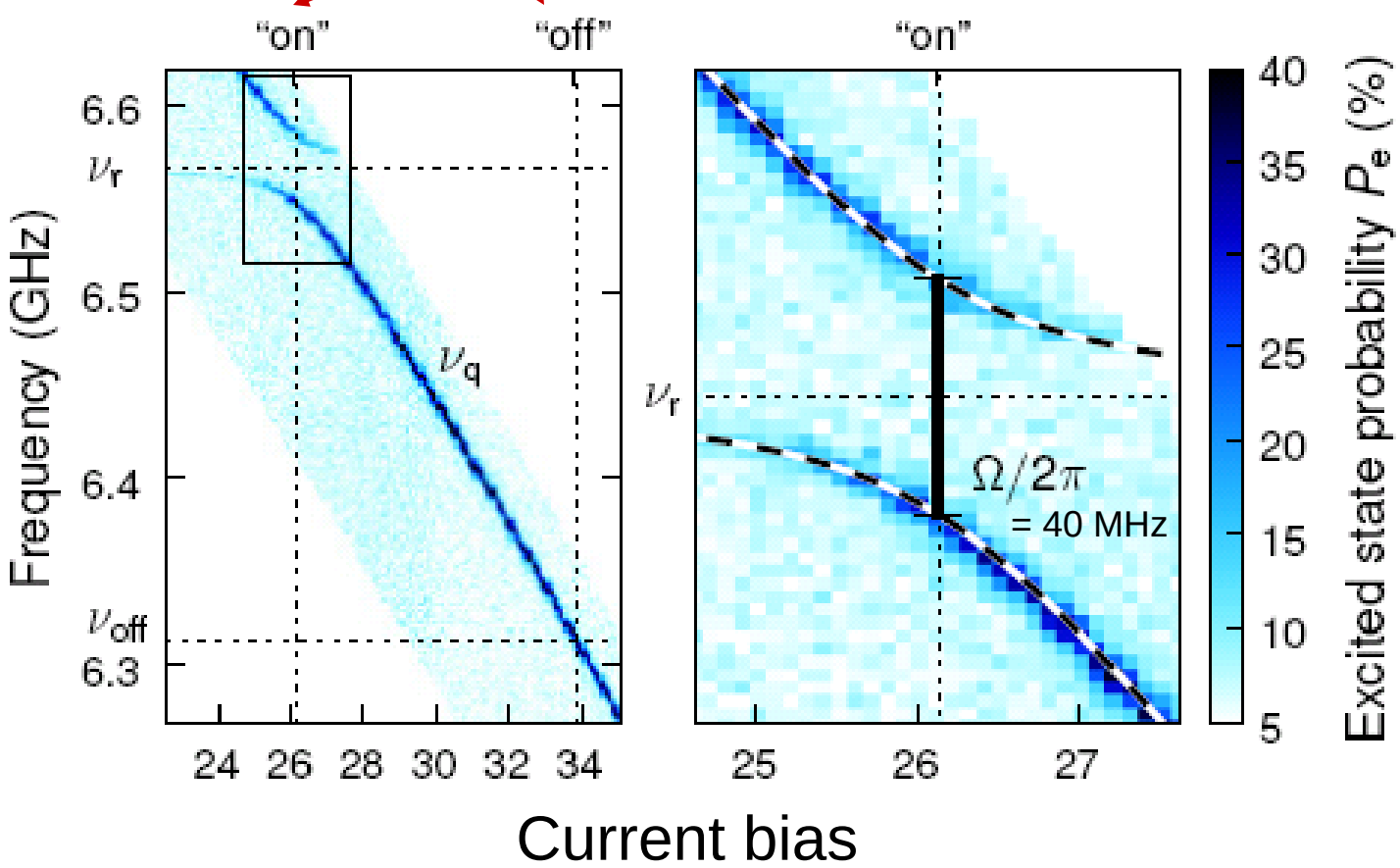




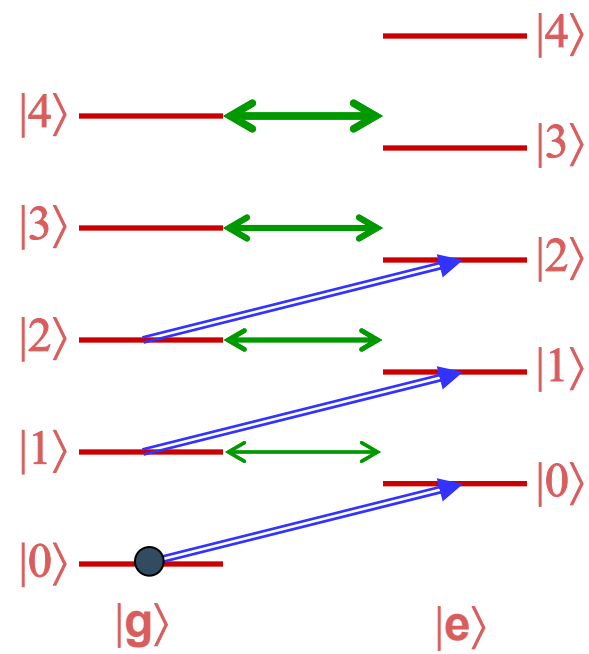
Calibration with Qubit Spectroscopy

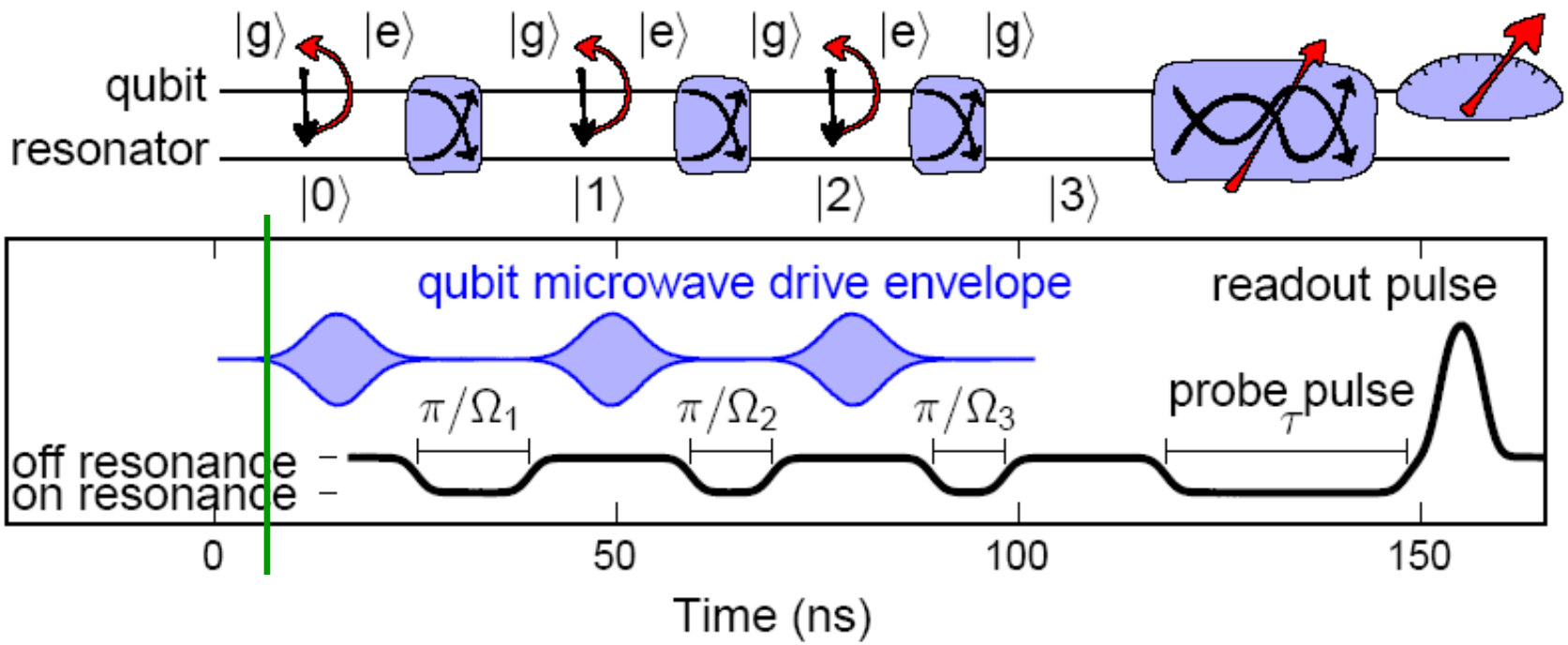


Coupling turned on & off via qubit bias

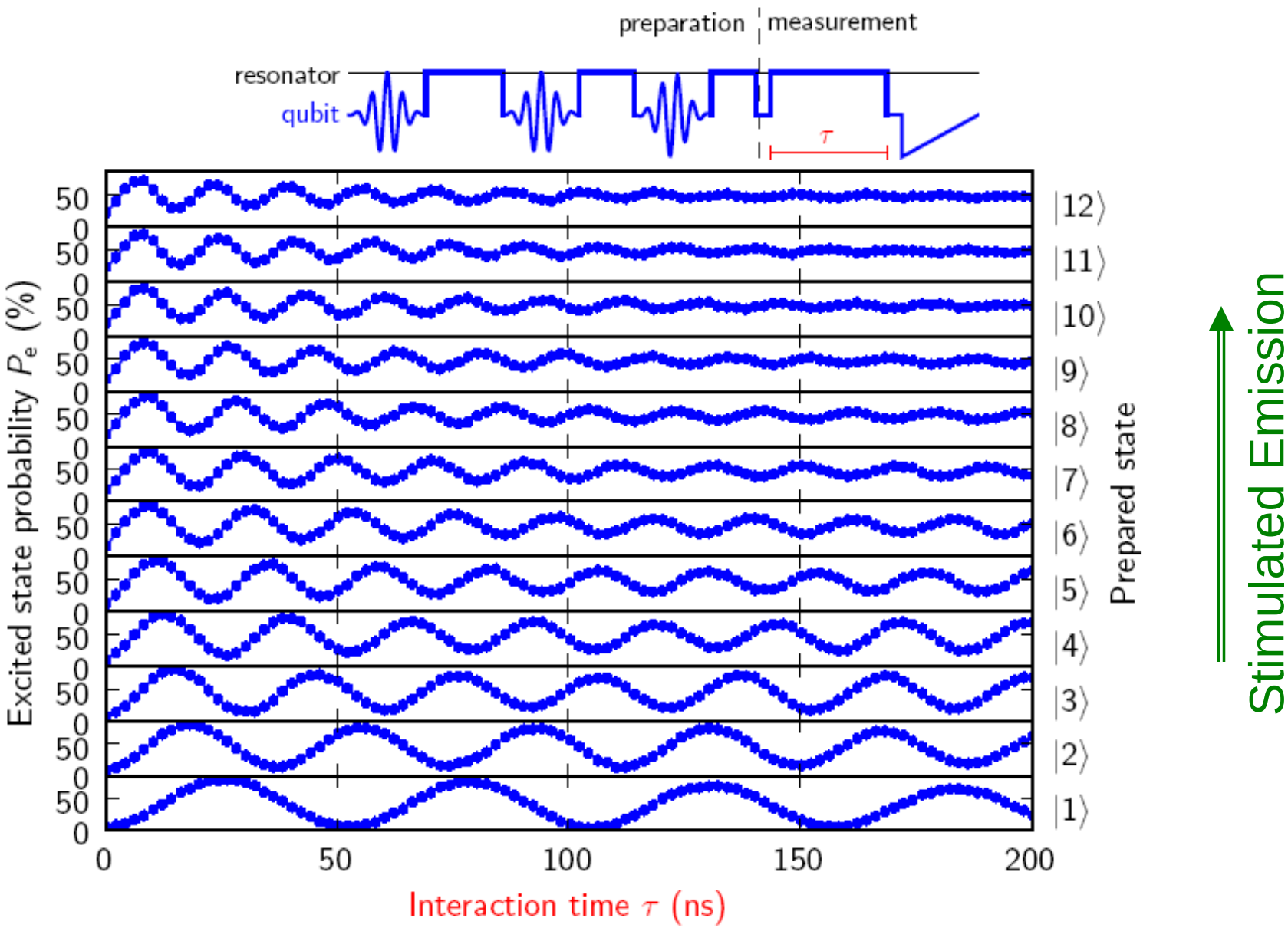


Generating Fock States: Pumping Photons One by One

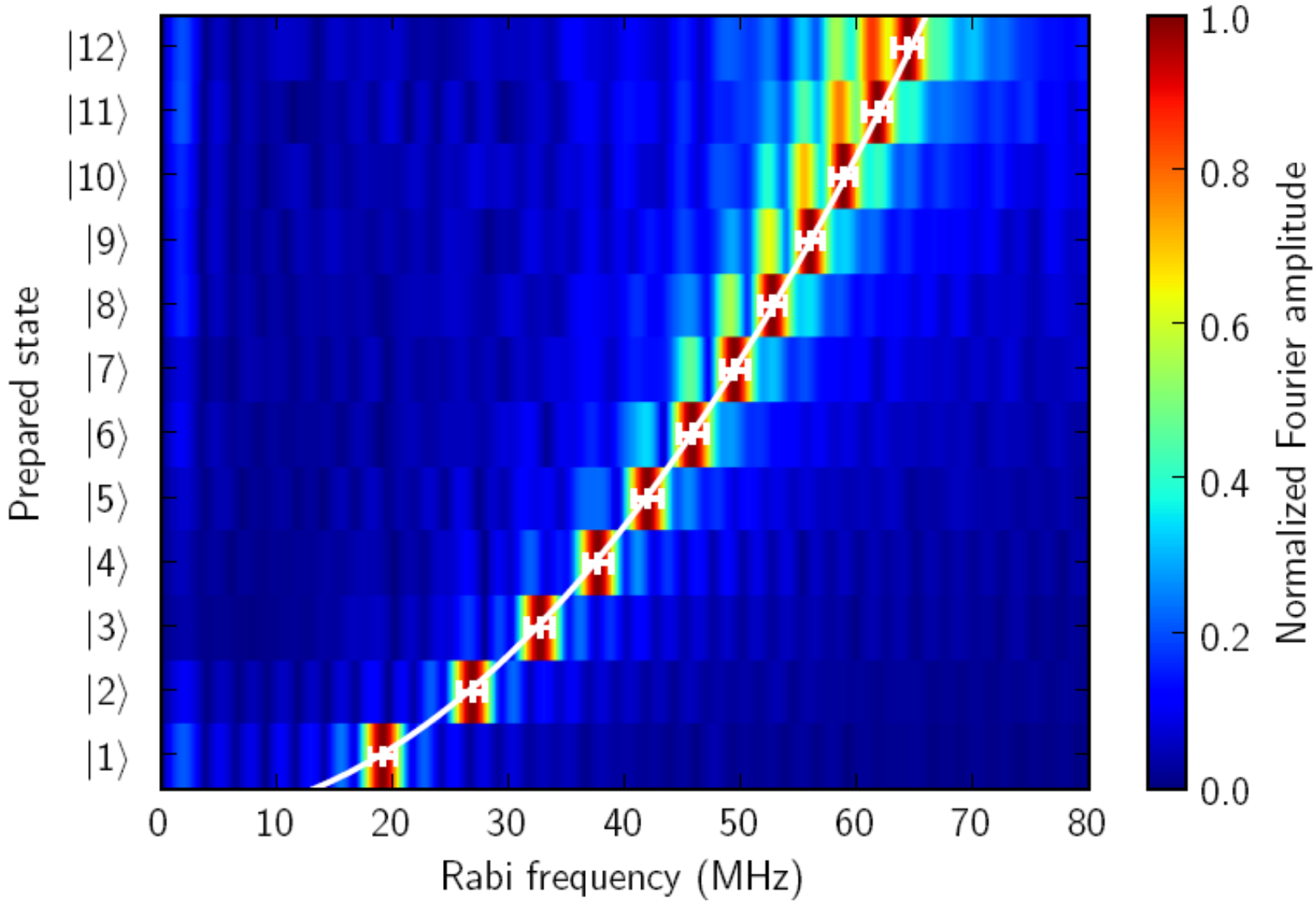




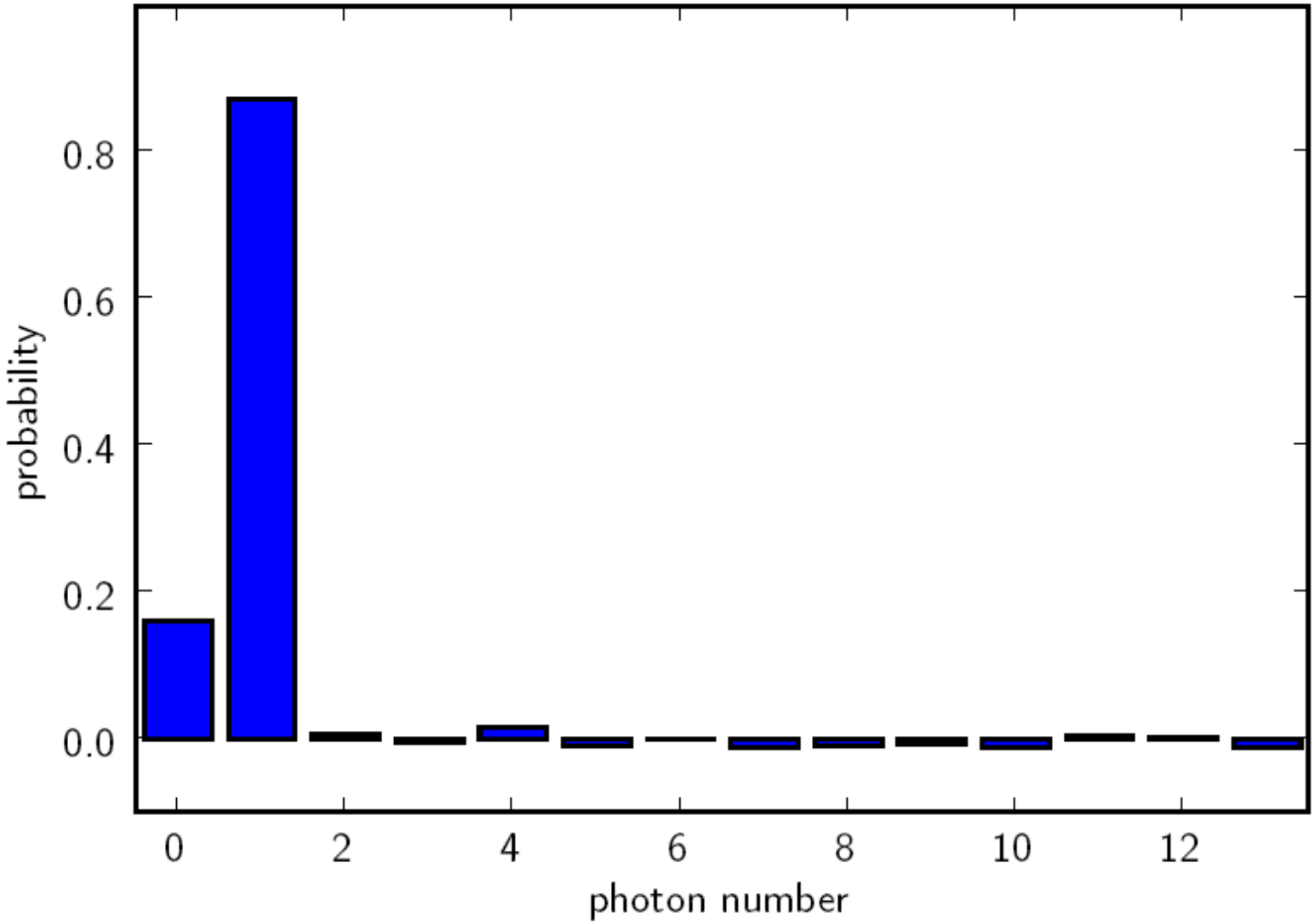
Fock States: Swap oscillations depend on |n⟩



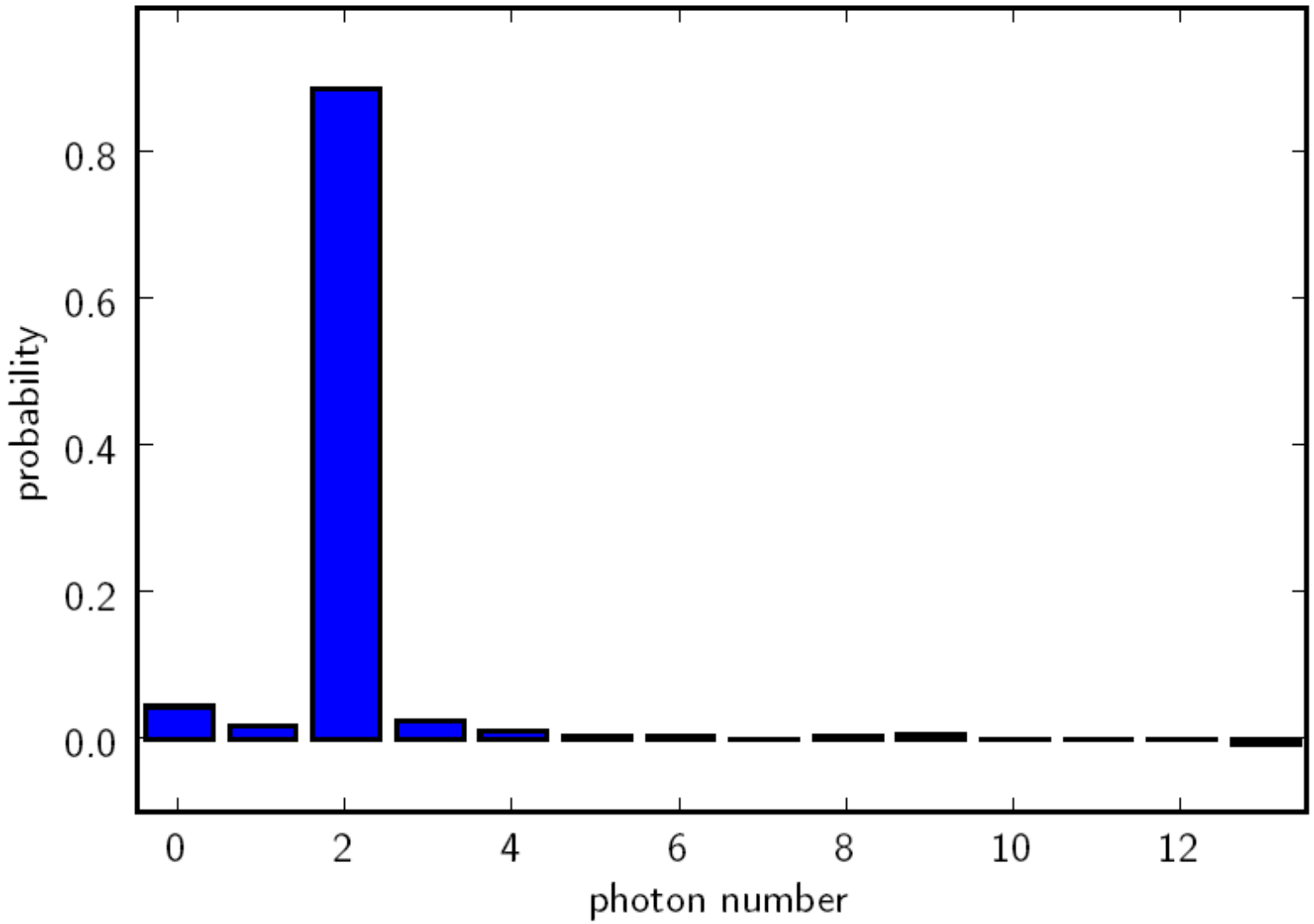
Fock States: Swap frequency scales as $n^{1/2}$



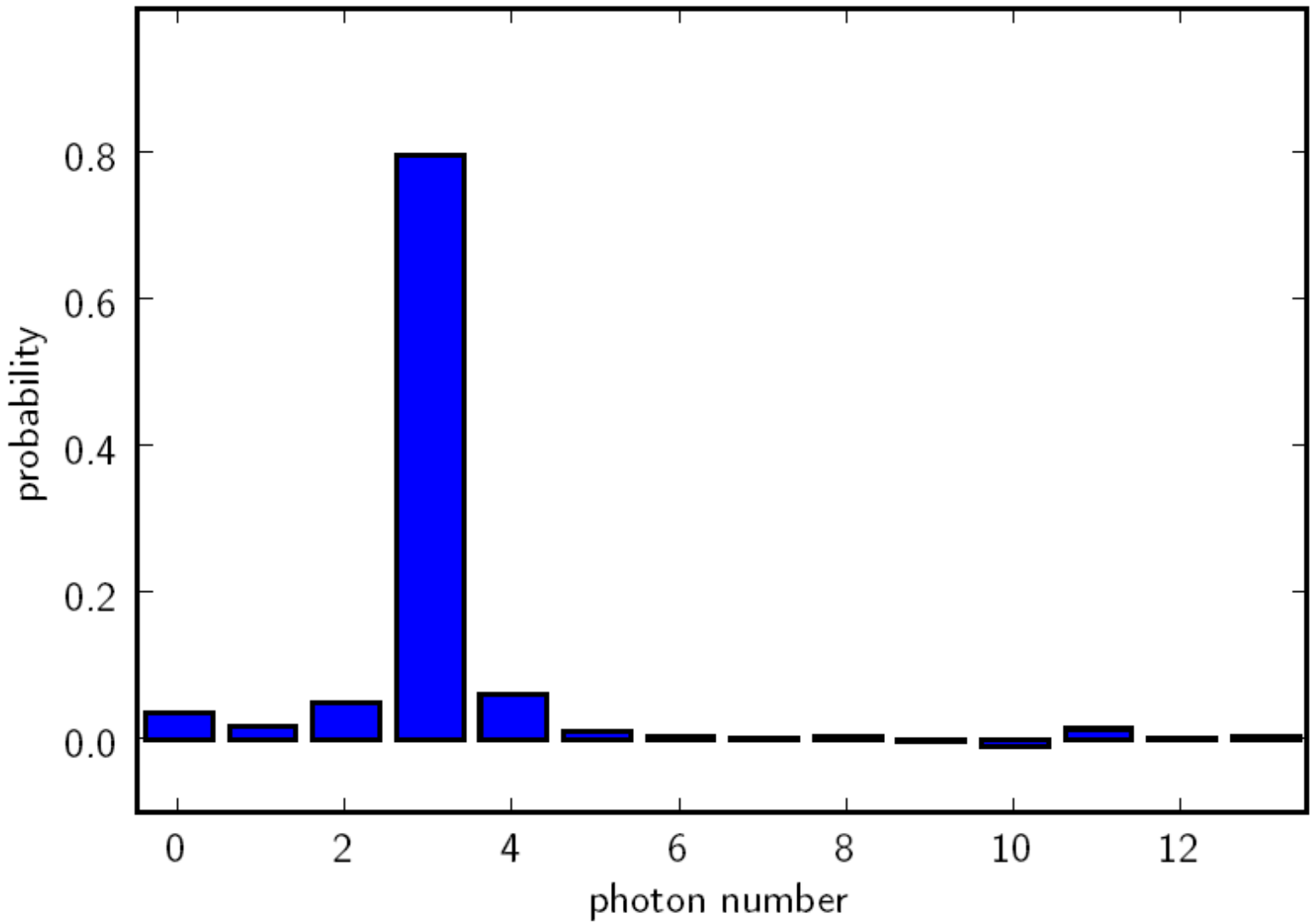
Fock States: Photon number distribution for $|1\rangle$



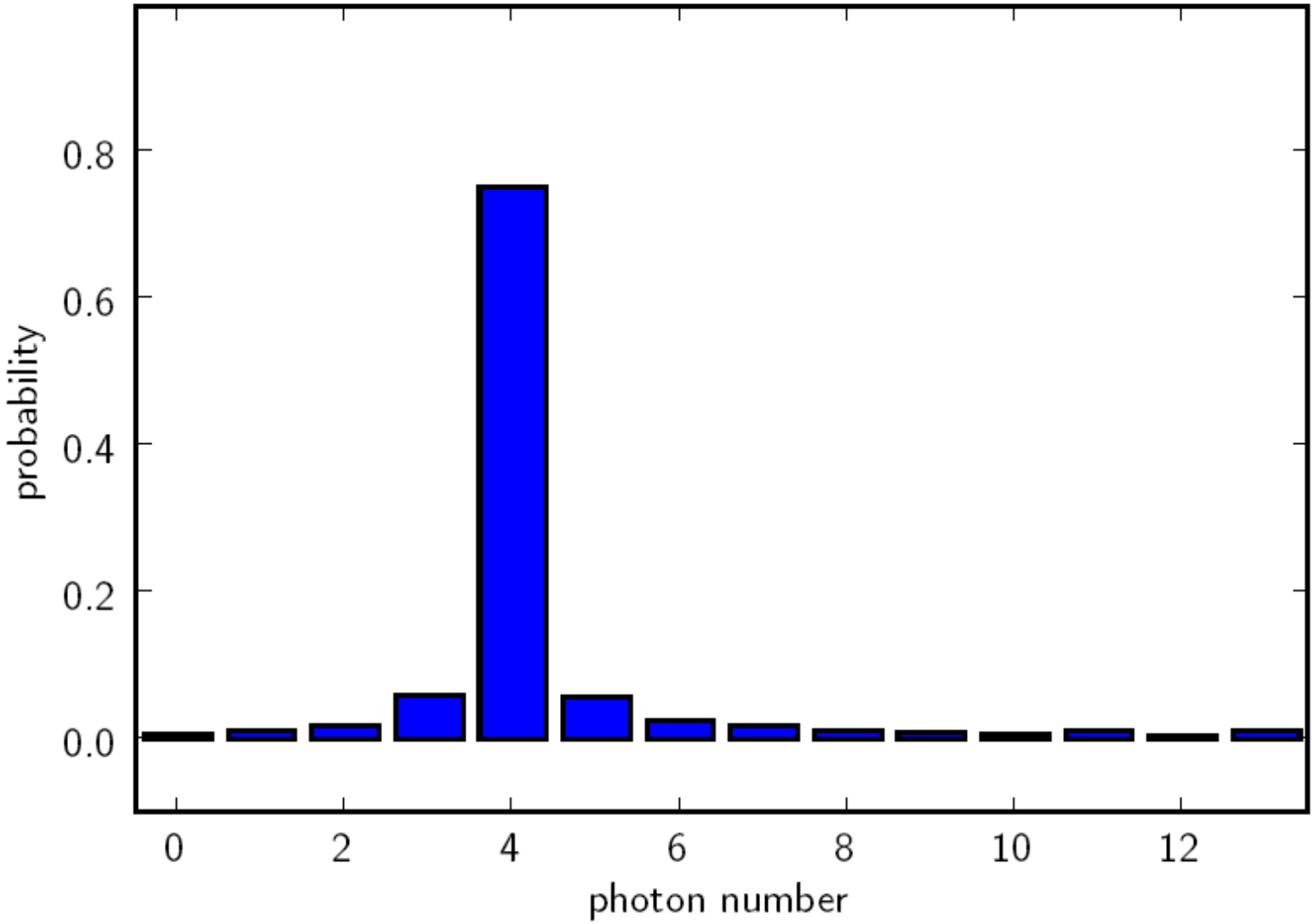
Fock States: Photon number distribution for $|2\rangle$



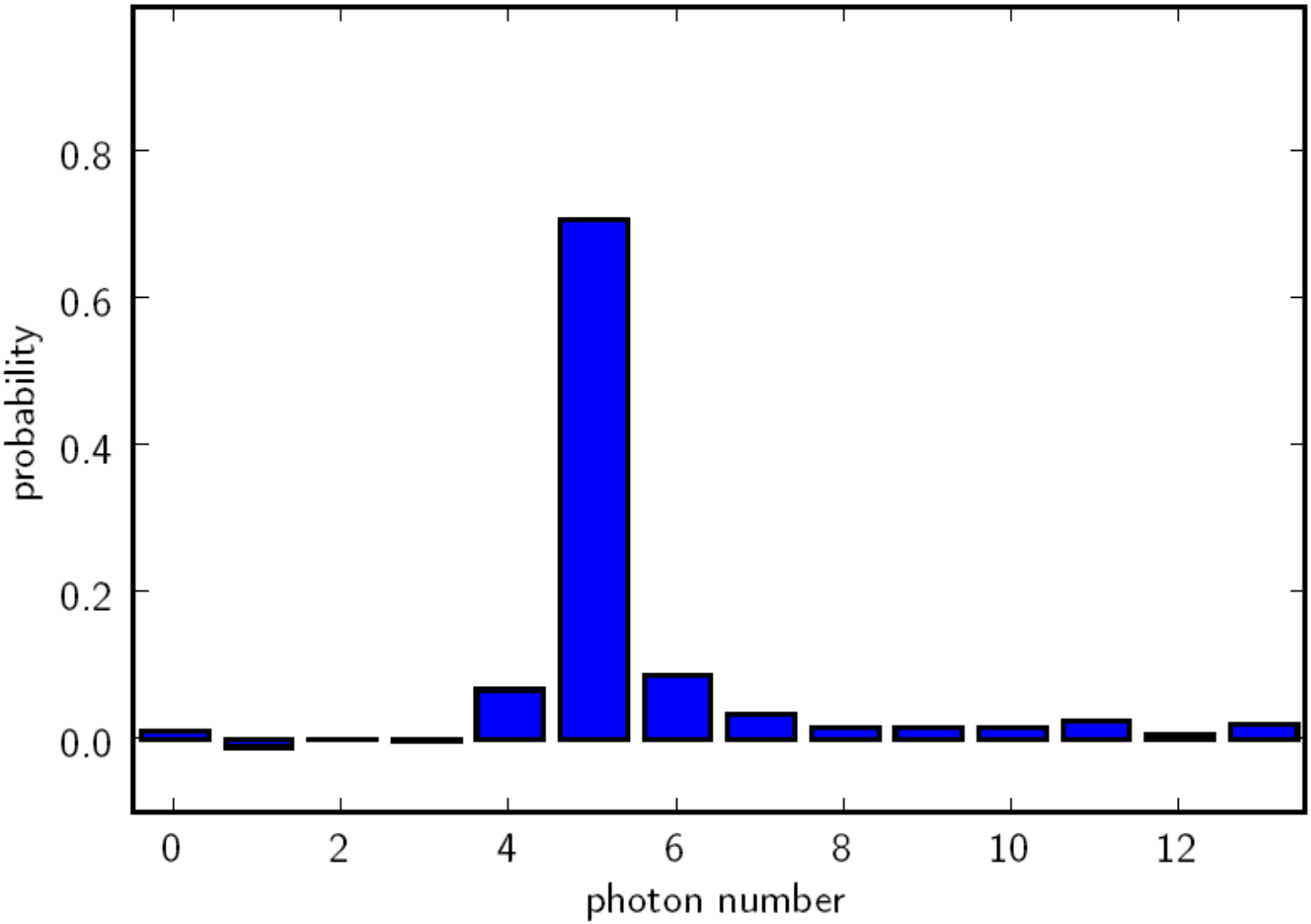
Fock States: Photon number distribution for $|3\rangle$



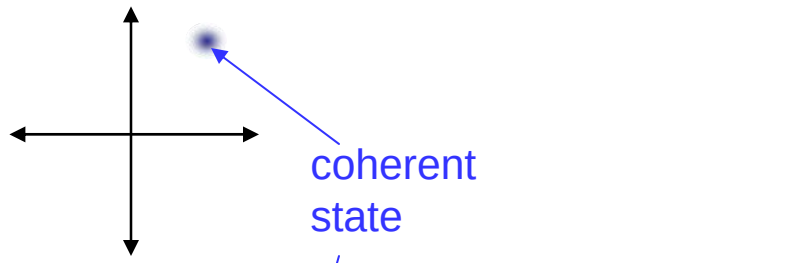
Fock States: Photon number distribution for $|4\rangle$



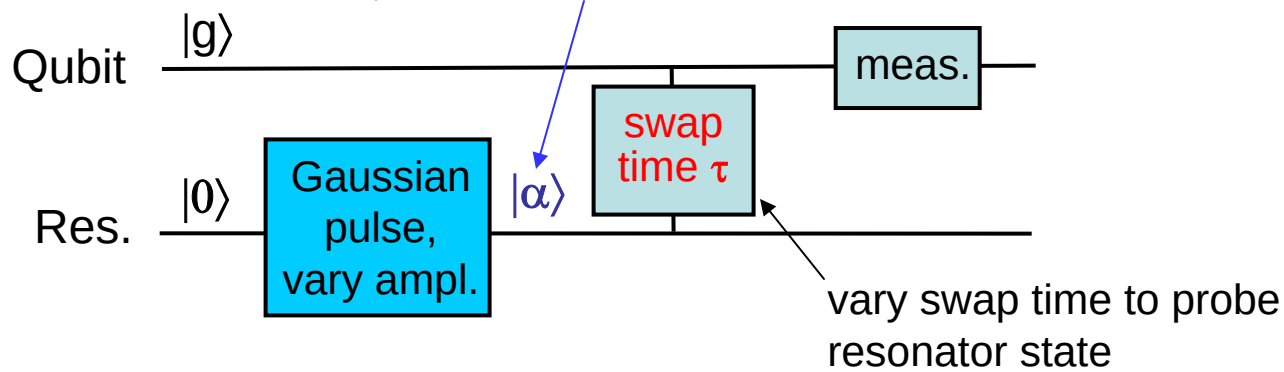
Fock States: Photon number distribution for $|5\rangle$

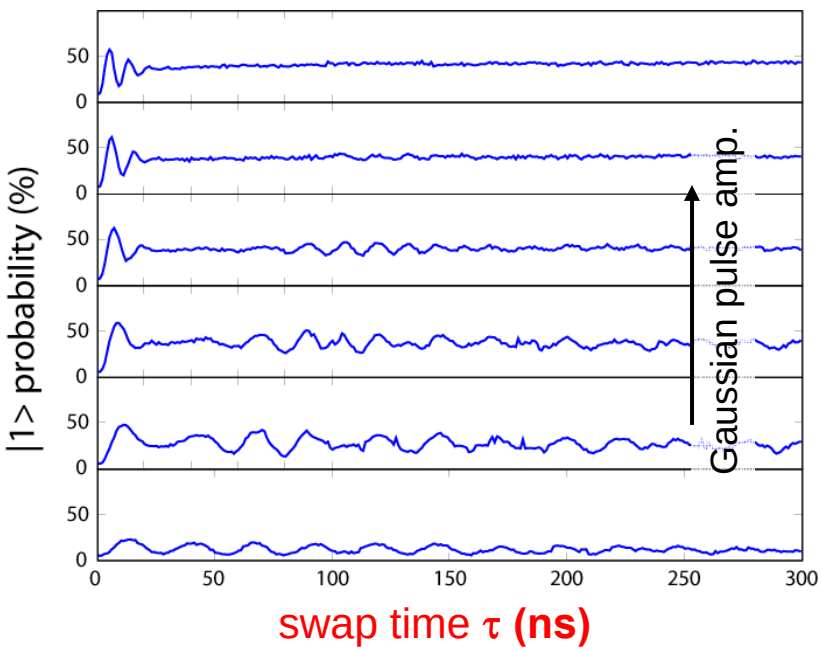


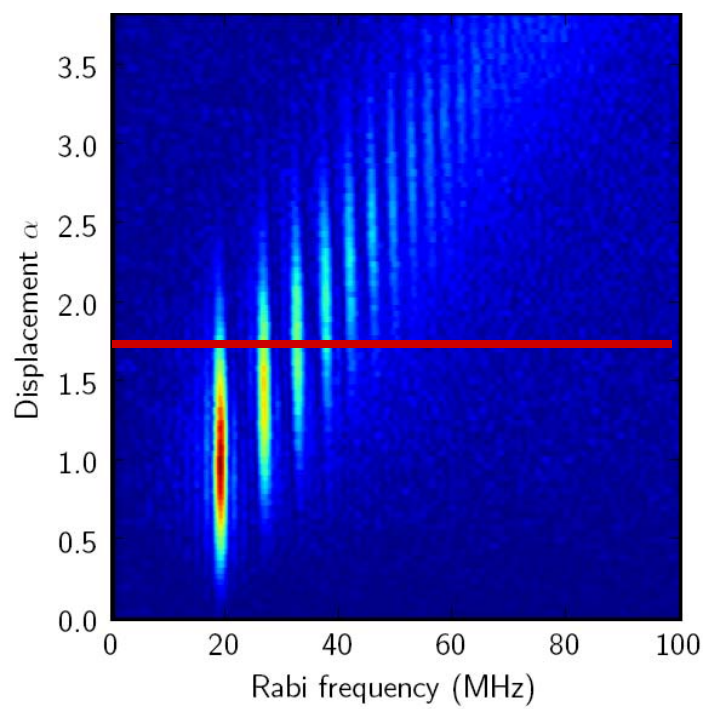
Coherent states

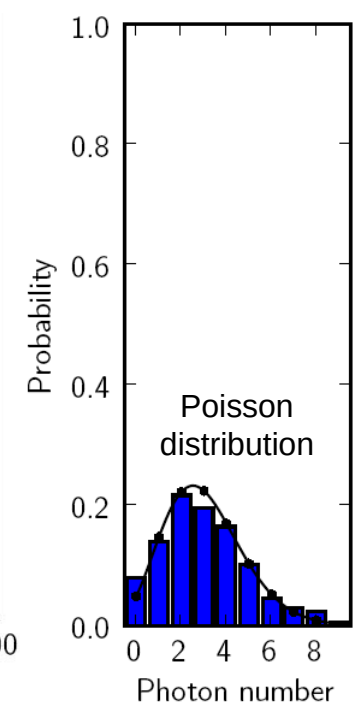


State preparation:







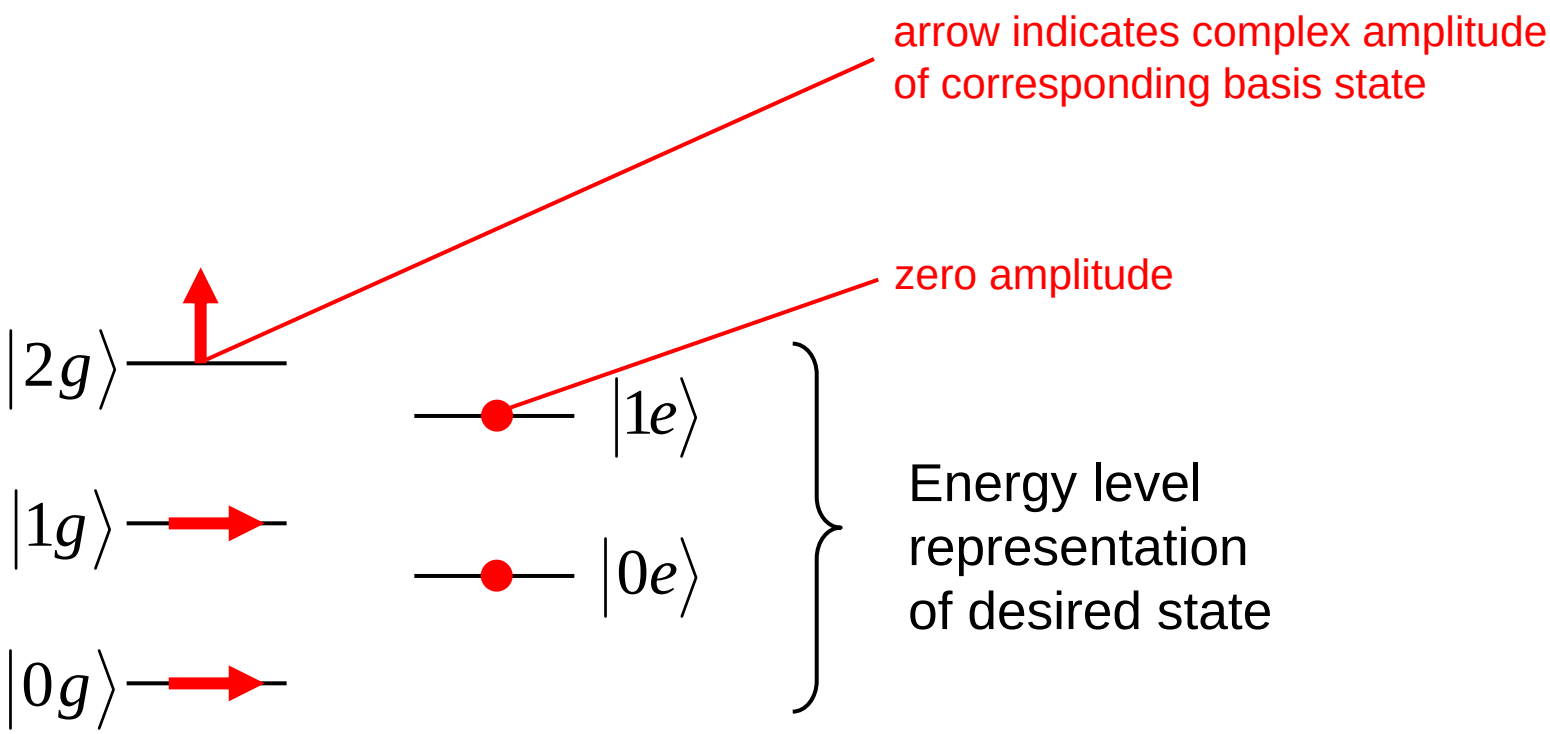


Generating Arbitrary States

Desired final state → $|\psi\rangle = \boxed{0.577\left(|0\rangle + |1\rangle + i|2\rangle\right)} \otimes \boxed{|g\rangle}$

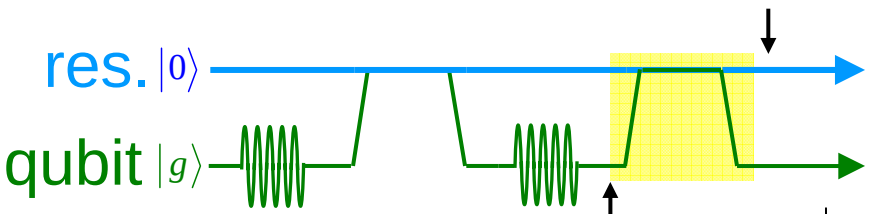
Law and Eberly, PRL (1996)

Reverse-engineer final state by building pulse sequence backwards



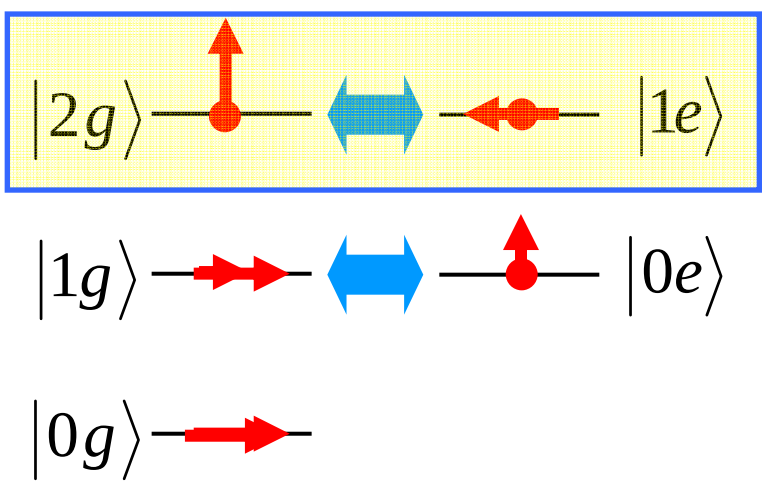
Generating Arbitrary States

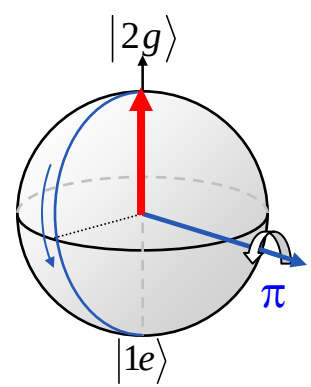
Desired final state $\rightarrow$ $|\psi\rangle = 0.577\left(|0\rangle + |1\rangle + i|2\rangle\right) \otimes |g\rangle$



$0.566|0g\rangle - (0.000 + 0.517i)|0e\rangle + 0.256|1g\rangle + 0.577|1e\rangle$

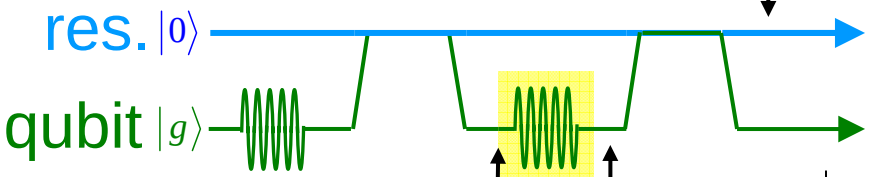
- Qubit and resonator into resonance
- Transfer $|2g\rangle$ amplitude into $|1e\rangle$
- $|2g\rangle$ amplitude now zeroed





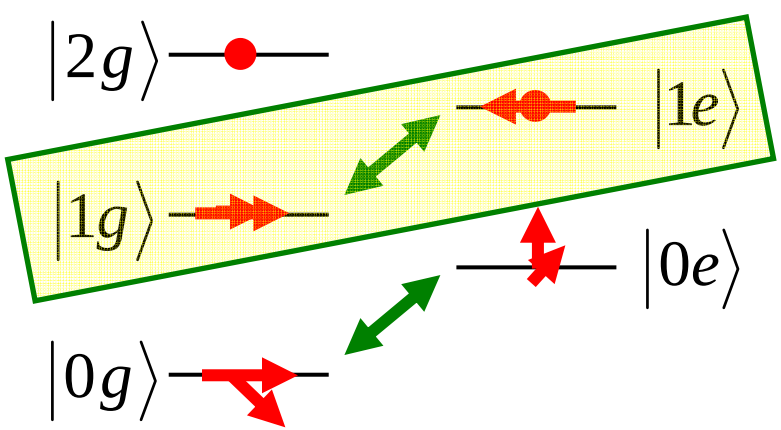
Generating Arbitrary States

Desired final state $\Rightarrow$ $|\psi\rangle = 0.577\left(|0\rangle + |1\rangle + i|2\rangle\right) \otimes |g\rangle$

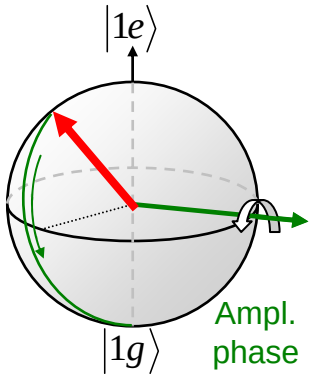


$0.566|0g\rangle - (0.000 + 0.517i)|0e\rangle + 0.256|1g\rangle + 0.577|1e\rangle$

$(0.234 - 0.473i)|0g\rangle + (0.528 + 0.210i)|0e\rangle + 0.632|1g\rangle$

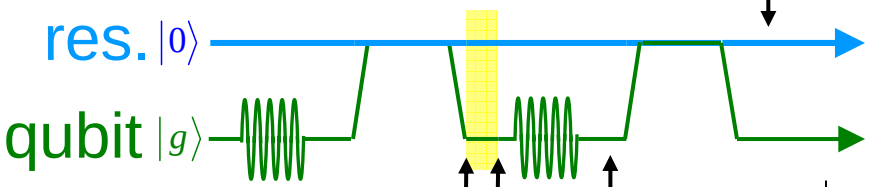


- Microwaves to transfer $|e\rangle$ to $|g\rangle$
- Transfer $|1e\rangle$ amplitude into $|1g\rangle$
- $|1e\rangle$ amplitude now zeroed



Generating Arbitrary States

Desired final state $\Rightarrow$ $|\psi\rangle = 0.577\left(|0\rangle + |1\rangle + i|2\rangle\right)\otimes|g\rangle$

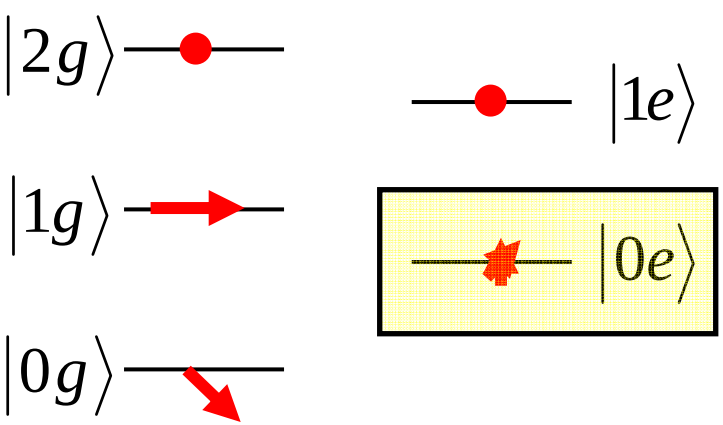


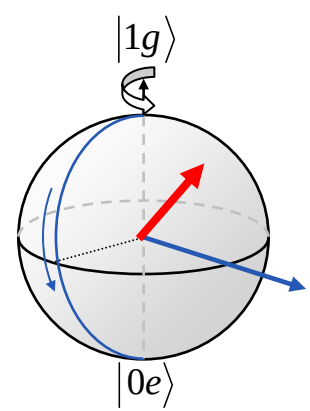
$0.566|0g\rangle - (0.000 + 0.517i)|0e\rangle + 0.256|1g\rangle + 0.577|1e\rangle$

$(0.234 - 0.473i)|0g\rangle + (0.528 + 0.210i)|0e\rangle + 0.632|1g\rangle$

$(0.234 - 0.473i)|0g\rangle + (0.000 + 0.568i)|0e\rangle + 0.632|1g\rangle$

- Correct phase by waiting





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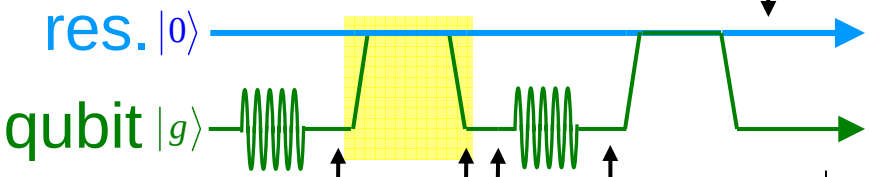
Desired final state $\Rightarrow$ $|\psi\rangle = 0.577\left(|0\rangle + |1\rangle + i|2\rangle\right) \otimes |g\rangle$

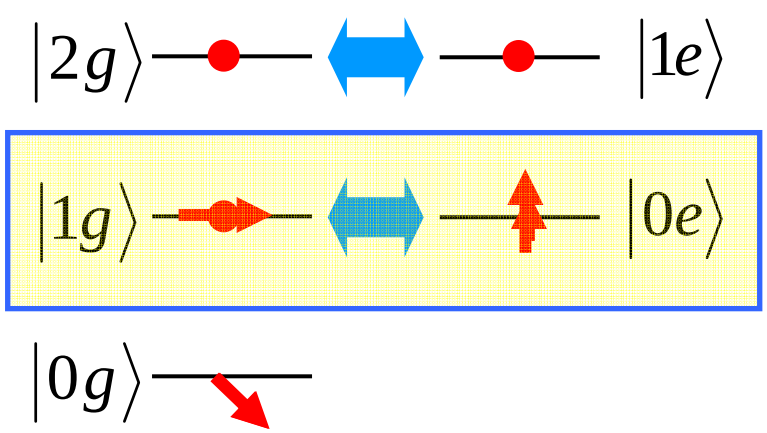


$0.566|0g\rangle - (0.000 + 0.517i)|0e\rangle + 0.256|1g\rangle + 0.577|1e\rangle$

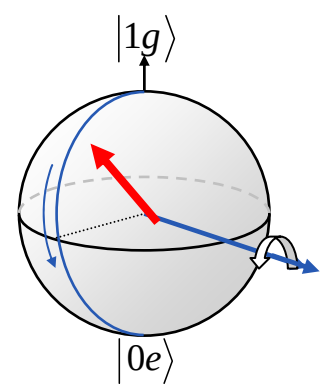
$(0.234 - 0.473i)|0g\rangle + (0.528 + 0.210i)|0e\rangle + 0.632|1g\rangle$

$(0.234 - 0.473i)|0g\rangle + (0.000 + 0.568i)|0e\rangle + 0.632|1g\rangle$

$(0.234 - 0.473i)|0g\rangle + (0.000 + 0.849i)|0e\rangle$

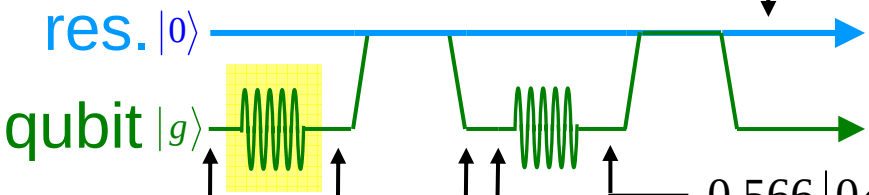


- Transfer $|1g\rangle$ amplitude into $|0e\rangle$



Generating Arbitrary States

Desired final state $\Rightarrow$ $|\psi\rangle = 0.577\left(|0\rangle + |1\rangle + i|2\rangle\right) \otimes |g\rangle$



$0.566|0g\rangle - (0.000 + 0.517i)|0e\rangle + 0.256|1g\rangle + 0.577|1e\rangle$

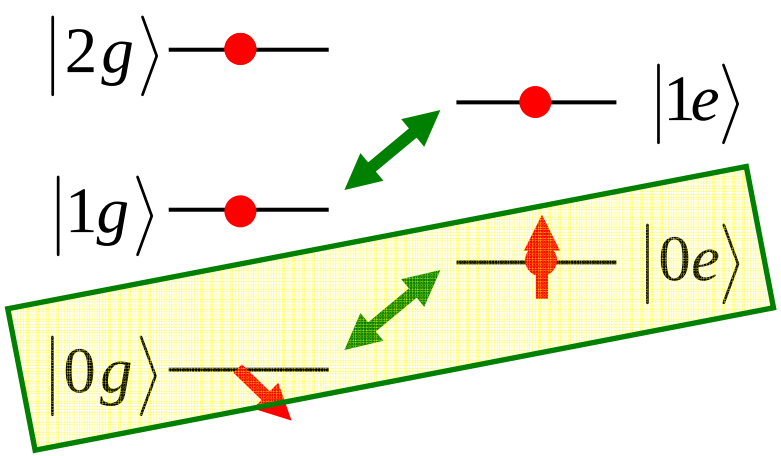
$(0.234 - 0.473i)|0g\rangle + (0.528 + 0.210i)|0e\rangle + 0.632|1g\rangle$

$(0.234 - 0.473i)|0g\rangle + (0.000 + 0.568i)|0e\rangle + 0.632|1g\rangle$

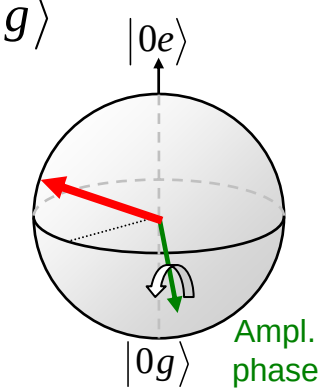
$(0.234 - 0.473i)|0g\rangle + (0.000 + 0.849i)|0e\rangle$

$|\psi\rangle = (0.444 - 0.896i)\boxed{|0g\rangle}$

initial ground state

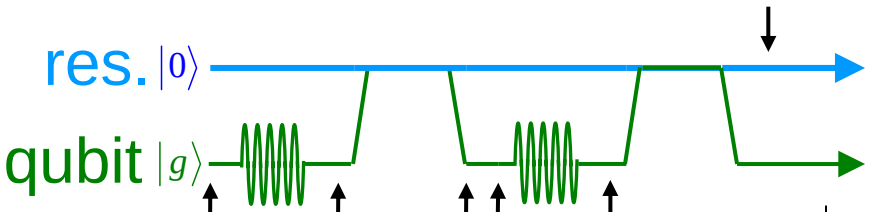


- Microwaves to transfer $|e\rangle$ to $|g\rangle$
- Transfer $|0e\rangle$ amplitude into $|0g\rangle$



Generating Arbitrary States

Desired final state → $|\psi\rangle = 0.577\left(|0\rangle + |1\rangle + i|2\rangle\right) \otimes |g\rangle$



$0.566|0g\rangle - (0.000 + 0.517i)|0e\rangle + 0.256|1g\rangle + 0.577|1e\rangle$

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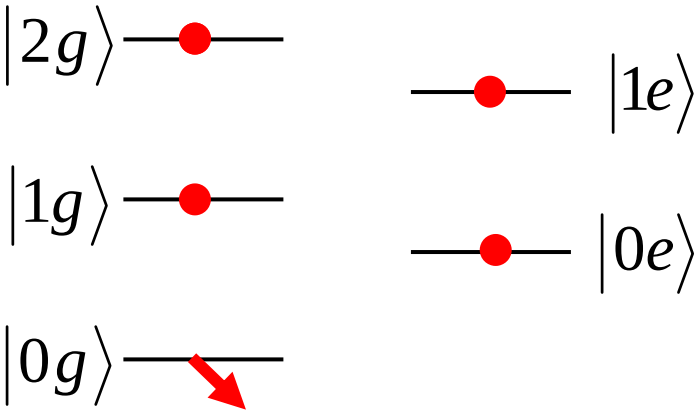
$|\psi\rangle = (0.444 - 0.896i)\boxed{|0g\rangle}$

initial ground state



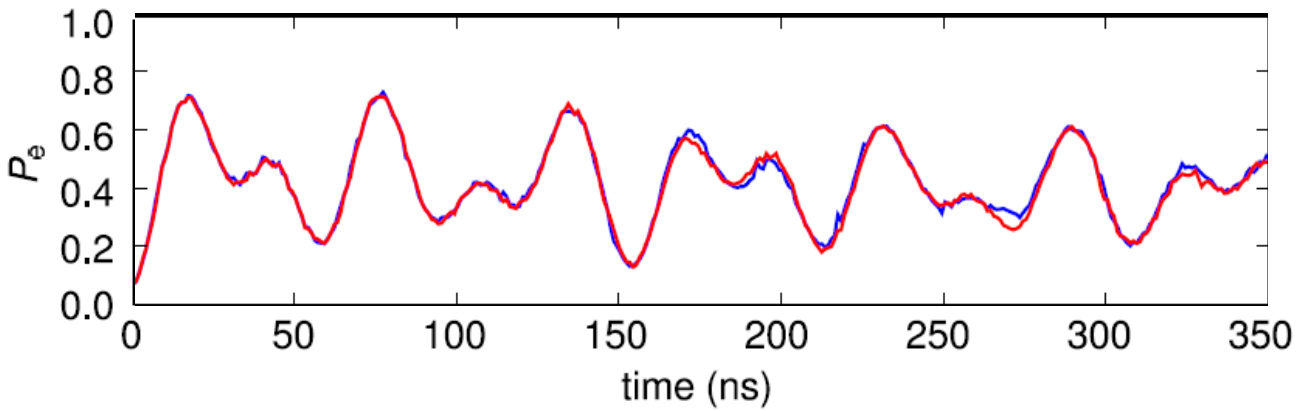
- Phase of $|0g\rangle$ does not matter
- Run sequence in reverse

Generating Arbitrary States: Quantum Digital to Analog Converter

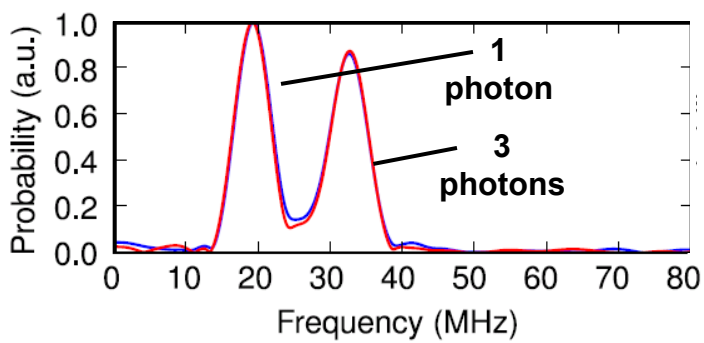
$$|\psi_a\rangle = |1\rangle + |3\rangle$$

Now try: $|\psi_b\rangle = |1\rangle + \boldsymbol{i}\,|3\rangle$

Measure using qubit and watching evolution (usual method):



Fourier transform yields number composition:



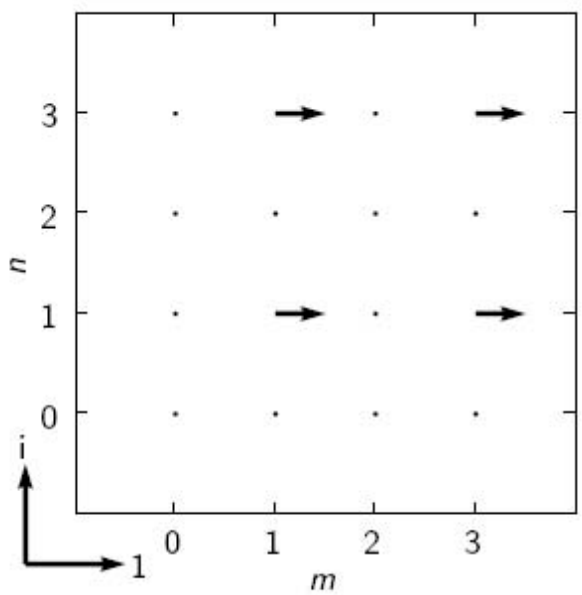
Cannot distinguish based on this measure!

State Tomography of Harmonic Oscillator

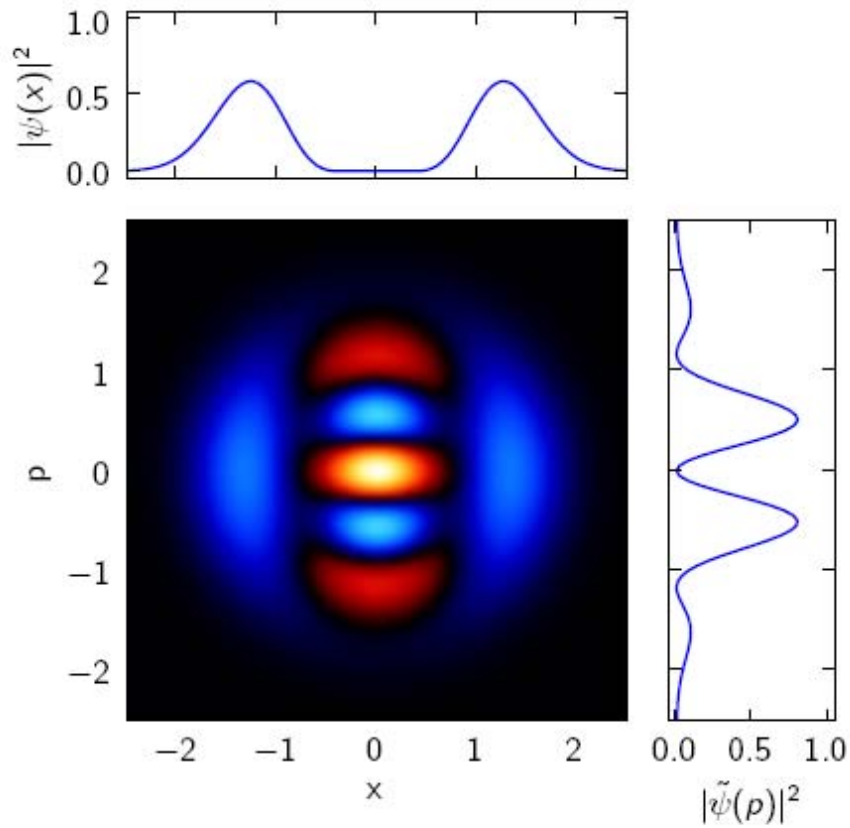
Example: $|\psi\rangle = |1\rangle + |3\rangle$

Density matrix:

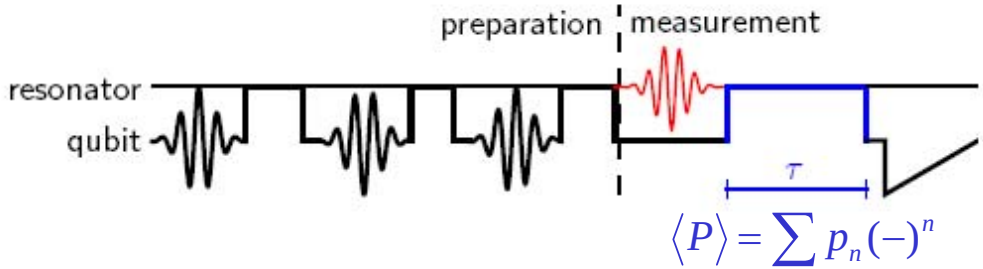
$$\rho = |\psi\rangle\langle\psi|$$



Wigner function:



State Tomography of Harmonic Oscillator



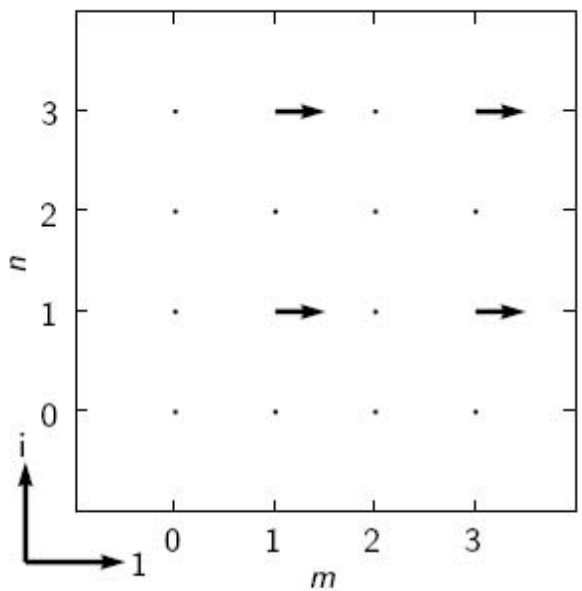
Example: $|\psi\rangle = |1\rangle + |3\rangle$

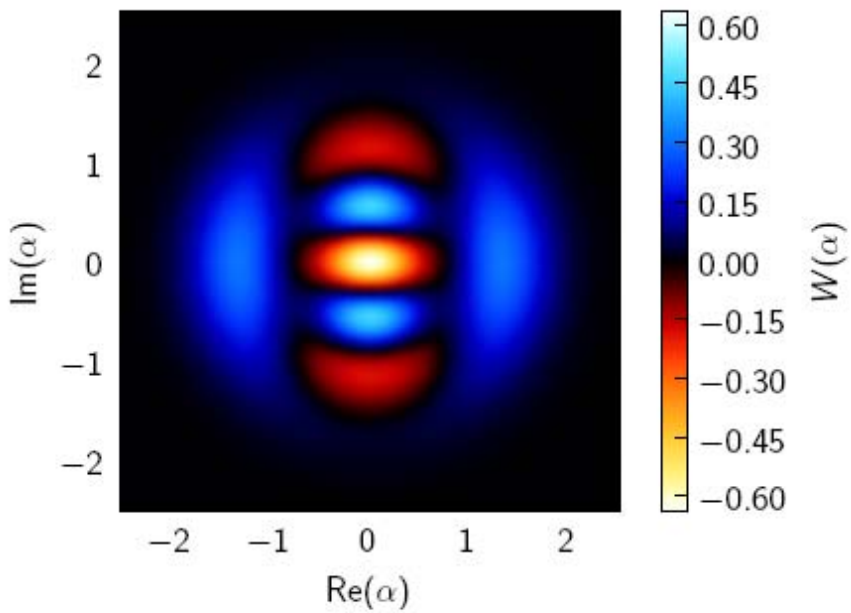
Density matrix:

$$\rho = |\psi\rangle\langle\psi|$$

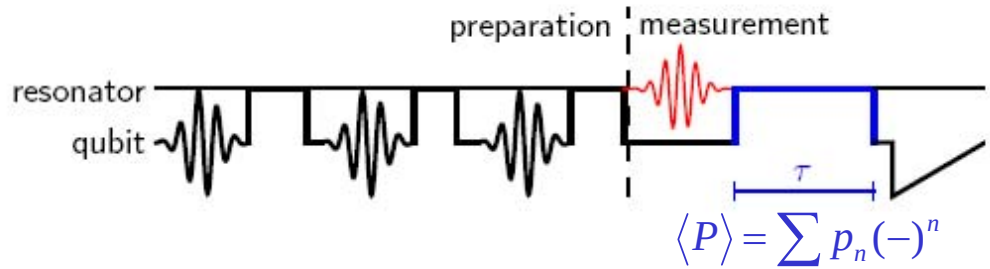
Wigner function:

$$W(\alpha) = \frac{2}{\pi}\langle\psi|D(\alpha)PD(-\alpha)|\psi\rangle$$





State Tomography of Harmonic Oscillator



Example: $|\psi\rangle = |1\rangle + i|3\rangle$

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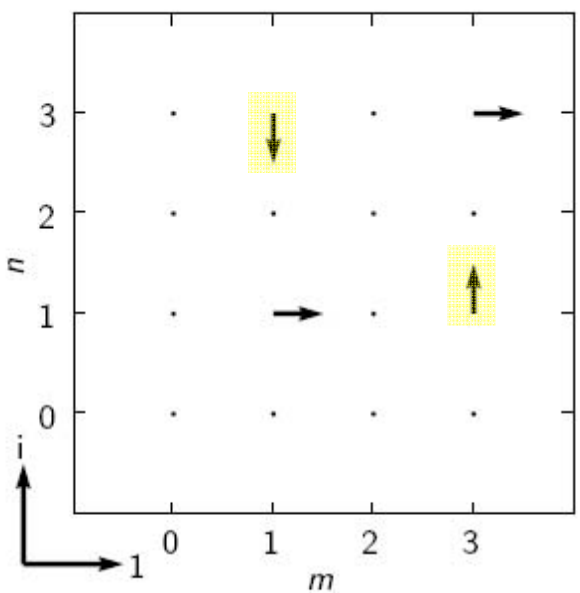
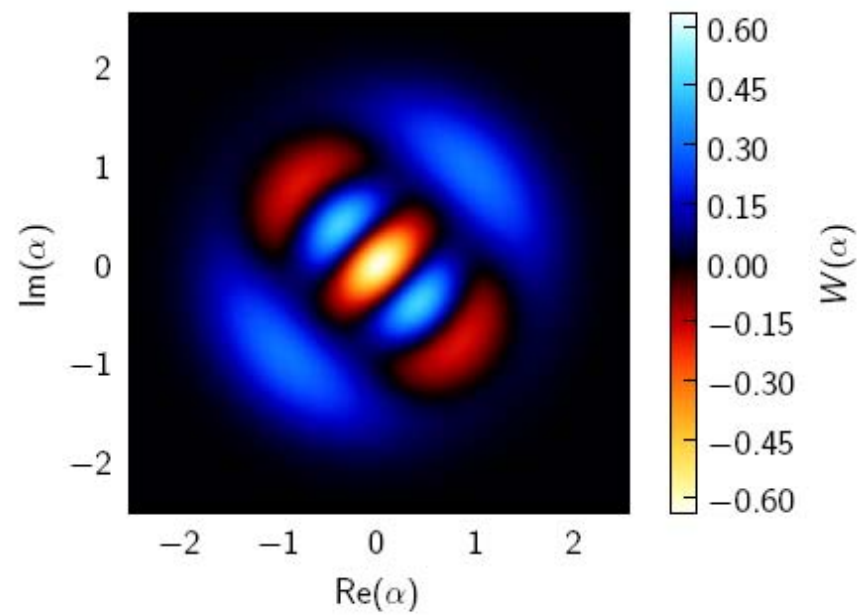
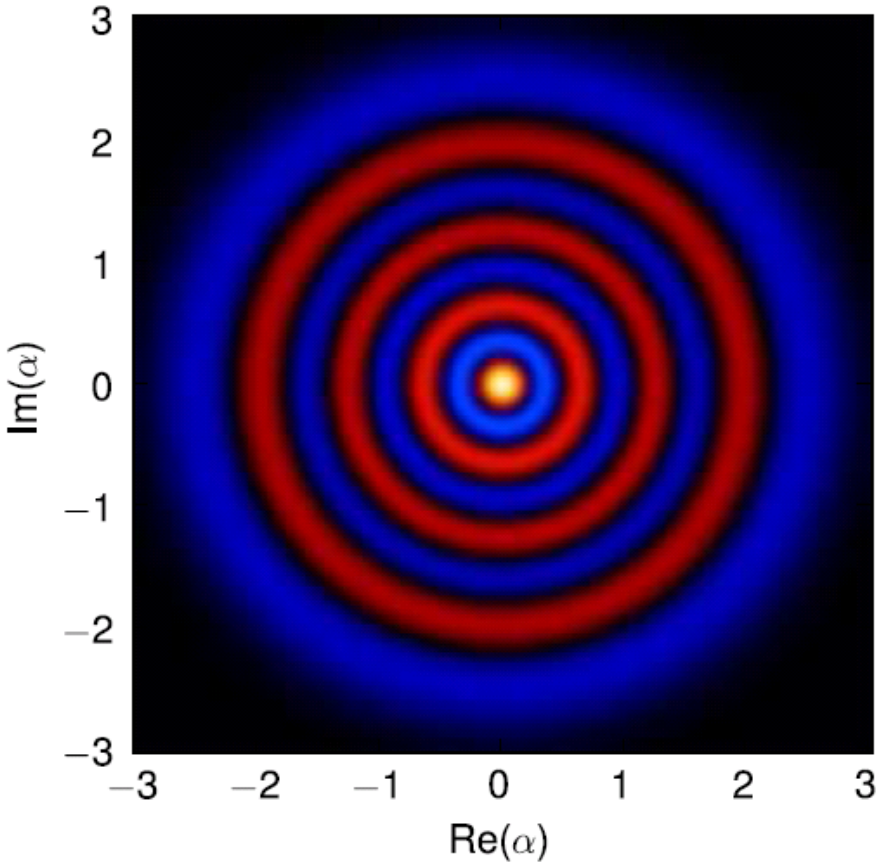


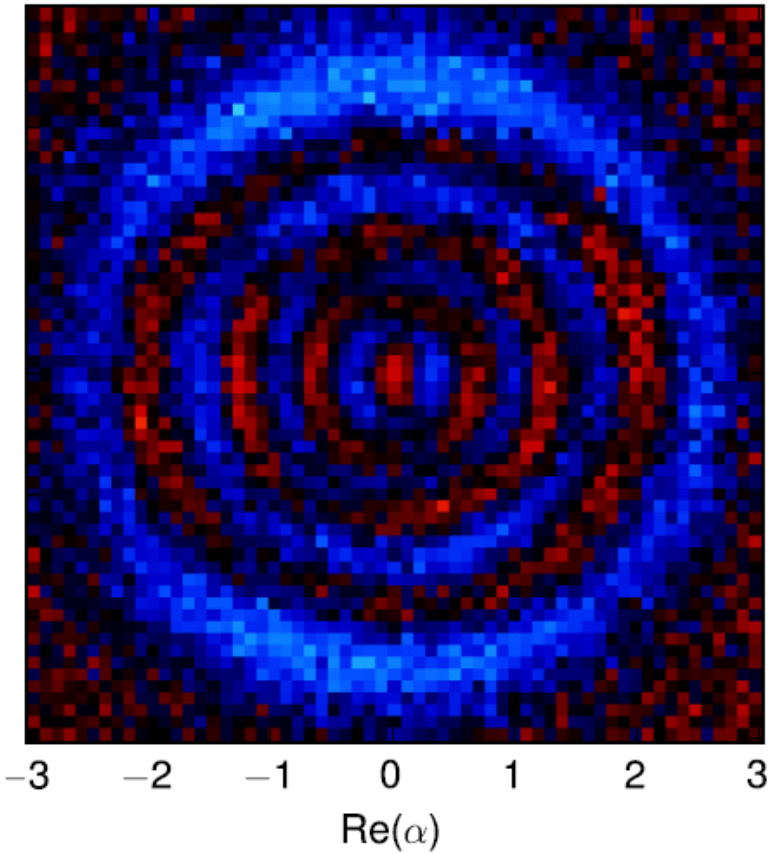


Wigner Tomography of Fock State $|7\rangle$

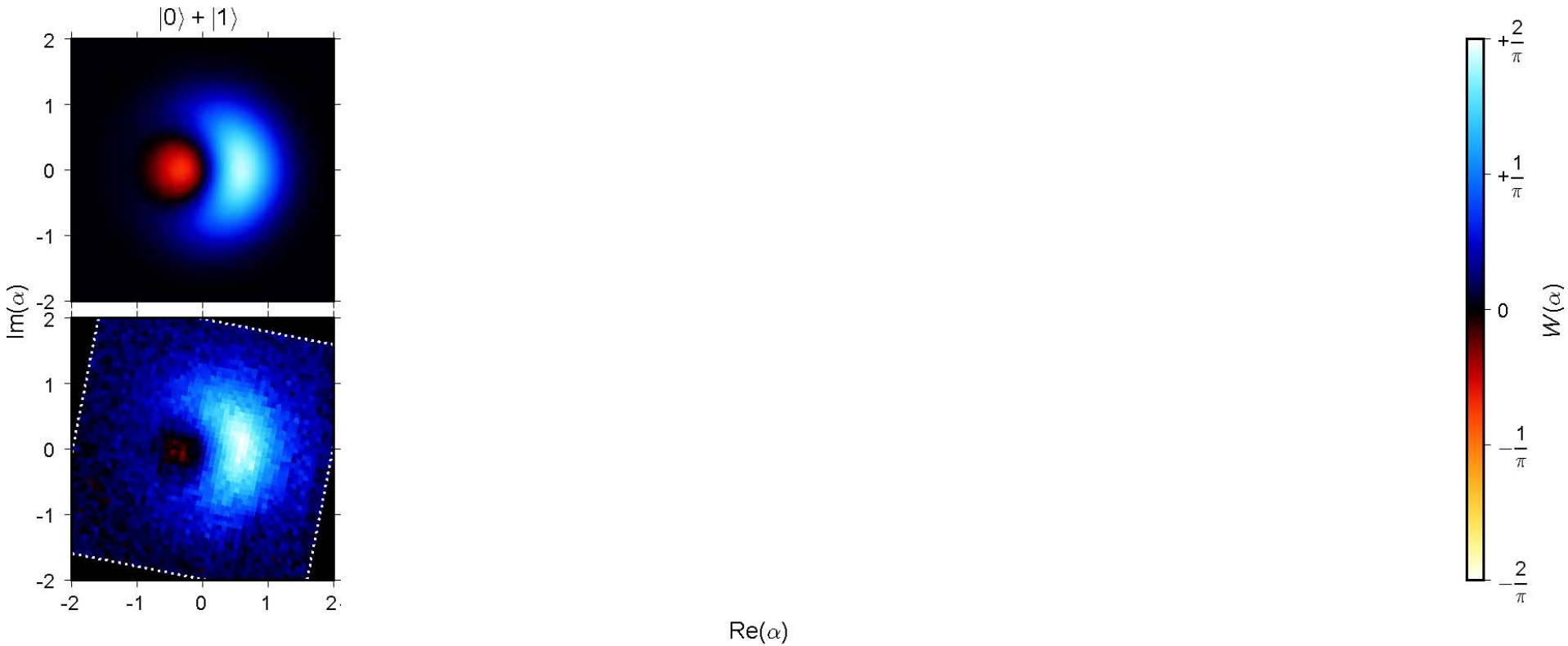
Theory



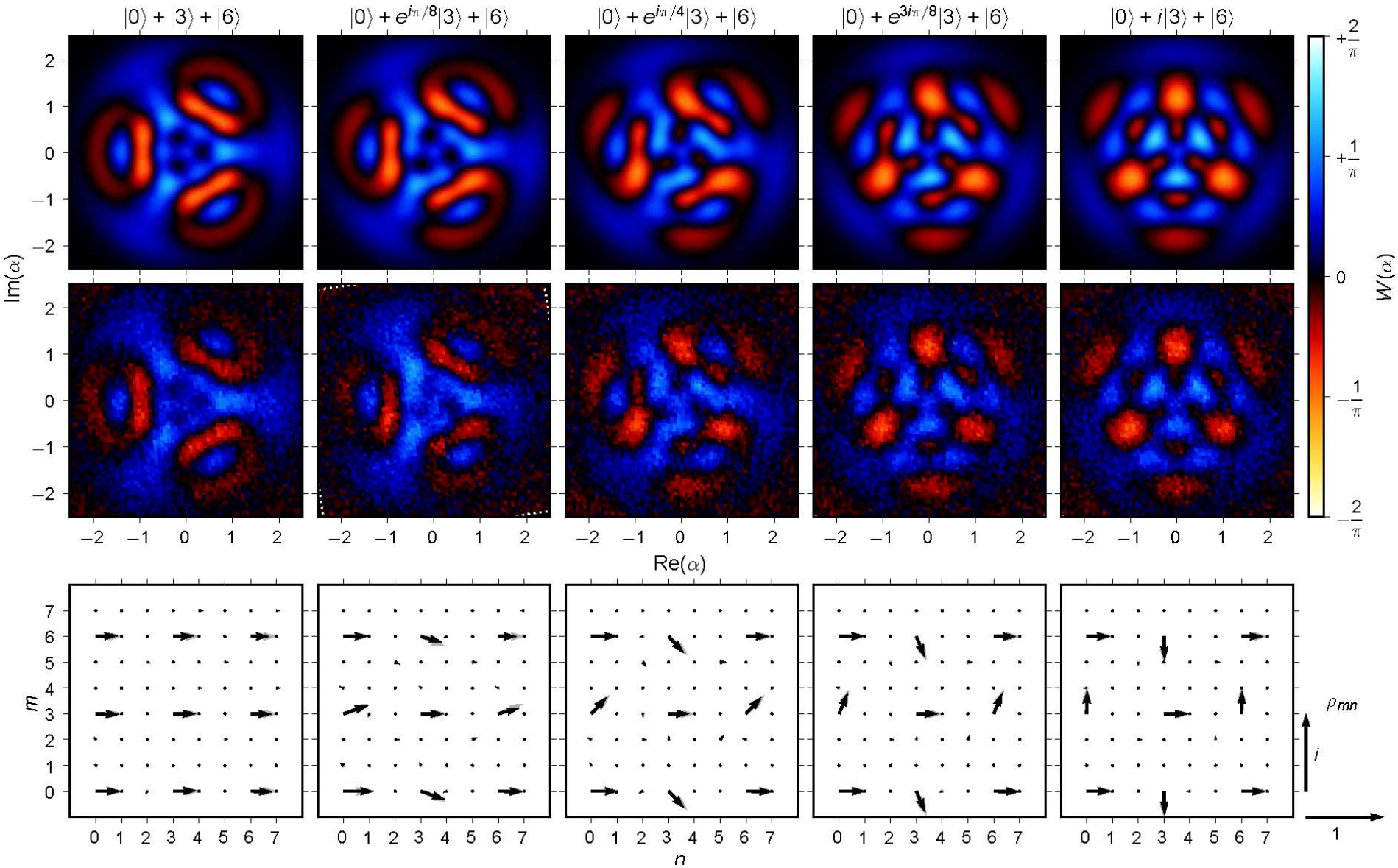
Experiment

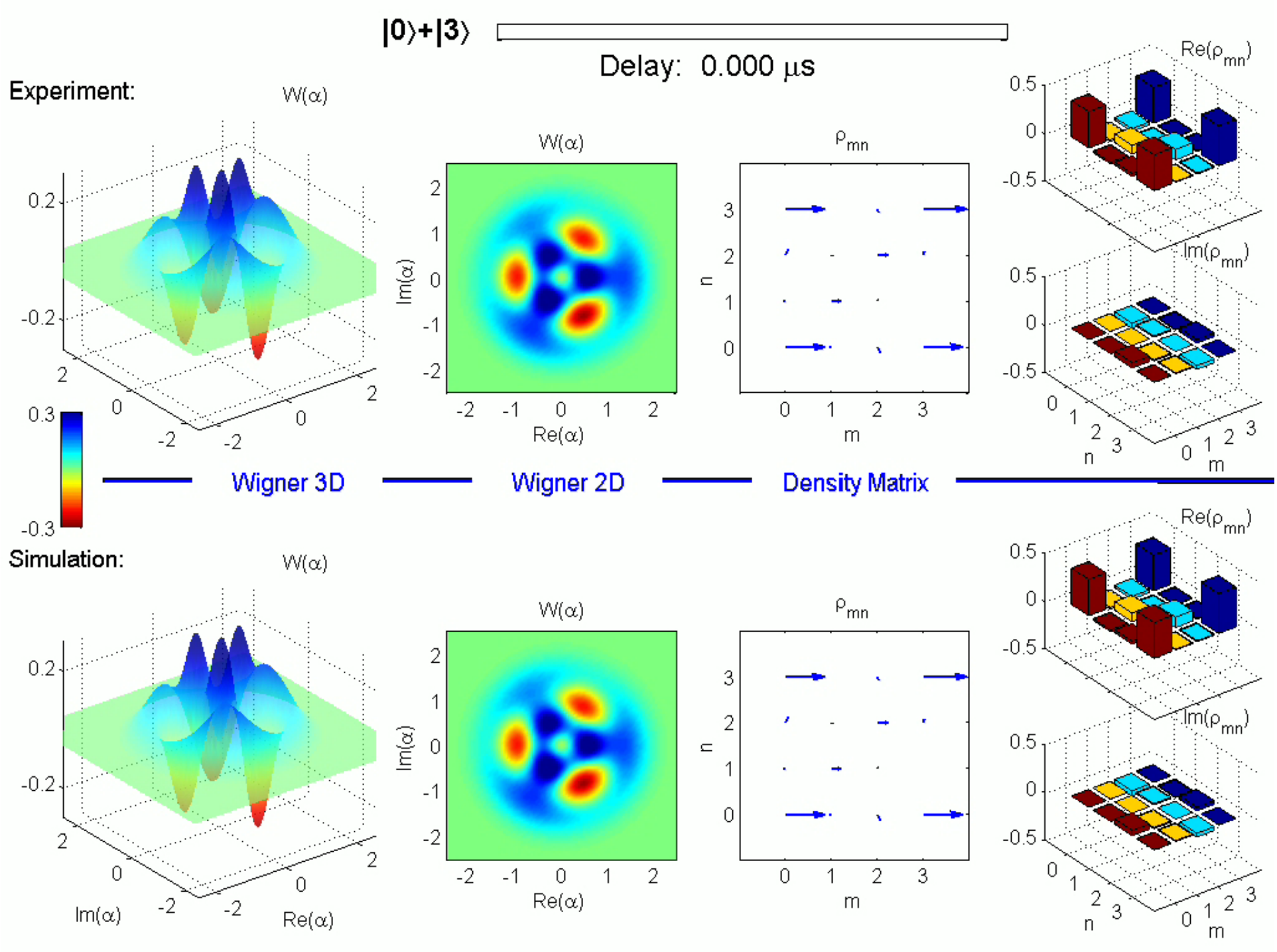


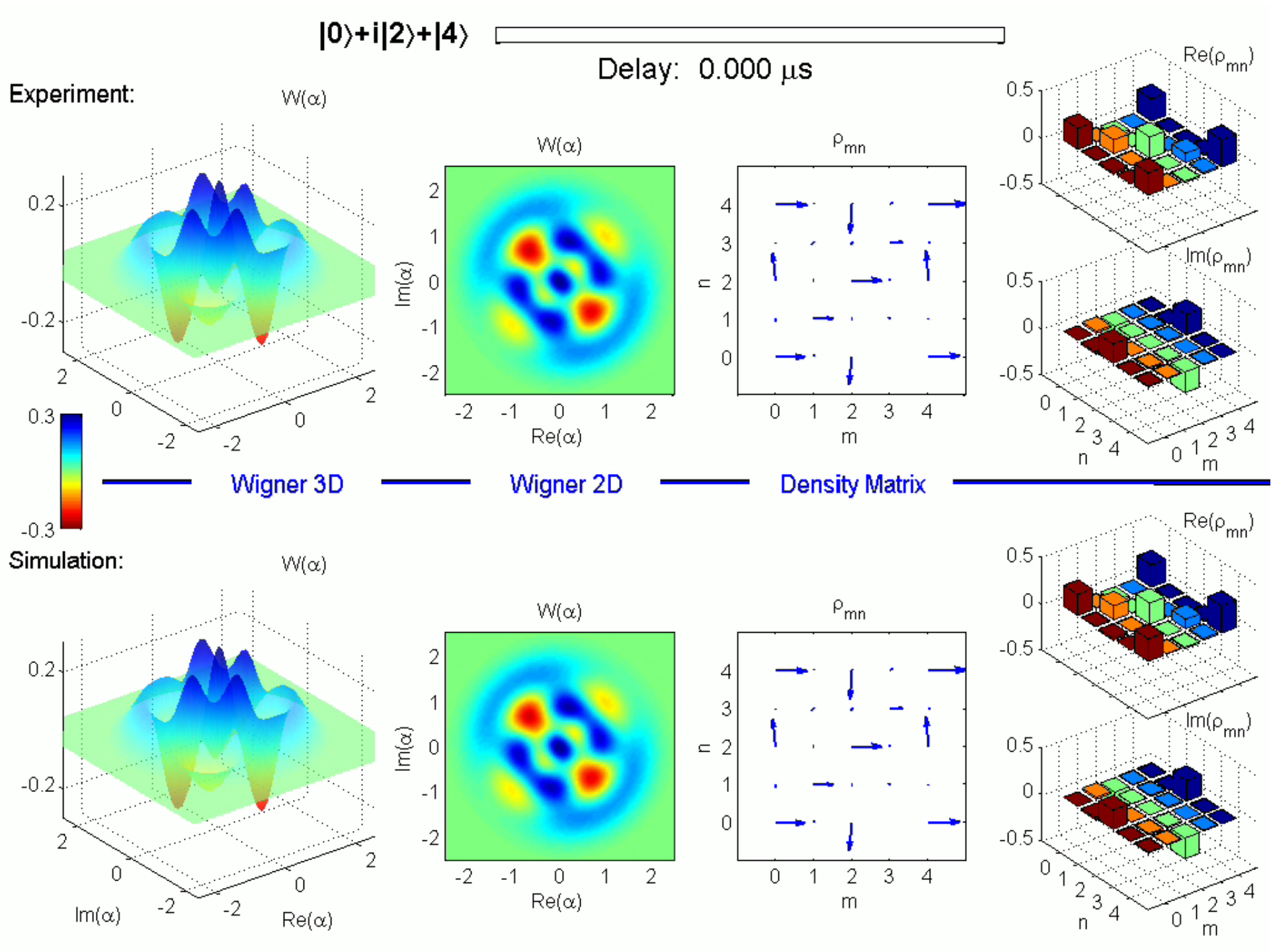
Wigner Tomography, $|0\rangle + |N\rangle$ states:



Wigner Tomography: States with phase

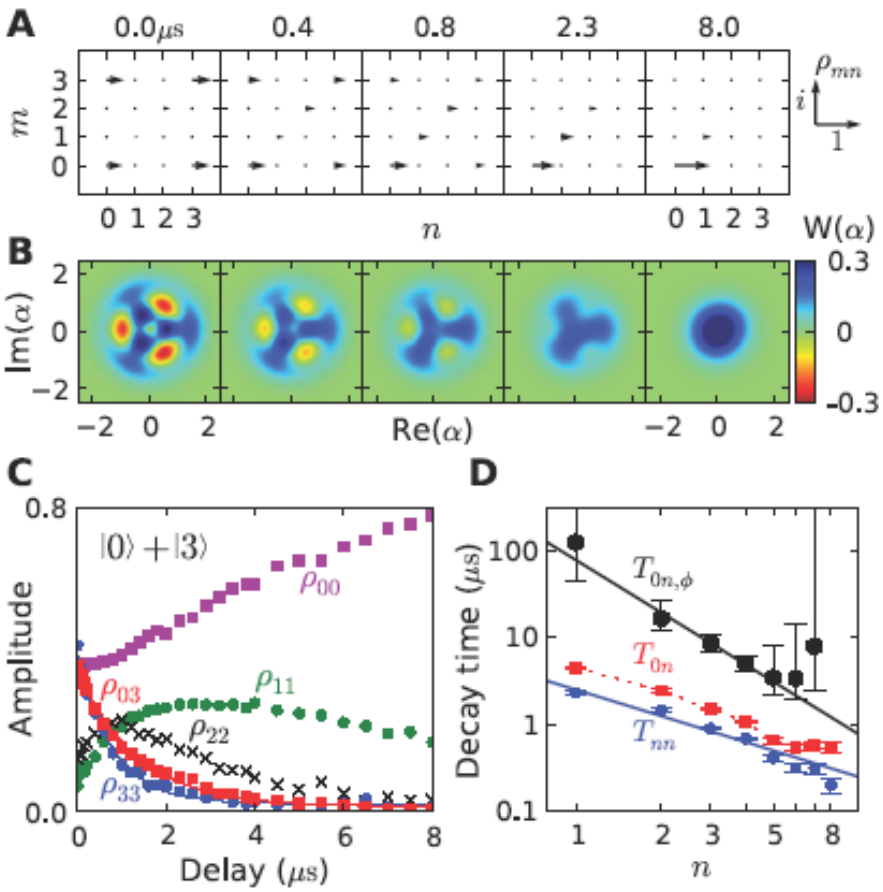



|0⟩+|3⟩
Delay: 0.000 μs
Experiment:
Simulation:
W(α)
Im(α)
Re(α)
ρmn
n
m
Re(ρmn)
Im(ρmn)
Wigner 3D
Wigner 2D
Density Matrix

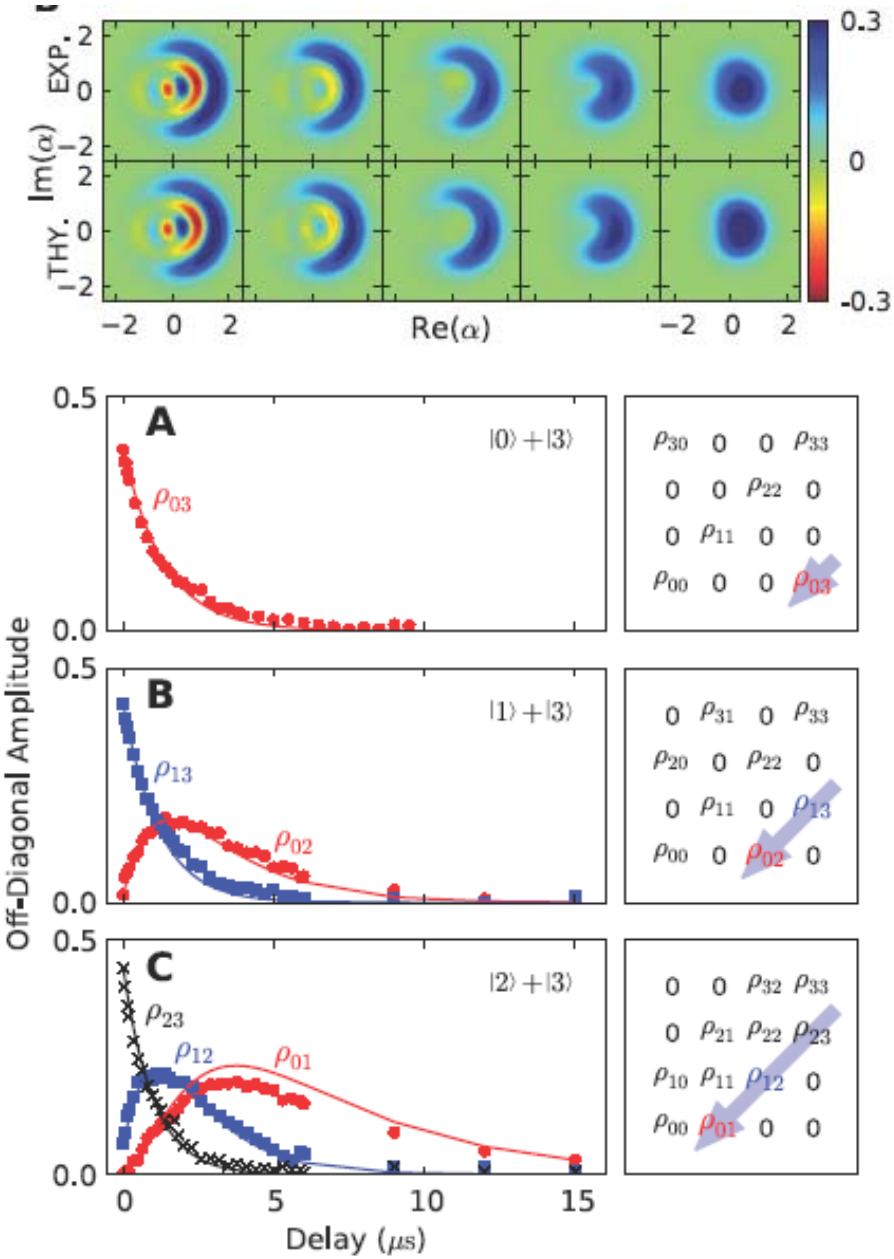

|0⟩+i|2⟩+|4⟩
Delay: 0.000 μs
Experiment:
Simulation:
W(α)
Wigner 3D
Wigner 2D
Density Matrix
ρmn
Re(ρmn)
Im(ρmn)
Re(α)
Im(α)
m
n

Off-Diagonal Decay of Density Matrix

$|0\rangle + |N\rangle$ (far-element decay)

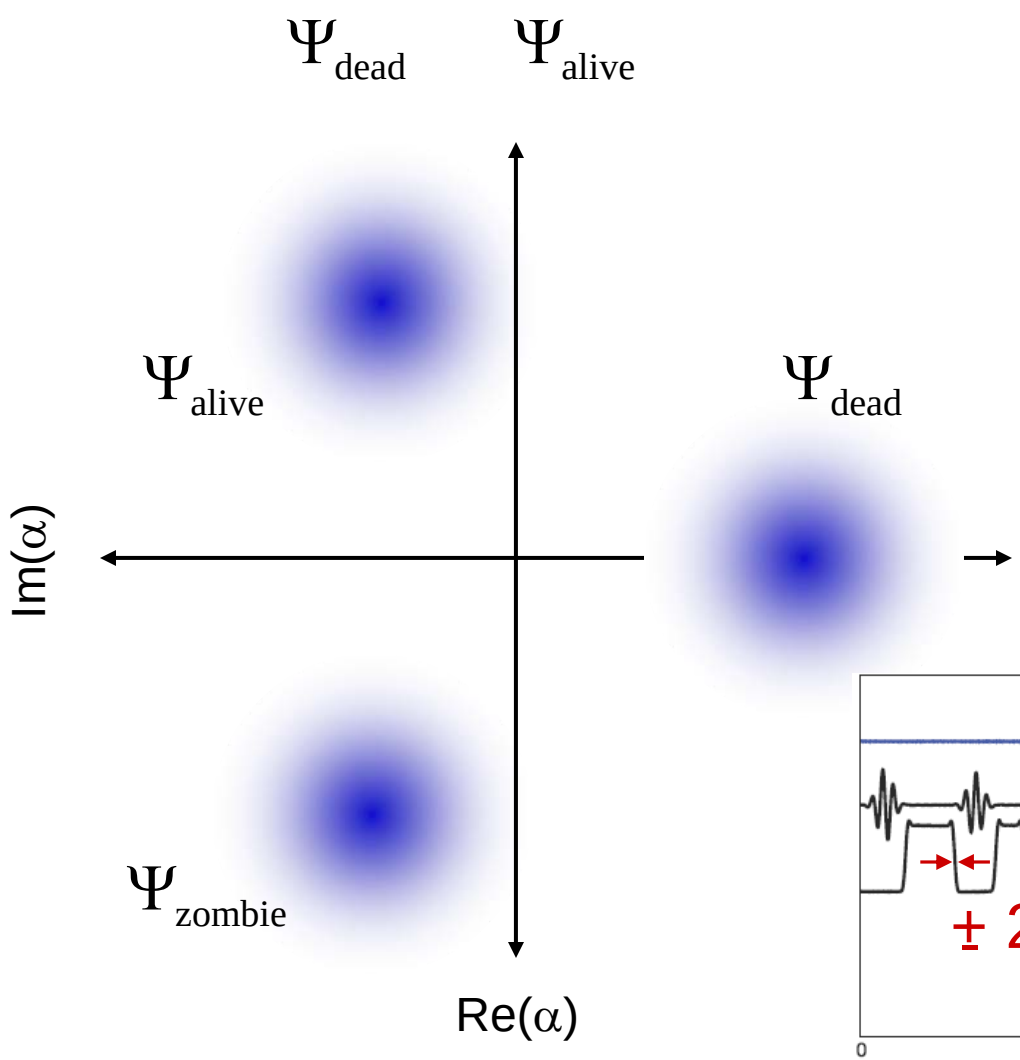


$|2\rangle + |3\rangle$ (decay cascade)

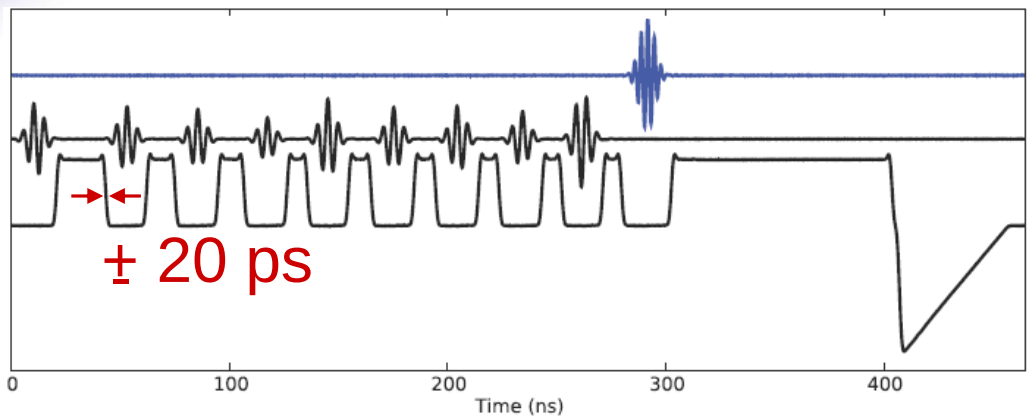


3-state Schrodinger Cat : “Voodoo Cat”

$$|\psi\rangle = |2e^{i0}\rangle + |2e^{i2\pi/3}\rangle$$



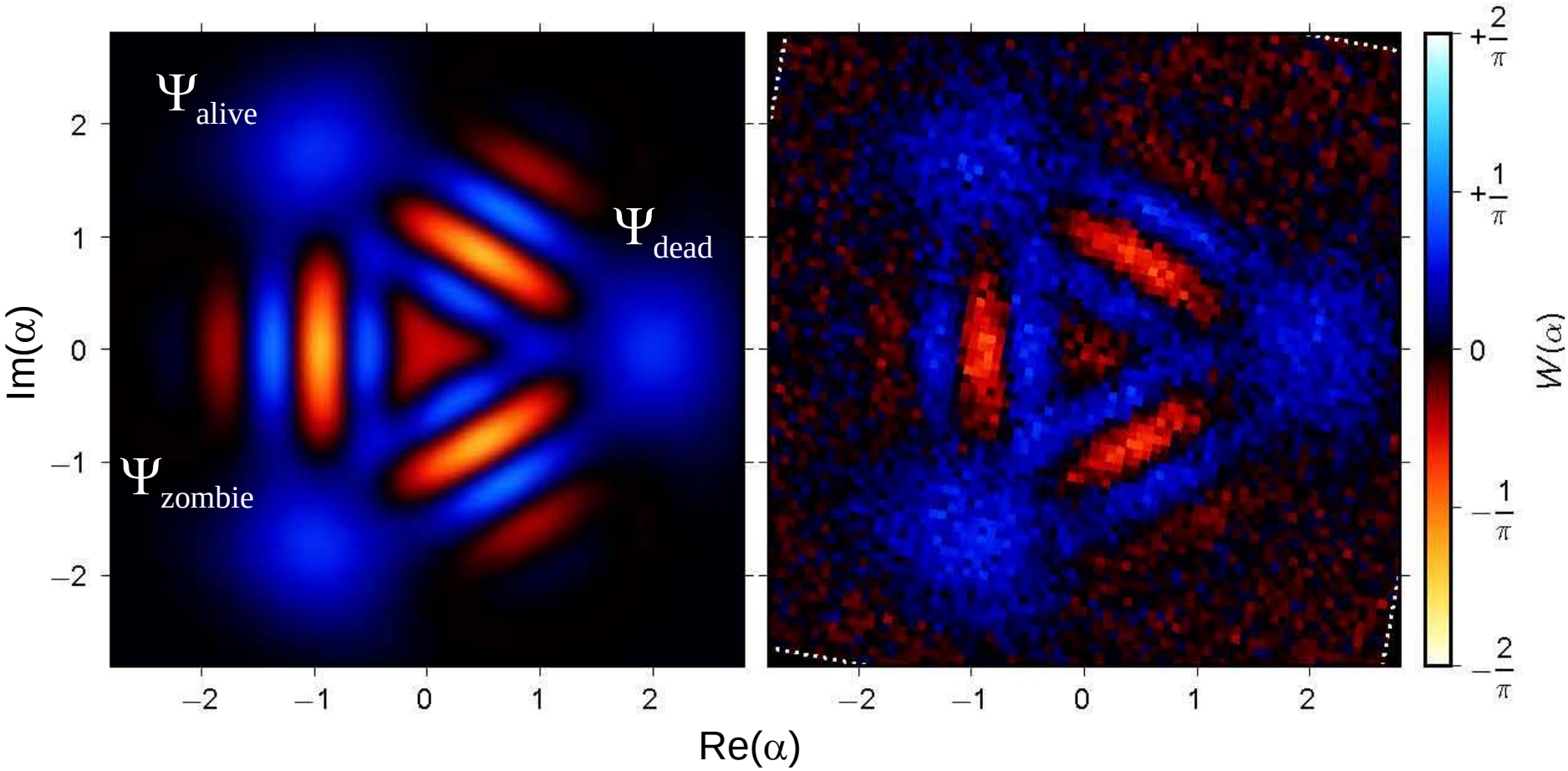
Digitally programmed cat!



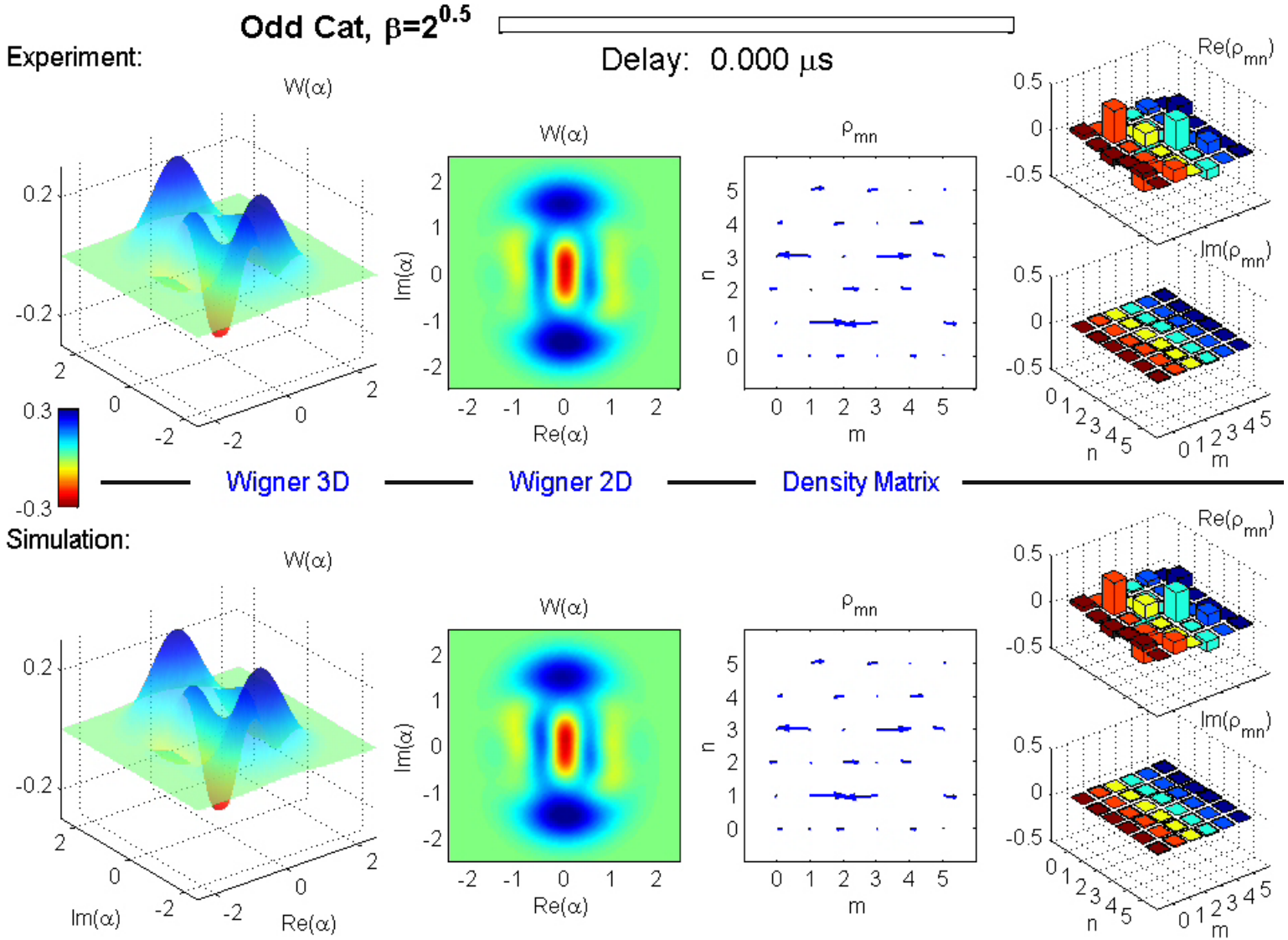
3-state Schrodinger Cat : “Voodoo Cat”

$$\left|\psi\right\rangle = \left|2e^{\mathrm{i}0}\right\rangle + \left|2e^{\mathrm{i}2\pi/3}\right\rangle + \left|2e^{\mathrm{i}4\pi/3}\right\rangle \quad = \sum_{n=0,3,6,9} \left(2^n / \sqrt{n!}\right) \left|n\right\rangle$$

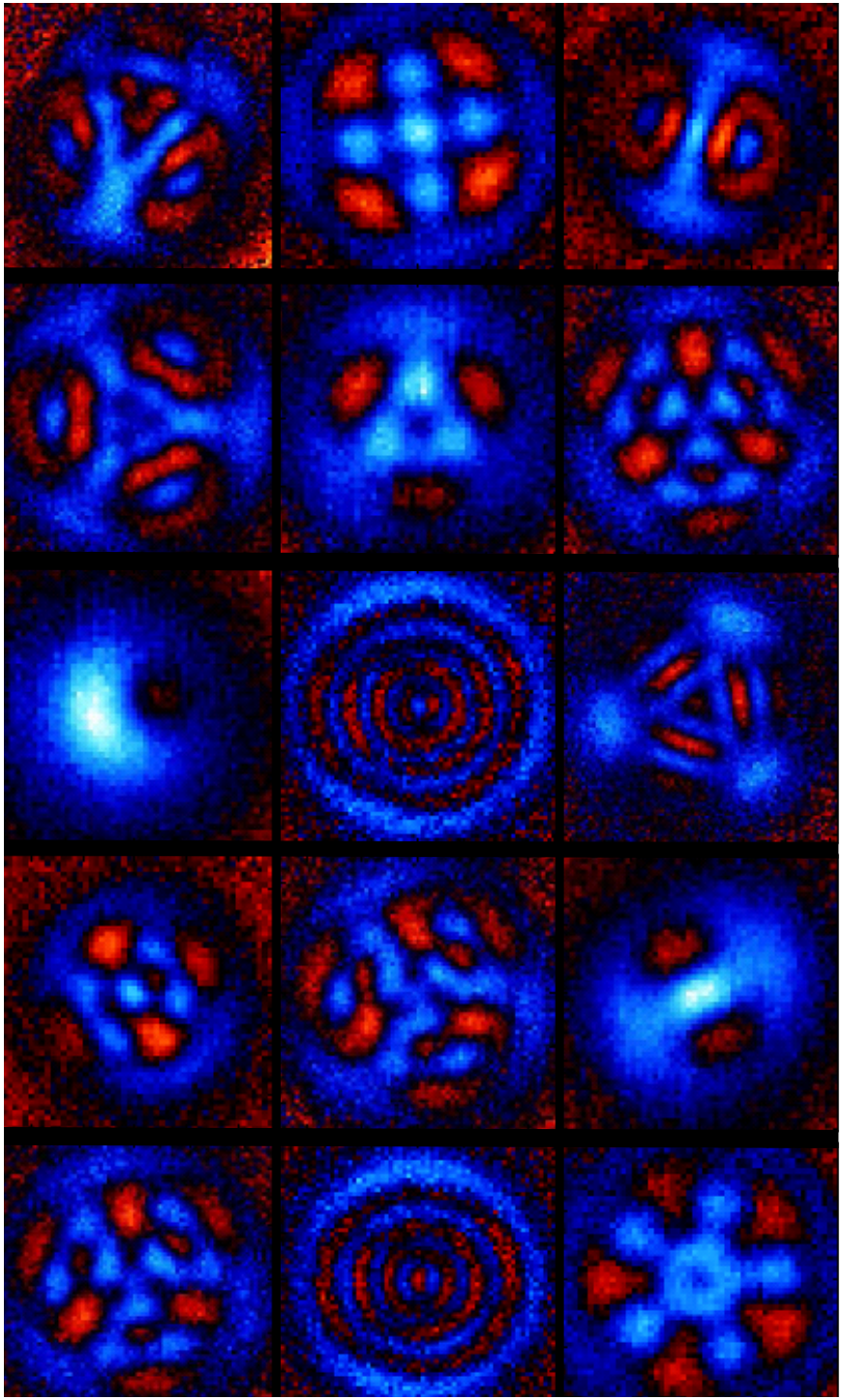
Ψ_{dead} Ψ_{alive} Ψ_{zombie}



Collapse of Schrodinger Cat State



Quantum feature disappears first, and the classical peaks merge later



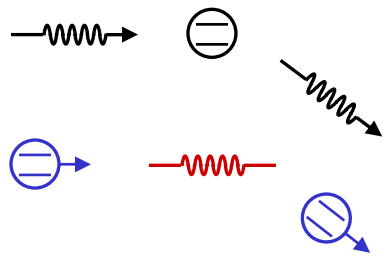
Conclusions

- Full quantum control over a harmonic oscillator
- <u>Deterministic</u> creation of arbitrary states (up to n = 9)
  Most complex states
- Direct Wigner tomography

➢ Demonstrate powerful idea:
Qubit for control, measure

Typical:

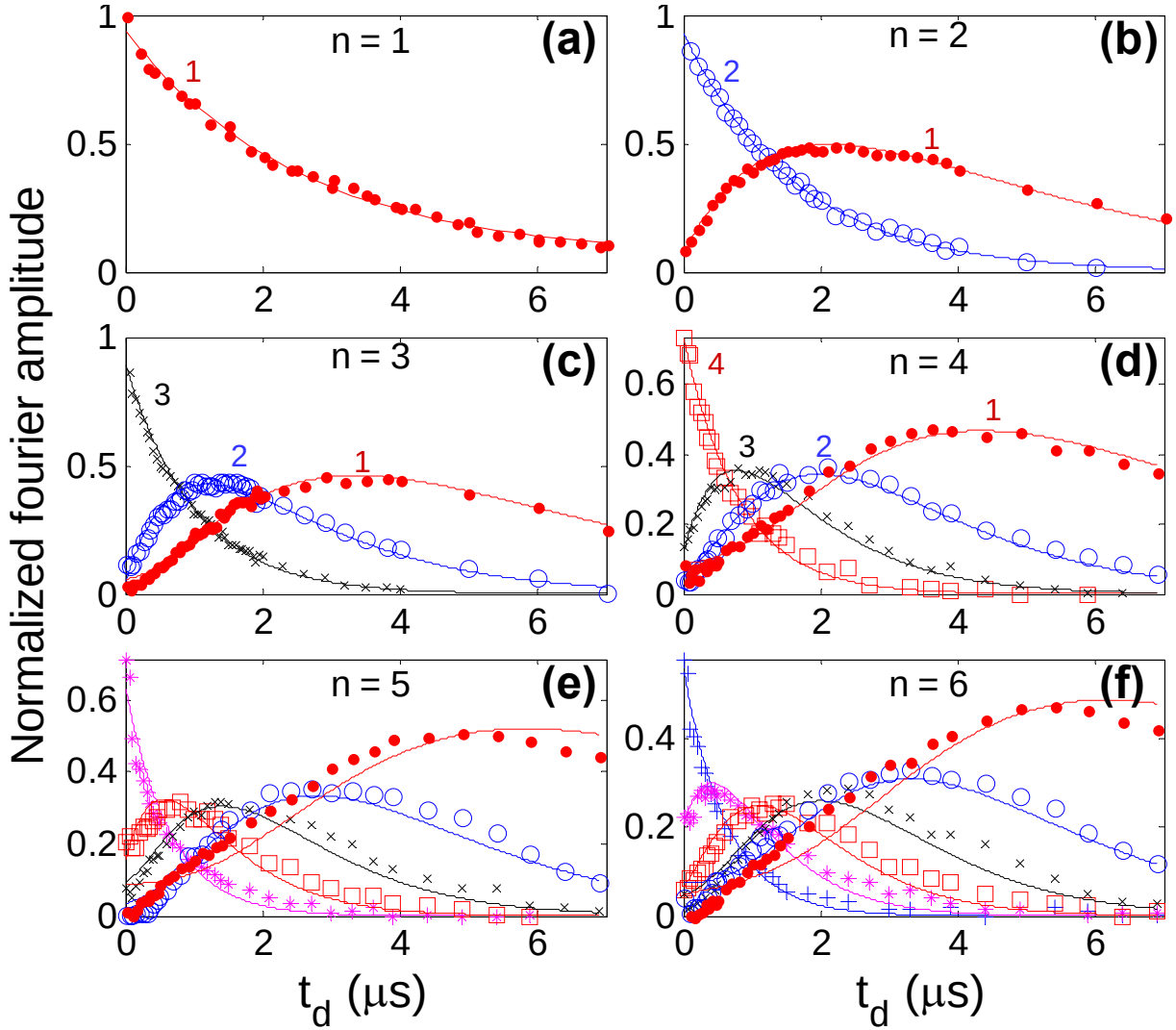
Here:



Decay of Fock States

Master Equation
(no phase info.)

$$\frac{dP_i(t)}{dt} = -\frac{P_i(t)}{T_i} + \frac{P_{i+1}(t)}{T_{i+1}}$$



	n	1	2	3	4	5	6	
(exp.)	T_n	3.25	1.65	1.02	0.79	0.65	0.64	(μs)
(thy.)	T_1/n	3.25	1.63	1.08	0.81	0.65	0.54	

Density matrix and Wigner tomography

State preparation

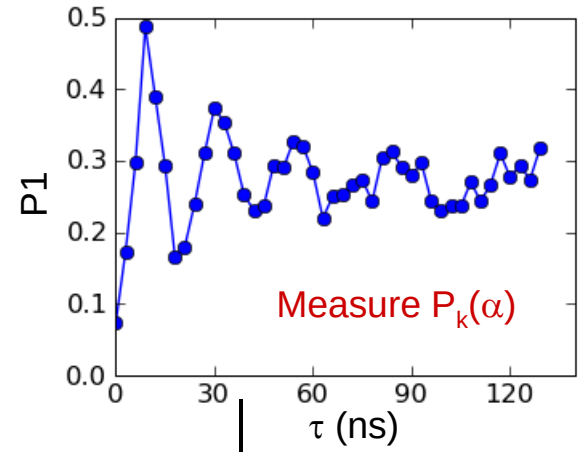
Delay

D(α)

State readout:

bring the qubit on resonance with the resonator for τ

Readout qubit



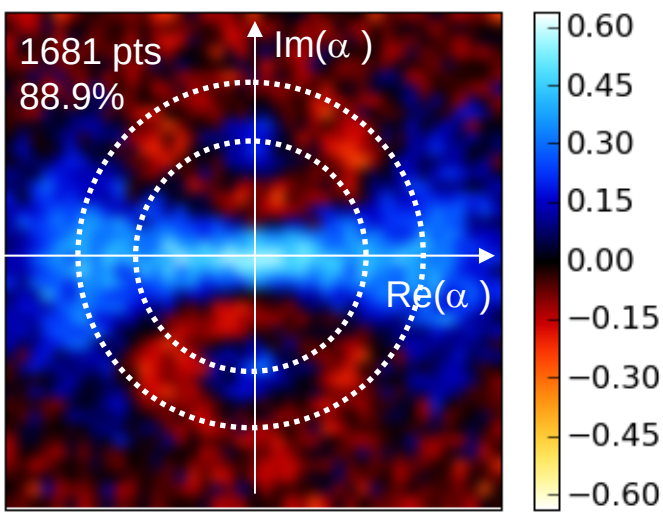
Direct Wigner tomography

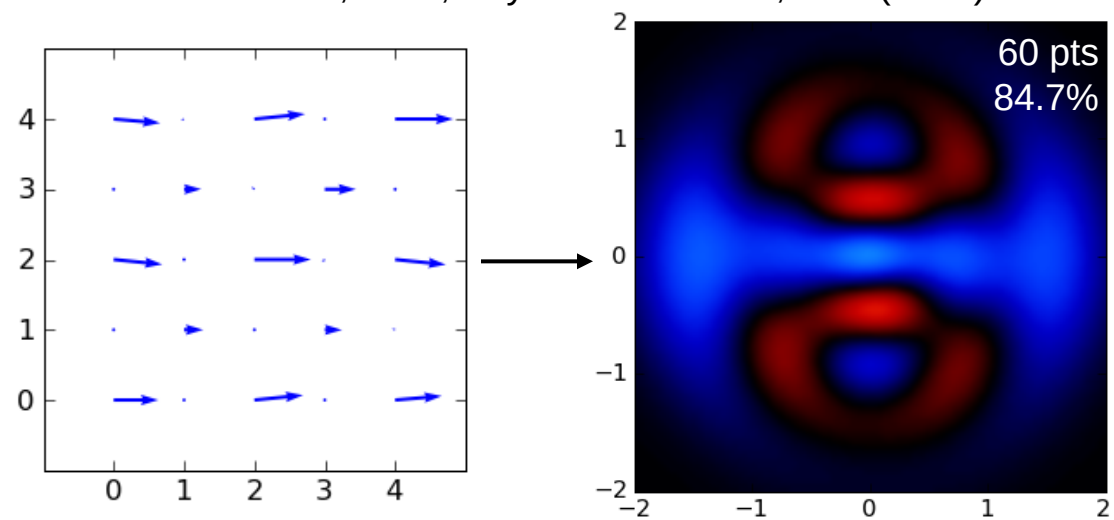
Matrix inversion

$$W(\alpha)=\frac{2}{\pi}\sum_k(-1)^k P_k(\alpha)$$

$$P_k(\alpha)=\sum_{n,m=0}^{\infty}\sum_{x=0}^{\min\{n,k\}}\sum_{y=0}^{\min\{m,k\}}(-1)^{x+y}\frac{\sqrt{n!m!}}{k!(n-x)!(m-y)!}\binom{k}{x}\binom{k}{y}e^{-|\alpha|^2}|\alpha|^{2(k-x-y)}(\alpha^*)^n\alpha^m\rho_{nm}$$

D. Leibfried, et al., Phys. Rev. Lett. **77**, 4281(1996)





Calibrations

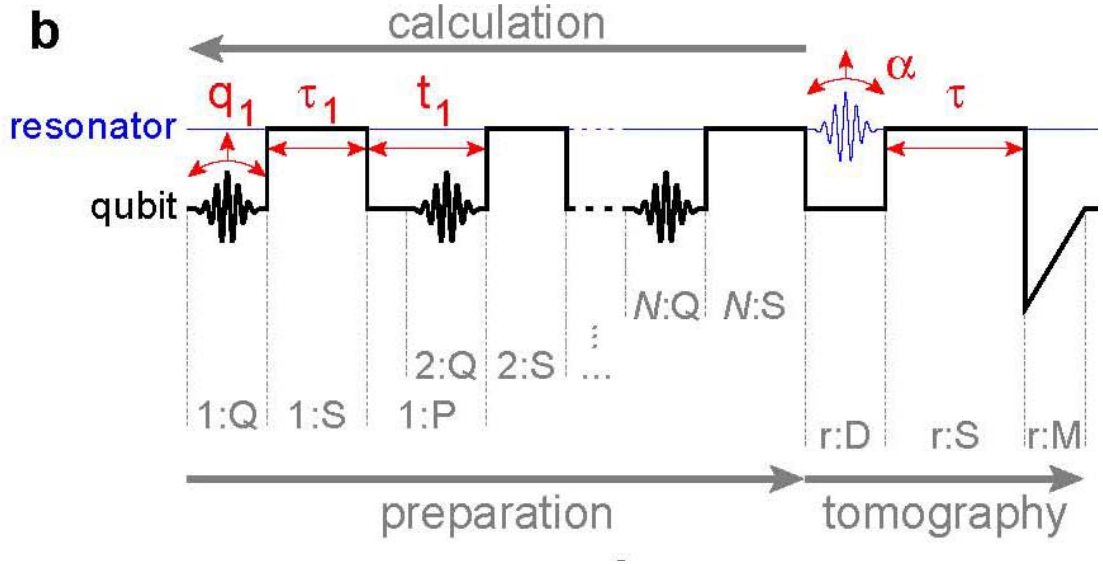
- impulse response (sampling scope)
- imperfections of IQ-mixer (spectrum analyzer)
- cable response (qubit as sampling scope)

- various parameters for readout SQUID
- π pulses (amplitude and frequency)
- measure pulse amplitude
- swap pulse amplitude
- swap pulse time for each photon number
- first order correction for finite rise time of swap pulses
- qubit/resonator dephasing rate when off resonance
- resonator displacement/drive amplitude ratio
- resonator drive phase
- readout visibility

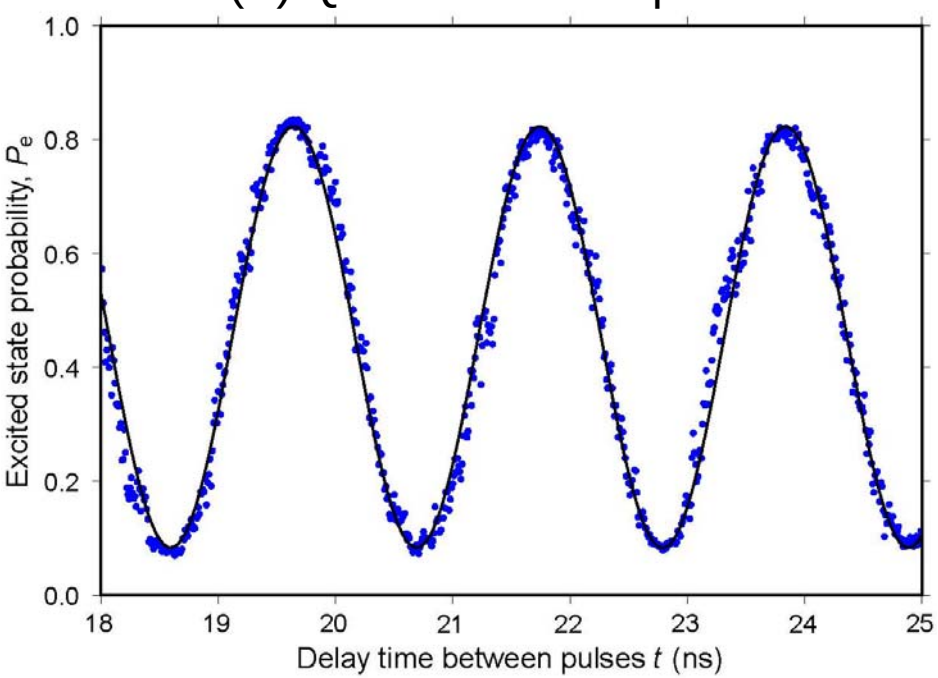
BUT:

- no per-state calibrations; just run the calculated sequence

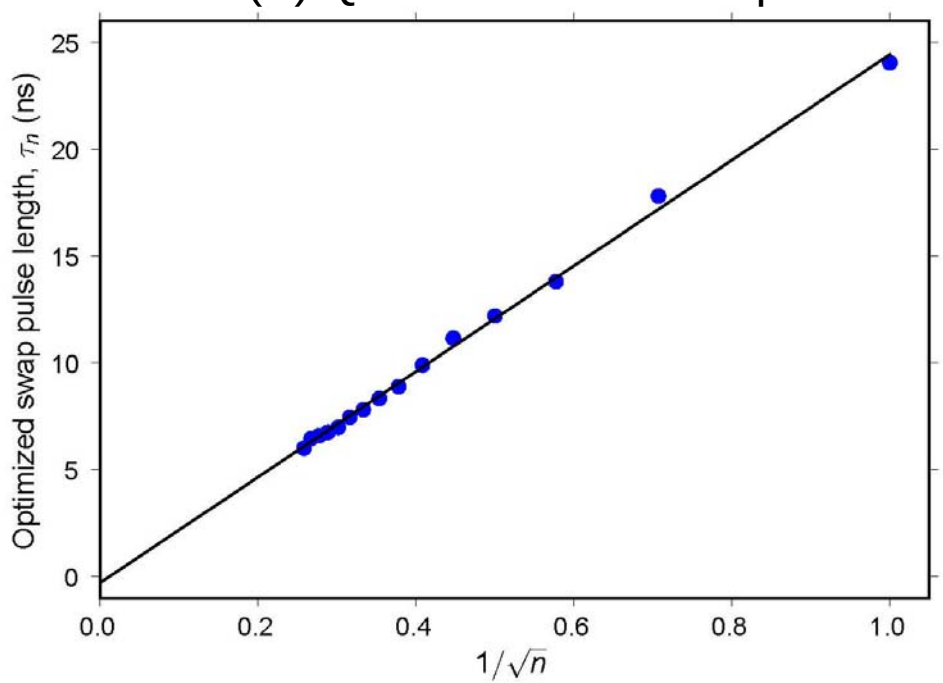
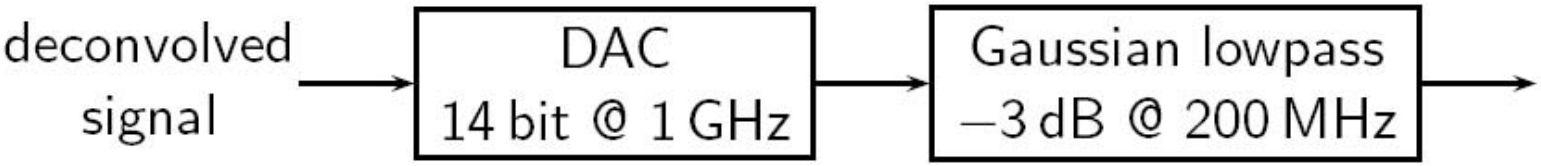
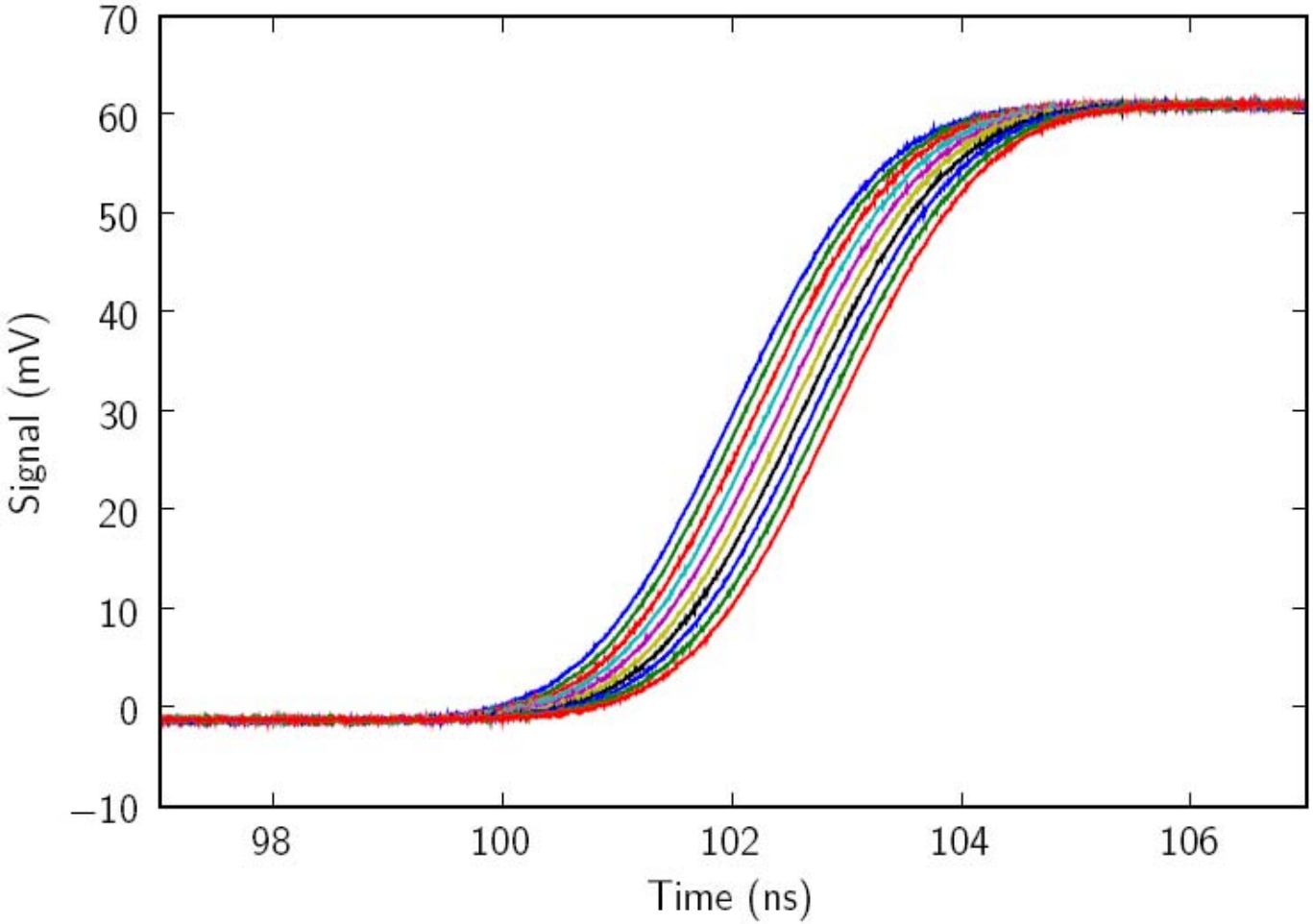
Calibration



(P) Qubit-resonator phase



(S) Qubit-resonator swap



Sub-ns timing resolution with a 1 GHz DAC