

More Games and the Quality of Outcomes

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Last two talks:

- Bounds on Quality of Nash (Price of Anarchy or Stability) via potential function or smoothness.
- Learning outcomes and smoothness bounds.
- Bounds via smoothness are tight even for pure Nash
- “Natural learning” (via Hedge algorithm) results almost always in pure Nash.

Today: Quality of Nash in other context: AdWords

Games considered are often congestion games

- **Routing:**

- routers choose path for packets though the Internet



- **Bandwidth Sharing:**

- routers share limited bandwidth between processes



- **Facility Location:**

- Decide where to host certain Web applications



- **Load Balancing**

- Balancing load on servers (e.g. Web servers)



- **Network Design:**

- Independent service providers building the Internet



Sponsored Search

Toronto - Google Search - Mozilla Firefox

File Edit View History Bookmarks Yahoo! Tools Help

http://www.google.com/search?hl=en&client=firefox-a&rls=org.mozilla%3Aen-US%3Aofficial&q=+Toronto&aq=f&soq=&aqi=g-c1g9

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ADS

Done

Model of Sponsored Search

Sponsored Links

Focus on ads on the side

Ordered slots, higher is better

Advertisers:

Hilton, RailEurope,
CentralBudapestHotels,
DestinationBudapest,
RacationRentals.com,
Travelzoo.com,
TravelYahhoo.com,
BudgetPlace.com

Hilton Hotel Budapest

Our best rates guaranteed online.
Book at the official Hilton site.
Hilton.com

Discover Budapest

And The Many Hungarian Cities.
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Budapest Hotel Apartments

Hotels, apartments, pension, hostel
Book & stay **Budapest** room from 25â,-!
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Budapest Vacation Listings

Thousands of **Vacation** Home Choices
Book Directly from Owners & Save!
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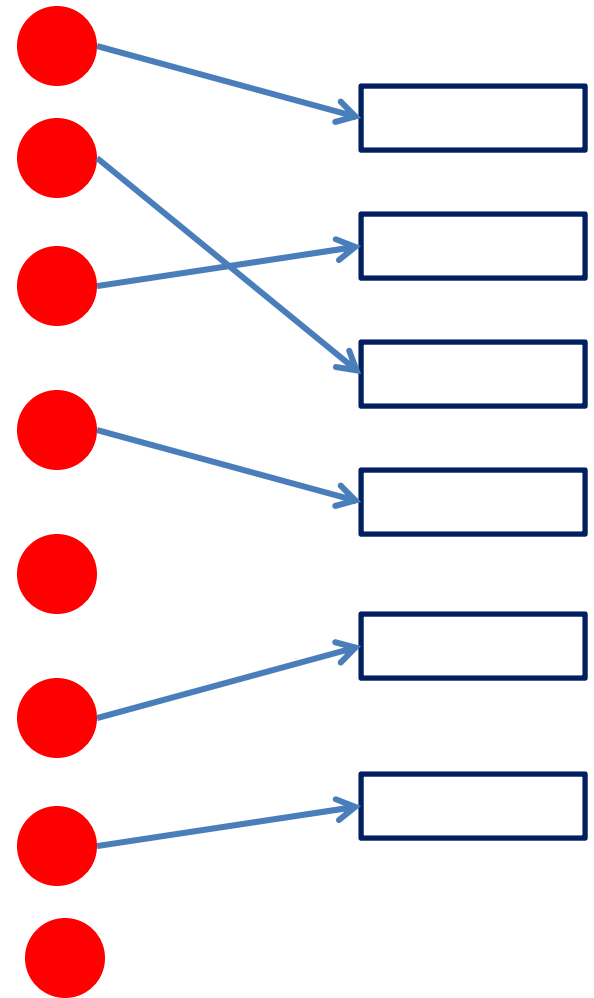
Model of Sponsored Search

Questions:

- which advertiser to assign to each slots, and
- how much to charge each

Advertisers

Slots



Value of Assignment

Value of a click for adv. i is v_i

At per-click price of p , value to advertiser is

$$(v_i - p) \cdot \text{click-rate}$$

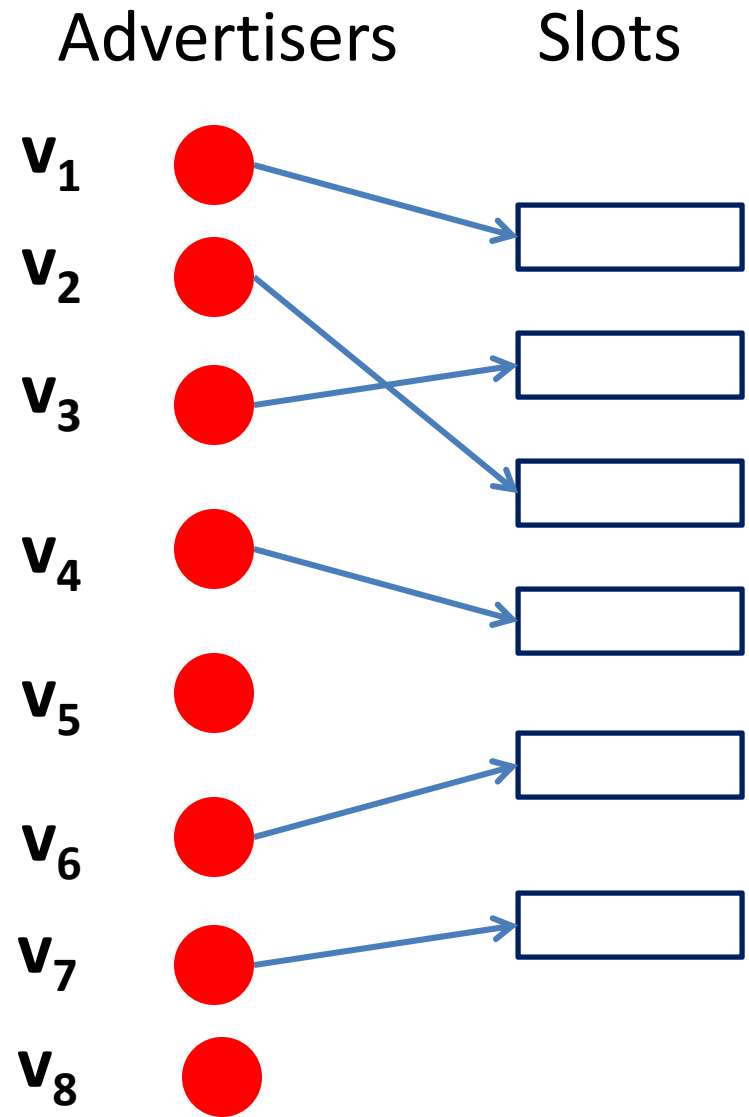
Value of search engine

$$p \cdot \text{click-rate}$$

Total of

$$v_i \cdot \text{click-rate}$$

But what is the click rate?



Click rate models

Simplest model:

click rate in slot ℓ is α_ℓ

Assume $\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4 > \dots$

Value of advertiser i in slot ℓ

$$\mathbf{v}_i \cdot \alpha_\ell$$

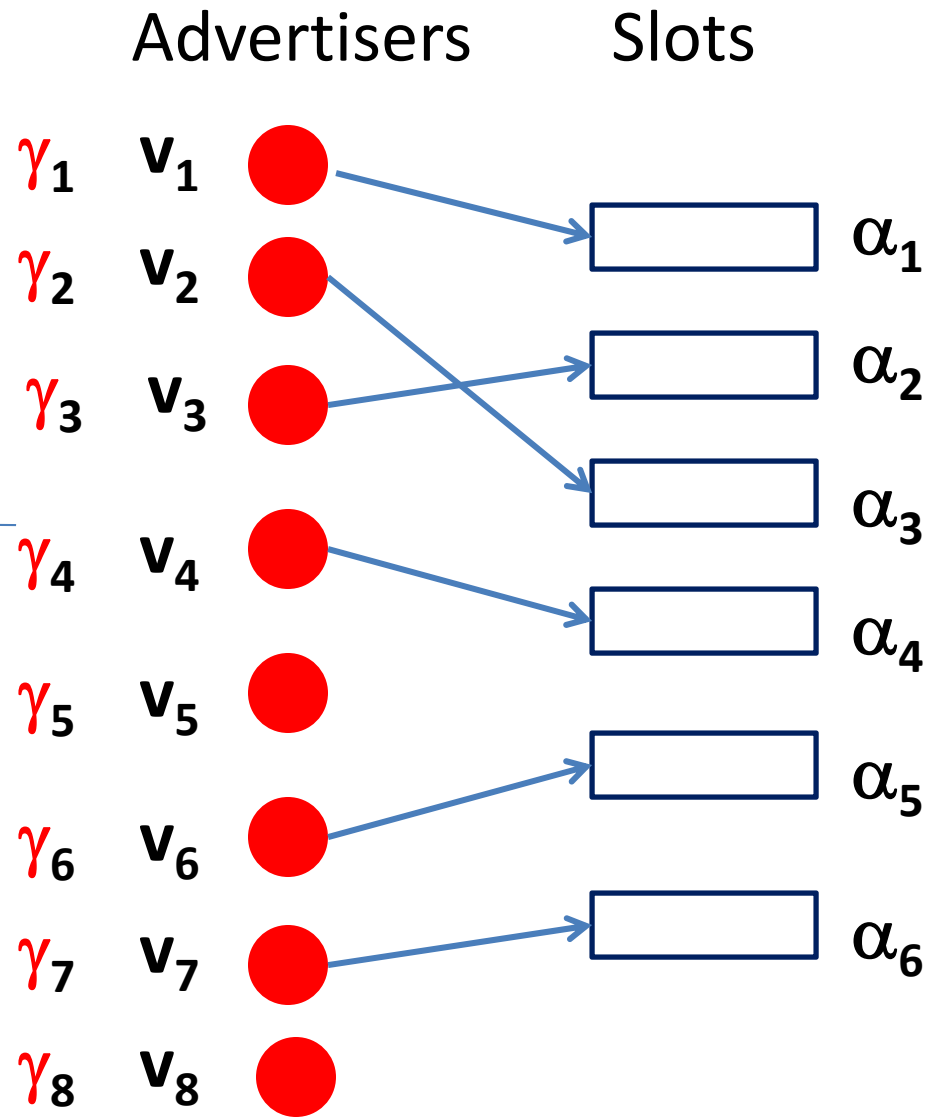
Better model: click rate

also depends on advertiser i

$$\alpha_\ell \cdot \gamma_i$$

Advertiser i value of slot j

$\mathbf{v}_i \cdot \gamma_i \cdot \alpha_\ell$
effective value



Social welfare

Talk: simplest model:

$$\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4 > \dots$$

Value of advertiser i in slot j

$$v_i \cdot \alpha_j$$

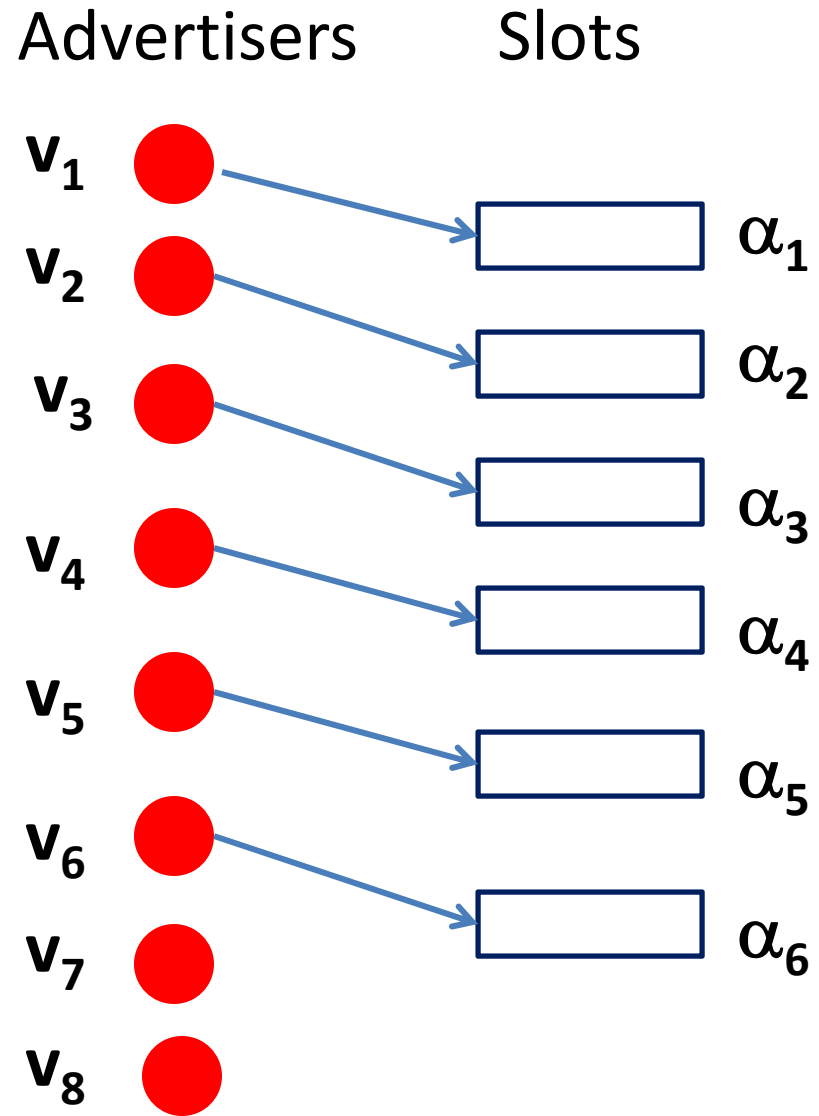
Social welfare = total value

$$\sum_{i \rightarrow j} v_i \alpha_j$$

Maximize social welfare:

Sort $v_1 > v_2 > v_3 > v_4 > \dots$

Welfare $\sum_i v_i \alpha_i$



Can we elicit values?

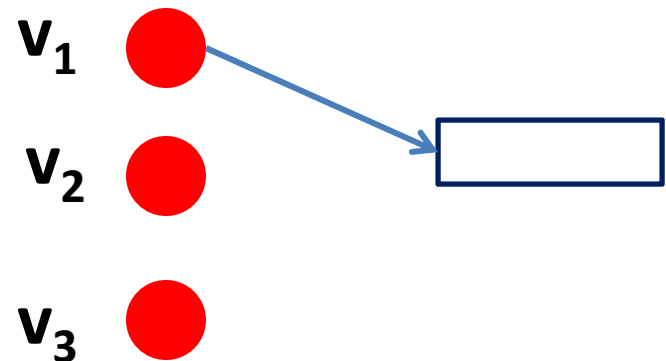
Trouble: Value v_i is private, only known to user i .

Single slot: Vickrey auction (second price)

- Winner highest value $\operatorname{argmax}_i v_i \rightarrow \text{say } 1$
- price $\max_{j \neq 1} v_j$

Result: truthful:

Players will bid their
true values $b_i = v_i$



Generalized Second Prize (GSP)

Sort by bid value

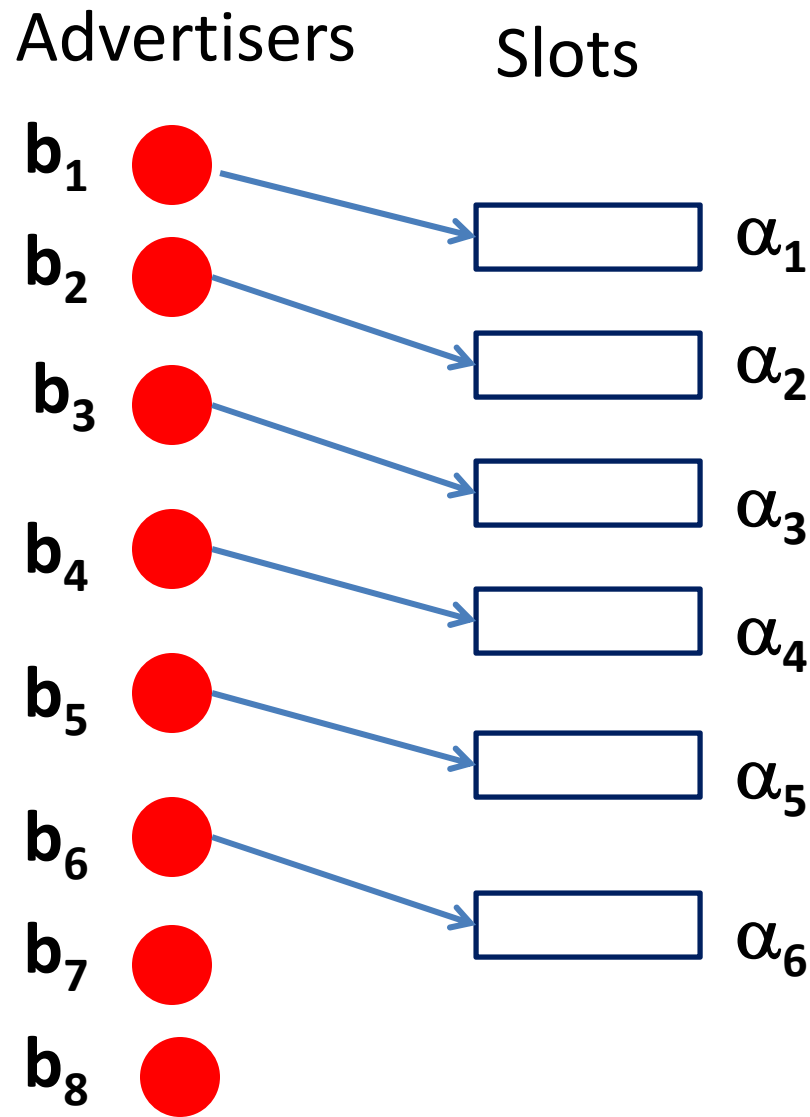
$$b_1 > b_2 > b_3 > b_4 > \dots$$

Assign bidder i to slot i

Charge next price $p = b_{i+1}$

Value to bidder i

$$(v_i - b_{i+1}) \cdot \alpha_i$$



Is GSP truthful?

Is bidding $\mathbf{b_i = v_i}$ Nash equilibrium for the bidders?

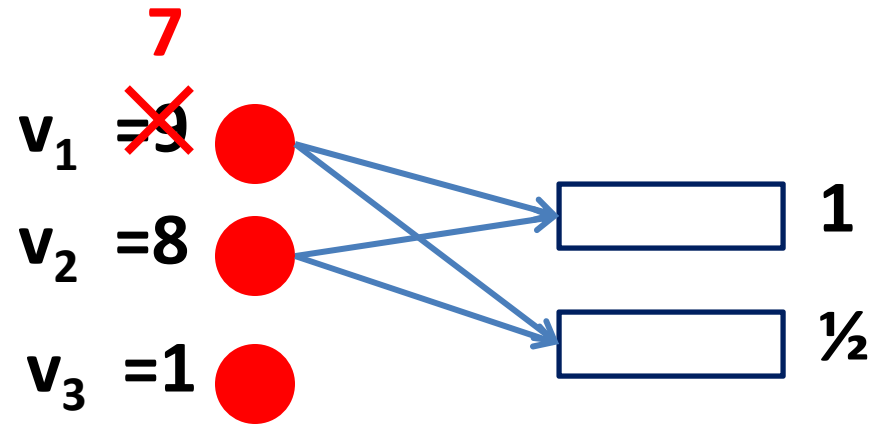
Example:

Bidder 1's value if telling the truth

$$(9-8) \cdot 1 = 1$$

If bidding $\mathbf{b_1 = 7}$

$$(9-1) \cdot \frac{1}{2} = 4$$



Sort by bid value

$$\mathbf{b_1 > b_2 > b_3 > b_4 > \dots}$$

Charge next price $\mathbf{p = b_{i+1}}$

Value to bidder i

$$(\mathbf{v_i - b_{i+1}}) \cdot \alpha_i$$

How good/bad are Nash equilibria?

Theorem: [Edelman & Ostrovsky & Schwarz,
Varian]

There exists a Nash equilibrium set of bids that result in socially optimal assignment

bids are sorted the same order as values

Note: There is also Vickrey-Clark-Groves (VCG) mechanisms, that is always truthful. This is not VCG

\exists Optimal Nash

Maximum social welfare:

Sort $v_1 > v_2 > v_3 > v_4 > \dots$

Welfare $\sum_i v_i \alpha_i$

Corresponding Nash?

user i to slot ℓ value: $v_i \alpha_\ell$

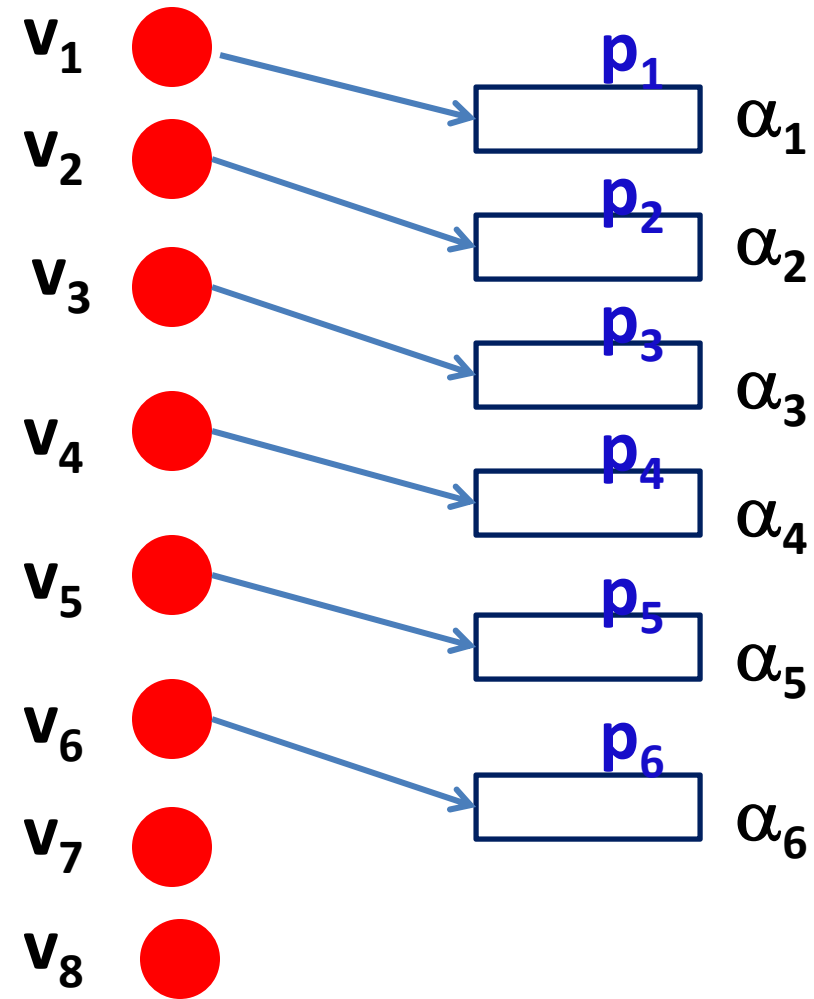
LP dual: \exists prices p_ℓ such that
user i prefers slot i

$$i = \arg\max_{\ell} v_i \alpha_\ell - p_\ell$$

Nash bids $b_i = p_{i-1} / \alpha_i$

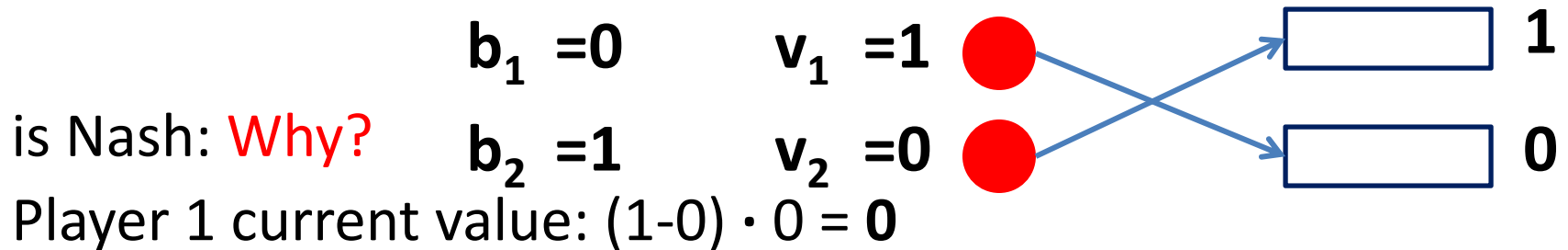
Why? Will pay exactly p_i

Advertisers



How bad can Nash equilibria be?

But also: There exists Nash equilibrium of very bad social value



alternate option: bid $b > 1$ and pay $p = 1$

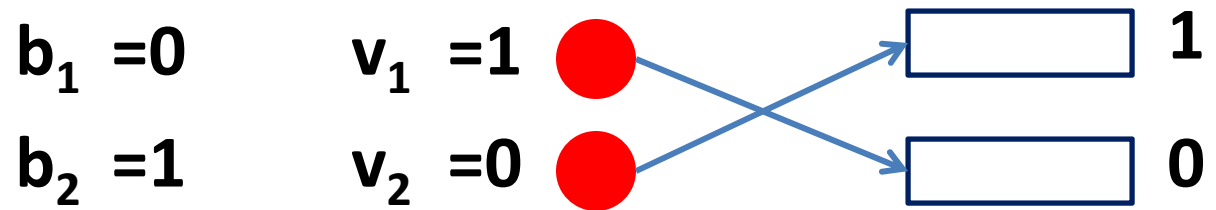
for a value of $(1-p) \cdot 1 = 0$

Social welfare **0**

Optimum social welfare 1

Note: this is Vickrey auction...

Is bad Nash realistic?



Notice: $b_2=1$ extremely dangerous... $b_2 \gg v_2$

- if a new bidder shows up between $b_i > b > v_i$
can be forced to pay above value
- bidding ... $b_2 = v_2$ is just as good
and not dangerous

Assumption: Conservative bidders

$b_i \leq v_i$ for all bidders i

How good are Nash equilibria with **conservative** bidders?



Theorem: [Renato Paes Leme & Tardos'09]

All Nash equilibria with conservative bids has social welfare at least 1.618 fraction of the maximum possible

Vickrey: Nash equilibria with conservative bids has optimal social welfare

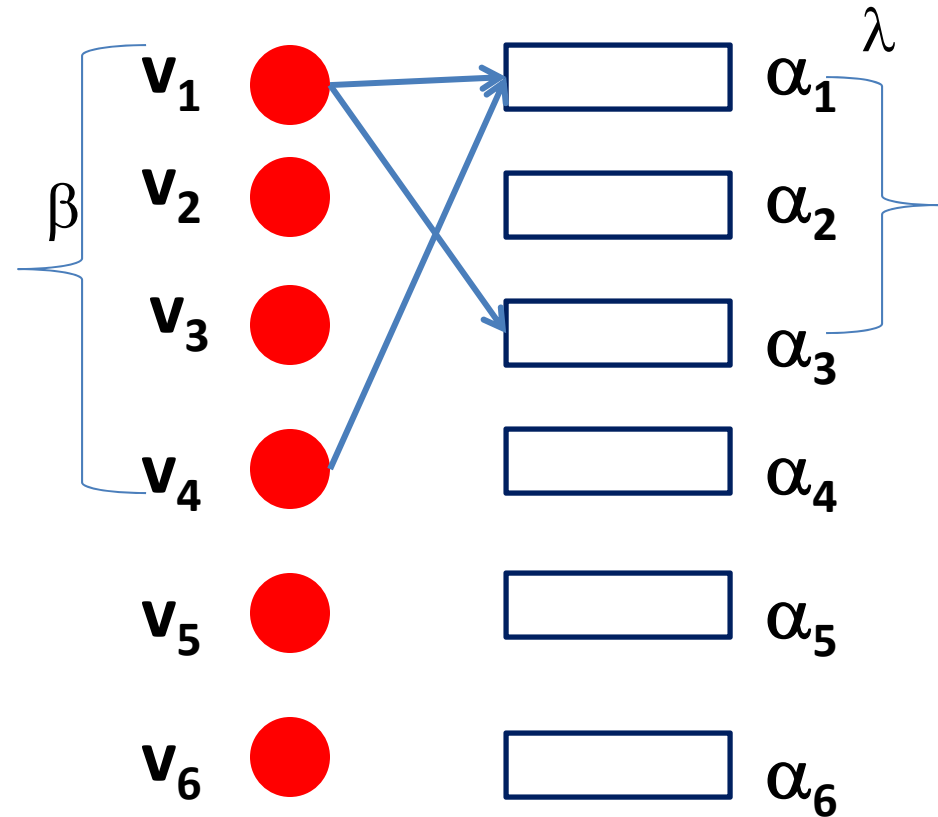
$$\begin{array}{l} + v_1 = \max_i v_i \\ + \max_{j \neq 1} v_j \end{array}$$

Proof: induction on number of slots

If top value assigned to top slot: use induction

Else let β and λ be the ratio in values as shown

Lemma $1/\beta + 1/\lambda \geq 1$

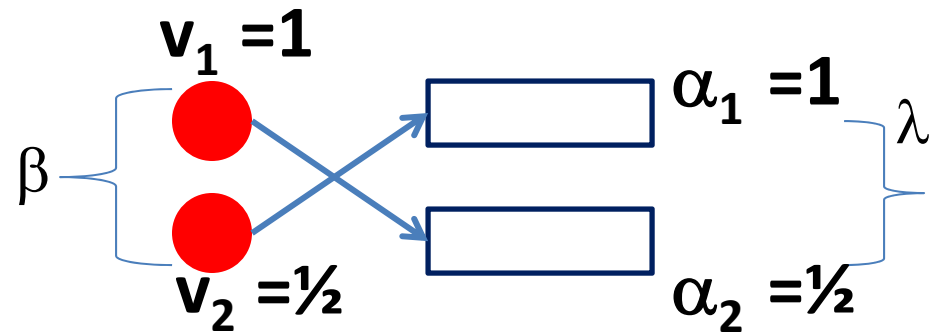


Worst case for two slots

Else let β and λ be the ratio in values as shown

Lemma $1/\beta + 1/\lambda \geq 1$

Worst case for two slots



Optimal value: $1 + 1/4$

Conservative Nash solution bids $b_1 = 0$ and $b_2 = 1/2$

total value: 1

Value for bidders: $(v_1 - 0)\alpha_2 = 1/2$ and: $(v_2 - 0)\alpha_1 = 1/2$

Alternative for bidder 1: $b_1 > 1/2$ for value $(v_1 - 1/2)\alpha_1 = 1/2$

Optimum $1 + 1/\beta\lambda$ Nash value $1/\beta + 1/\lambda$

Proof: simple bound of 2

induction on number of slots

If top value assigned to top slot: use induction

Else let β and λ be the ratio in values as shown

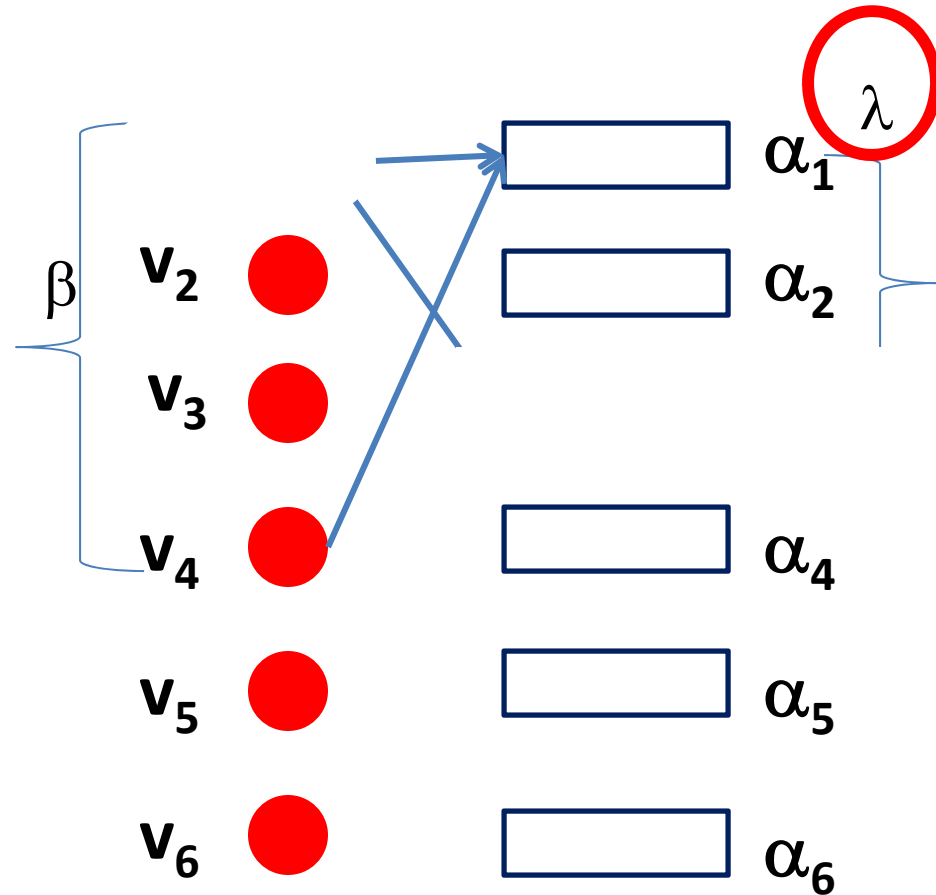
Lemma $1/\beta + 1/\lambda \geq 1$

hence either β or $\lambda \leq 2$

We get total value

at least $\frac{1}{2} \mathbf{v_1} \cdot \mathbf{\alpha_1}$ (as $\lambda \leq 2$)

+ $\frac{1}{2}$ **rest** by induction



Proof: Improved bound:

$$\gamma = (1 + \sqrt{5})/2 \approx 1.618$$

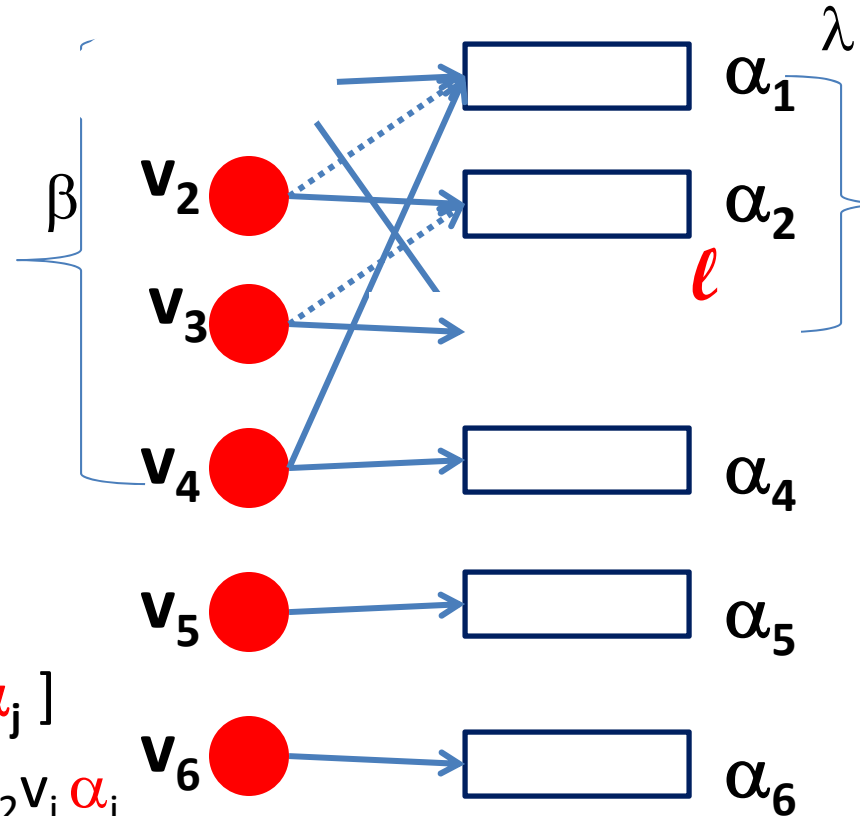
If top value \rightarrow to top slot: use induction

Else β and λ : gap as shown

Lemma $1/\beta + 1/\lambda \geq 1$

We get total value at least

$$\begin{aligned} & 1/\lambda \mathbf{v}_1 \cdot \alpha_1 + \gamma \text{ rest} \\ \geq & 1/\lambda \mathbf{v}_1 \cdot \alpha_1 + \gamma[(\alpha_1 - \alpha_\ell) \mathbf{v}_\ell + \sum_{j \geq 2} \mathbf{v}_j \alpha_j] \\ \geq & 1/\lambda \mathbf{v}_1 \cdot \alpha_1 + \gamma(1/\beta) (\alpha_1 - \alpha_\ell) \mathbf{v}_1 + \sum_{j \geq 2} \mathbf{v}_j \alpha_j \\ \geq & 1/\lambda \mathbf{v}_1 \cdot \alpha_1 + \gamma(1/\beta)(1 - 1/\lambda) \alpha_1 \mathbf{v}_1 + \sum_{j \geq 2} \mathbf{v}_j \alpha_j \\ \geq & (1/\lambda + \gamma(1 - 1/\lambda)^2) \alpha_1 \mathbf{v}_1 + \gamma \sum_{j \geq 2} \mathbf{v}_j \alpha_j \end{aligned}$$



Main Lemma

Lemma $1/\beta + 1/\lambda \geq 1$

Proof: bidder 1 is happy

Current value

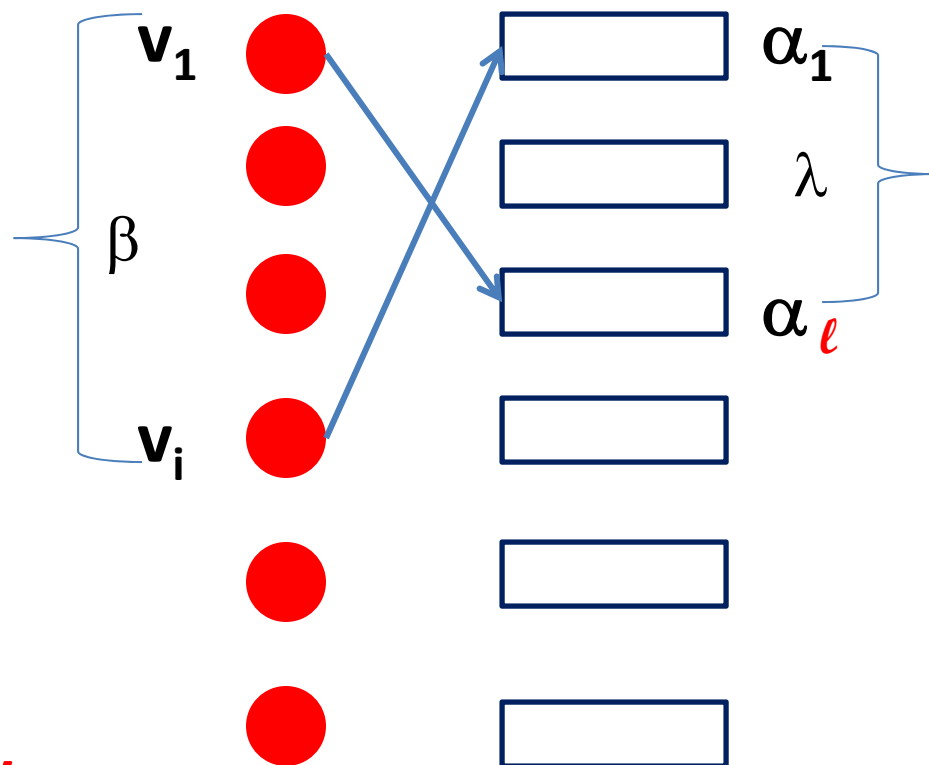
$$(\mathbf{v}_1 - \mathbf{b}_{\ell+1}) \cdot \alpha_{\ell} \leq \mathbf{v}_1 \cdot \alpha_{\ell}$$

Alternate option: $> \mathbf{b}_i$

$$(\mathbf{v}_1 - \mathbf{b}_i) \cdot \alpha_1 \geq (\mathbf{v}_1 - \mathbf{v}_i) \cdot \alpha_1$$

conservative

$$\text{Nash} \Rightarrow \mathbf{v}_1 \cdot \alpha_{\ell} \geq (\mathbf{v}_1 - \mathbf{v}_i) \cdot \alpha_1$$



Conclusion

On line advertisement: fun assignment problem

Mechanism used GSP (generalized second price). Simple, but not always good.

- There exists Nash with optimal value
- Resulting social value not too bad (assuming conservative bidders)

Questions: which Nash is realistic?

Learning in context with many identical bid options

Other models of click-rate? (externalities?)

Summary

Network Games and Quality of Nash

- Examples routing and cost-sharing
- Outcome : Nash & Price of Anarchy /Stability
- Smoothness: a common and powerful proof technique

Learning in Network Games

- No-regret learning
- (coarse) correlated equilibrium & Total Price of Anarchy
- Smoothness bounds also valid
- Natural learning leads to good outcome

Quality in other games: Ad-Auctions

Many natural questions:

- Other classes of games and other learning methods?