How Much Quantum Noise is Detrimental to Entanglement

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I examine the effect of quantum noise on entanglement. The source of noise could be either an attenuator or even an amplifier which one would presumably use in quantum communication protocols. I present quantitative results on the survival of entanglement as a result of various types of quantum noise. I consider entanglement for both continuous variables [1] and qubits [2].

1. **G. S. Agarwal and S. Chaturvedi, arXiv:0906.2743**
2. **Sumanta Das and G. S. Agarwal , arXiv:0901.2114**

Phase Insensitive Amplification

$$\begin{aligned} \varepsilon_{out} &= G\varepsilon_{in} \\ a_{out} &= Ga_{in} \times \end{aligned}$$

Commutation relation violation

$$\begin{aligned} a_{out} &= Ga_{in} + c^\dagger \\ &[a_{out}, a^\dagger_{out}] = 1 \Rightarrow \\ &[c, c^\dagger] = (|G|^2 - 1) \end{aligned}$$

Individual correlations ? , Statistics of c ?

Attenuator

$$a_{out} = \gamma \; a_{in} + c \quad , \gamma < 1$$

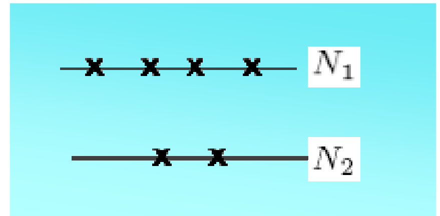
No mixing of annihilation, creation operators

$$\langle a^{\dagger}_{out} a_{out} \rangle = \gamma^2 \; \langle a^{\dagger}_{in} a_{in} \rangle$$

No addition of noise photons
(perfect attenuator)

Microscopic Model $N_1 > N_2$

Atoms relax much faster than field



The Master equation in the interaction picture

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\kappa N_1 (a a^\dagger \rho - 2 a^\dagger \rho a + \rho a a^\dagger) \\
&- \kappa N_2 (a^\dagger a \rho - 2 a \rho a^\dagger + \rho a^\dagger a)
\end{aligned}
$$

Time dependent operators

$$
\begin{aligned}
a(t) &= G a(0) + c^\dagger \\
G &= \exp\{k(N_1 - N_2)t\}
\end{aligned}
$$

The Noise operators

$$
\begin{aligned}
< c c^\dagger > &= (1+\eta)(|G|^2 - 1) \\
< c^\dagger c > &= \eta(|G^2| - 1) \\
\eta &= N_2/(N_1 - N_2)
\end{aligned}
$$

c : Gaussian

Nonclassical Features : Sub Poissonian Statistics and Squeezing survive if

$$|G|^2 < \frac{2N_1}{N_1 + N_2} < 2$$

P function does not necessarily become classical

Other measures of nonclassicality

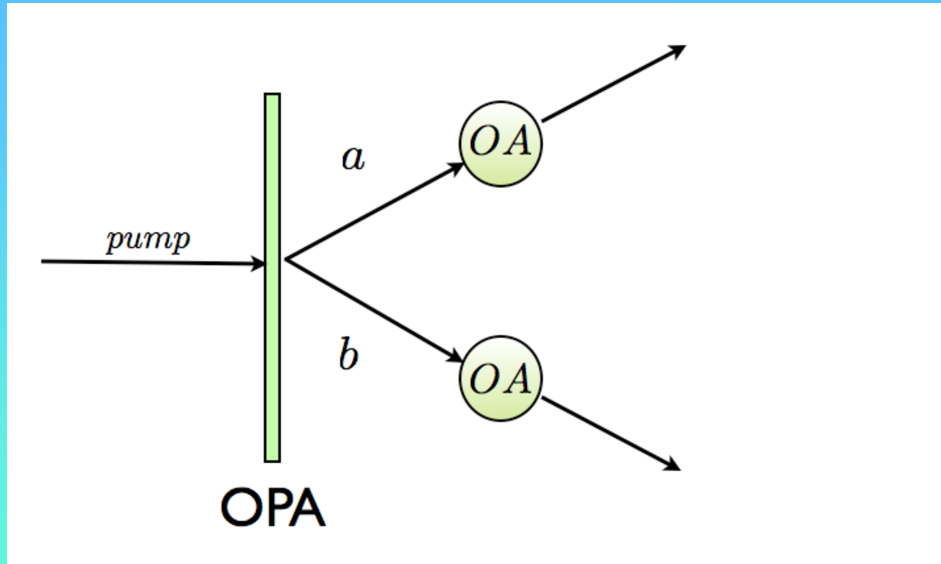
Quantum Entanglement

Characterization well understood for Gaussian States

Master Equation for Amplifier →

If input state has Gaussian Wigner function, then output is also Gaussian

Model: Phase Insensitive Amplification



The two mode squeezed vacuum

$$|\Psi\rangle = S(z)|0,0>, S(z) = \exp[za^\dagger b^\dagger - z^* ab], \; z = re^{i\theta}$$

Wigner Function

$$\frac{4}{\pi^2}\exp\{-2|\alpha\cosh(r) - \beta^*\sinh(r)e^{i\theta}|^2 - 2|\beta\cosh(r) - \alpha^*\sinh(r)e^{i\theta}|^2\}$$

Quantum Entanglement of a Gaussian Sate

The Wigner Distribution

$$W(X) = \frac{e^{-(X-<\hat{X}>)\sigma^{-1}(X-<\hat{X}>)^T/2}}{(2\pi)^n\sqrt{\mathrm{Det}(\sigma)}}$$

$$X \equiv (x_1, p_1, \cdots, x_n, p_n)$$

The elements of covariance matrix

$$\sigma_{ij} = \frac{1}{2} < (\hat{X}_i\hat{X}_j + \hat{X}_j\hat{X}_i) > - < \hat{X}_i >< \hat{X}_j >$$

The covariance matrix form

$$\sigma = \begin{pmatrix} \alpha & \gamma \\ \gamma^T & \beta \end{pmatrix}$$

Two symplectic eigenvalues

$$\tilde{\nu}_{\mp} = \sqrt{\frac{\tilde{\Delta}(\sigma) \pm \sqrt{\tilde{\Delta}(\sigma)^2 - 4\mathrm{Det}(\sigma)}}{2}}$$

$$\tilde{\Delta}(\sigma) = \mathrm{Det}(\alpha) + \mathrm{Det}(\beta) - 2\mathrm{Det}(\gamma)$$

The necessary and sufficient condition for ρ to be entangled

$$\tilde{\nu}_{<} < \frac{1}{2}$$

Logarithmic Negativity

$$E_{\mathcal{N}}(\rho) = \max[0, -\ln(2\tilde{\nu}_{<})]$$

Input State $\longrightarrow$ $E_{\mathcal{N}} = 2r$

The Symmetric Case :

$$a \longrightarrow Ga + c^\dagger, b \longrightarrow Gb + d^\dagger,$$

$$< cc^\dagger >= (1+\eta)(|G|^2 - 1), < c^\dagger c >= \eta(|G^2| - 1),$$

$$< dd^\dagger >= (1+\eta)(|G|^2 - 1), < d^\dagger d >= \eta(|G^2| - 1)$$

$$\tilde{\nu}_< = [|G|^2(e^{-2r} + (1+2\eta)) - (1+2\eta)]/2$$

Entanglement survives if

$$|G|^2 < \left(\frac{2+2\eta}{1+2\eta+e^{-2r}} \right)$$

$$\eta \rightarrow 0 \qquad |G|^2 < \frac{2}{(1+e^{-2r})} = \frac{2}{(1+e^{-E_N})}$$

The Asymmetric case :

$$a \longrightarrow Ga + c^{\dagger}, b \longrightarrow b$$

$$\begin{aligned} \tilde{\nu}_{<} &= \frac{1}{4}[(|G^2| + 1)\cosh 2r + (1 + 2\eta)(|G|^2 - 1) \\ &\quad - \sqrt{(|G|^2 - 1)^2(\cosh 2r + 1 + 2\eta)^2 + 4|G|^2 \sinh^2 2r}] \end{aligned}$$

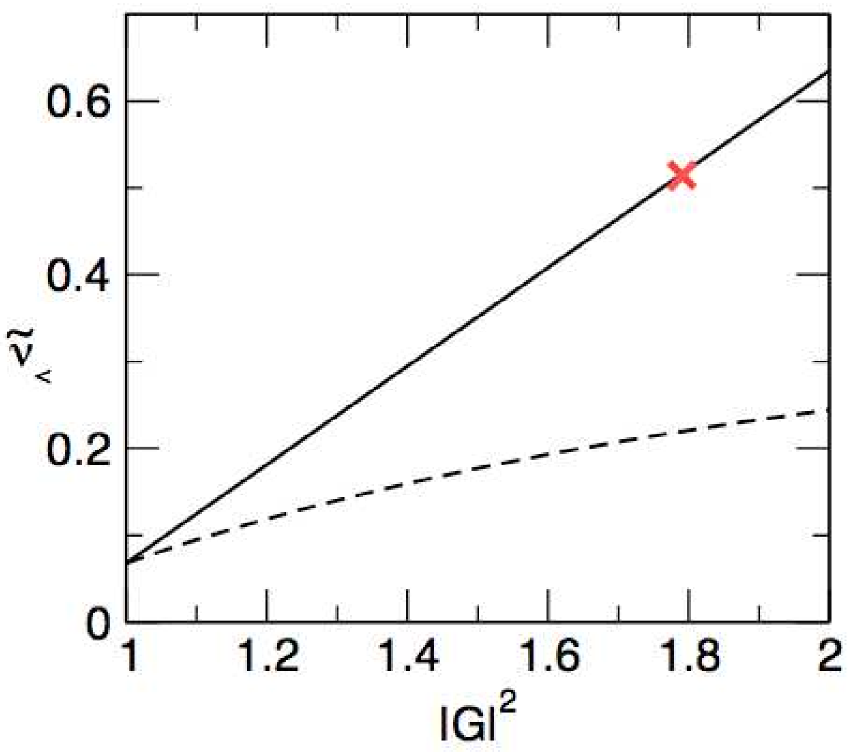
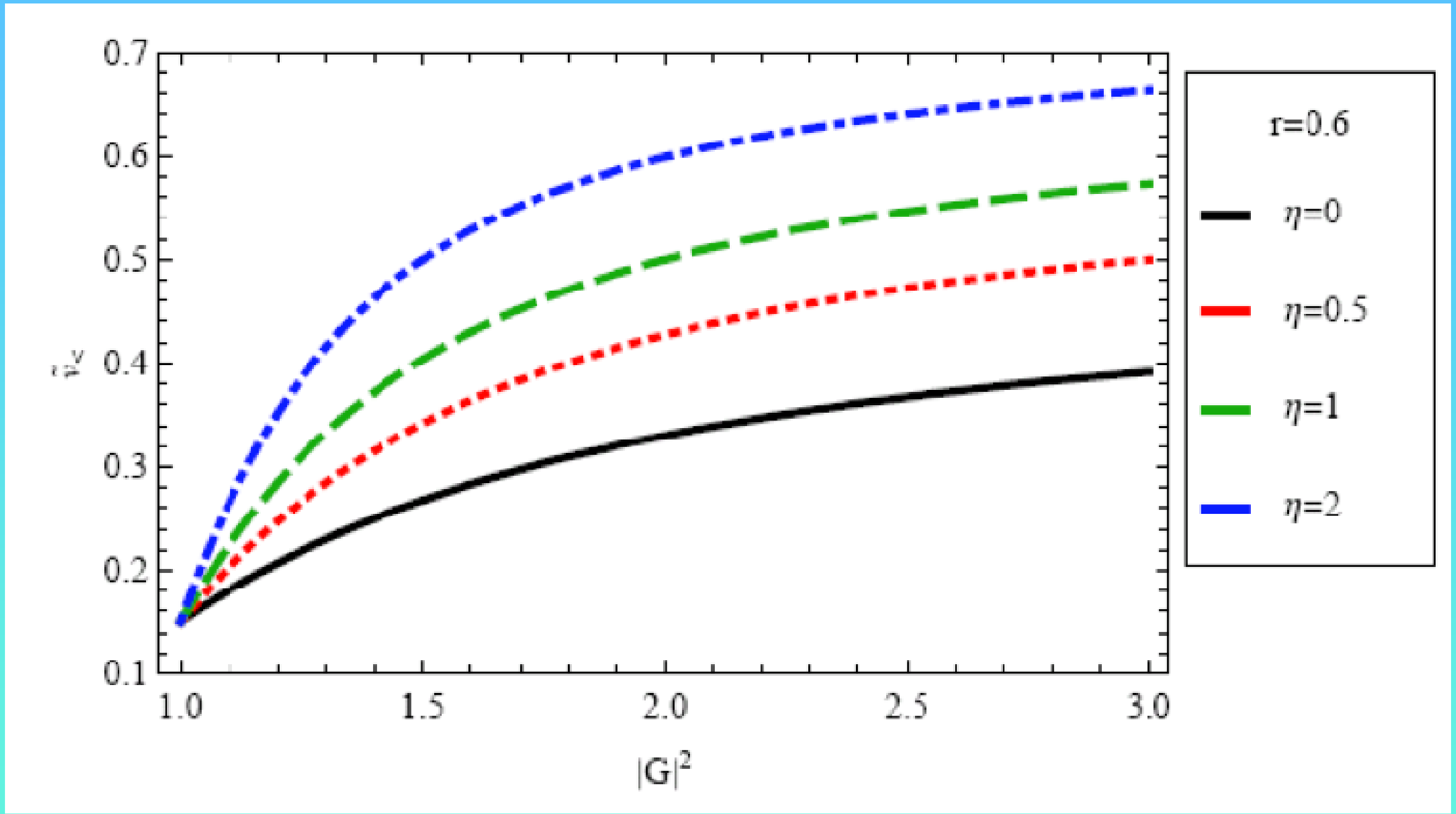


FIG. 1: Schematic diagram for the amplification of a two mode entangled Gaussian state by a phase insensitive amplifier. The optical parametric amplifier (OPA) produces a two mode squeezed vacuum state of a and b. In the symmetric case, both the optical amplifiers (OA) are present. In the asymmetric case the OA from the b arm is removed.

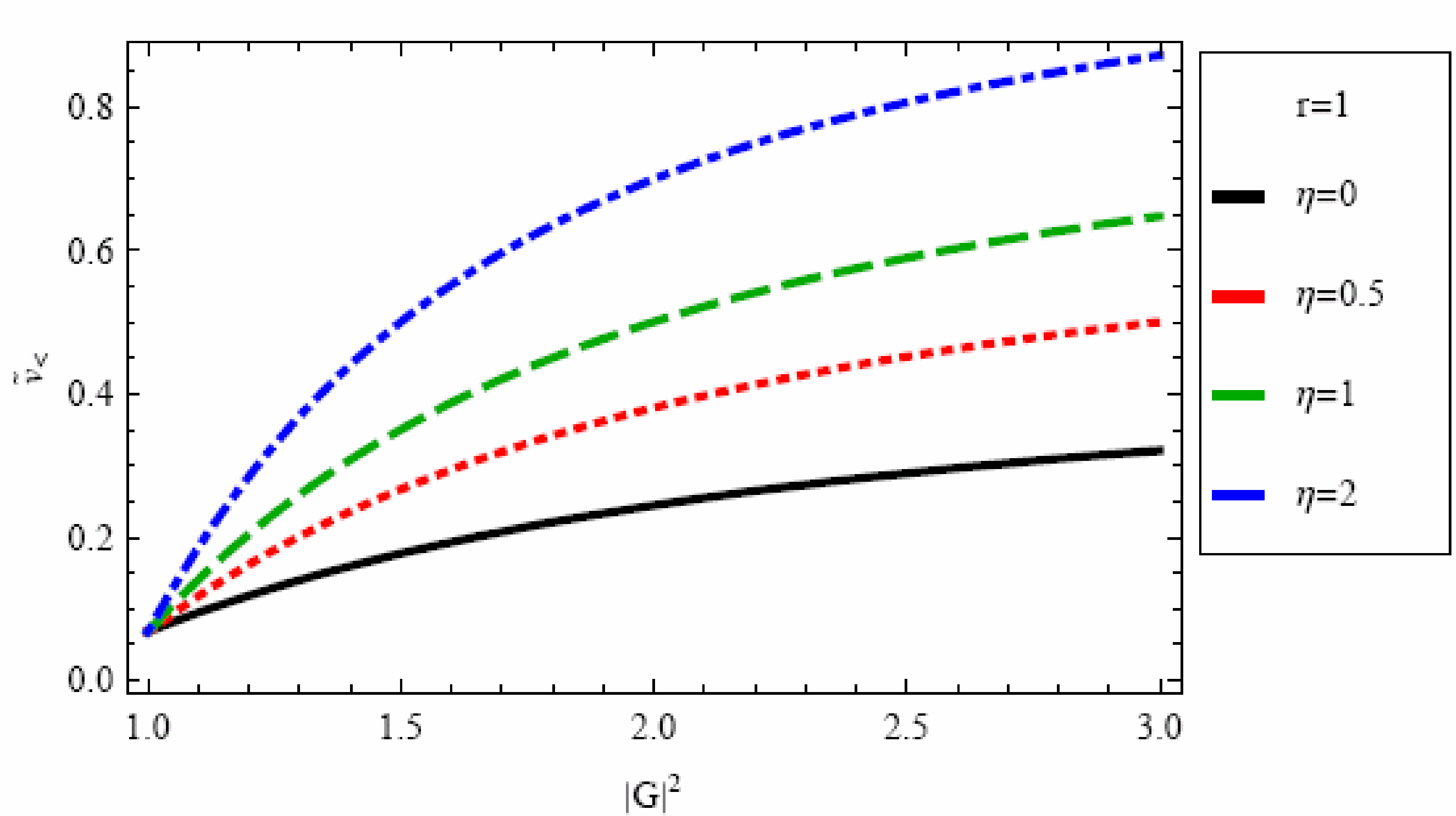
Entanglement Robust

$\eta \neq 0, N_2 \neq 0$



Critical Value of |G| η dependent

Larger Entanglement in the Input State



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Low-Noise Amplification of a Continuous-Variable Quantum State

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We present an experimental realization of a low-noise, phase-insensitive optical amplifier using a four-wave mixing interaction in hot Rb vapor. Performance near the quantum limit for a range of amplifier gains, including near unity, can be achieved. Such low-noise amplifiers are essential for so-called quantum cloning machines and are useful in quantum information protocols. We demonstrate that amplification and "cloning" of one half of a two-mode squeezed state is possible while preserving entanglement.

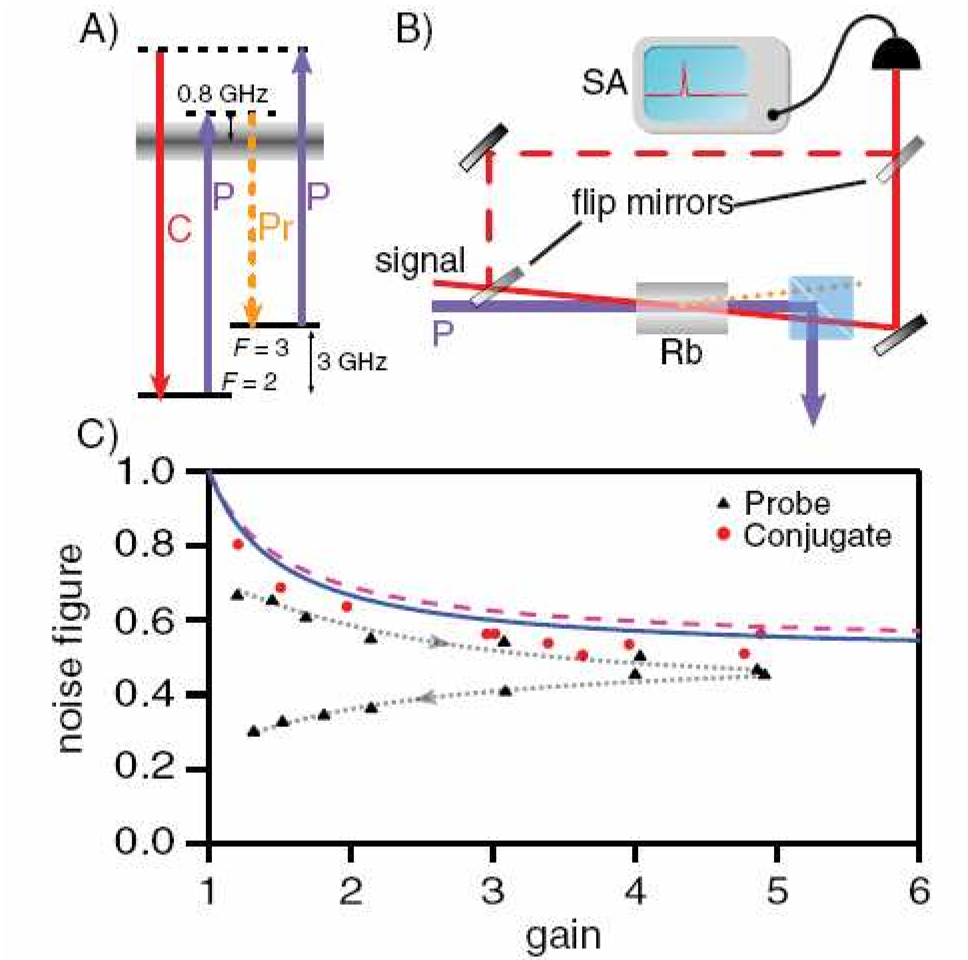

FIG. 1 (color online). (a) Energy level diagram for the 4WM process. Pr, probe; C, conjugate; P, pump. (b) The configuration used to verify the noise figure of the amplifier. SA, spectrum analyzer. The dotted (orange) line represents the unused ancillary beam generated by the amplifier. (c) The noise figure of the amplifier for various gains. The solid (blue) curve represents the ideal noise figure. The dashed (purple) curve shows the noise figure that would be measured if an ideal amplifier were monitored with a 95% efficient detector. The dotted line shows the change in gain as the pump frequency is moved from blue to red through the gain maximum for amplification of the probe beam.

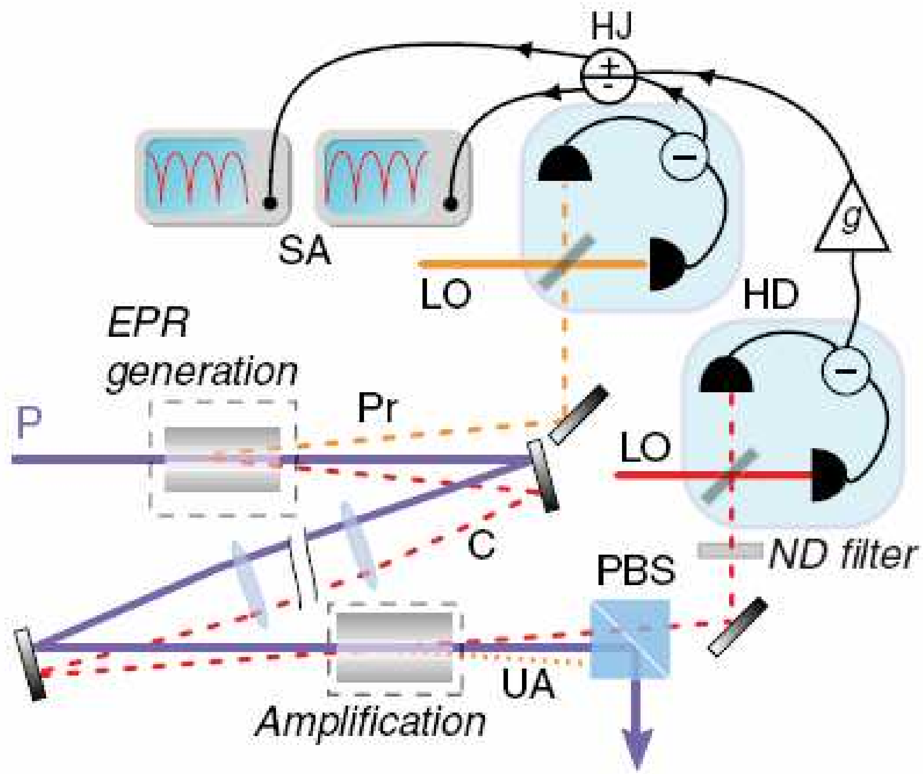

FIG. 2 (color online). Experimental setup: C, conjugate beam; Pr, probe beam; SA, spectrum analyzer; PBS, polarizing beam splitter; g, electronic attenuator; HJ, hybrid junction; LO, local oscillator; HD, homodyne detector; UA, unused ancilla. The LOs follow almost identical beam paths to those of the EPR beams (dashed lines).

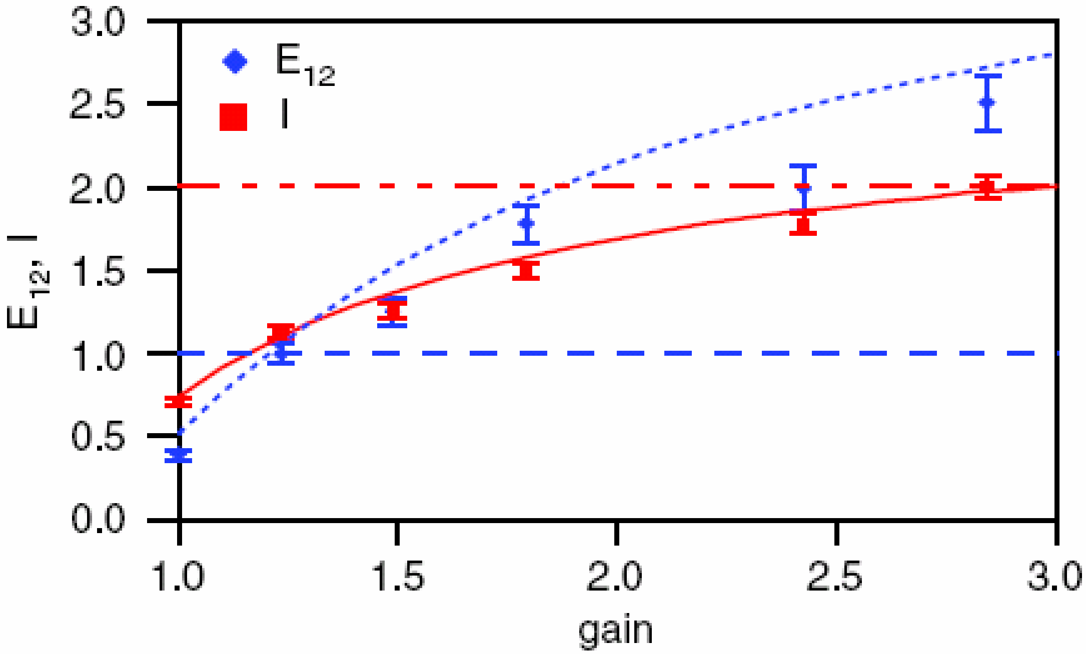

FIG. 4 (color online). E_{12} and I as a function of gain. The gain-loss product for each point was unity. The dash-dotted horizontal line and the dashed horizontal line represent the upper bounds to the inseparability and EPR criteria, respectively. The solid curve is the theoretical prediction for the inseparability, assuming an ideal amplifier beam splitter and accounting for detection efficiency and the measured input state. The dotted curve shows the theoretical prediction for the EPR parameter, calculated following the method in [25]. The error bars are propagated from a combined statistical and systematic uncertainty of 0.2 dB in the noise reduction measurements. The disagreement between E_{12} and the theory is due to experimental difficulty in ascertaining the purity of the input state.

Non-Gaussian States more Robust against Quantum Noise Addition ?

Typical Non-Gaussian States :

(I) Photon added / subtracted states

$$ab\rho_0 a^\dagger b^\dagger \quad , \quad a^\dagger b^\dagger \rho_0 ab$$

$\rho_0 \Rightarrow$ Two mode squeezed state

(II) NOON States

$$\frac{1}{2N!}((a^\dagger)^N + (b^\dagger)^N)|00\rangle\langle 00|(a^N + b^N)$$

NOON State Amplification

$$\rho^{pt} = \frac{1}{2}\{|N,0\rangle\langle N,0| + |0,N\rangle\langle 0,N| + |N,N\rangle\langle 0,0| + |0,0\rangle\langle N,N|\}$$

$$\begin{aligned} |N,N\rangle\langle 0,0| + |0,0\rangle\langle N,N| &= \frac{1}{2}(|N,N\rangle + |0,0\rangle)(\langle N,N| + \langle 0,0|) \\ &- \frac{1}{2}(|N,N\rangle - |0,0\rangle)(\langle N,N| - \langle 0,0|) \end{aligned}$$

Negative eigenvalue -1/2

Fully Inverted Amplifier

Q function; motion on trajectories (amplifiers add noise !)

$$Q_{out}(\alpha, \beta) = \frac{1}{|G|^4} Q_{in}(\frac{\alpha}{G}, \frac{\beta}{G})$$

Q function (Hushimi; Mehta+ Sudarshan; Kano)

$\langle \alpha, \beta | \rho | \alpha, \beta \rangle$: Projection on Coherent States

Nonclassical Nature Revealed by zeroes of Q function

Zeroes of Q_{out} same as of Q_{in}

$$|\alpha^N + \beta^N| = 0$$

Nonclassical Q_{out}

$$\rho_{out} = \frac{1}{2\ N!\ G^{2N}}\{(a^\dagger)^N + (b^\dagger)^N\}\rho_{th}\{a^N + b^N\}$$

ρ_{out} Never factorizes !

ρ_{out}: 2 mode photon added thermal state -- either mode has added photon

single mode version : Agarwal and Tara, Phys. Rev. A 46,485,92

Amplification of Photon subtracted 2 mode squeezed vacuum state

$$Q_{out}(\alpha,\beta) \rightarrow \underbrace{|\alpha|^2|\beta|^2}$$ → 2-mode squeezed states

↓

Nonclassicality Intact

Entanglement lost if $|G| > |\tilde{G}|$

Model: Phase sensitive amplification

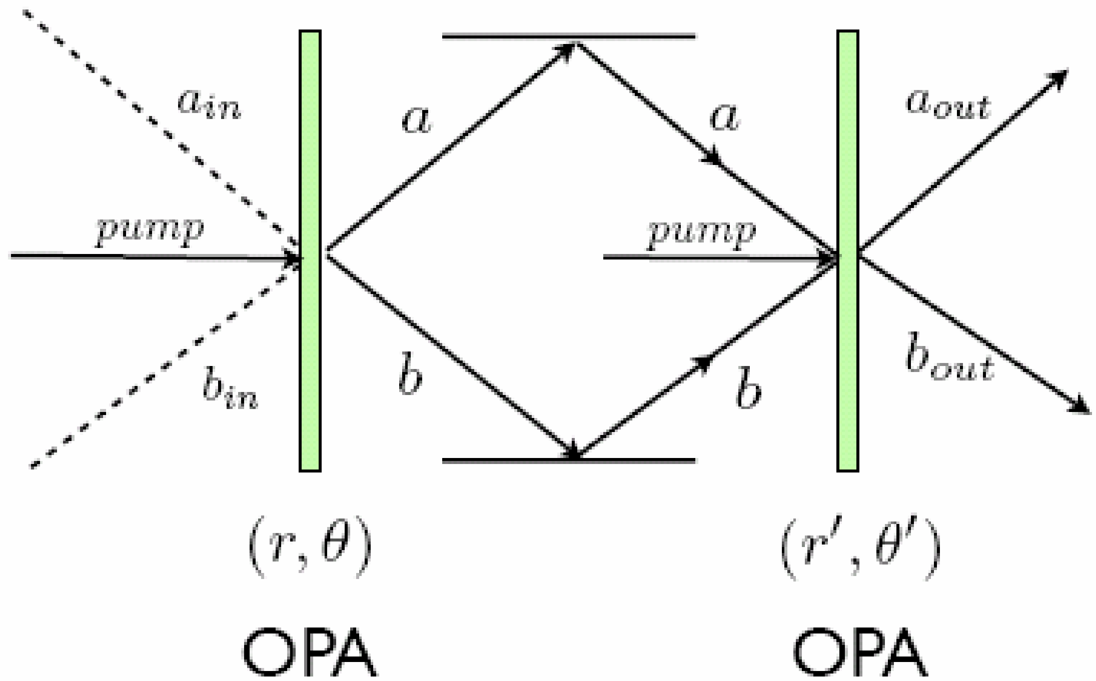

FIG. 3: Schematic diagram of a phase sensitive amplifier. The first OPA produces a squeezed vacuum state of the modes (a, b) and the second OPA acts as an amplifier.

Amplifier Chains

Phase sensitive Amplification

$$S(z_1)S(z_2) = S(z_3)e^{i(a^\dagger a + b^\dagger b + 1/2)\Phi}, \quad z_i = r_i e^{i\theta_i}$$

$$\zeta_3 = \frac{\zeta_1 + \zeta_2}{1 + \zeta_1^* \zeta_2}, \zeta_i = \tanh r_i e^{i\theta_i}$$

$$\Phi = \frac{1}{2i} \ln \left(\frac{1 + \zeta_1 \zeta_2}{1 + \zeta_1^* \zeta_2} \right)$$

$$\cosh(2r'') = \cosh(2r)\cosh(2r') + \sinh(2r)\sinh(2r')\cos\alpha$$

Quantum Entanglement is not degraded (r'' > r) if

$$0 \leq |\theta - \theta'| \leq \alpha_0$$

$$\cos\alpha_0 = \begin{array}{l} -\coth 2r \tanh r' \text{ if } r \geq r', \\ -\coth 2r' \tanh r \text{ if } r \leq r' \end{array}$$

Conclusions

- Entanglement in Gaussian states under Quantum noise addition well understood quantitatively

- Non-Gaussian states -- obtained via photon addition/ subtraction on Gaussian States completely Characterised

- NOON states -- Remain entangled quantitative measures like negativity parameter

- Results applicable to both active and passive amplifiers

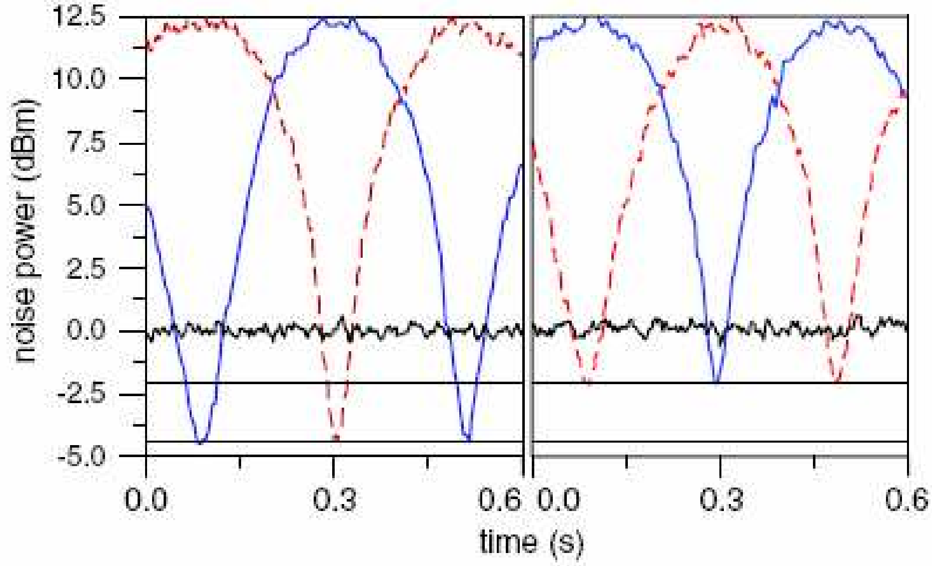

FIG. 3 (color online). Squeezing traces at 1 MHz (zero span, 10 kHz resolution bandwidth, and 300 Hz video bandwidth) for the amplitude difference and phase sum quadratures, normalized to the shot noise level, for two different amplifier gains as a function of HD phase. The HD phases are scanned synchronously in time so that they always measure the same quadratures for each beam at a given time [20]. The traces on the left show the squeezing level with the amplifier turned off and no ND filter in the conjugate beam path. The right traces show the squeezing when the amplifier gain is ≈ 1.8. The minima of the solid (blue) traces represent $\Delta\hat{X}_{-}^{2}$, while the minima of the dashed (red) traces represent $\Delta\hat{Y}_{+}^{2}$.