



Combinatorial Optimization in Canadian Forces Airlift Modeling

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Berlin Blockade



- 1st major cold war international crisis
- Multinational occupation of post-WII Germany
(24 June 1948 – 11 May 1949)
- Soviets blocked rail and road
- Req. supply of 5,000 tons/day
- The Berlin Airlift





The Berlin Airlift



- 2,326,406 tons supplies
- 278,228 flights
- C-47s and C-54s
- 92 million flying miles
- 101 crash fatalities
- Cost: 224\$ million
= (~ 2 billion 2008 \$)





Recent Canadian Airlifts



- Operation APOLLO 2001-2003
 - Deployment of 750 troops
 - Initial airlift:
 - 450 large items (vehicles, pallets, etc.)
 - ~1.7 million kgs
- Operation ATHENA 2003
 - International Security Assistance Force in Afghanistan
 - Deployment of 900 personnel
 - Initial airlift:
 - 648 large items ~4 million kgs



Canadian Airlift Assets



- C-130
- C-5
- C-17
- Il-76
- AN-124





Problem Statement



- Input:
 - N items
 - M types of aircraft
- Optimize:
 - minimize a function of the # of flights (sorties)
- Output:
 - Pareto optimal set of solutions
 - Solution = set of aircraft & their cargo



GALAHAD: GENETIC ANNEALING FOR LOADING AIRCRAFT, A HEURISTIC AIDING DEPLOYMENT

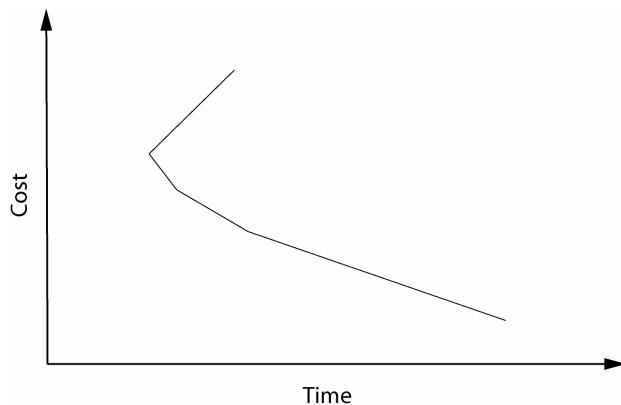
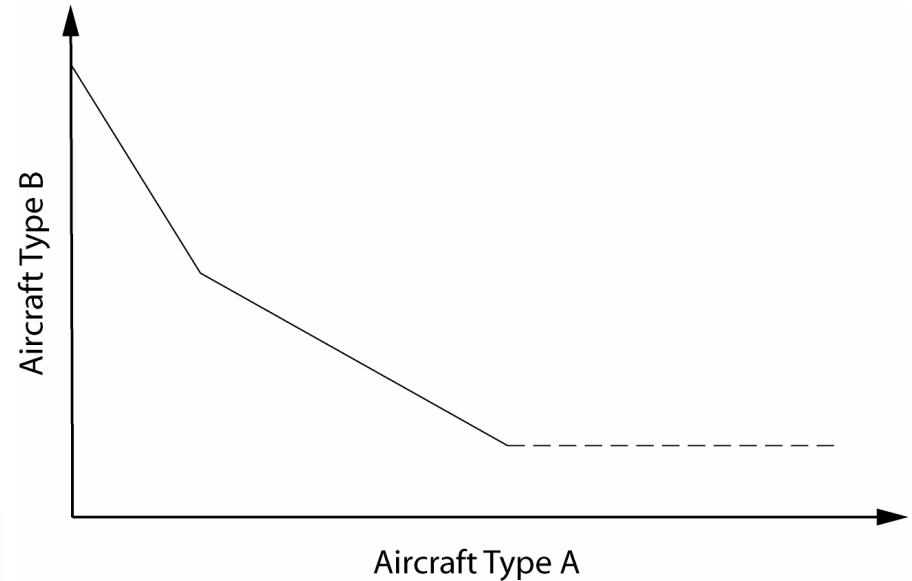
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Multi-Objective Optimization



- More than one Aircraft type
 - No single “optimal” solution
 - Family of equally good solutions
- GALAHAD
 - Find solutions within family of good solutions





Background



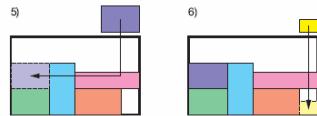
- Supplements currently available tools:
 - NATO Allied Deployment and Movements System (ADAMS)
 - USAF Automated Air Load Planning System (AALPS)
- Operates at the item level:
 - use dimensions and weights of individual items



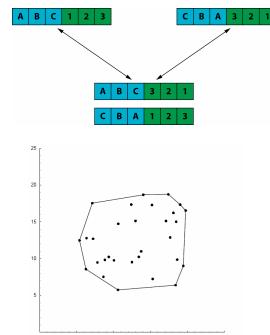
Components of GALAHAD



Four components of GALAHAD:

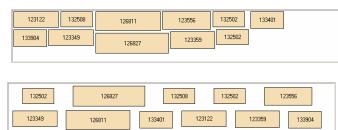


- Bin packing heuristic



- Genetic annealing engine

- Convex-hull-based fitness function



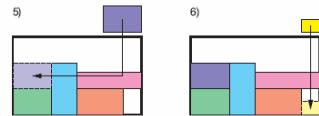
- MILP load balancing optimizer



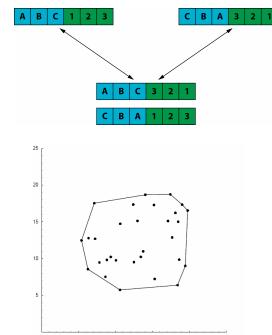
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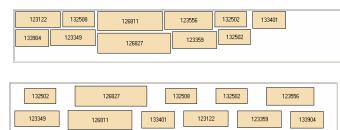


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- MILP load balancing optimizer



Aircraft Loading — Bin Packing



- NP-hard problem
 - Finding a provably optimal solution generally impossible
 - Heuristics exist to find good solutions rapidly
- “2.5D” problem
 - No stacking
 - Treat bins (aircraft) and items as 2D rectangles
 - Solve 2D layout, after checking that other constraints (height, weight, inter-item spacing, etc.) are not violated



Requirements for bin-packing heuristic



- To operate within our genetic annealing framework, our heuristic must be:
 - Deterministic
 - Identical input gives identical output
 - Computationally simple
 - Fast execution (called for every new individual)
 - Relatively efficient
 - Produce reasonably tight packings of items



A List-Based Heuristic



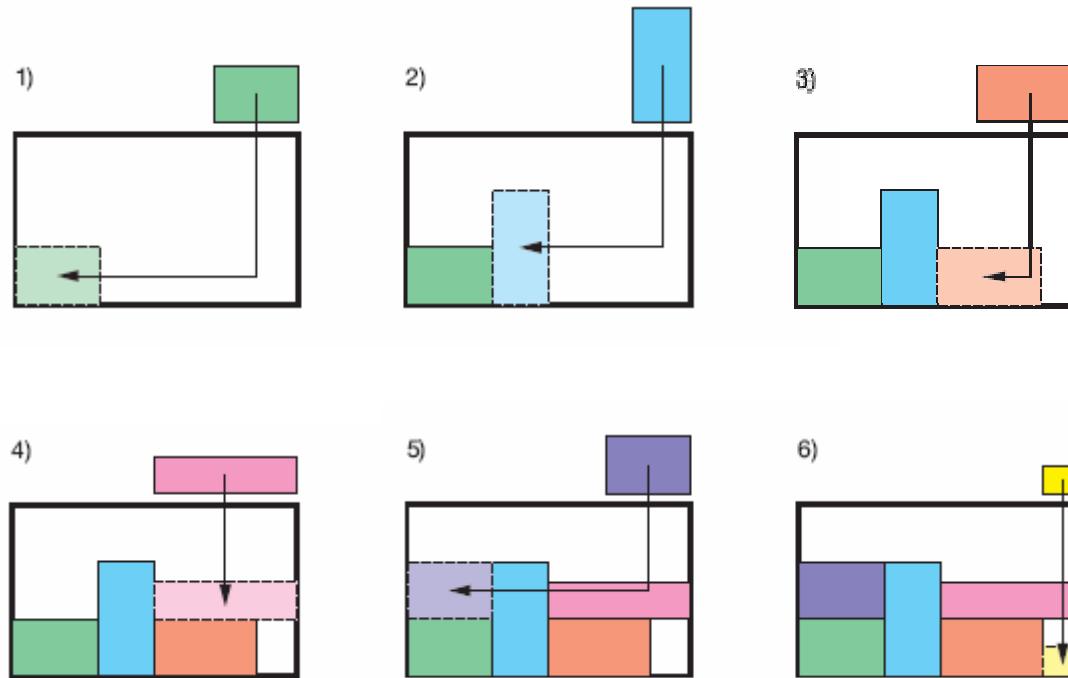
- List-based heuristic approach to loading aircraft
 - Given:
 - An ordered list of bins (aircraft cargo bays)
 - An ordered list of items
 - Move through the list of items in sequence
 - Each item is packed into the first available bin into which it fits
 - Placement of items using Bottom-Left (Fill)



Bottom-Left (Fill) Bin Packing



*Baker et al (1980)
Lui & Teng (1999)*





Bin-Packing Summary



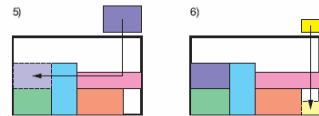
- Changing the order of items (or bins) in the list will change the resulting aircraft loading solution
- Problem comes down to finding “the right list(s)”
- Exhaustive search is generally not feasible:
 - 100 items: $100! = 9.33262 \times 10^{157}$ possible permutations
 - $O(10^{85})$ elementary particles in the observable universe
 - $O(10^{120})$ possible chess games
- We require an efficient means of searching through only the most “interesting” solutions



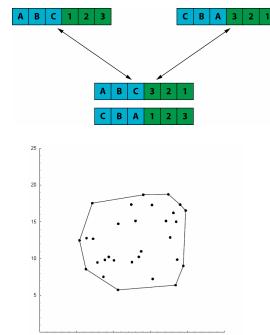
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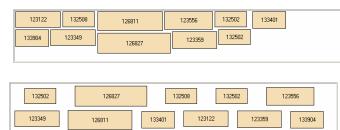


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Genetic Algorithm Basics



- Mimics Darwinian evolution
 - A population of random individuals is created
 - The goal-specific “fitness” of each individual is determined
 - Elements of the **fittest** solutions are combined to generate new individuals, which are then added to the population
 - Additional genetic operators (mutations, immigration, etc.) are applied
 - The fitness of the new population is evaluated
 - ...
- Over many “generations” the population “evolves” towards optimality



Representing Individuals — DNA



- **Recall:** The listed order of items and bins determines the resulting load plan
- For the genetic algorithm:
 - Individual is represented by “DNA” consisting of:
 - Ordered list of assets
 - Ordered list of items
- For each individual:
 - Use the bin packing heuristic to find the number of non-empty assets of each type after loading

Bins (Aircraft)

Items (Cargo)

A	2
B	1
B	3
A	4
C	3
A	3
B	2
	5

B	1
C	5
A	3
A	3
A	4
B	2
	2



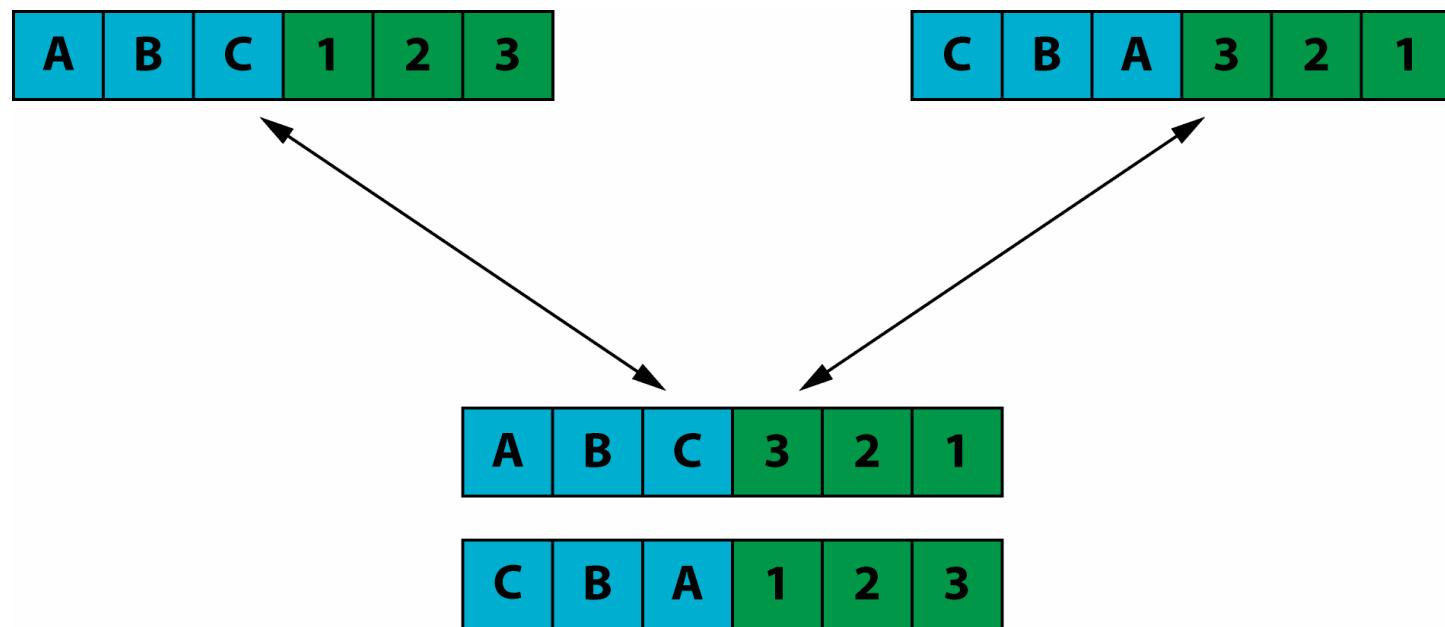
Genetic Operations



- For each generation, we act on the population using a variety of genetic operators:
 - Reproduction
 - Mutation
 - Immigration
 - Aging
 - Extinctions (load balancing)
- Since our individuals are not represented as binary strings, implementation of these operations is different from standard treatments



Reproduction Type I: “Chromosome Exchange”



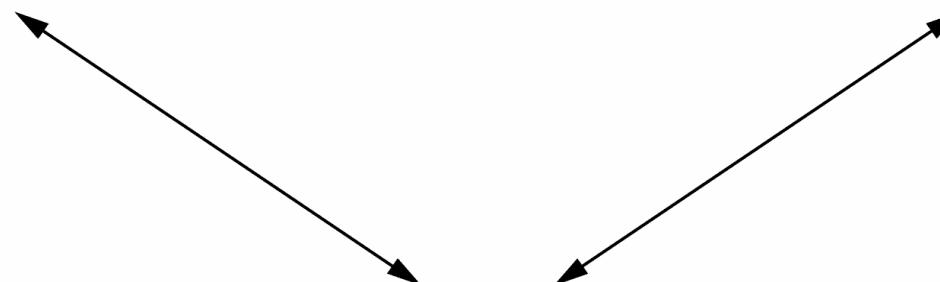


Reproduction Type II: k-point Crossovers



1	2	6	5	4	3
---	---	---	---	---	---

1	4	5	2	3	6
---	---	---	---	---	---



1	4	2	5	6	3
---	---	---	---	---	---

1	2	4	6	5	3
---	---	---	---	---	---

1	2	4	5	6	3
---	---	---	---	---	---

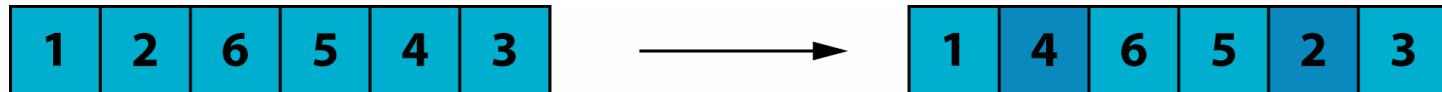
1	4	2	6	5	3
---	---	---	---	---	---



Mutation and Immigration



- Mutation:



- Immigration:
 - After each generation, we randomly create a few individuals and add them to the population
 - Adds new information to the mix; prevents “inbreeding,” or convergence to a local (non-global) minimum



Aging and Extinction



- Aging
 - Individuals are only allowed to exist for set number of generations
 - Exception: Current “best” solutions are rendered immortal
- Extinction
 - Offers a means of introducing constraints without raising the dimensionality of the problem
 - Infrequent culling of solutions which do not respect the constraint
 - Currently under experimentation...



Selection?



- Which individuals to select for “breeding”:
 - Reproduction
 - Mutation
 - Immigration



Genetic Annealing



- A hybrid of **genetic algorithms** and **simulated annealing**
(Pakhira 2003)
- Implementation uses a **local temperature concept**
(Cho et al. 1998)
- Each individual has its own, fitness-based temperature

$$T(x) = \alpha^{F(x)} T_{reference}$$

- Since $0 < \alpha < 1$
 - Fitter individuals have very low temperatures
 - Unfit individuals have very high temperatures



Using local temperatures



- We wish to use an elitist breeding strategy, so:
 - Randomly select an individual from the population
 - Accept or reject for breeding with probability

$$P = \begin{cases} 1 & F(x) \geq F_{threshold} \\ \exp\left(\frac{F(x) - F_{threshold}}{T(x)}\right) & \text{otherwise} \end{cases} \quad (\text{Boltzmann prob. dist.})$$

- Fitness threshold can be fixed or selected randomly
- Somewhat analogous procedure for mutations
 - Higher local temperature \Rightarrow more likely to mutate and more mutations.



Survival of the fittest...



- Local temperature based strategy effects:
 - More fit individuals reproduce more frequently
 - Fitter individuals tend to reproduce amongst themselves
 - Fit individuals are more immune to disruption by mutations, while unfit individuals are used to explore larger regions of phase space
- How to determine an appropriate “fitness” measure?



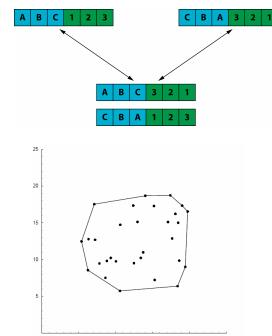
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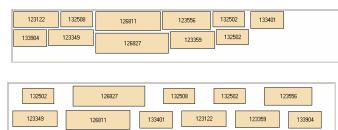


- Bin packing heuristic



- Genetic annealing engine

– Convex-hull-based fitness function



- MILP load balancing optimizer



Developing a fitness function

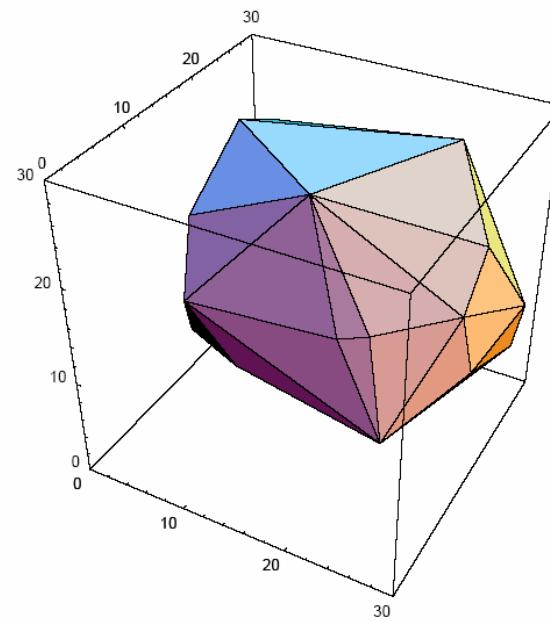
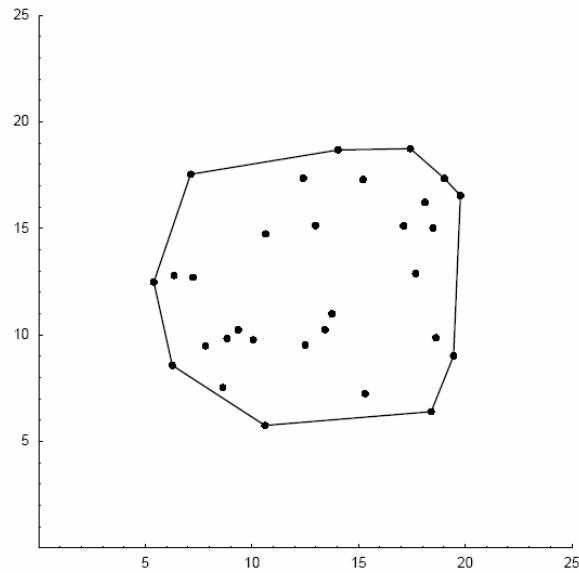


- Given M aircraft types
- An individual represented by a M -dimensional point in space (# of flights required of each asset type)
- Multi-objective optimization problem:
 - minimize the usage of each asset type
 - Solution can be improved by movement in a large number of directions

Convex hull

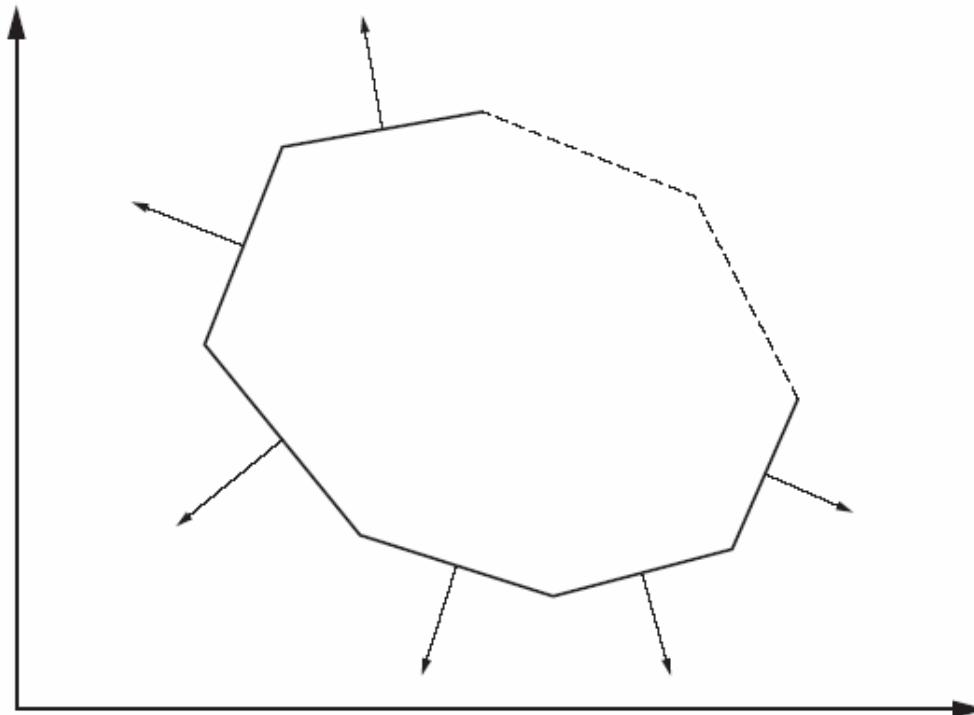


Convex Hulls — 2D and 3D Examples





The Frontier



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Convex Hull-Based Fitness Function



- After each generation:
 - Determine the convex hull and extract the frontier
 - **Fitness** of each individual
 - distance to the nearest facet of the frontier
 - Use this fitness in genetic selection
- After many generations, the “final frontier” should closely correspond to the optimal set of solutions



Example: Op ATHENA



Operation Details:

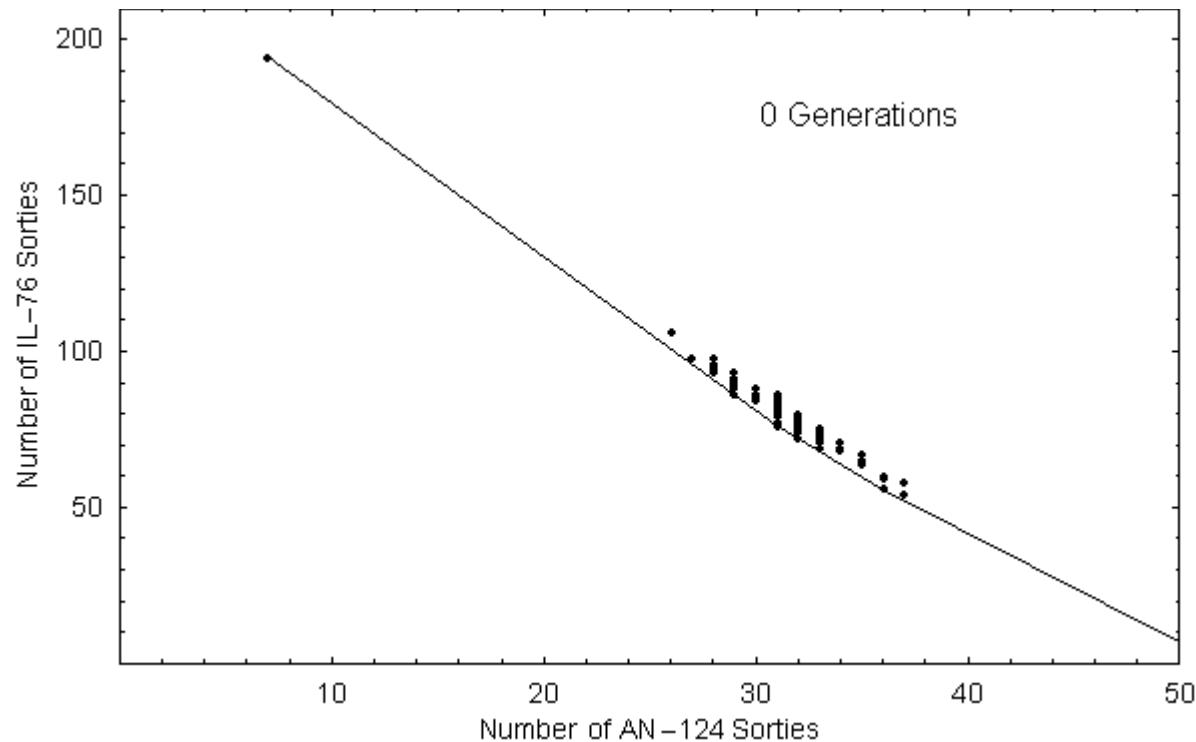
- Cargo:
 - 350 Vehicles
 - 300 Sea-Containers
- Historical Assets:
 - AN-124
 - IL-76

Optimization Parameters:

- Population:
 - Initial Population: 100
 - Maximum Population: 5000
 - Maximum Age: 3
- Genetic Operations:
 - Reproduction: 80
 - Random Mutations: 10
 - Immigrations: 10

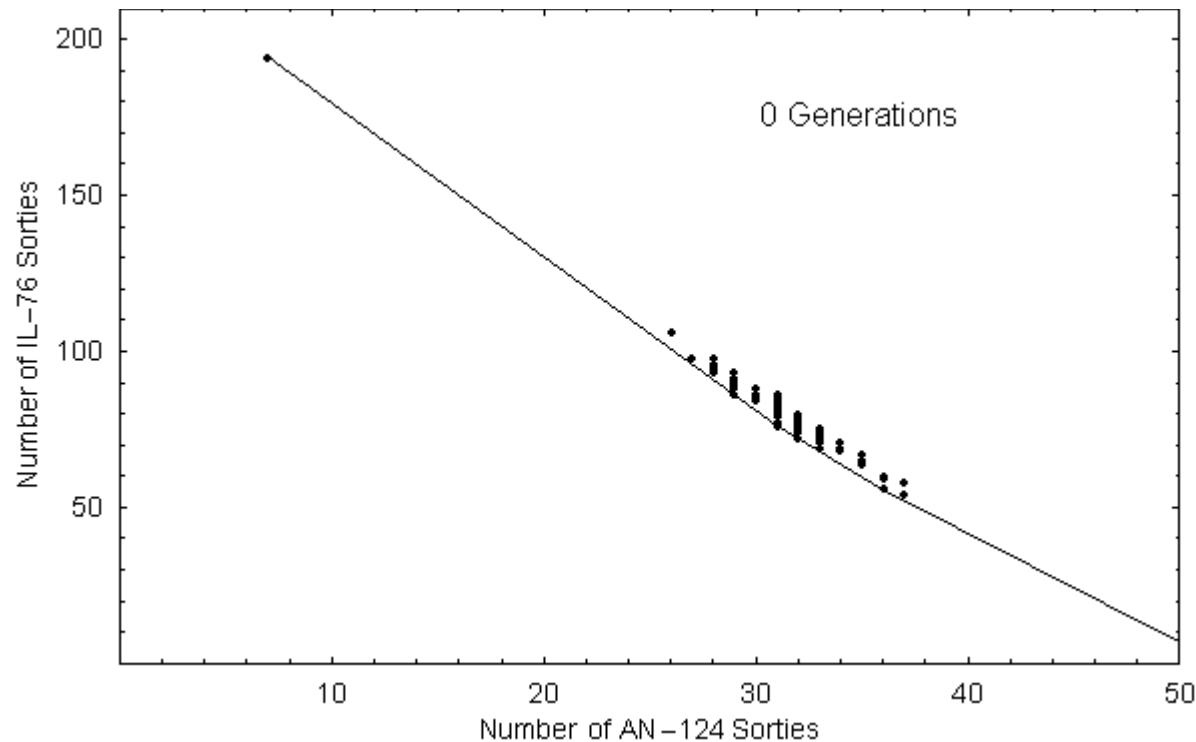


Example: Op ATHENA



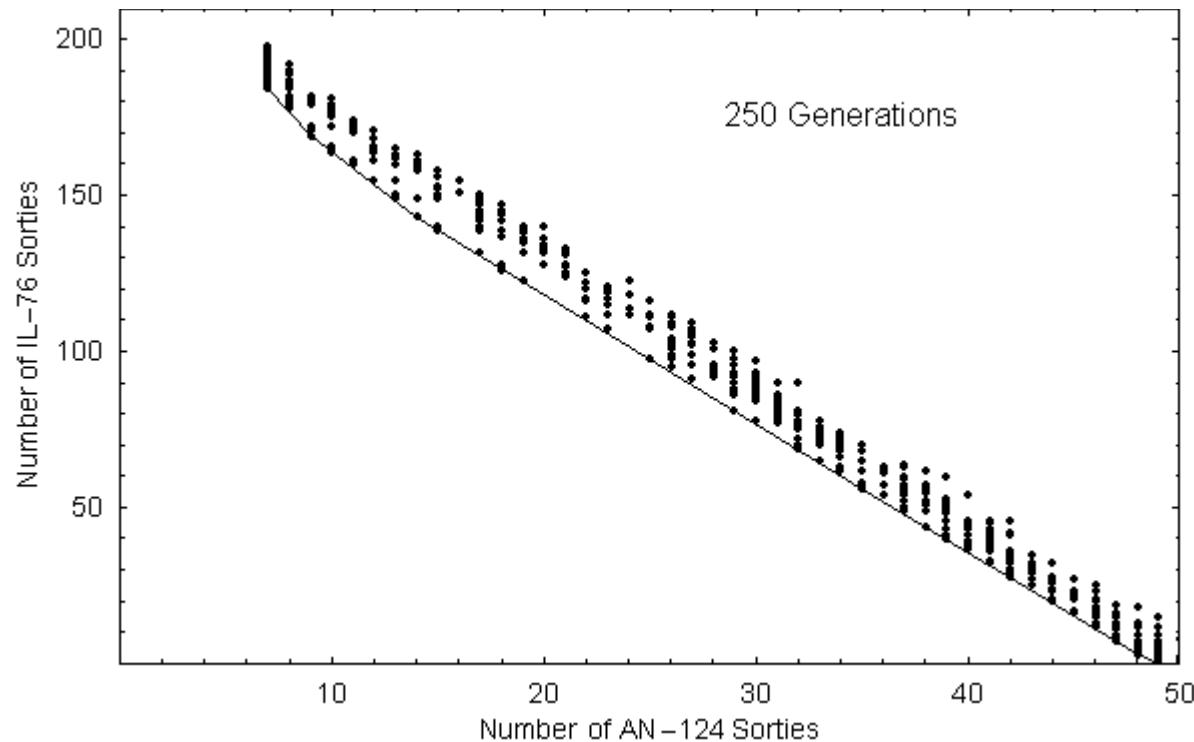


Example: Op ATHENA





Example: Op ATHENA





Performance



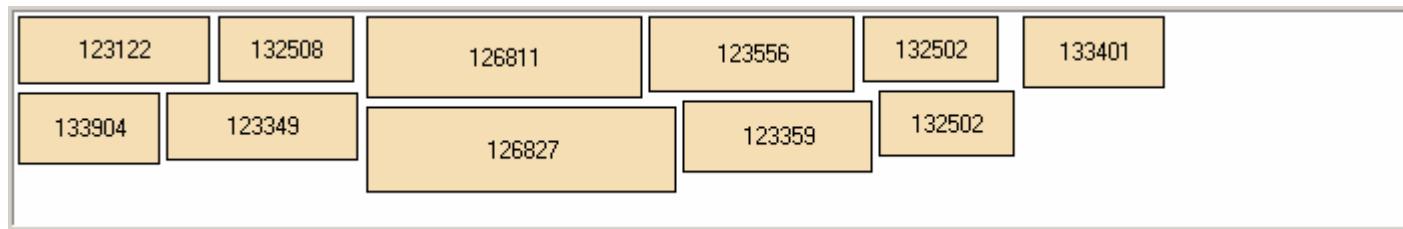
- Op ATHENA examples:
 - Two airlift assets
 - 1000 generations in about 10 minutes
- Performance strongly dependent on the number of airlift asset types.
 - Increased memory requirements and running time
- Most strenuous test so far:
 - Op STRUCTURE (response to 2004 tsunami)
 - Approx 75 cargo items
 - Six airlift asset types



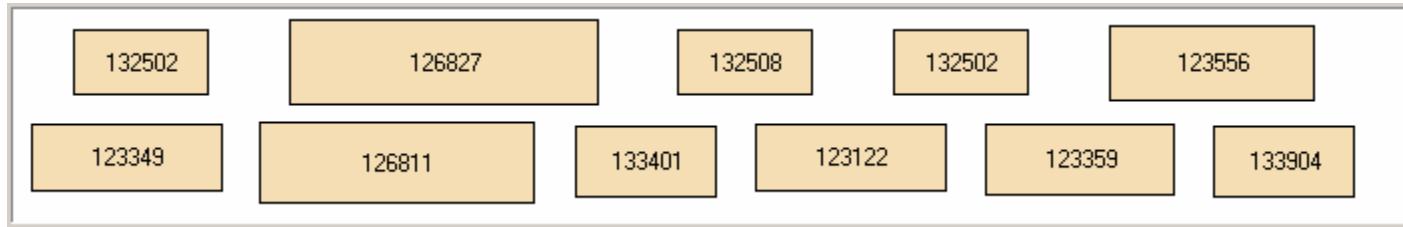
Realistic aircraft layouts?



- Heuristic generates feasible packings, but not necessarily realistic...



- Need to apply balancing and other constraints





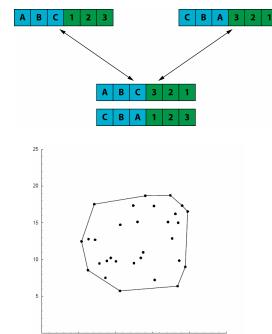
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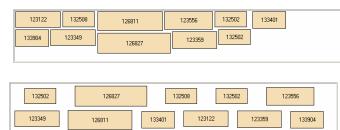


- Bin packing heuristic



- Genetic annealing engine

- Convex-hull-based fitness function



- **MILP load balancing optimizer**



Load Balancing



Problem:

Position items in an aircraft to balance the load.



Why?

Aircraft safety and efficiency



Safety



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Load Balancing



Problem:

Position items in an aircraft to balance the load.



Airbus A340-300 example:

“Displacement from center of gravity of < 75cm over a 10,000km flight saves 4,000kg of fuel” - Mongeau & Bes (2003)



Background



Current practice:

- Experienced load masters
- Historical load manifests
- Heuristic Methods
- Canadian Forces acquired AALPS in 2007
- Lack of trained load masters



Related Work



Bin Packing:

- Pack as many items as possible
- Minimize wasted space

Set Partition:

- split a set of numbers into two: A and B
- $\sum A = \sum B$

Load Balancing: NP-Complete

Amiouny et al. (1992)



Related Work



Heuristics:

- Amiouny et al. (1992) 1D
- Wodziak & Fadel (1994) 2D, *Genetic Algorithm*
- Mathur (1998) 1D

Integer Linear Programming:

- Homogeneous Items, Fixed locations, 2D
 - Thomas et al. (1998)
 - Mongeau & Bes (2003)
- Heterogeneous Items, Free placement, 2D, 3D
 - Chen et al. (1995)
 - Fasano (1998) (*Optimize: bin packing*)
 - Padberg (2000)



Integer Linear Programming



$$\begin{aligned} & \text{Maximize} && \mathbf{c}x + \mathbf{d}y \\ & \text{subject to:} && \mathbf{Ax} + \mathbf{By} \leq \mathbf{b} \\ & && x \text{ is integer} \end{aligned}$$

Why model as an ILP?

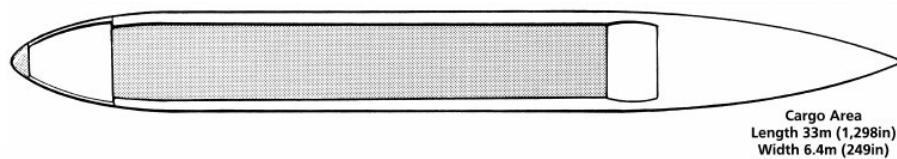
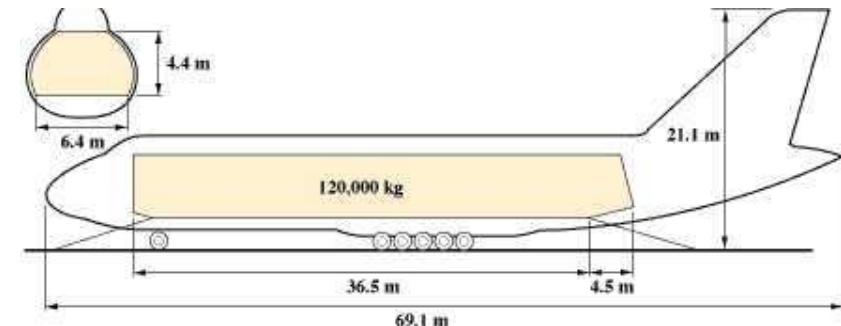
- Sophisticated solvers exist: *exact / heuristic*
 - CPLEX 11.0 (2007)
 - SYMPHONY (2004)
 - Feasibility Pump (2004)
- New user-friendly modeling languages:
 - ZIMPL (2004)
- Modeling flexibility



Problem Definition



Type	Length	Width	Height	Max Payload
C-17	27	5.2	3.6	35000
C-5	44	5.8	4.1	89000



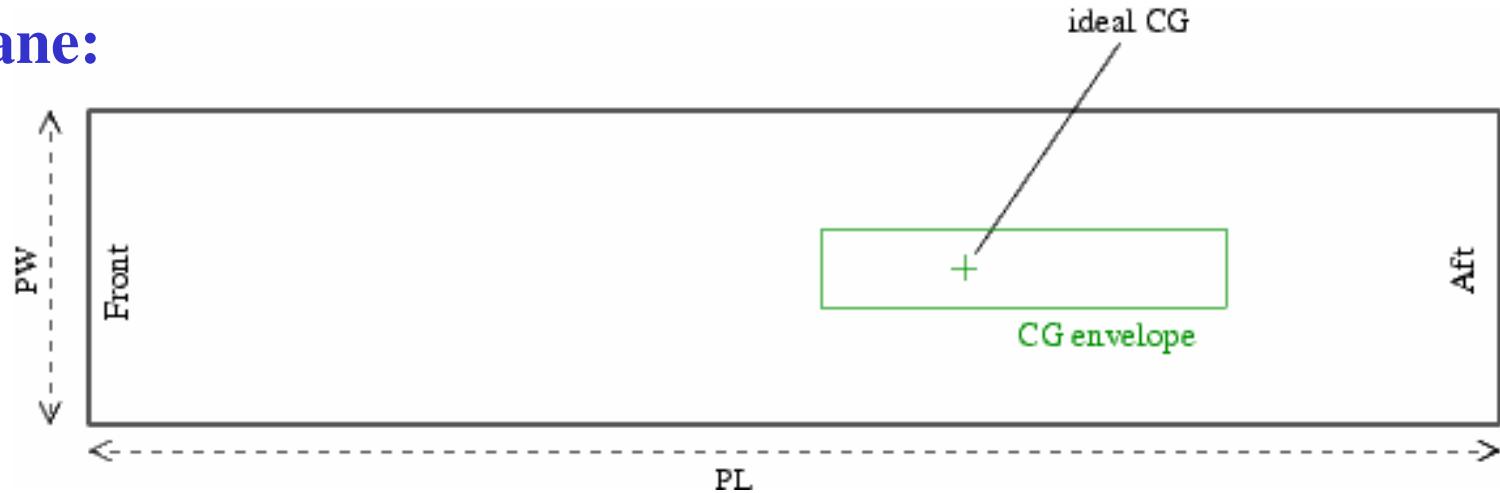
Payload	C-17	C-5
0-2272	8.89 - 35.56 (35.56)	10.16 - 25.4(25.4)
2273-4545	8.89 - 26.16 (26.16)	10.16 - 25.4(25.4)
4546-6818	8.89 - 25.4 (25.4)	10.16 - 25.4 (25.4)
6819-9090	8.89 - 25.15 (24.38)	10.16 - 34.8 (26.42)
9091-11363	9.14 - 25.02 (24.13)	10.16 - 34.8 (26.42)
11364-13636	11.43 - 24.89 (23.88)	17.02 - 35.05 (29.29)
13637-18182	14.22 - 24.13 (23.88)	21.21 - 35.05 (30.66)
18183-22727	15.75 - 23.88 (23.75)	23.75 - 35.05 (31.55)
22728-27272	16 - 23.88 (23.75)	25.4 - 35.052 (32.13)
27273-31818	17.53 - 23.75 (23.37)	26.67 - 35.28 (32.512)
31819-36363	18.29 - 23.75 (23.24)	27.56 - 35.31 (32.84)
36364-40909		28.32 - 35.31 (33.25)
40910-45454		28 - 35.31 (33.53)
45455-54545		28.83 -35.31 (33.63)
54546-68181		30.48 - 35.31 (33.83)
68182-79545		31.12 - 35.31 (33.99)
79546-111363		32.51 - 35.31 (34.16)



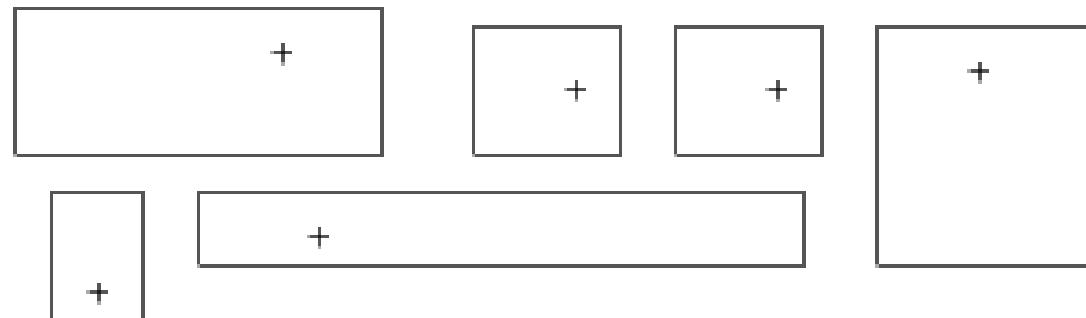
Input



Plane:



Set *I* of *n* Items:



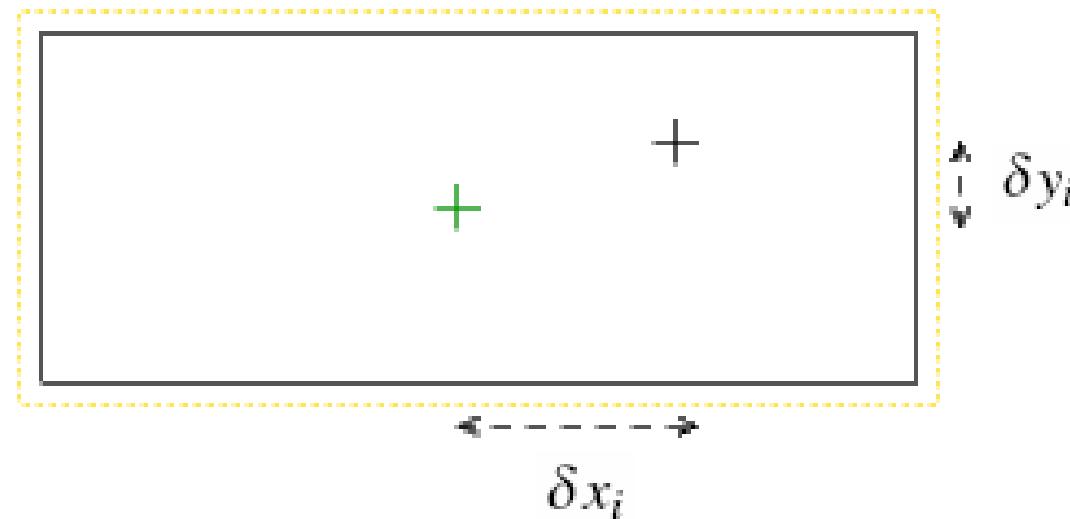


Input



Item i Specifications:

- Weight: m_i
- Dimensions: $w_i \times l_i$
- CG offsets: δx_i and δy_i
- Spacing req.: sp_i

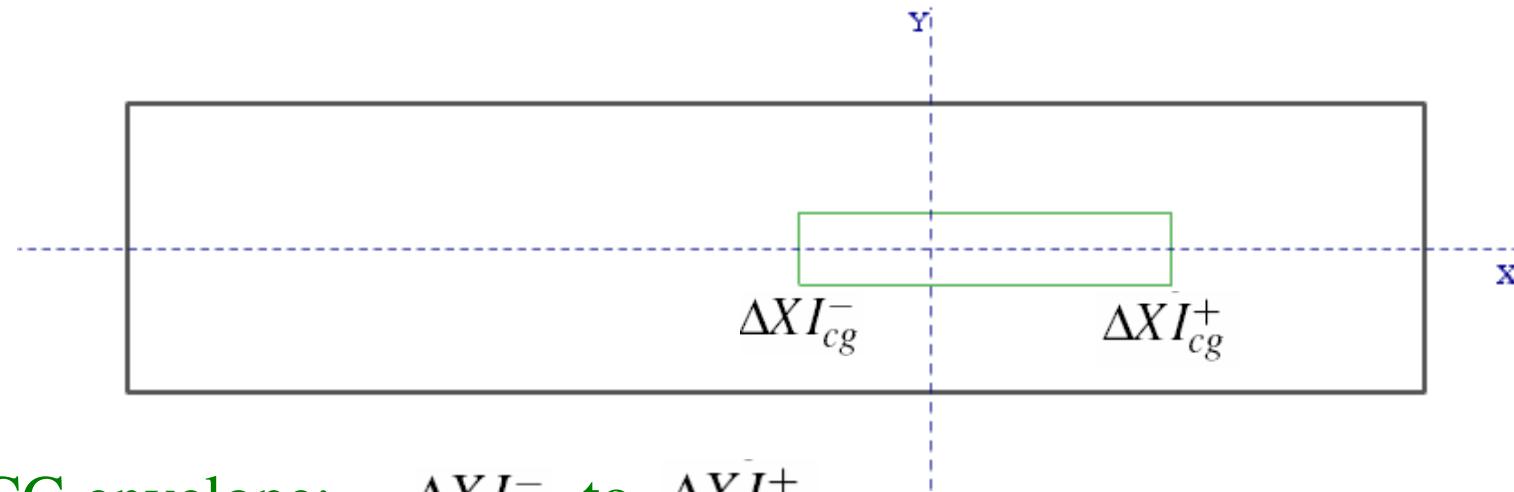




ILP Setup



Coordinate System:



CG envelope: ΔXI_{cg}^- to ΔXI_{cg}^+
 $-\Delta YI_{cg}$ to ΔYI_{cg}

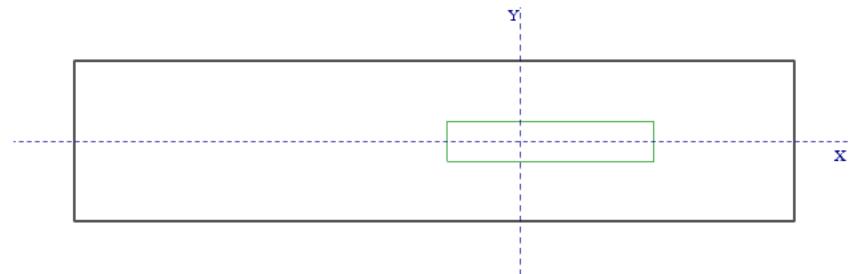
- Main decision variables: (x_i, y_i) for all items
(placement of physical center)



ILP Setup



Main decision variables: (x_i, y_i) for all items



Real CG computation:

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i) + M_P \cdot X}{\sum_{i \in I} m_i + M_P}$$
$$YR_{cg} = \frac{\sum_{i \in I} m_i \cdot (y_i + \delta y_i)}{\sum_{i \in I} m_i + M_P}$$

M_p = Mass of plane X = CG of empty plane

Real CG minimization:

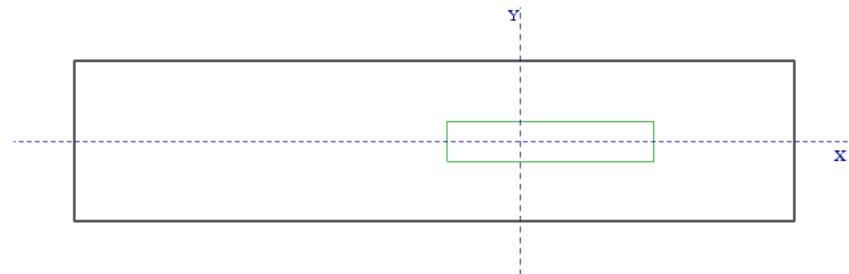
$$\text{minimize } \sqrt{(XR_{cg})^2 + (YR_{cg})^2}$$



ILP Setup



Main decision variables: (x_i, y_i) for all items



Real CG computation:

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i) + M_P \cdot X}{\sum_{i \in I} m_i + M_P}$$

$$YR_{cg} = \frac{\sum_{i \in I} m_i \cdot (y_i + \delta y_i)}{\sum_{i \in I} m_i + M_P}$$

M_p = Mass of plane X = CG of empty plane

Real CG minimization:

$$\cancel{\text{minimize } \sqrt{(XR_{cg})^2 + (YR_{cg})^2}}$$

$$\text{minimize } |XR_{cg}| + |YR_{cg}|$$



ILP Objective



$$\text{minimize} \quad |XR_{cg}| + |YR_{cg}|$$

= Shanno & Weil (1971)

$$\text{minimize} \quad XR_{cg}^+ + XR_{cg}^- + YR_{cg}^+ + YR_{cg}^-$$

$$XR_{cg} = XR_{cg}^+ - XR_{cg}^-$$

$$XR_{cg}^+ \geq 0$$

$$YR_{cg} = YR_{cg}^+ - YR_{cg}^-$$

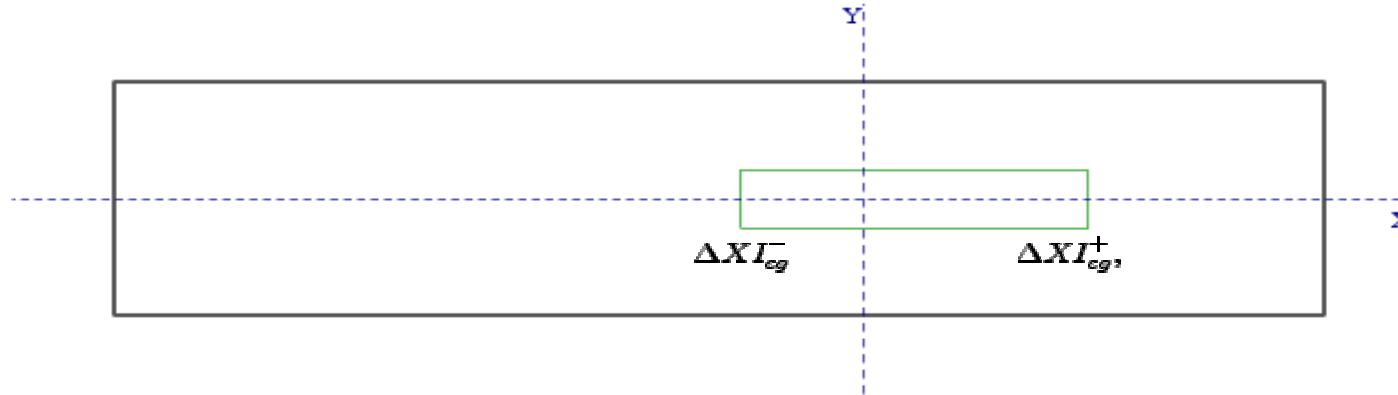
$$XR_{cg}^- \geq 0$$

$$YR_{cg}^+ \geq 0$$

$$YR_{cg}^- \geq 0$$



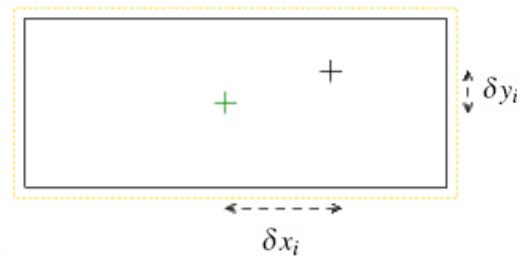
ILP Constraints



Respect CG envelope:

$$\begin{aligned}\Delta XI_{cg}^- \leq XR_{cg} &\leq \Delta XI_{cg}^+, \\ -\Delta YI_{cg} \leq YR_{cg} &\leq \Delta YI_{cg}.\end{aligned}$$

Each item i within boundary:



$$\begin{aligned}PL^- + sp_i + \frac{l_i}{2} \leq x_i &\leq PL^+ - sp_i - \frac{l_i}{2}, \\ PW^- + sp_i + \frac{w_i}{2} \leq y_i &\leq PW^+ - sp_i - \frac{w_i}{2}.\end{aligned}$$



ILP Constraints



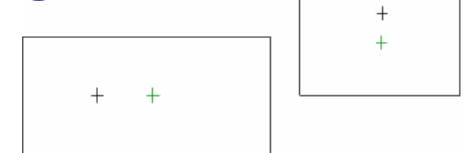
Prevent item overlap:

$$|x_i - x_j| \geq \frac{l_i}{2} + \frac{l_j}{2} + \max(spi, spj)$$

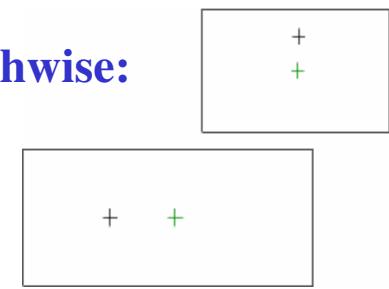
or

$$|y_i - y_j| \geq \frac{w_i}{2} + \frac{w_j}{2} + \max(spi, spj)$$

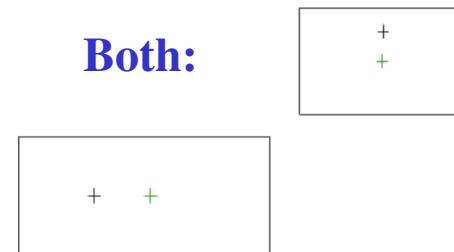
Lengthwise:



Widthwise:



Both:





ILP Constraints



Prevent item overlap:

For every pair of items $\{i,j\}$ with $i < j$: m_{ij} *binary*



$$|x_i - x_j| \geq \frac{l_i}{2} + \frac{l_j}{2} + \max(sp_i, sp_j) - M \cdot m_{ij}$$

and

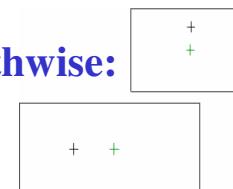
for some large M

$$|y_i - y_j| \geq \frac{w_i}{2} + \frac{w_j}{2} + \max(sp_i, sp_j) - M \cdot (1 - m_{ij})$$

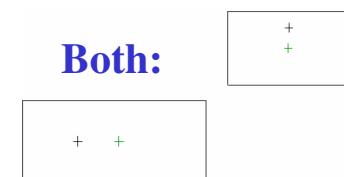
Lengthwise:



Widthwise:



Both:





ILP Constraints



Prevent item overlap:

For every pair of items $\{i,j\}$ with $i < j$: $m_{ij} \ q_{ij} \ r_{ij}$ binary

$$x_i - x_j \geq \frac{l_i}{2} + \frac{l_j}{2} + \max(sp_i, sp_j) - M \cdot (m_{ij} + q_{ij}),$$

$$x_j - x_i \geq \frac{l_i}{2} + \frac{l_j}{2} + \max(sp_i, sp_j) - M \cdot (m_{ij} + 1 - q_{ij}), \quad \text{for some large } M$$

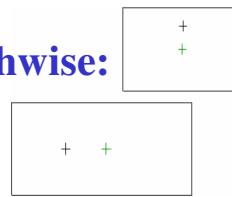
$$y_i - y_j \geq \frac{w_i}{2} + \frac{w_j}{2} + \max(sp_i, sp_j) - M \cdot (1 - m_{ij} + r_{ij}),$$

$$y_j - y_i \geq \frac{w_i}{2} + \frac{w_j}{2} + \max(sp_i, sp_j) - M \cdot (1 - m_{ij} + 1 - r_{ij}).$$

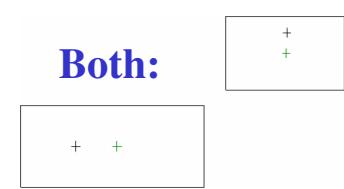
Lengthwise:



Widthwise:

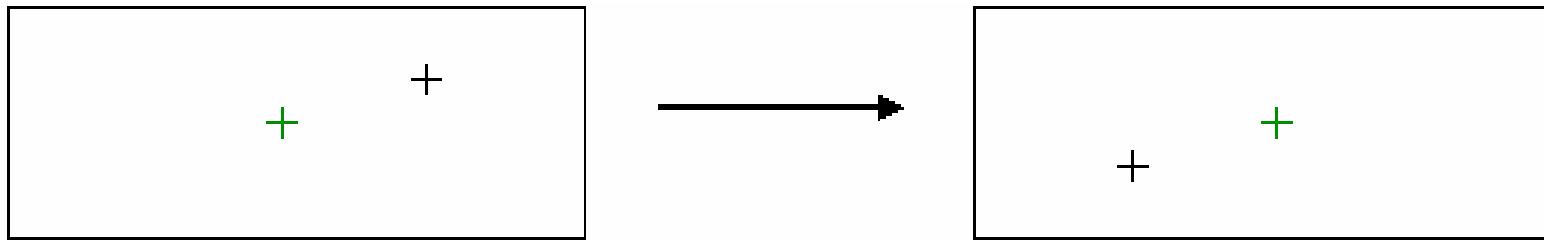


Both:





Item Orientation: Item Flips (180 degree rotation)



New binary variable: $flip_i = 0$ (initial orientation)
 $= 1$ (rotated 180 degrees)

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i - 2 \cdot \delta x_i \cdot flip_i)}{\sum_{i \in I} m_i},$$

$$YR_{cg} = \frac{\sum_{i \in I} m_i \cdot (y_i + \delta y_i - 2 \cdot \delta y_i \cdot flip_i)}{\sum_{i \in I} m_i}.$$

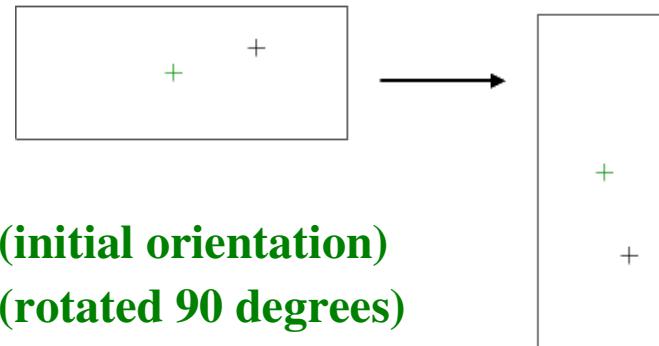


Item Orientation: Orthogonal Rotations (90 degrees)

clockwise



Rotate 90 degrees:



New binary variable: $rot_i = 0$ (initial orientation)
 $= 1$ (rotated 90 degrees)

Need to modify **boundary** and **overlap** constraints

$$X_{overlap_i} = \frac{w_i}{2} \cdot rot_i + \frac{l_i}{2} \cdot (1 - rot_i),$$

$$Y_{overlap_i} = \frac{l_i}{2} \cdot rot_i + \frac{w_i}{2} \cdot (1 - rot_i), \quad \text{for } i \in I.$$

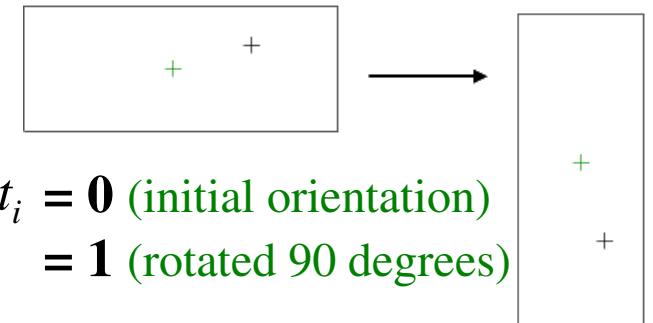


Item Orientation: Orthogonal Rotations (90 degrees)

clockwise



$$Xoverlap_i = \frac{w_i}{2} \cdot rot_i + \frac{l_i}{2} \cdot (1 - rot_i),$$



$rot_i = 0$ (initial orientation)
 $= 1$ (rotated 90 degrees)

Each item i within boundary :

$$PL^- + sp_i + Xoverlap_i \leq x_i \leq PL^+ - sp_i - Xoverlap_i,$$

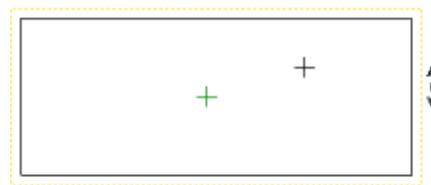
Prevent item overlap, for all pairs $\{i,j\}$ with $i < j$:

$$x_i - x_j \geq Xoverlap_i + Xoverlap_j + \max(sp_i, sp_j) - M \cdot (m_{ij} + q_{ij}),$$

$$x_j - x_i \geq Xoverlap_i + Xoverlap_j + \max(sp_i, sp_j) - M \cdot (m_{ij} + 1 - q_{ij}),$$



Item Orientation: Orthogonal Rotations



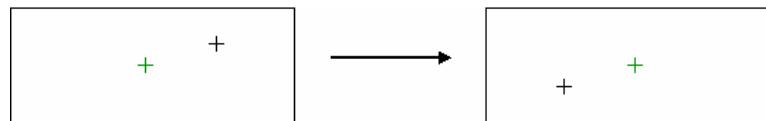
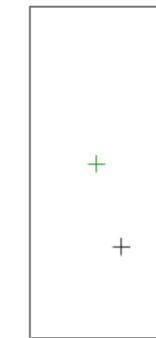
$rot_i = 0$ and $flip_i = 0$

0°



$rot_i = 1$ and $flip_i = 0$

90°



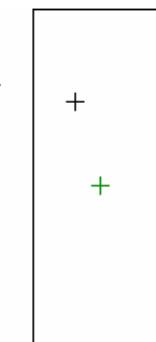
$rot_i = 0$ and $flip_i = 1$

180°

270°

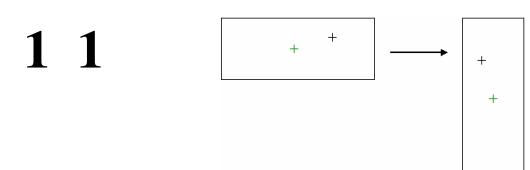


$rot_i = 1$ and $flip_i = 1$





Item Orientation: Orthogonal Rotations



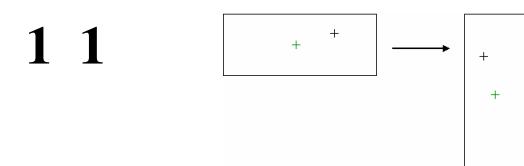
CG computation:

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i + (\delta y_i - \delta x_i) \cdot rot_i - 2 \cdot \delta x_i \cdot flip_i + 2 \cdot (\delta x_i - \delta y_i) \cdot flip_i \cdot rot_i)}{\sum_{i \in I} m_i},$$

$$YR_{cg} = \frac{\sum_{i \in I} m_i \cdot (y_i + \delta y_i - (\delta x_i + \delta y_i) \cdot rot_i - 2 \cdot \delta y_i \cdot flip_i + 2 \cdot (\delta y_i + \delta x_i) \cdot flip_i \cdot rot_i)}{\sum_{i \in I} m_i}.$$



Item Orientation: Orthogonal Rotations



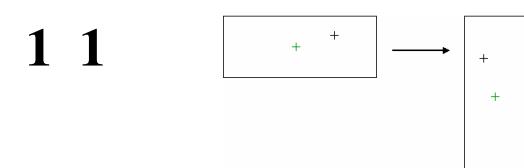
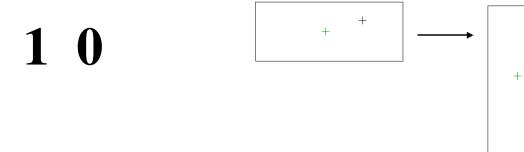
CG computation:

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i)}{\sum_{i \in I} m_i},$$

$$YR_{cg} = \frac{\sum_{i \in I} m_i \cdot (y_i + \delta y_i - (\delta x_i + \delta y_i) \cdot rot_i - 2 \cdot \delta y_i \cdot flip_i + 2 \cdot (\delta y_i + \delta x_i) \cdot flip_i \cdot rot_i)}{\sum_{i \in I} m_i}.$$



Item Orientation: Orthogonal Rotations



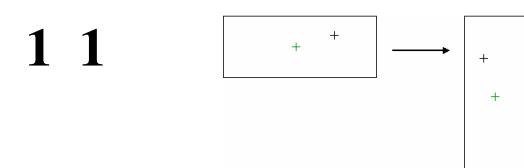
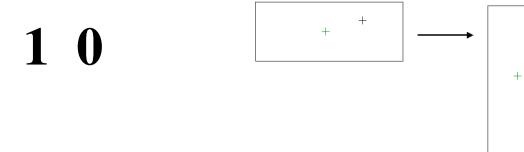
CG computation:

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i + (\delta y_i - \delta x_i) \cdot rot_i)}{\sum_{i \in I} m_i},$$

$$YR_{cg} = \frac{\sum_{i \in I} m_i \cdot (y_i + \delta y_i - (\delta x_i + \delta y_i) \cdot rot_i - 2 \cdot \delta y_i \cdot flip_i + 2 \cdot (\delta y_i + \delta x_i) \cdot flip_i \cdot rot_i)}{\sum_{i \in I} m_i}.$$



Item Orientation: Orthogonal Rotations



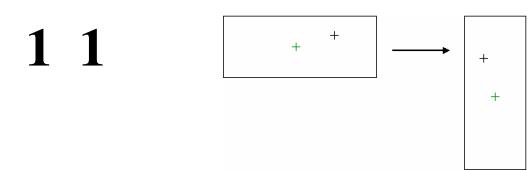
CG computation:

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i \cdot \text{[gray box]} - 2 \cdot \delta x_i \cdot flip_i \text{[gray box]})}{\sum_{i \in I} m_i},$$

$$YR_{cg} = \frac{\sum_{i \in I} m_i \cdot (y_i + \delta y_i - (\delta x_i + \delta y_i) \cdot rot_i - 2 \cdot \delta y_i \cdot flip_i + 2 \cdot (\delta y_i + \delta x_i) \cdot flip_i \cdot rot_i)}{\sum_{i \in I} m_i}.$$



Item Orientation: Orthogonal Rotations



CG computation:

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i + (\delta y_i - \delta x_i) \cdot rot_i - 2 \cdot \delta x_i \cdot flip_i + 2 \cdot (\delta x_i - \delta y_i) \cdot flip_i \cdot rot_i)}{\sum_{i \in I} m_i},$$

$$YR_{cg} = \frac{\sum_{i \in I} m_i \cdot (y_i + \delta y_i - (\delta x_i + \delta y_i) \cdot rot_i - 2 \cdot \delta y_i \cdot flip_i + 2 \cdot (\delta y_i + \delta x_i) \cdot flip_i \cdot rot_i)}{\sum_{i \in I} m_i}.$$



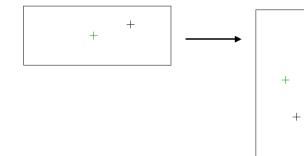
Item Orientation: Orthogonal Rotations



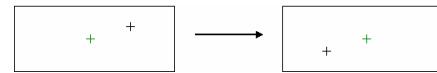
0 0



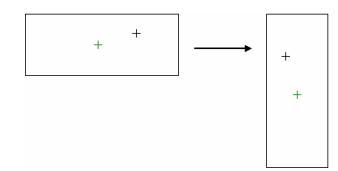
1 0



0 1



1 1



CG computation:

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i + (\delta y_i - \delta x_i) \cdot rot_i - 2 \cdot \delta x_i \cdot flip_i + 2 \cdot (\delta x_i - \delta y_i) \cdot flip_i \cdot rot_i)}{\sum_{i \in I} m_i},$$

$$fr_i = flip_i \cdot rot_i$$

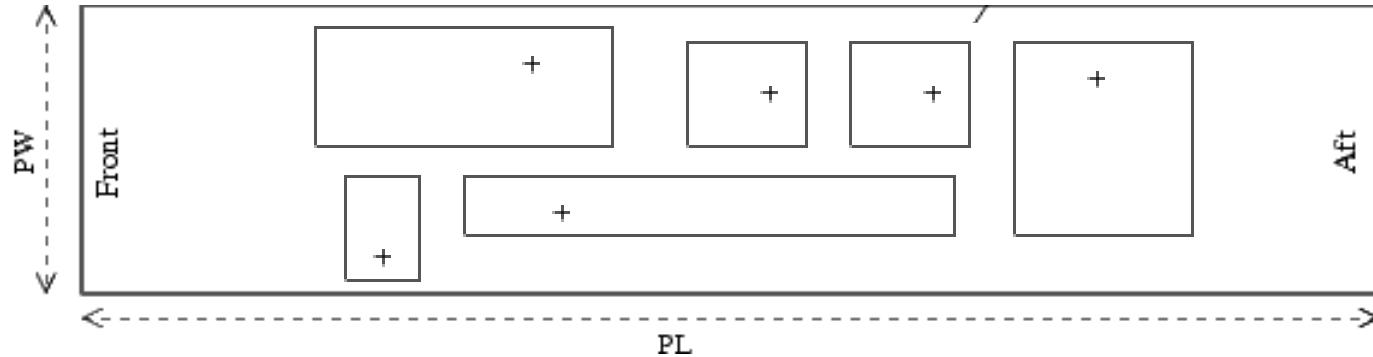
$$\iff$$

$$fr_i \leq \frac{flip_i + rot_i}{2},$$
$$fr_i \geq flip_i + rot_i - 1.$$

binary



Valid Inequalities



Transitivity: *i to the left of j to the left of k* \implies *i to the left of k*

Fitting Cuts: pre-compute sets of items that

- don't fit lengthwise
- don't fit widthwise

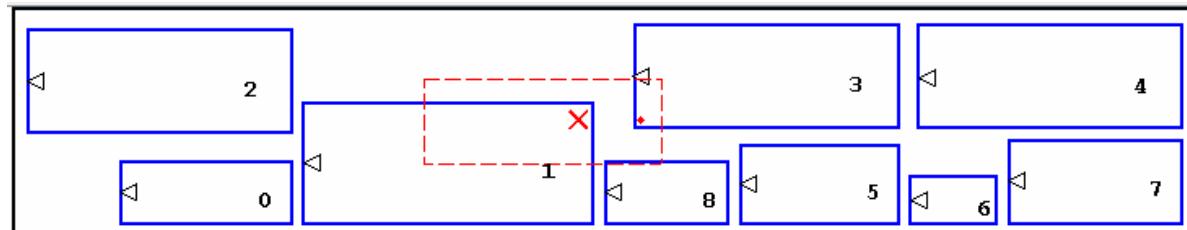
Common Items Cuts: remove solution symmetries
using placement by order of indices



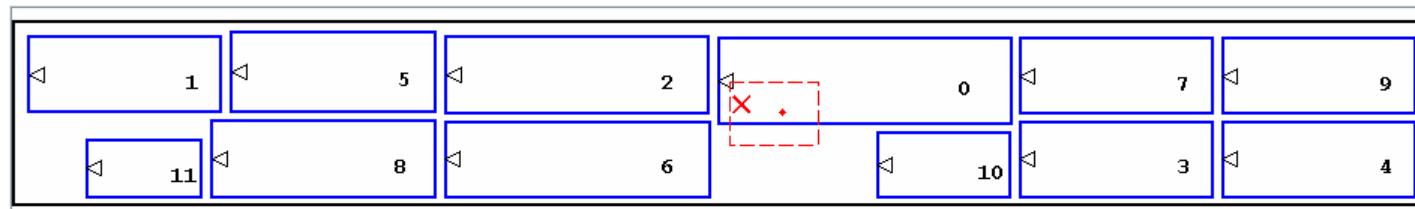
Solution Times (sec.)



Asset	# Items	# Unique	Payload	No Cuts	All	No Trans	No Fit	No Comm	Fit Pairs
C5	27	3	85238	600.06	600.08	600.06	600.05	600.08	600.05
C5	27	6	78893	600.03	16.77	19.61	1.28	52.22	13.59
C5	26	6	85695	600.03	90.73	14.06	600.06	600.09	600.06
C5	25	5	82017	600.05	60.78	211.42	600.06	600.08	15.16
C5	25	8	75033	599.41	541.56	2.41	600.05	600.08	21.63
C5	25	5	73804	600.03	22.80	197.08	10.64	487.22	3.81
C5	24	6	85369	6.08	14.67	405.92	1.64	39.78	6.66
C5	24	6	78647	9.27	27.61	77.88	2.86	475.53	8.73
C5	24	6	71845	0.98	10.31	2.11	1.39	99.14	1.23
C5	23	7	75261	13.66	5.53	61.38	1.39	7.08	0.72
C5	22	6	88406	1.30	3.98	10.25	3.42	3.19	1.08
C5	21	6	77880	142.89	15.80	14.80	1.69	20.84	76.14
C5	21	7	71923	4.05	0.66	1.44	1.91	19.47	1.88
C5	18	6	84854	0.45	1.20	0.84	0.36	2.34	1.78
				3778	1412	1619	2427	3607	1353



	Type Name	X Position	Y Position	Taken	
0	TRUCK,UTILITY,LIGHT,4X...	-9.918	-1.64	<input checked="" type="checkbox"/>	Aft
1	CAR, ARMD, AMBULANCE,...	-4.374	-0.971	<input checked="" type="checkbox"/>	Aft
2	20 FT ISO CONTAINER SU...	-10.977	0.9	<input checked="" type="checkbox"/>	Aft
3	20 FT ISO CONTAINER SU...	2.876	1.011	<input checked="" type="checkbox"/>	Aft
4	20 FT ISO CONTAINER SU...	9.399	1	<input checked="" type="checkbox"/>	Aft
5	TRAILER,CARGO, 850 KG ...	4.109	-1.46	<input checked="" type="checkbox"/>	Aft
6	ALL TERRAIN VEHICLE,4 ...	7.179	-1.82	<input checked="" type="checkbox"/>	Aft
7	TRAILER CARGO 850 KG ...	10.429	-1.41	<input checked="" type="checkbox"/>	Aft
8	TRAILER,CARGO,1/4 TON,...	0.624	-1.64	<input checked="" type="checkbox"/>	Aft



Item Index	Type Name	X Position	Y Position	Taken	Orientation
0	TRUCK, PALLETIZED LOA...	2.587	1.005	<input checked="" type="checkbox"/>	Aft
1	20 FT ISO CONTAINER SU...	-20.453	1.21	<input checked="" type="checkbox"/>	Aft
2	TRAILER,PALLETIZED LO...	-6.373	1.195	<input checked="" type="checkbox"/>	Aft
3	20 FT ISO CONTAINER SU...	10.447	-1.48	<input checked="" type="checkbox"/>	Aft
4	20 FT ISO CONTAINER SU...	16.747	-1.48	<input checked="" type="checkbox"/>	Aft
5	CAR,ARMD, CP, WHLED, 8...	-13.963	1.285	<input checked="" type="checkbox"/>	Aft
6	TRUCK,CARGO,2-1/2 TON...	-6.383	-1.48	<input checked="" type="checkbox"/>	Aft
7	20 FT ISO CONTAINER SU...	10.447	1.16	<input checked="" type="checkbox"/>	Aft
8	TRUCK,CARGO,2-1/2 TON...	-14.288	-1.455	<input checked="" type="checkbox"/>	Aft
9	20 FT ISO CONTAINER SU...	16.747	1.16	<input checked="" type="checkbox"/>	Aft
10	TRAILER, CARGO, 1-1/2 T...	5.087	-1.645	<input checked="" type="checkbox"/>	Aft
11	TRAILER,CARGO, 850 KG ...	-19.873	-1.76	<input checked="" type="checkbox"/>	Aft



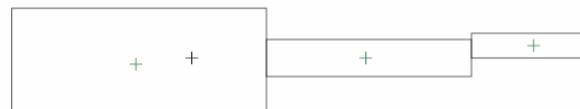
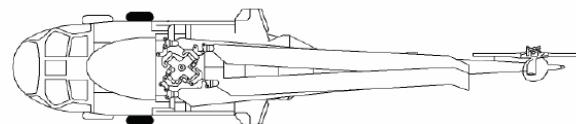
Enhancements



- Forced placement
- Load Ordering
- Macro Items:



- Obstacles
- Height Restrictions
- Incompatible-Items spacing
- Complex Items:
- etc.





Example: DART Chalk



- Canadian Forces Disaster Assistance Response Team to Sri Lanka

(2004 tsunami in South-East Asia)

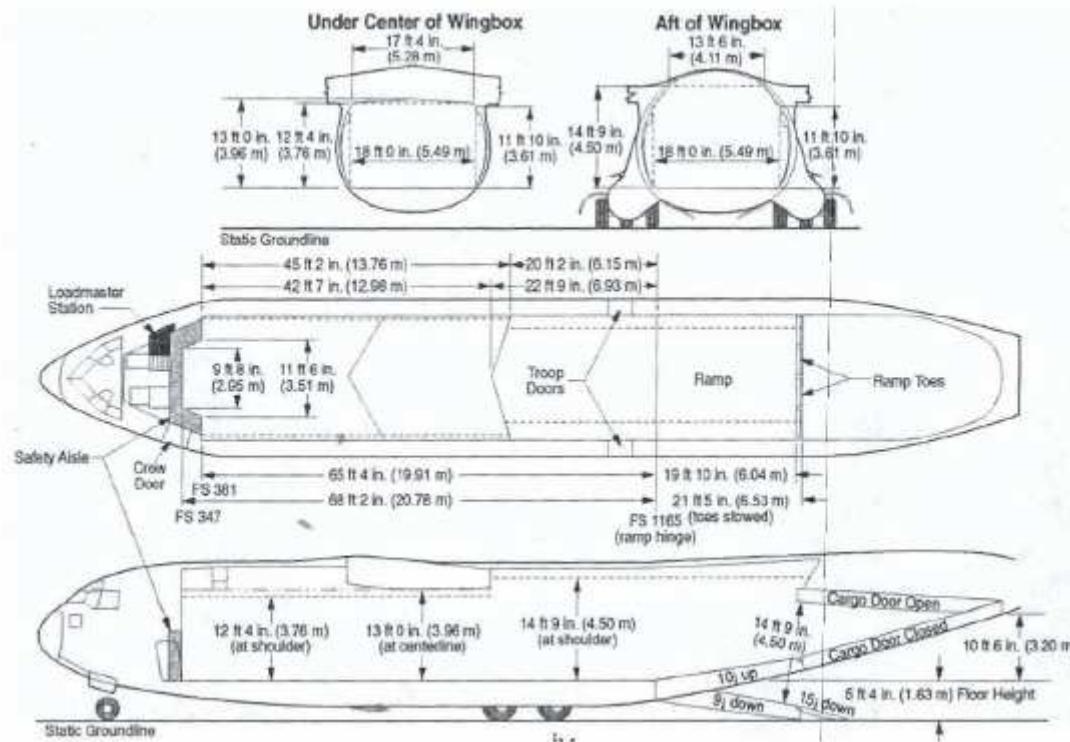
ID	Nomenclature	Weight (kg)	Length (mm)	Width (mm)	Height (mm)
0	60KW GEN ON TRLR	4064	4597	2591	2489
1	TRLE LSVW	575	3962	2032	2032
2	LSVW CGO	4053	5588	2007	2565
3	COY MLVW HIAB	11458	7976	2489	2692
4	SKID-STEER LOADE	3674	3759	1829	2134
5	COY CC DUAL	4055	6477	2438	1829
6	LSVW AMBULANCE	4787	5588	2007	2565
7	7500 LB/PALLET 5-4	2313	2743	2235	1575
8	4500 LB/PALLET 5-2	1624	2743	2235	1575
9	7500 LB/PALLET 4-4	1775	2743	2235	1575
10	4500 LB/PALLET 5-3	1860	2743	2235	1575



Example: DART Chalk



- Boeing C-17 Globemaster

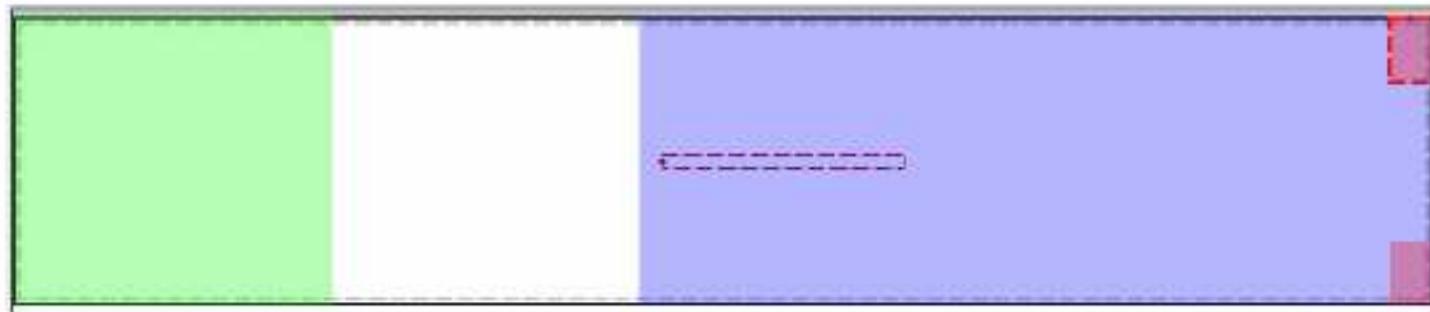




Example: DART Chalk



- Boeing C-17 Globemaster

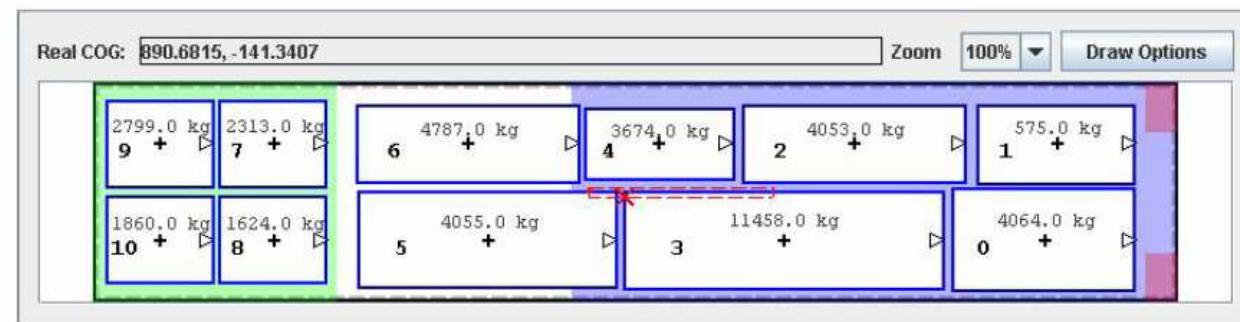




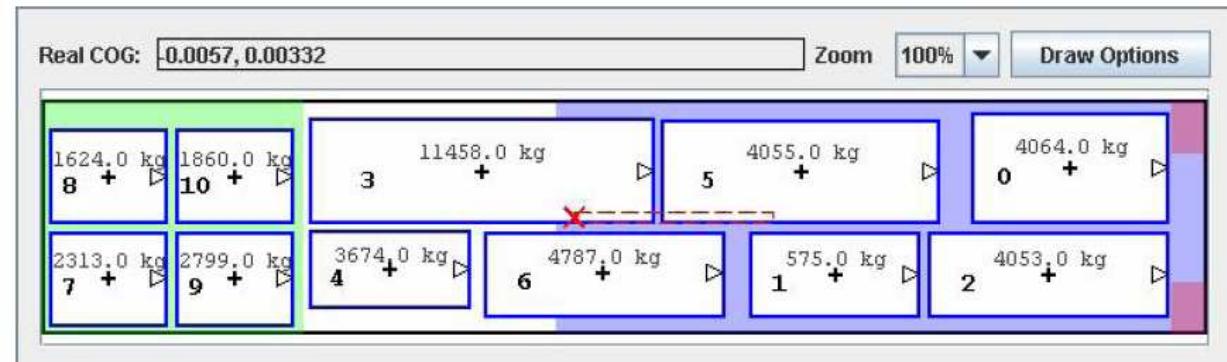
Example: DART Chalk



- CF Disaster Assistance Response Team to Sri Lanka post 2004 Tsunami in South-East Asia
- Boeing Packing:



- Proposed Packing:



RO



Example: DART Chalk



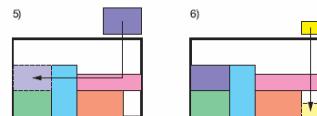
- CF Disaster Assistance Response Team to Sri Lanka post 2004 Tsunami in South-East Asia
- Comparison (millimeters from ideal CG)

	Load Planner			Chalk 4		
	ΔX	ΔY	$\sqrt{\Delta X^2 + \Delta Y^2}$	ΔX	ΔY	$\sqrt{\Delta X^2 + \Delta Y^2}$
Boeing	890	141	901			
CABAL	0	0	0			

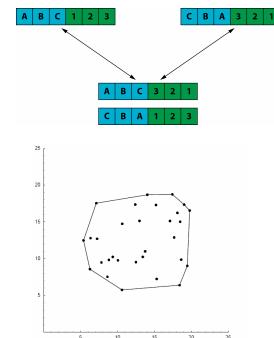
“Displacement from center of gravity of < 75cm over a 10,000km flight saves 4,000kg of fuel” - Mongeau & Bes (2003)



Summary of GALAHAD

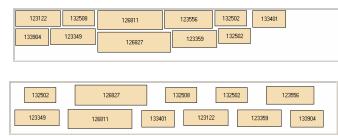


- Bin packing heuristic



- Genetic annealing engine

- Convex-hull-based fitness function



- MILP load balancing optimizer



Continuing Work...



- Current work focuses on:
 - Load Balancing as Extinction Phase
 - Optimizing center-of-gravity location
 - Cargo compatibility constraints
 - Incorporating item priorities into loading solutions
 - Performance improvements (both speed and memory)
- Future areas of research can/will include:
 - Full 3-D representations of cargo bays and manifest items
 - Detailed floor strength and tie-down planning





Variations



- **Maximize # of items taken:**

$$t_i = \begin{cases} 0 & \text{item } i \text{ left on ground,} \\ 1 & \text{item } i \text{ loaded onto plane.} \end{cases}$$

$$\max \sum_{i \in I} t_i$$

- **Hybrid Objective:**

$$\text{minimize} \quad \lambda \cdot (|XR_{cg}| + |YR_{cg}|) - (1 - \lambda) \cdot \sum_{i \in I} t_i,$$



Maximize Items Taken



$$t_i = \begin{cases} 0 & \text{item } i \text{ left on ground,} \\ 1 & \text{item } i \text{ loaded onto plane.} \end{cases}$$

$$\max \sum_{i \in I} t_i \quad \sum_{i \in I} m_i \cdot t_i \leq MAX_PAYLOAD,$$

Modified Constraints:

$$x_i - x_j \geq X_{overlap_i} + X_{overlap_j} + \max(sp_i, sp_j) - M \cdot (2 + m_{ij} + q_{ij} - t_i - t_j),$$
$$x_j - x_i \geq X_{overlap_i} + X_{overlap_j} + \max(sp_i, sp_j) - M \cdot (3 + m_{ij} - q_{ij} - t_i - t_j),$$



Maximize Items Taken



$$t_i = \begin{cases} 0 & \text{item } i \text{ left on ground,} \\ 1 & \text{item } i \text{ loaded onto plane.} \end{cases}$$

CG computation:

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i) \cdot t_i}{\sum_{i \in I} m_i \cdot t_i},$$



Maximize Items Taken


$$t_i = \begin{cases} 0 & \text{item } i \text{ left on ground,} \\ 1 & \text{item } i \text{ loaded onto plane.} \end{cases}$$

CG computation:

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i) \cdot t_i}{\sum_{i \in I} m_i \cdot t_i}$$





Maximize Items Taken



$$t_i = \begin{cases} 0 & \text{item } i \text{ left on ground,} \\ 1 & \text{item } i \text{ loaded onto plane.} \end{cases}$$

CG computation:

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i) \cdot t_i}{\sum_{i \in I} m_i \cdot t_i},$$

Trick:

$$x_i + \delta x_i \leq M \cdot t_i,$$

$$x_i + \delta x_i \geq -M \cdot t_i,$$

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i)}{\sum_{i \in I} m_i \cdot t_i},$$



Maximize Items Taken



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CG computation:

$$XR_{cg} = \frac{\sum_{i \in I} m_i \cdot (x_i + \delta x_i)}{\sum_{i \in I} m_i \cdot t_i},$$

$$\Delta XI_{cg}^- \leq XR_{cg} \leq \Delta XI_{cg}^+,$$

\Leftrightarrow

$$\Delta XI_{cg}^- \cdot \sum_{i \in I} m_i \cdot t_i \leq \sum_{i \in I} m_i \cdot (x_i + \delta x_i) \leq \Delta XI_{cg}^+ \cdot \sum_{i \in I} m_i \cdot t_i, \quad (\text{linear})$$