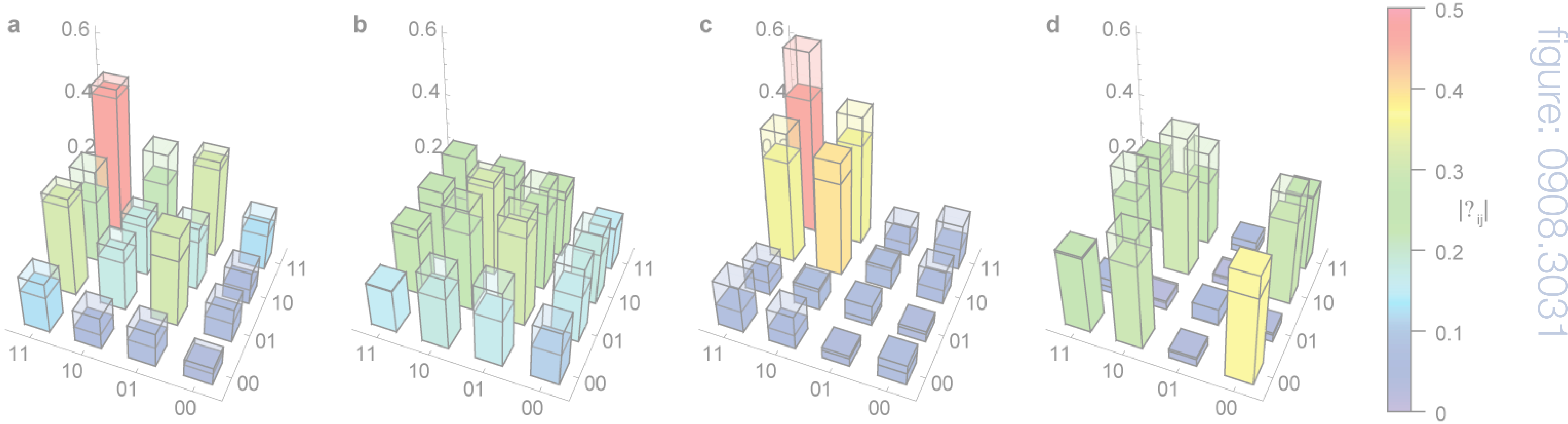

Quantum State Tomography

Steve Flammia
Perimeter Institute

CQIQC: Aug 24, 2009

Joint work with:
1) S Becker, J Eisert, D Gross, Y-K Liu &
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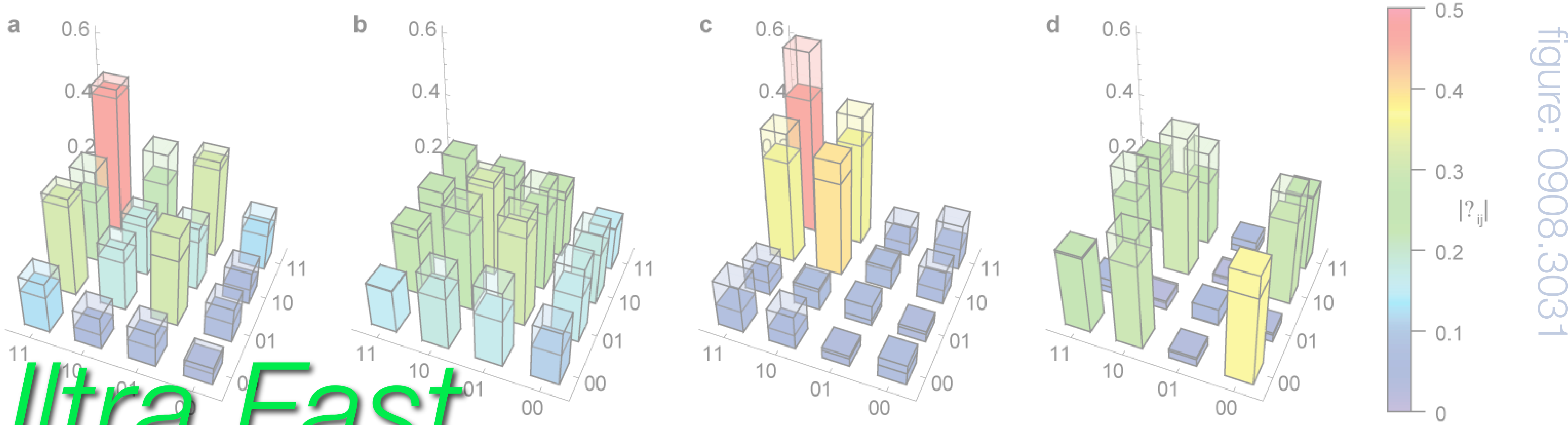

Ultra Fast Quantum State Tomography

Steve Flammia
Perimeter Institute
CQIQC: Aug 24, 2009

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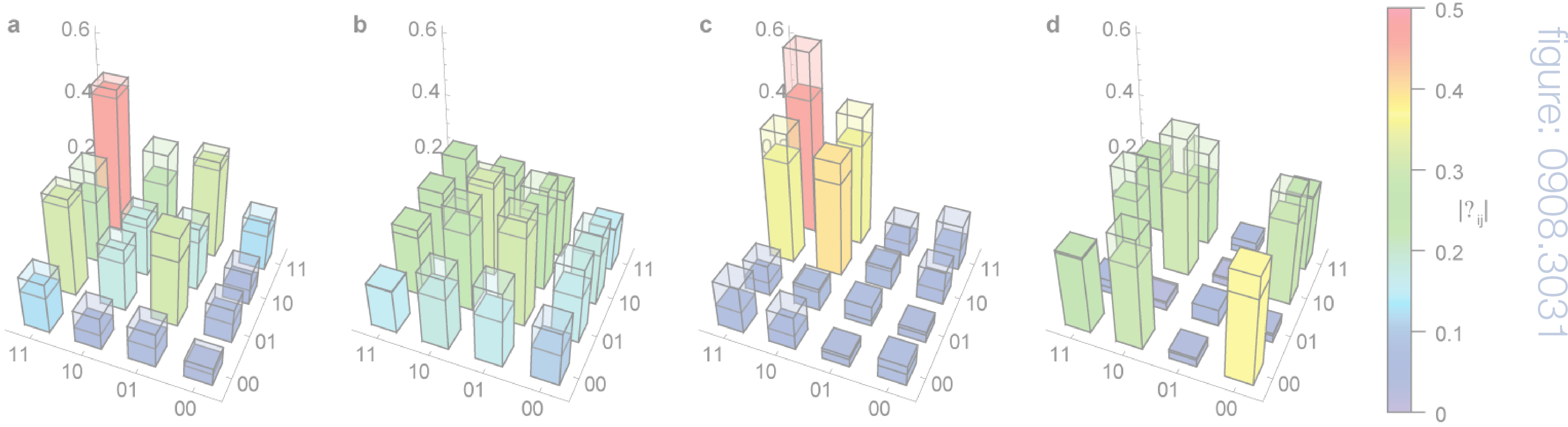

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Quantum State Tomography

Q: What is it?

- Reconstructing a classical description of a quantum state from single-copy measurements and classical post-processing.

Q: Why is it so difficult?

- the dimension $d=2^n$ is exponentially large, so one must measure an exponential number of observables....

- which determine exponentially small parameters, so you need to make exponentially many measurements...

- and once you have all the data, finding a compatible density operator takes an exponential amount of classical computing!

Why do we care about tomography?

- Short answer: we don't. What we really care about is certifying state preparations...
- ...but full tomography could still be useful to characterize noise processes (see also quantum process tomography).

# of qubits needed to do something "interesting"

n<10

n>100

Pure State Tomography

- The most interesting quantum states are pure; can we do tomography of pure states with fewer measurement settings? [1]

Yes! For states of rank r, O(r d $\log^2$(d)) measurements suffice.

- This result is quite robust and similar performance holds even if...
  - ...the measurements are corrupted by stochastic, deterministic, or adversarial noise.
  - ...the state is full rank, but has large purity.
  - ...the efficiency of the classical post-processing is O(poly(d)).
- Moreover, the scheme requires just Pauli measurements.
- The result is heralded, so no *a priori* promise of large purity or low entropy is necessary.

[1] Kaznady & James, PRA 2009

What is Compressed Sensing?

Pioneered by: Candès, Tao, Donoho, ...

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Shoot,
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~~Shoot, then compress~~

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What is Compressed Sensing?

~~Shoot, then compress~~

Take only as many samples as you need, then post-process to reconstruct the image!

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Finding the sparsest vector consistent with the sampling is NP-hard

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Use a convex relaxation!

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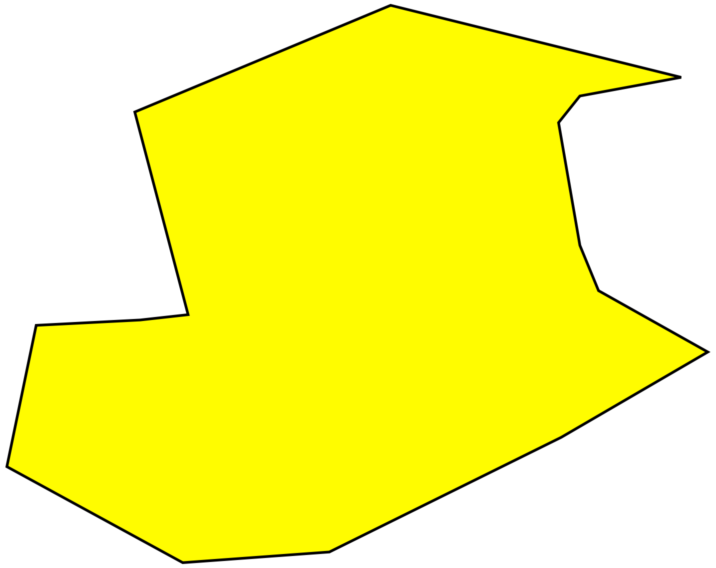

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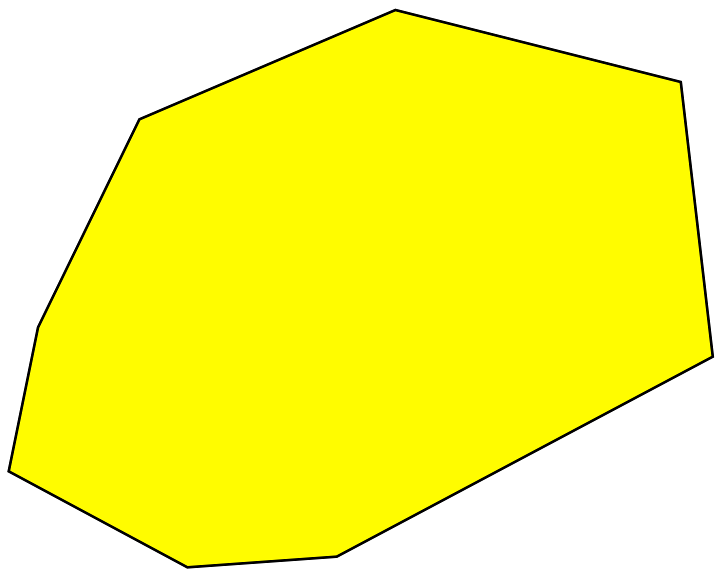

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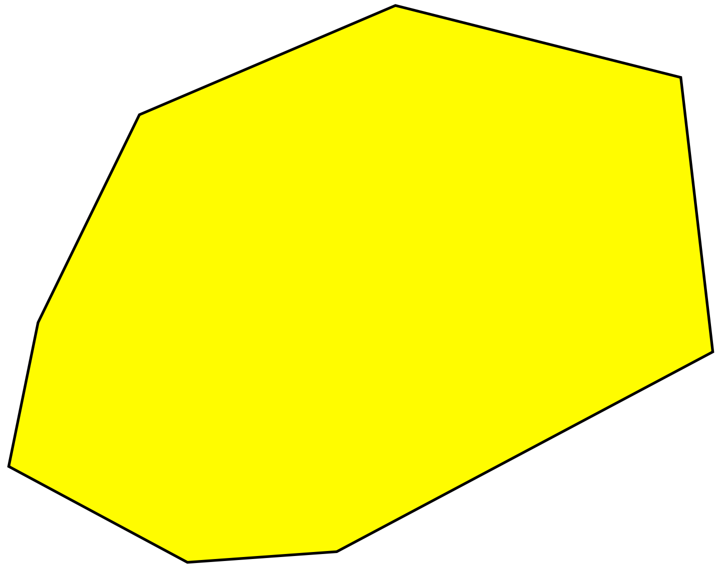

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Unlike classical info., quantum information contains a *basis*.

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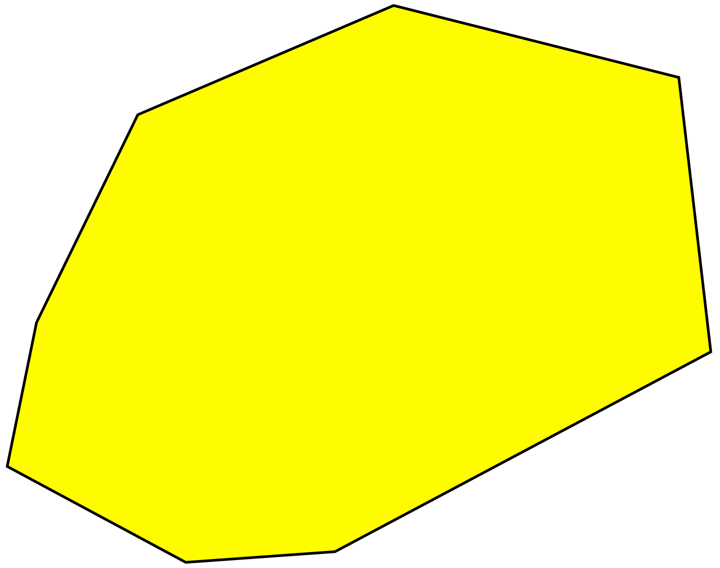


Problem:
Unlike classical info., quantum information contains a *basis*.

Solution:
Use *rank minimization* instead (and its convex relaxation, of course.)

Tomography of low-rank states

Measure observables A_k to get expectations b_k, and collect them each into vectors of length m, with $\boldsymbol{m=O(r\,d\,\log^2(d))}$.

Now we can solve the convex optimization

$$\min\ \mathrm{Tr}(X)\ :\ \|\mathcal{A}(X)-b\|_2 \leq \delta\,,\ X \geq 0$$

Theorem 1: If the A_k are randomly chosen Paulis, then this convex optimization has a unique solution as δ vanishes, with overwhelming probability.

This is essentially optimal, since $m=\Omega(rd)$ follows from simple parameter counting.

Robustness to noise

Suppose instead that we measure a different state, but still pretty close (as measured by the 2-norm), $\|\rho - \sigma\|_2 < \delta$

Solving the same optimization as before,

$$\min \ \mathrm{Tr}(X) \ : \ \|\mathcal{A}(X) - b\|_2 \leq \delta \, , \ X \geq 0$$

we obtain **Theorem 2**, a result about robustness

$$\boxed{\|\rho_{\mathrm{opt}} - \rho\|_2 = \delta \ O\left(\frac{d}{\log d}\right)}$$

As we will see from the numerics, there is probably room for improvement in this bound.

Heralding the success of the protocol

Theorem 1 holds under the promise of low rank, and theorem 2 says that it is robust under small perturbations. Can this be improved to a **heralded** scheme?

Yes!

Using the Pauli measurements, we can estimate the **purity** of the state.

If the purity is large enough, then theorems 1 and 2 apply. Otherwise, we can abort, or do e.g. maximum likelihood estimation.

Testing the algorithm

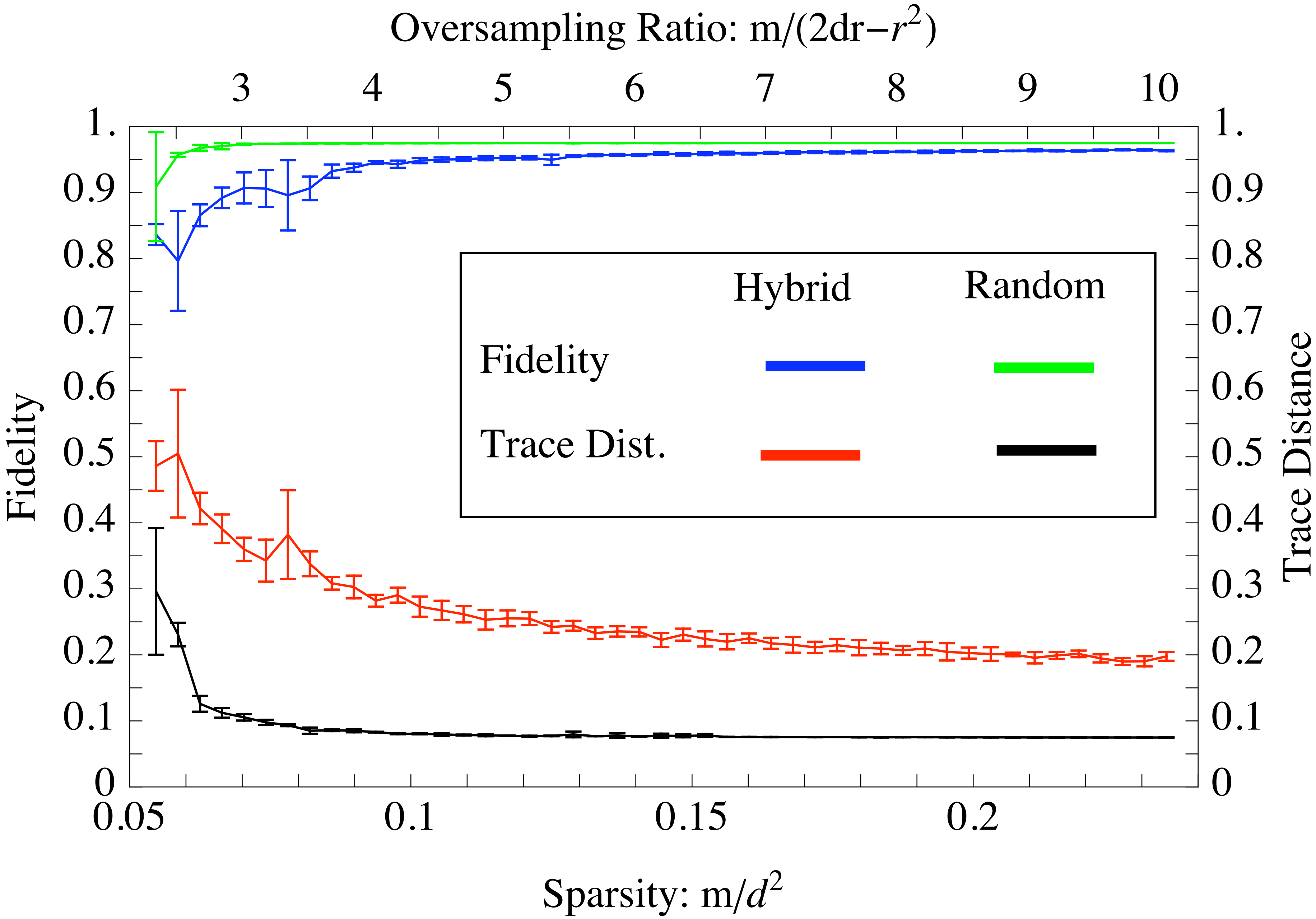
Choose a random pure state from a $d \times r$ system and trace out the r-dimensional ancilla.

Add statistical noise and additional depolarizing noise.

Solve the convex program to get the estimate of the state. (ask me about the details!)

Matlab interlude

Testing the algorithm



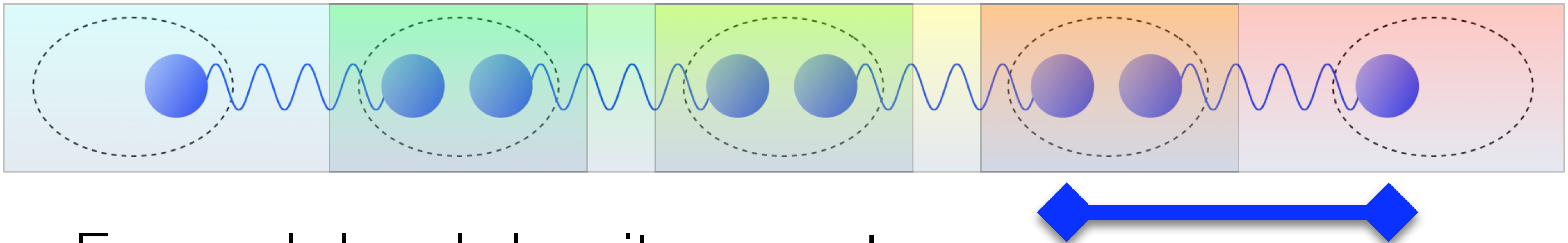
Matrix Product State Tomography

- Many interesting quantum states can be well approximated by a matrix product state with a small bond dimension. Can we use this to our advantage?

Yes, we can do certified tomography of certain n-qubit quantum states in time poly(n)

- The idea is to produce efficient classical descriptions of large quantum states by doing tomography of the MPS description directly, rather than the exponentially large description as a vector in Hilbert space.

State certification for non-degenerate MPS



For each local density operator,
we do complete local tomography.

ρ_j

$$\|\rho_j - \sigma_j\|_1 \leq \epsilon_0$$

$$(\mathrm{Prob}:\ 1-\delta_0)$$

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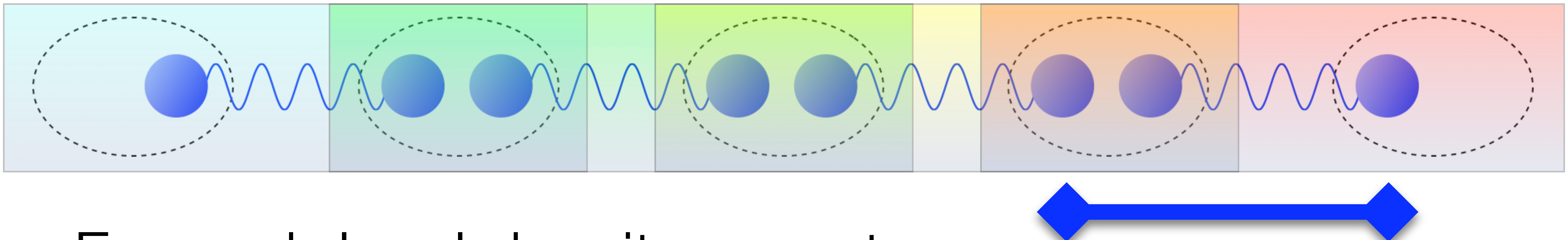

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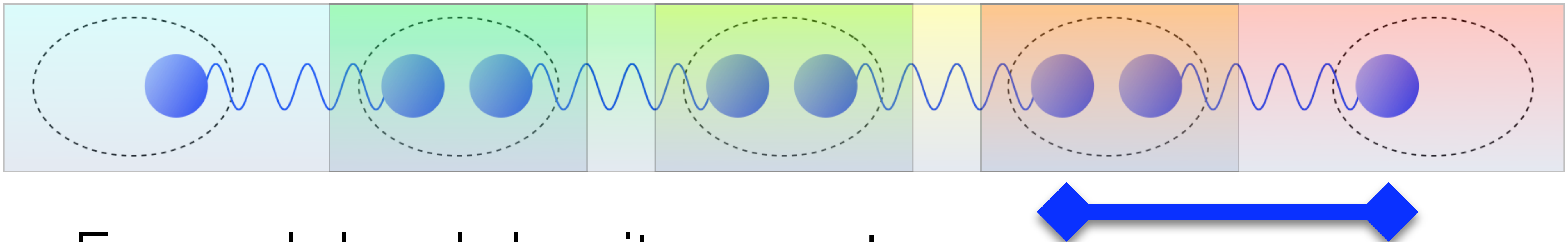

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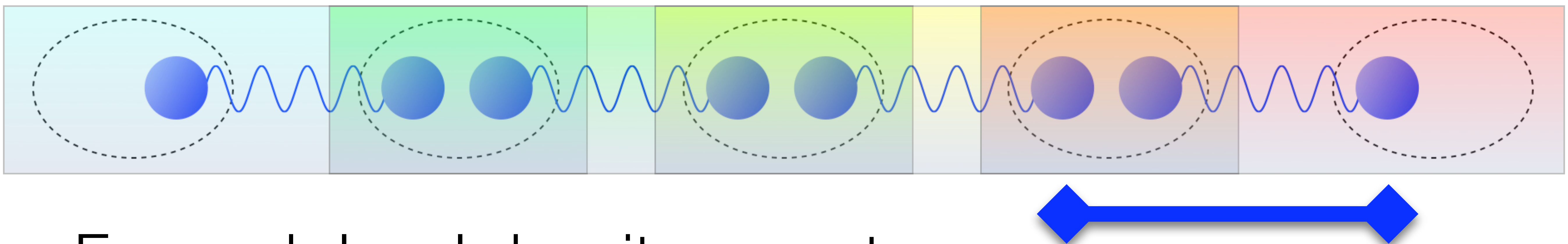

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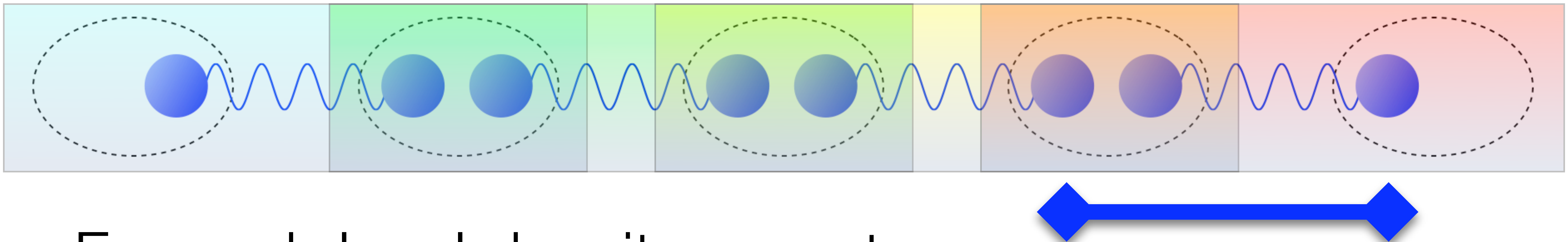

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...but how can we find it?

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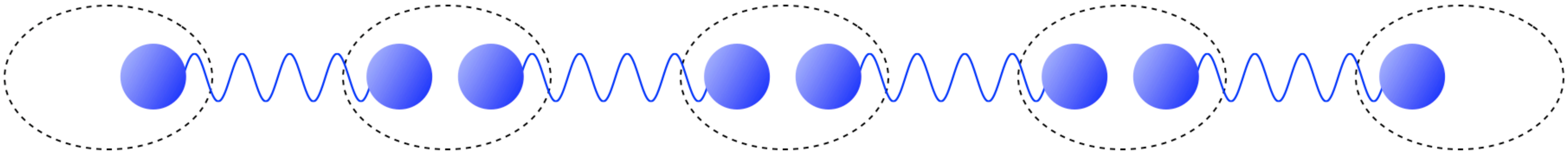

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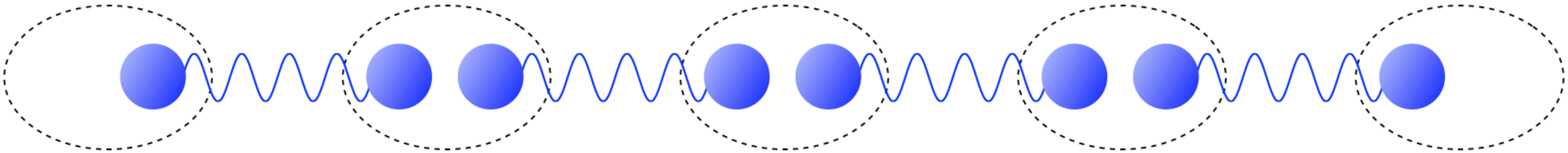

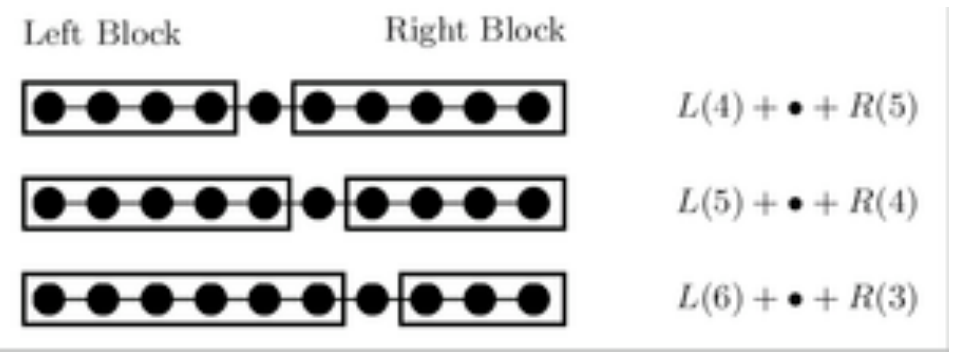

Use DMRG!

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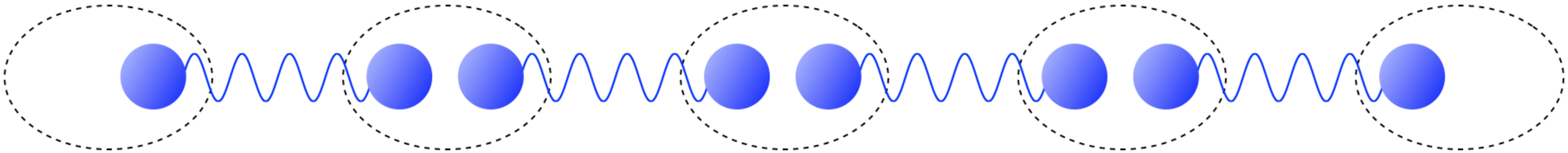

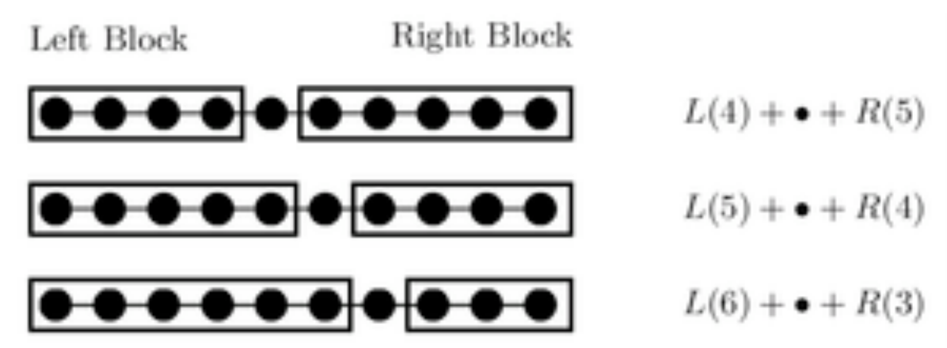

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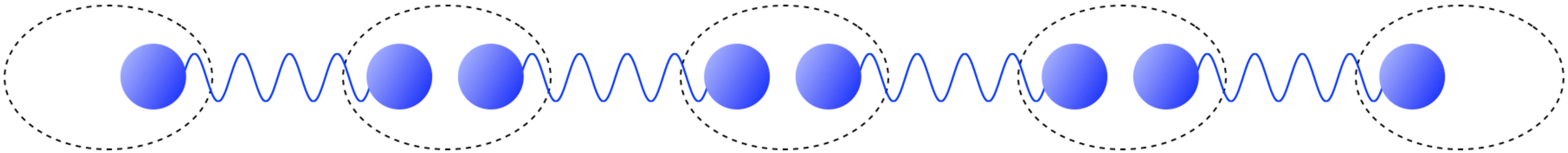

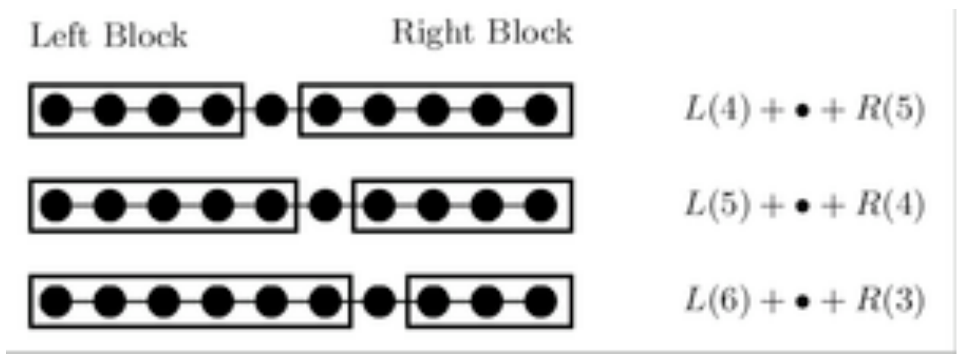

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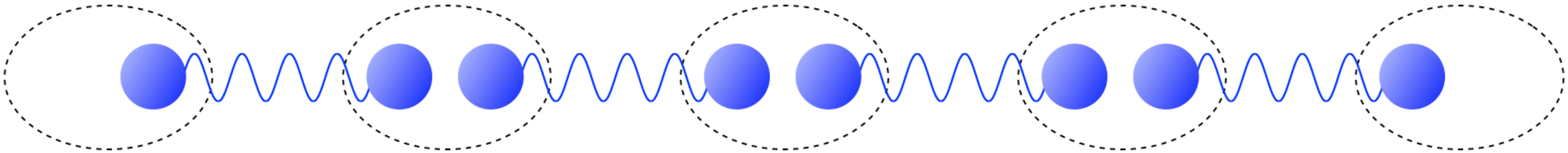

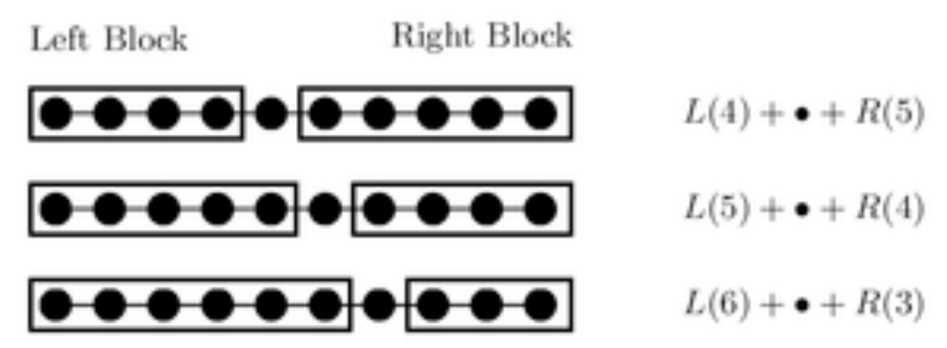

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STF, T Osborne 2009: Parent Hamiltonians can be solved in polynomial time

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candidate ground state in MPS form

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$$G = \sum_j Q_j$$

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We can bound the fidelity now by first introducing a new error parameter

$$\xi = \max_j \mathrm{Tr}(Q_j \sigma_j)$$

Non-degenerate MPS

$$\Delta(1 - \langle\phi|\rho|\phi\rangle) \leq \sum_j \lambda_j \langle g_j|\rho|g_j\rangle = \mathrm{Tr}(G\rho)$$

$$\mathrm{Tr}(G\rho) \leq \sum_j [\mathrm{Tr}(Q_j\sigma_j) + \epsilon] \leq \sum_j [\xi + \epsilon] \leq n(\xi + \epsilon)$$

$$\boxed{1 - \langle\phi|\rho|\phi\rangle \leq n(\epsilon + \xi)/\Delta}$$

Thus, we have a fidelity bound with an extensive error in terms of the truncation error and the statistical error of the local estimates

Conclusions

- Tomography of arbitrary states with high purity can be done with roughly O(*d*) measurements, and the scheme is fully certified and heralded, is robust to errors, and works very well in practice.

- States that are well-approximated by a matrix product state can be learned exponentially faster.

- Lots of open questions. Channels, better error bounds, better code, other types of measurements...