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Optimization of Surgery Delivery Systems

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Summary

- Optimization models for surgery planning and scheduling
- Stochastic Programming:
    - Problem 1: Single OR scheduling
    - Problem 2: Multi-OR Surgery Allocation
- Simulation Optimization:
    - Problem 3: Bi-criteria patient appointment scheduling
- Future research

Surgery Process

- <u>Patient Intake:</u> administrative activities, pre-surgery exam, gowning, site prep, anesthetic
- <u>Surgery:</u> incision, one or multiple procedures, pathology, closing
- <u>Recovery:</u> post anesthesia care unit (PACU), ICU, hospital bed

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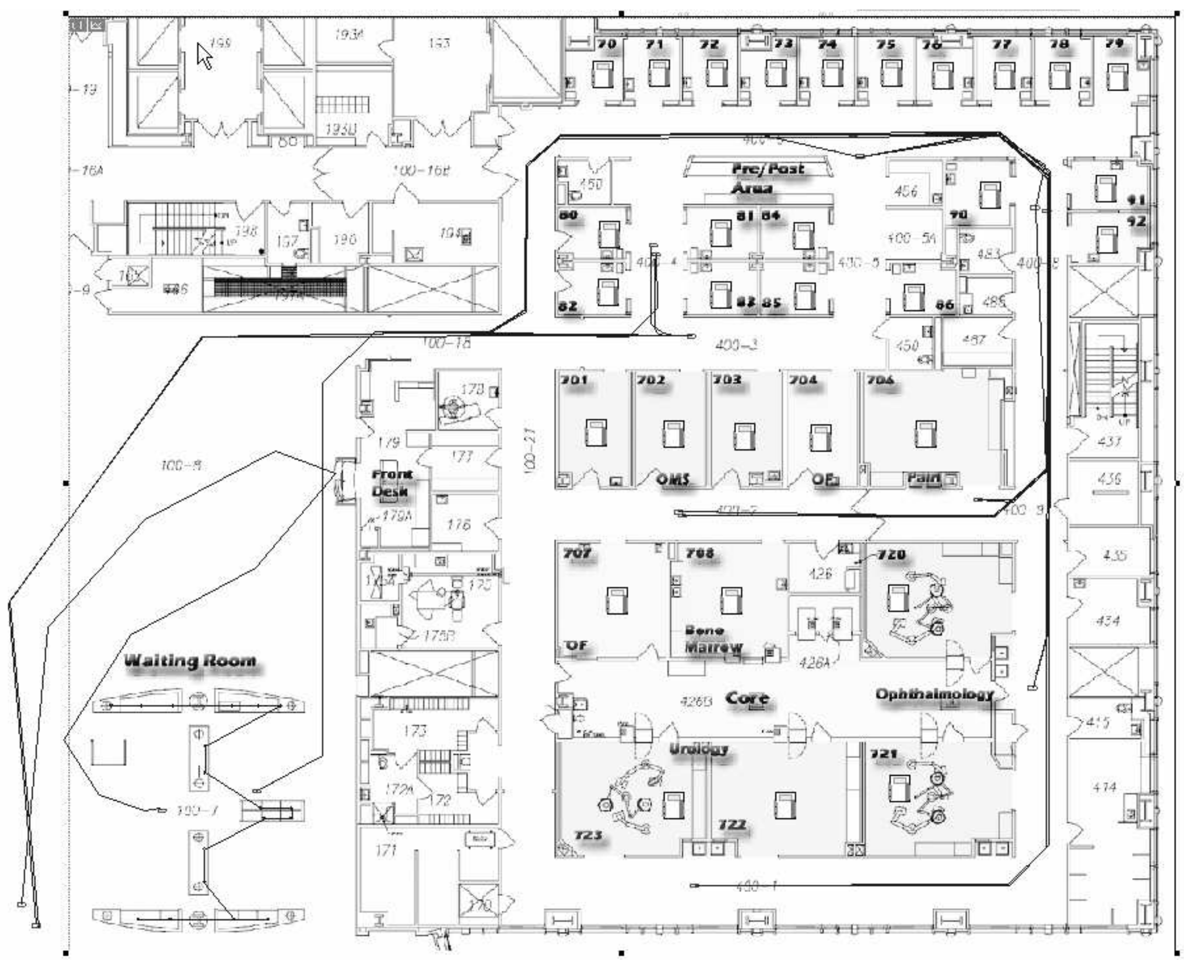
Outpatient Procedure Center



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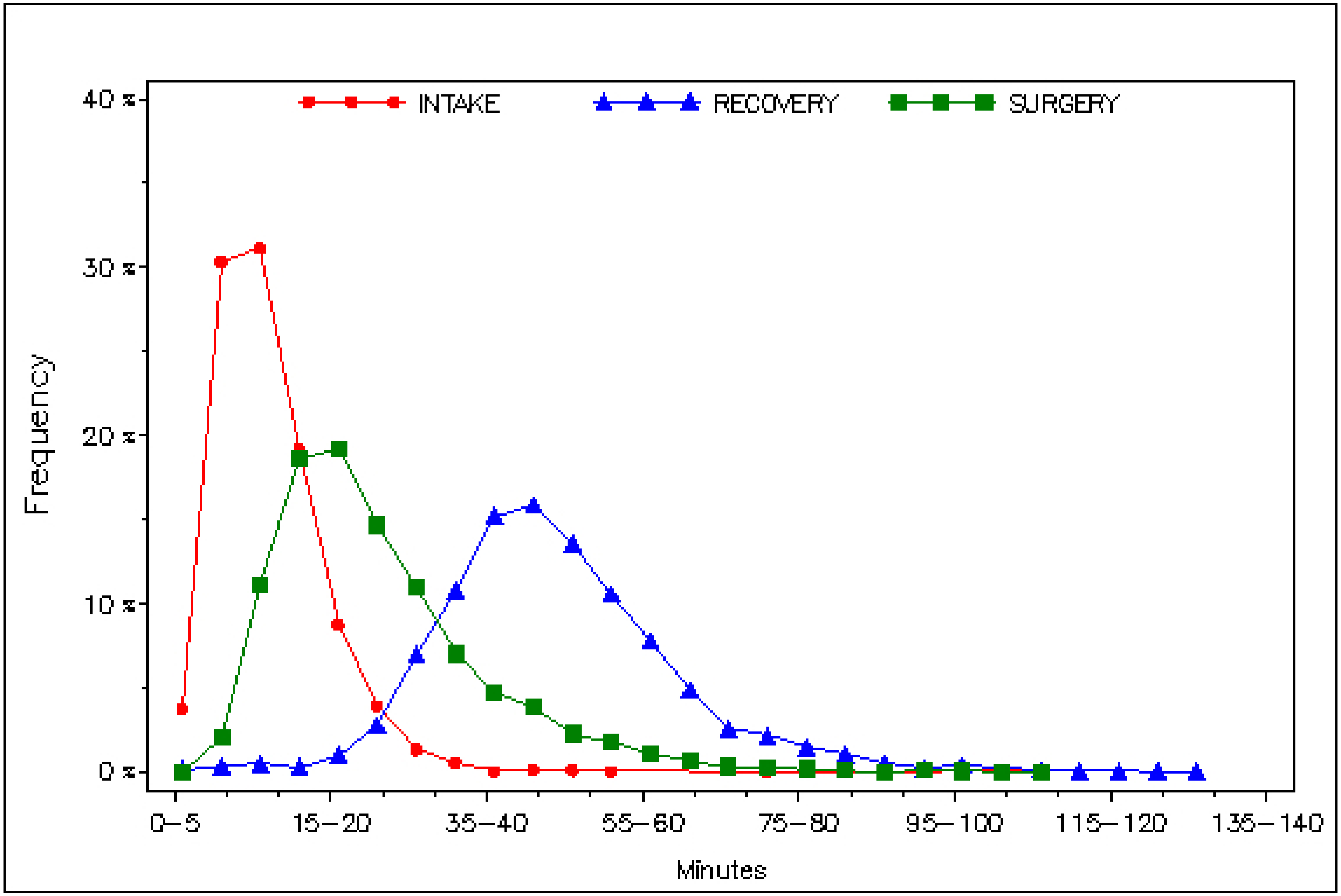
Complicating Factors

- Many types of resources to be scheduled: OR team, equipment, materials
- High cost of resources and fixed time to complete activities
- Large number of activities to be coordinated in a highly constrained environment
- Uncertainty in duration of activities
- Many competing criteria

Intake, Surgery, and Recovery



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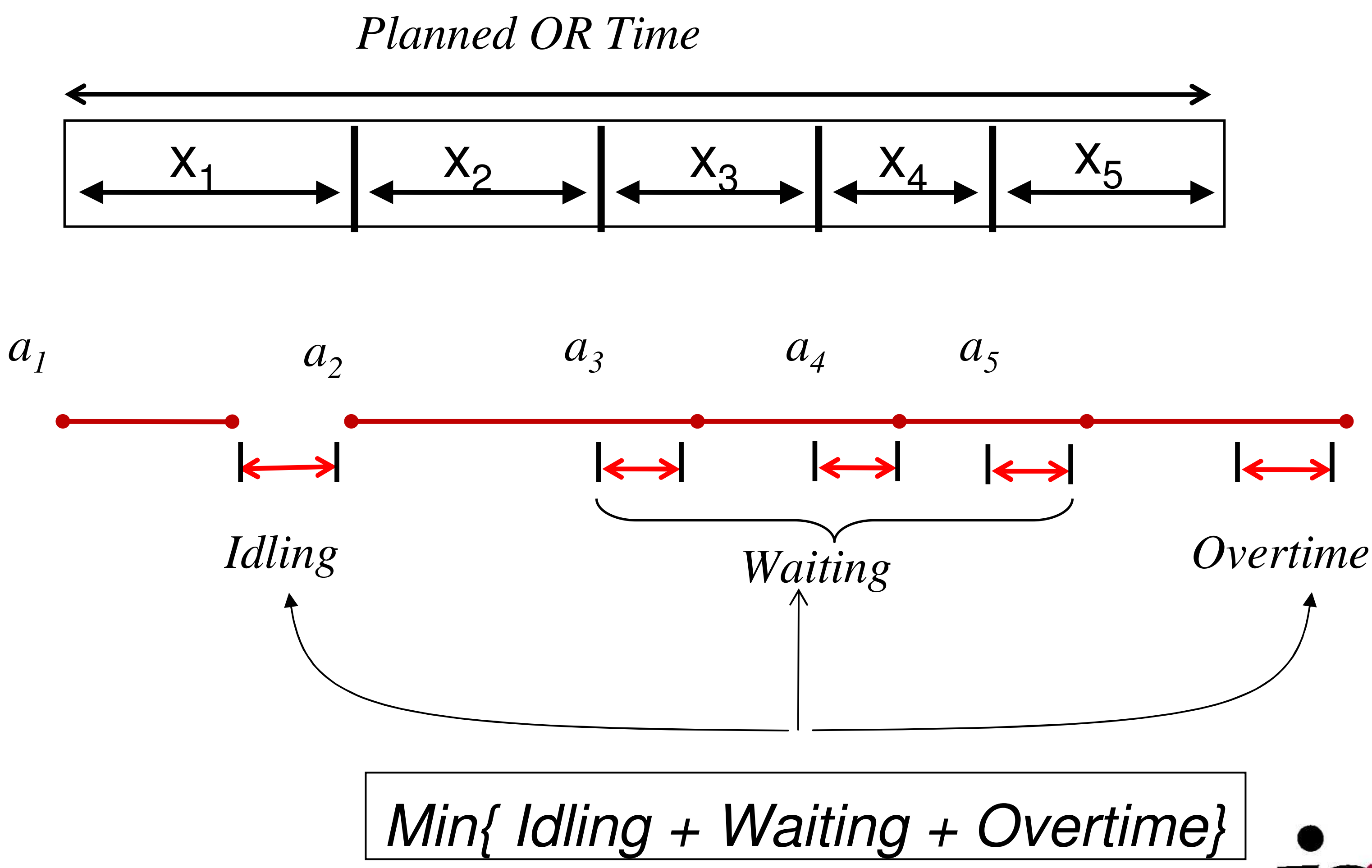
Problem 1: Single OR Scheduling

Single OR Scheduling - S(n)/G(n)/1



Stochastic Optimization Model

Cost of Waiting — Cost of Idling — Cost of Overtime

$$\min\{\sum_{i=1}^{n} C_i^w * E_{\mathbf{Z}}[W_i] + \sum_{i=1}^{n} C^s * E_{\mathbf{Z}}[S_i] + C^L * E_{\mathbf{Z}}[L]\}$$

$$W_i = \max(W_{i-1} + Z_{i-1} - x_{i-1}, 0)$$

$$S_i = \max(-W_{i-1} - Z_{i-1} + x_{i-1}, 0)$$

$$L = \max(W_n + Z_n + \Sigma x_i - d, 0)$$

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Stochastic Linear Program

$$\min\{E_Z[\sum_{i=2}^{n} c_i^w w_i + \sum_{i=2}^{n} c^s s_i + c^L l]\}$$

s.t.

$$\begin{array}{llll} w_2 & - s_2 & & = Z_1 - x_1 \\ -w_2 + w_3 & - s_3 & & = Z_2 - x_2 \\ \ddots & \ddots & & \\ -w_n & - s_n + l - g & & = Z_n - d + \sum_{j=1}^{n-1} x_i \end{array}$$

$$x_i \geq 0, w_i \geq 0, s_i \geq 0, i = 1,\ldots,n, \quad l, g \geq 0$$

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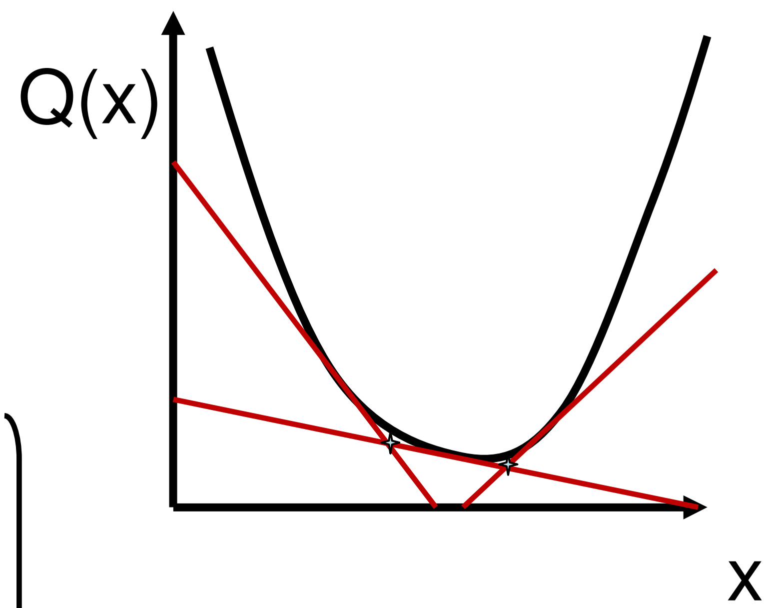
Two Stage Recourse Problem

Initial Decision (x) Uncertainty Resolved Recourse (y)

$$\min\{Q(\mathbf{x}) = E_{\mathbf{Z}}[Q(\mathbf{x},\mathbf{Z})]\}$$

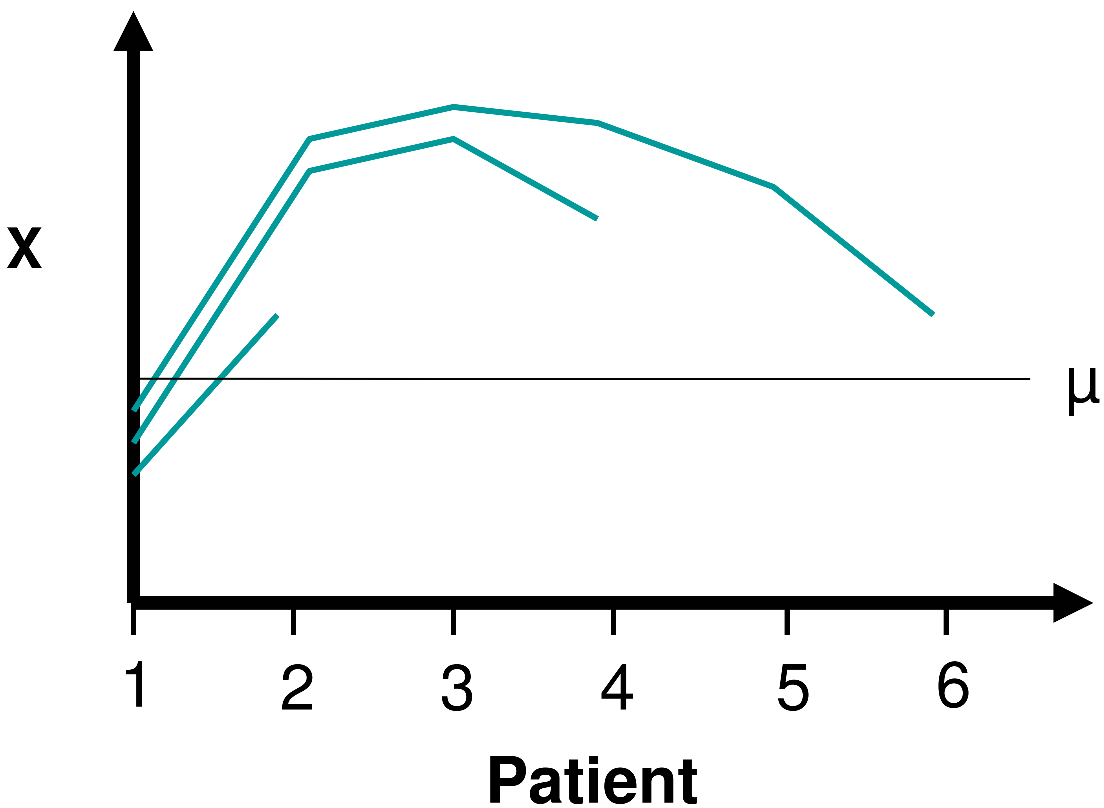
$$Q(\mathbf{x},\mathbf{Z}^k) = \min\{\mathbf{c}\cdot\mathbf{y}^k \mid T\mathbf{x} + W\mathbf{y}^k = \mathbf{h}^k, \mathbf{y}^k \geq 0\}$$

$$\begin{pmatrix} T & W^1 & & & \\ T & & W^2 & & \\ T & & & W^3 & \\ \vdots & & & & \ddots \\ T & & & & & W^K \end{pmatrix}$$



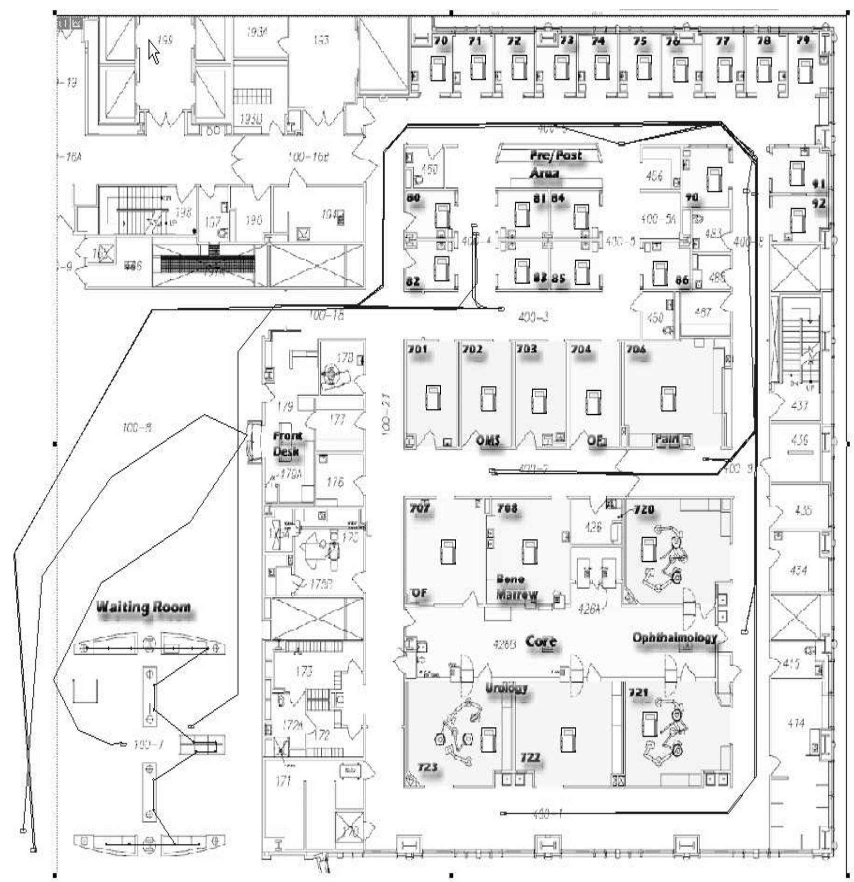
Example

- Comparison of surgery allocations for n=3, 5, 7 with i.i.d. distributions with U(1,2):



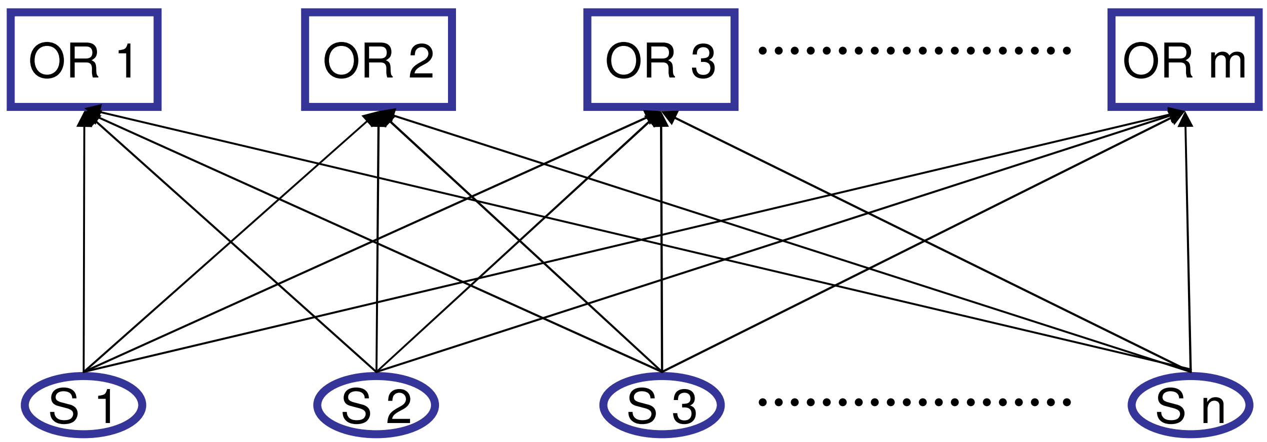
Surgical Suite Decisions

- How to design the suite (intake rooms, recovery rooms, ORs)
- Number of cases to schedule
- Number of ORs and staff to activate each day
- Surgery-to-OR assignment decisions
- Scheduling patients arrivals



Problem 2: Multi-OR Surgery Allocation

Multi-Operating Room Scheduling



Decisions:

- How many operating rooms (ORs) to open?
- Which OR to schedule each surgery in?

Performance Measures:

- Cost of operating rooms opened
- Overtime costs for operating rooms

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Extensible Bin Packing

$$x_j = \begin{cases} 1 \ \ if \ OR \ \ j \ open \\ 0 \ \ if \ \ OR \ \ j \ closed \end{cases} \qquad y_{ij} = \begin{cases} 1 \ \ if \ Surgery \ \ i \ assigned \ \ to \ OR \ \ j \\ 0 \ \ \ Otherwise \end{cases}$$

$$Z = \min\{\sum_{j=1}^{m} c^f x_j + c^v o_j\}$$

$$s.t. \qquad y_{ij} \leq x_j \quad \forall(i,j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \ \ \forall(i)$$

$$\sum_{i=1}^{n} d_i y_{ij} - o_j \leq T_j x_j \quad \forall(i,j)$$

$$y_{ij}, x_j \in \{0,1\}, \quad o_j \geq 0$$

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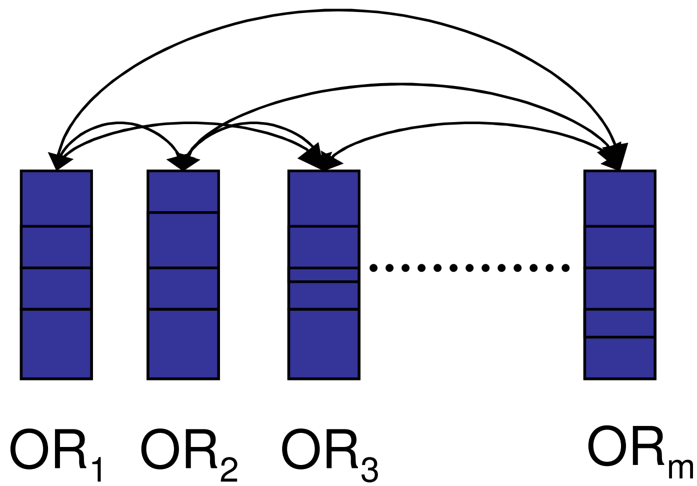
Symmetry

- m! optimal solutions:



- Anti-symmetry constraints:

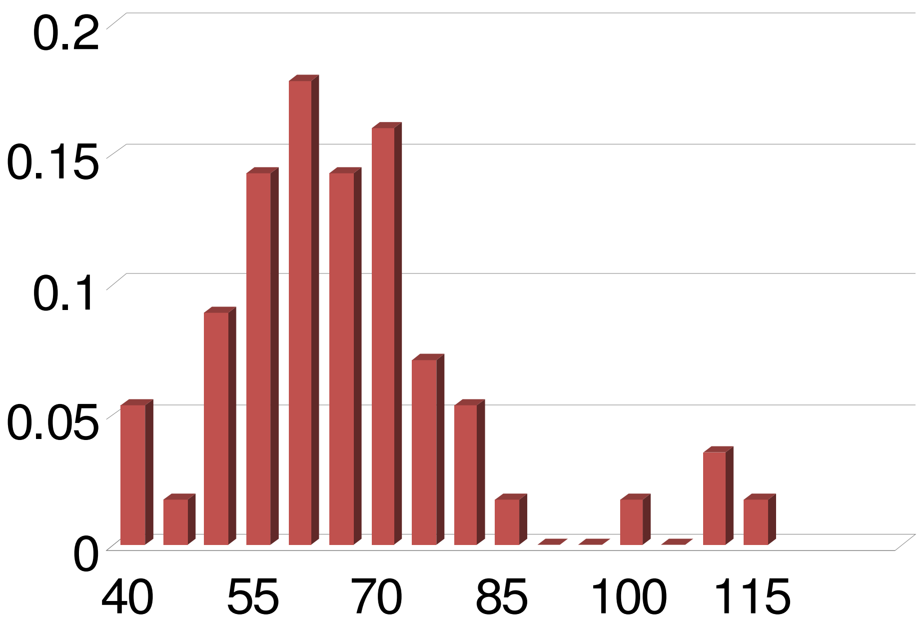
$$x_1 \geq x_2$$
$$x_2 \geq x_3$$
$$\vdots$$
$$x_m \geq x_{m-1}$$

OR Ordering

$$y_{11} = 1$$
$$y_{21} + y_{22} = 1$$
$$\vdots$$
$$\sum_{j=1}^{m} y_{mj} = 1$$

Surgery Assignment

Two-Stage Stochastic MIP



$$Q(\mathrm{x}) = \min\{\sum_{j=1}^{m} c^{f} x_j + c^{v} E_{\omega}[o_j(\omega)]\}$$

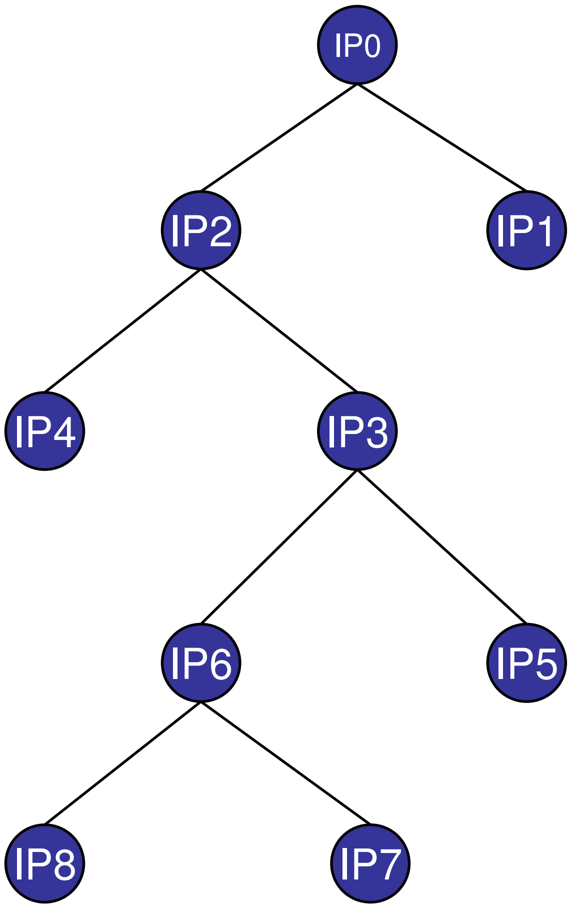
$$s.t. \quad y_{ij} \leq x_j \quad \forall (i, j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i)$$

$$\sum_{i=1}^{n} d_i(\omega) y_{ij} - o_j(\omega) \leq T x_j \quad \forall (i, j, \omega)$$

$$y_{ij}, x_j \in \{0,1\}, \quad o_j(\omega) \geq 0, \forall \omega$$

Integer L-Shaped Method



Master Problem:

$$Z = \min\{ \sum_{j=1}^{m} c^{f} x_{j} + \Theta \}$$

$$s.t. \quad y_{ij} \le x_{j} \quad \forall (i, j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i)$$

$$y_{ij}, x_{j} \in \{0,1\}, \Theta \ge 0$$

$$\Theta \ge E_{\omega}[\pi(h - Tx)]$$

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Heuristic and Bounds

Dell'Ollmo (1998) – 13/12 approximation algorithm for bin packing with extensible bins

EBP Heuristic:

$$n \leftarrow LB;$$
$$repeat;$$
$$\quad LPT(n);$$
$$\quad if\,(o_j = 0, \forall j)\;\; Stop;$$
$$\quad n \leftarrow n+1;$$
$$end(repeat);$$

$$LB = \left\lceil \frac{\sum_{i=1}^{n} d_i}{T\left(1+\frac{c^f}{c^v T}\right)} \right\rceil$$

- Sort surgeries from longest to shortest
- Sequentially apply surgeries to emptiest room

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Robust Formulation

$$Z = \min\{ \sum_{j=1}^{m} c^f x_j + F(x, y) \}$$

$$s.t. \quad y_{ij} \leq x_j \quad \forall (i, j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i)$$

$$y_{ij}, x_j \in \{0,1\} \geq 0$$

$$F(x, y) = \begin{cases} \max_{\delta} \{ \sum_{j=1}^{m} \eta_j \} \\ s.t. \quad \eta_j = c_j^v \max\{0, \sum_{i: y_{ij}=1} \delta_{ij} y_{ij} - T_j x_j\}, \quad \forall j \\ \sum_{(i,j): y_{ij}=1}^{m} \frac{\delta_{ij} - \underline{d}_i}{\overline{d}_i - \underline{d}_i} y_{ij} \leq \tau \\ \underline{d}_i \leq \delta_{ij} \leq \overline{d}_i, \forall (i, j) : y_{ij} = 1 \end{cases}$$

	15 surgery instances									
	Variable Cost = 0.033					Variable Cost = 0.0083				
			Robust IP					Robust IP		
Instance	MV_IP	LPT_Heu	Tau=2	Tau=4	Tau=6	MV_IP	LPT_Heu	Tau=2	Tau=4	Tau=6
1	0.808	0.806	0.892	0.906	0.933	0.999	0.998	0.880	0.948	0.948
2	0.953	0.966	0.898	0.896	0.970	0.999	0.999	0.999	0.999	0.980
3	0.854	0.852	0.936	0.937	0.970	0.999	0.999	0.929	0.952	0.944
4	0.925	0.972	0.911	0.971	0.917	0.999	0.998	0.930	0.930	0.929
5	0.896	0.946	0.831	0.916	0.892	0.990	0.996	0.932	0.938	0.924
6	0.862	0.853	0.923	0.931	0.938	0.989	0.990	0.886	0.881	0.881
7	0.930	0.936	0.810	0.930	0.817	0.973	0.993	0.844	0.974	0.927
8	0.888	0.966	0.876	0.903	0.904	0.966	0.966	0.966	0.987	0.939
9	0.962	0.966	0.964	0.969	0.964	0.975	0.993	0.847	0.960	0.957
10	0.860	0.924	0.910	0.893	0.918	0.997	0.996	0.900	0.901	0.903
average	0.894	0.919	0.895	0.925	0.922	0.988	0.993	0.916	0.951	0.933
stdev	0.046	0.057	0.047	0.028	0.046	0.013	0.010	0.059	0.045	0.028
max	0.962	0.972	0.964	0.971	0.970	0.999	0.999	1.042	1.040	0.980
min	0.808	0.806	0.810	0.893	0.817	0.966	0.966	0.844	0.881	0.881

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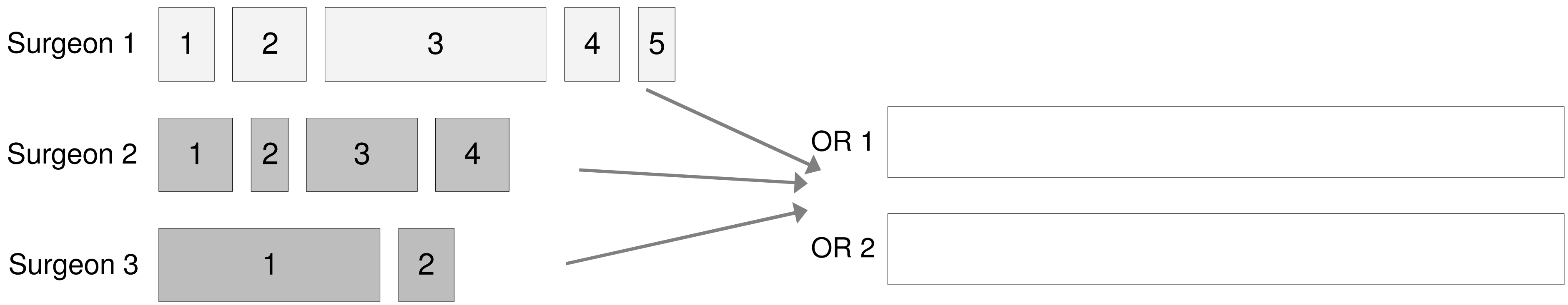
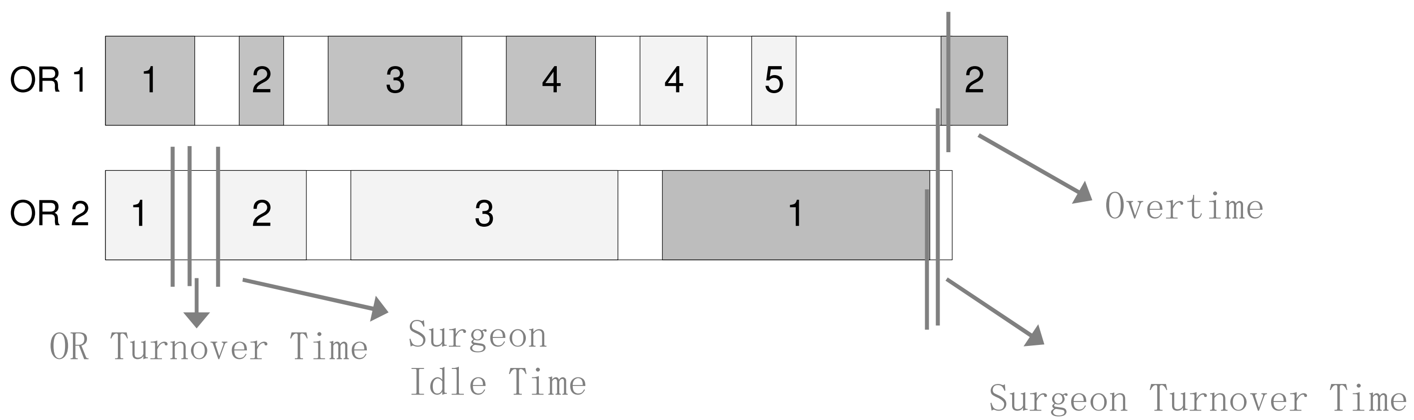
General Insights

- The fast LPT based heuristic works (fairly) well on a large number of instances
  - LPT works very well when overtime costs are low
  - LPT is better (and easier) than solving MV problem in most cases
- Robust IP is better than LPT but worse than Stochastic IP

Current Research: Setups and Parallel Processing





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Problem 3: Patient Arrival Scheduling

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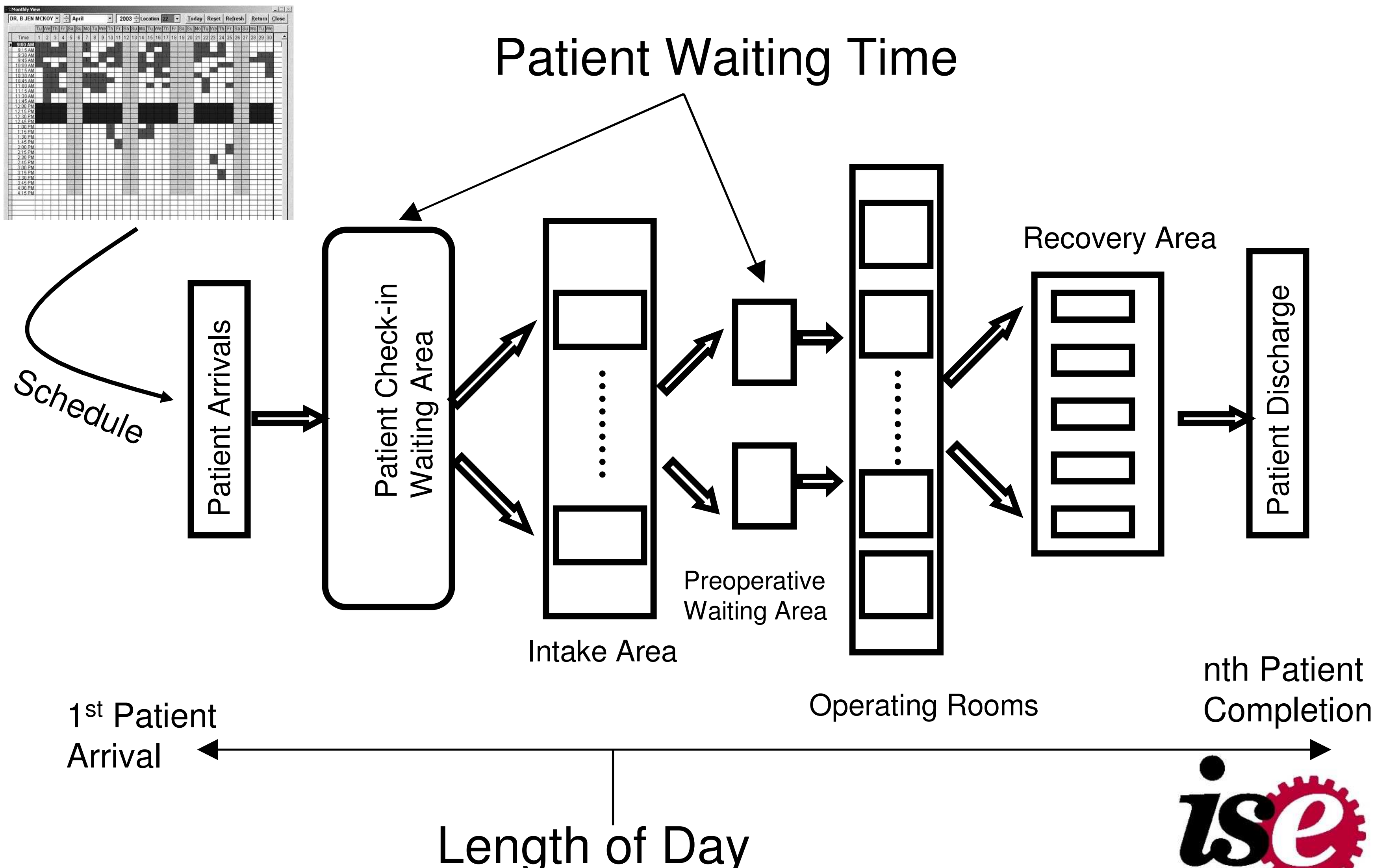
Endoscopy Suite

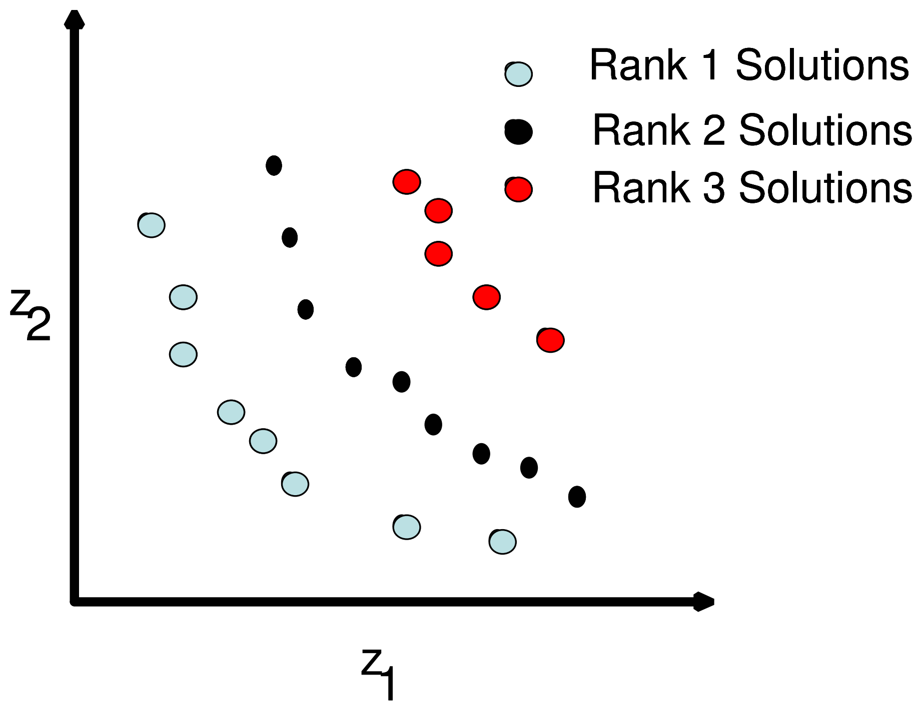


Simulation-optimization Summary

- Decision variables: scheduled start times to be assigned to *n* patients each day
- Goal: Generate the set of non-dominated schedules to understand tradeoffs between waiting and session length
- Schedules generated using a genetic algorithm (GA)
- Non-dominated sorting used to identify the Pareto set and feedback into GA

Pareto Set

- Non-dominated sorting genetic algorithm of Deb *et al.*(2000) is used for ranking



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Selection Procedure

- Sequential two stage indifference zone ranking and selection procedure of Rinott (1978) is used to compute the number of runs sufficient to determine whether a solution *i* "dominates" *j*

- Solution *i* "dominates" *j* if:

$$E[W_i] < E[W_j] \quad \text{and} \quad E[L_i] < E[L_j]$$

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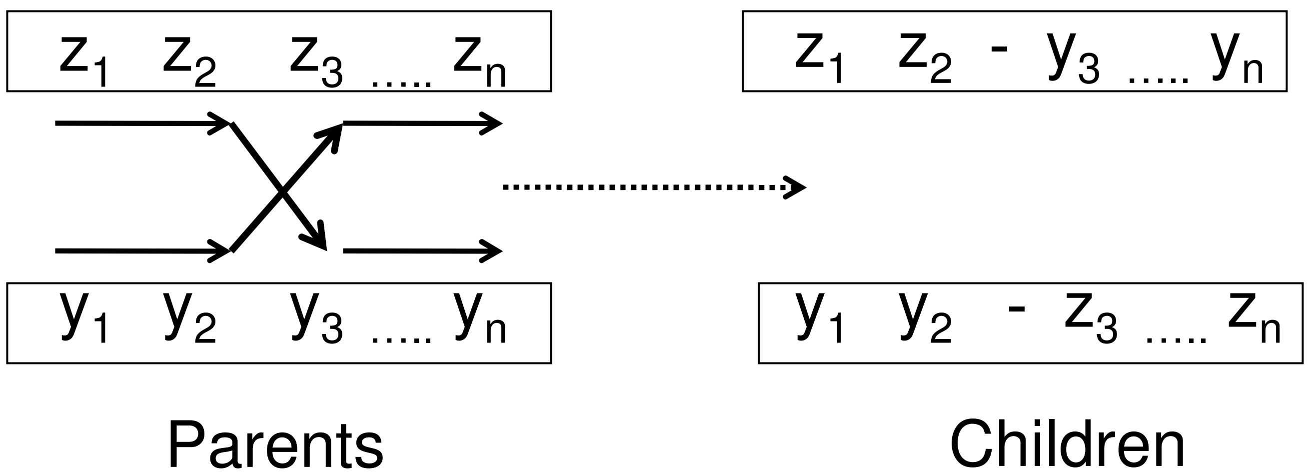
Genetic Algorithm

- Main features of the GA:
  - Randomly generated initial population of schedules
  - Single point crossover:



  - Mutation
  - Selection based on 1) ranks and 2) crowding distance

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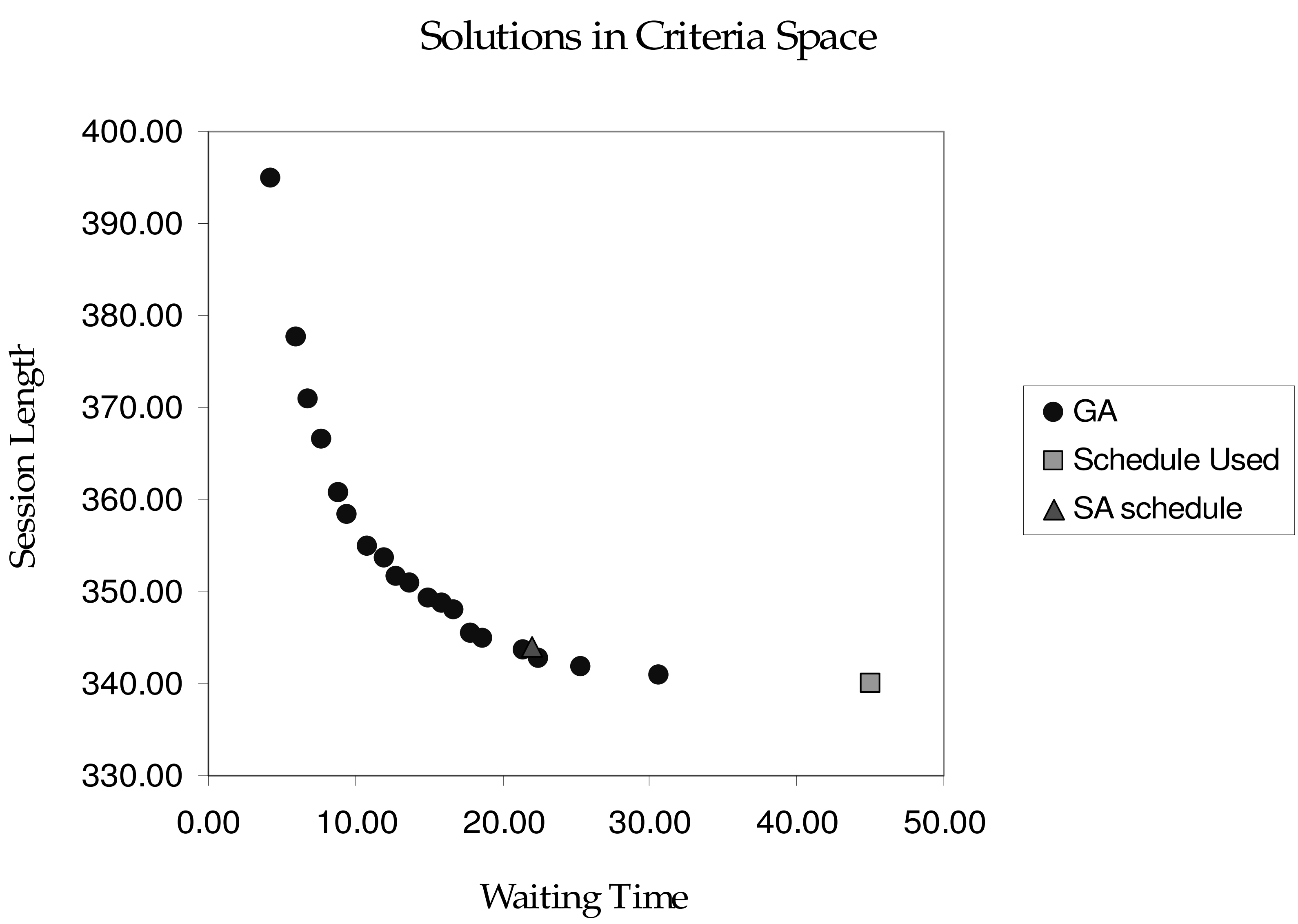
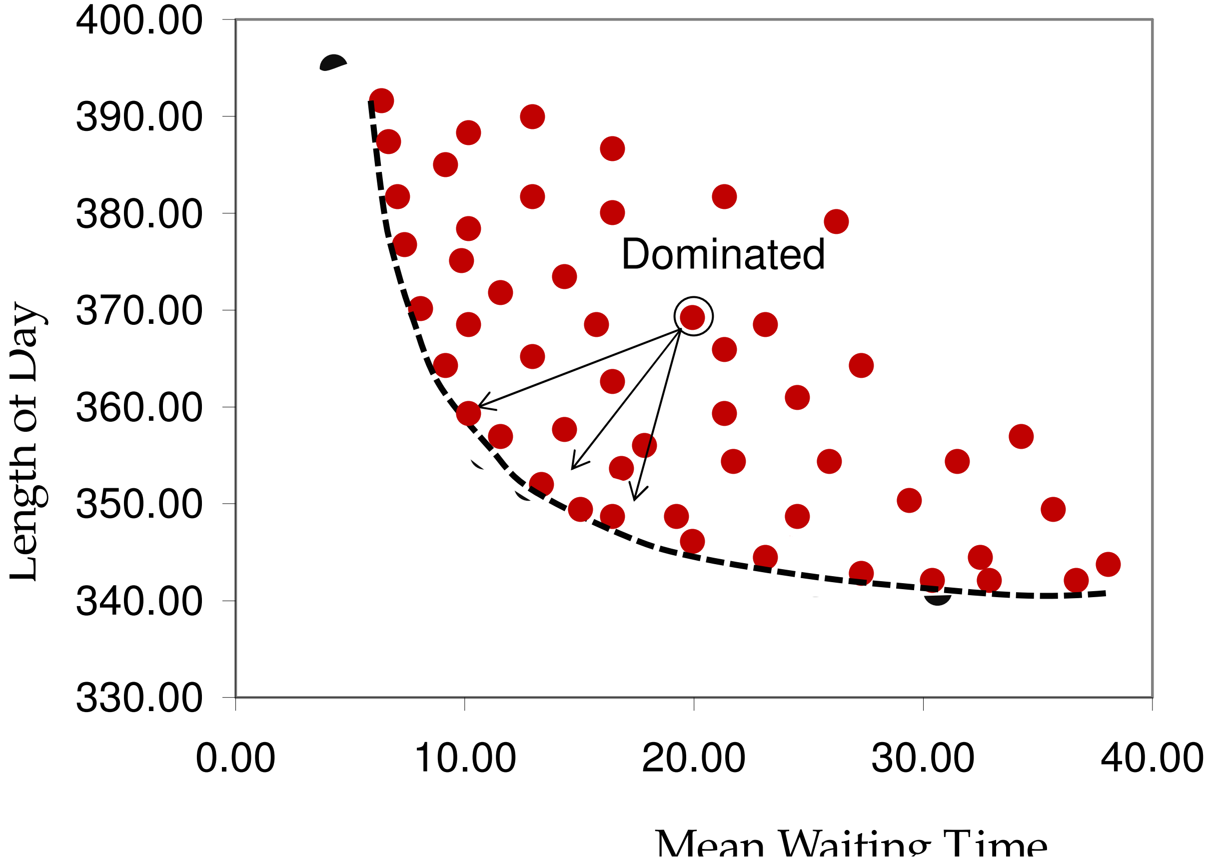
Example



Numerical Results



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Current and Future Research

- Investigating new stochastic programming and robust optimization formulations and methods
- Dynamic (online) scheduling problems
- Surgical suite design and re-configuration

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Questions?