Quantum computers: A new state of matter?

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Phys. Rev. Lett. 103, 020506 (2009)
and
arXiv:0807.4797

Quantum computing with a *cluster state*

- Quantum computing can proceed through *measurements* rather than unitary evolution
- Measurements are strong and incoherent: easier?

Uses a *cluster state*:

- a universal circuit board
- a 2-d lattice of spins in a specific *entangled* state

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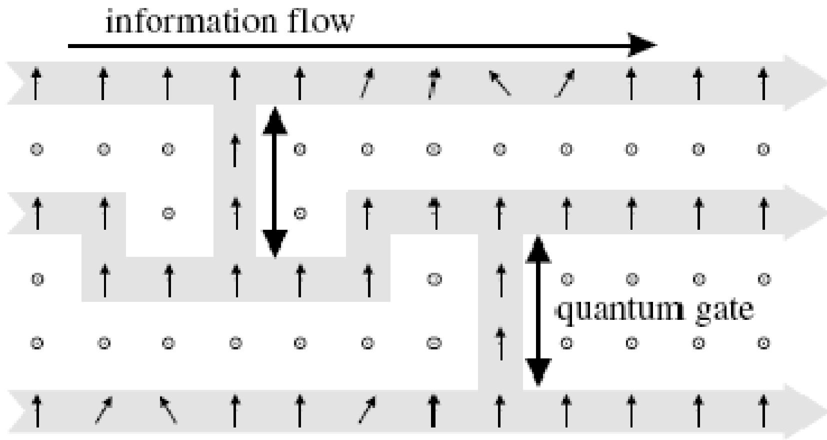
A One-Way Quantum Computer

Robert Raussendorf and Hans J. Briegel
Theoretische Physik, Ludwig-Maximilians-Universität München, Germany
(Received 25 October 2000)

We present a scheme of quantum computation that consists entirely of one-qubit measurements on a particular class of entangled states, the cluster states. The measurements are used to imprint a quantum logic circuit on the state, thereby destroying its entanglement at the same time. Cluster states are thus one-way quantum computers and the measurements form the program.

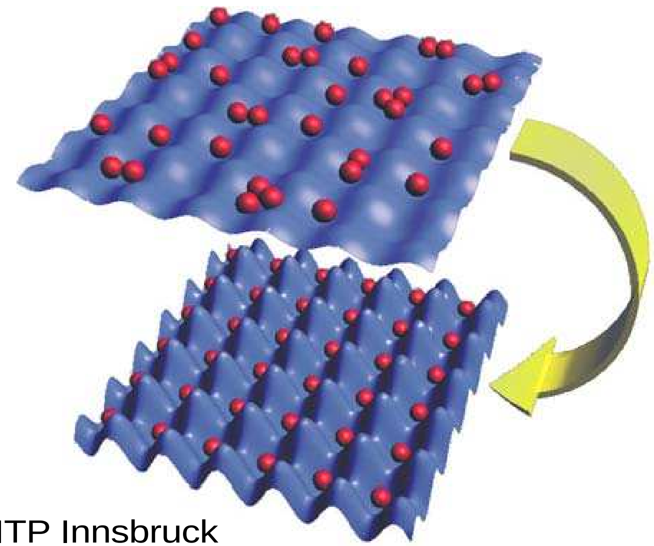


Q: What resource states allow for measurement-based QC?

Q: What physical systems are ‘natural’ for creating such states?

Q: How robust are these states to the relevant errors in these systems?

Resource states & Hamiltonians



ITP Innsbruck
www.uibk.ac.at/th-physik/qo/research/

$$H_{\text{cluster}} = -\sum_{\text{sites}} \begin{array}{ccc} & Z & \\ Z & X & Z \\ & Z & \end{array}$$

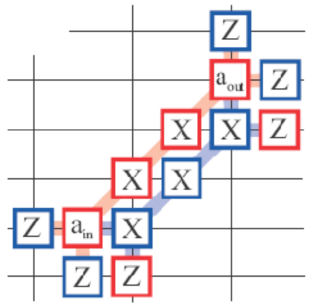
- Resource states can be:
  - constructed with unitary gates
  - the ground state of a coupled quantum many-body system
- Approach: cluster state is ground state of a model Hamiltonian H_{cluster}
- Error model: What if our Hamiltonian was only “close” to the desired cluster Hamiltonian?

$$H = H_{\text{cluster}} + \sum \text{local terms}$$

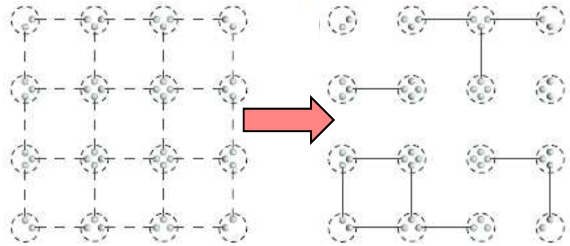
- Is the system fragile or robust to local perturbations, or finite temperature?

Summary of results

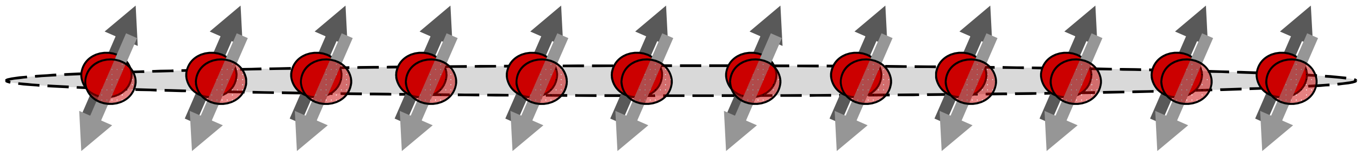
- Cluster Hamiltonian and a local X field:
  - there exists a cluster phase in 1-D and 2-D at T=0
  - quantum gates, as correlation functions, can serve as order parameters to identify universal phases



- Cluster Hamiltonian and a local Z field:
  - there is no phase transition
  - but local filtering in 3-D at finite temperature, there exists a transition between a universal region, and a region which is classically simulatable



- Spin-1 Hamiltonians in the Haldane phase:
  - local filtering can renormalize the state, removing defects



Identifying phases, part 1

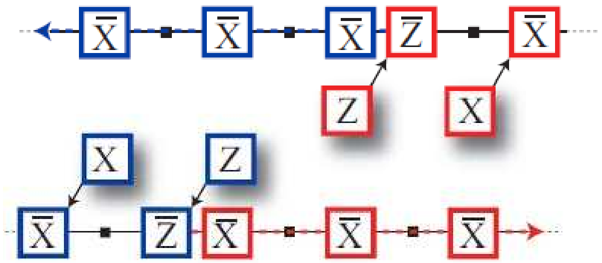
$$H = H_{\mathrm{cluster}} + B \sum\nolimits_{\mathrm{sites}} X$$

Duality transformations to known models

- Cluster Hamiltonian with local X field is unitarily related to some known models

- 1-D:

$$H = H_{\text{cluster}} + B \sum X$$

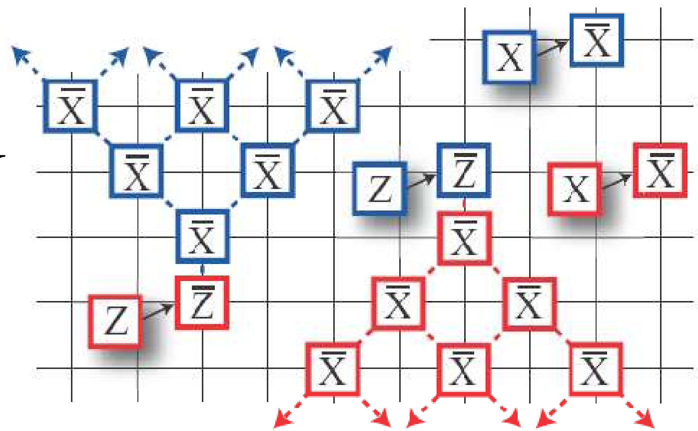


$$\bar{H} = -\sum \bar{Z}\bar{Z} + B \sum \bar{X}$$

Transverse field Ising model
• phase transition at B=1
• correlation functions B<1

- 2-D:

$$H = H_{\text{cluster}} + B \sum X$$

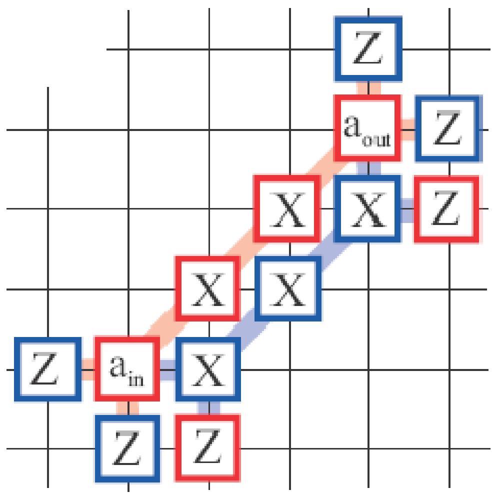


$$\tilde{H} = -\sum \tilde{Z}\tilde{Z} - B \sum \begin{matrix}\tilde{X} \\ \tilde{X}\end{matrix}$$

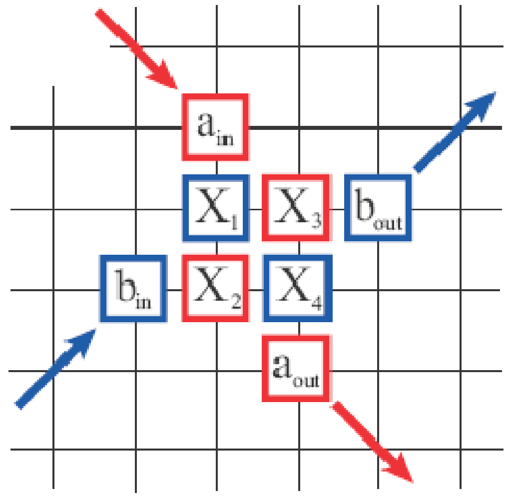
Anisotropic quantum compass model
• phase transition at B=1
• Ising-like + more correlation functions B<1

Correlation functions and quantum gates

- Anisotropic quantum compass model has long-ranged Ising order parameters in the ordered B<1 phase



String-like order parameter
Quantifies fidelity as of
“Quantum wire”
(Teleportation + 1 qubit gates)



Interacting strings order parameter
Quantifies CNOT gate fidelity

Conclusions 1.0

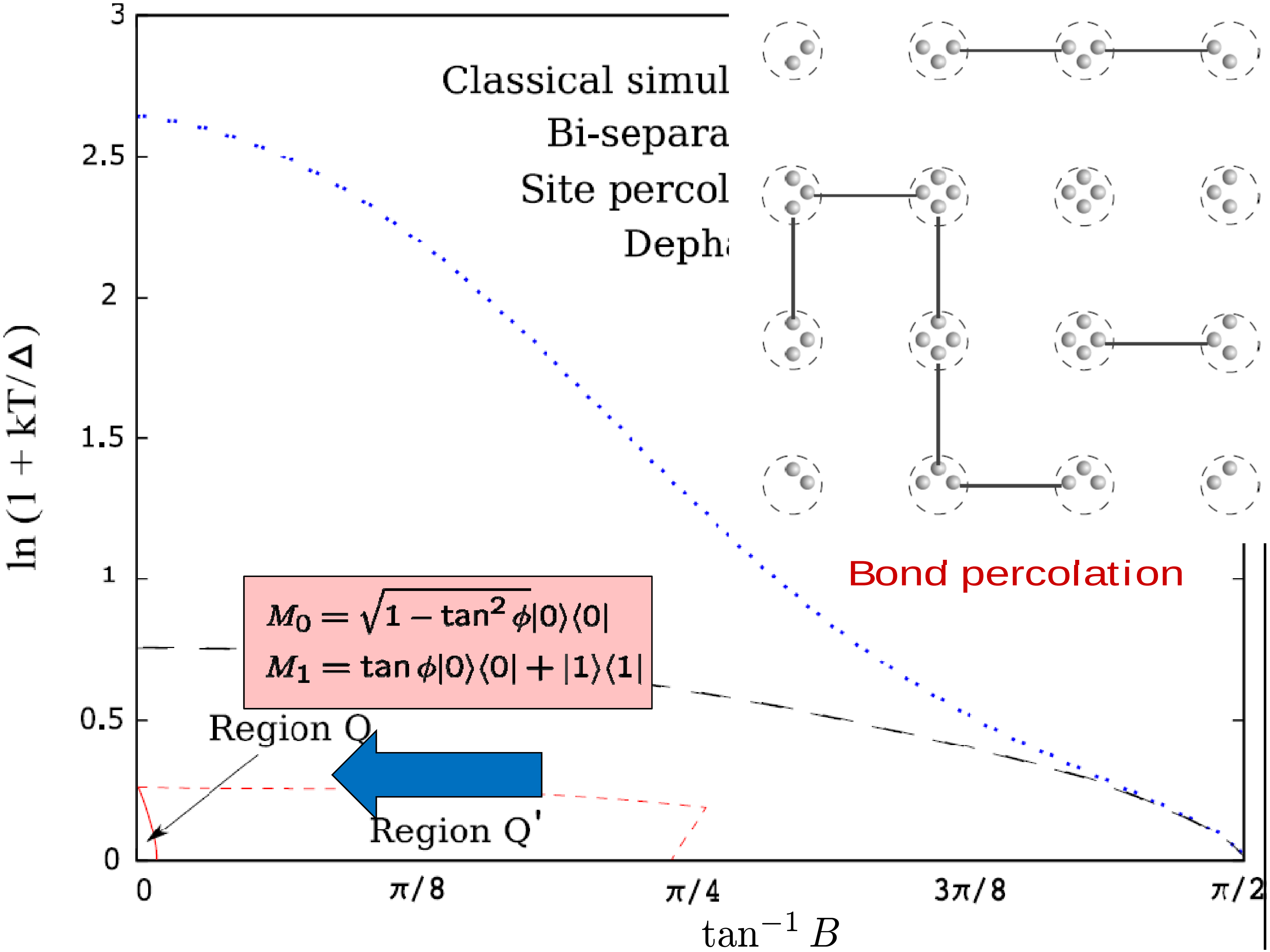
- Quantum gates – correlation functions – order parameters to identify universal phases for MBQC
- MBQC is a new type of long range "string" order

Identifying phases, part 2

$$H = H_{\mathrm{cluster}} + B \sum_{\mathrm{sites}} Z$$

- No phase transitions in T, no quantum phase transitions in B
- Investigate ground and thermal states for MBQC

Example: Cubic lattice (3-D)



$$M_0 = \sqrt{1-\tan^2\phi}\,|0\rangle\langle 0|$$

$$M_1 = \tan\phi\,|0\rangle\langle 0| + |1\rangle\langle 1|$$

Conclusions 2.0

- Quantum gates – correlation functions – order parameters to identify universal phases for MBQC
- MBQC is a new type of long range "string" order

- No phase transitions – is MBQC order destroyed?
- Local filtering reveals paths for "strings" to retain order

Identifying phases, part 3

$$H = \sum_i \cos\theta(\vec{S}_i \cdot \vec{S}_{i+1}) + \sin\theta(\vec{S}_i \cdot \vec{S}_{i+1})^2$$

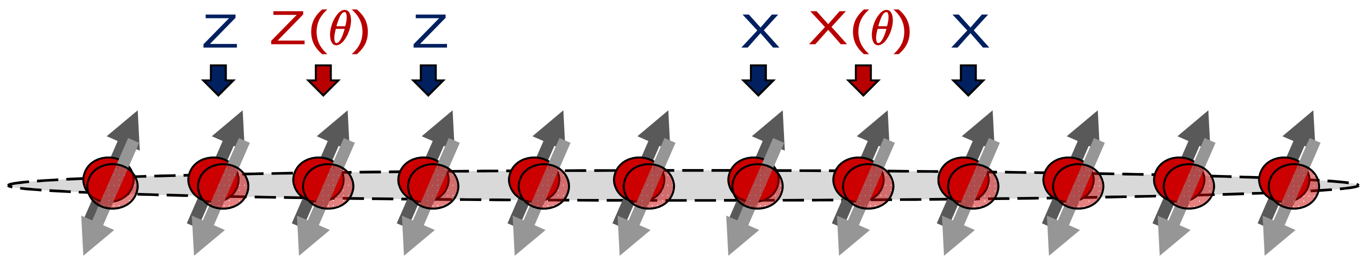
in collaboration with:

Gavin Brennen (Macquarie)

Akimasa Miyake (Perimeter)

Joseph Renes (Darmstadt)

Spin-1 chains in Haldane gap phase



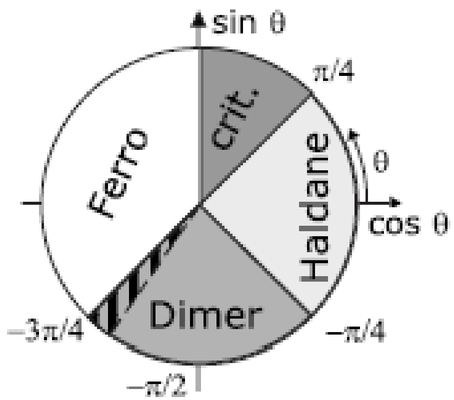
$$H = \sum_i \cos\theta(\vec{S}_i \cdot \vec{S}_{i+1}) + \sin\theta(\vec{S}_i \cdot \vec{S}_{i+1})^2$$

- AKLT state: "quantum wire" for measurement-based QC

  $$\tan\theta_{\mathrm{AKLT}} = 1/3$$

Brennen and Miyake
PRL 101, 010502 (2008)

- Any state in Haldane phase: "quantum wire" but 1-qubit gates have errors



Romero-Isart, Eckert, Sanpera
PRA 75, 050303(R) (2007)

"Buffering" measurements remove errors probabilistically

Renormalization! AKLT is fixed point

Removes short-ranged "errors", leaves long-ranged order

Conclusions 3.0

- Quantum gates – correlation functions – order parameters to identify universal phases for MBQC
- MBQC is a new type of long range “string” order

- No phase transitions – is MBQC order destroyed?
- Local filtering reveals paths for “strings” to retain order

- Local filtering can renormalize the system
- Short-ranged “errors” removed (probabilistically?)
- Long-ranged MBQC order retained