The Impact of Financial Constraints on Individual Asset Allocations: Under-Diversification and Asset Selection

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- Financial constraints are an important market friction
- Want to capture the impact of financial constraints on different types of investors: e.g., young vs. old, investors vs employees, etc.
- Want to quantify the asset pricing implications of financing constraints
- Empirical facts: largely attributed to irrational behavior
  - young people hold undiversified portfolios
  - diversification increases with age, wealth
- Can financial constraints provide a rational explanation for portfolio under-diversification?
- Can financial constraints explain observed deviations from the Capital Asset Pricing Model?

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- Asset Allocation with Frictions:
  - Cox & Huang (89), Karatzas, Lehoczky, Shreve & Xu (87), Cvitanic & Karatzas (92,93), He & Pearson (91), Shreve & Xu (92)
- Asset Allocation with Non-negative Wealth:
  - He & Pagès (93), Duffie, Fleming, Soner & Zariphopoulou (97), El Karoui & Jeanblanc-Picqué (98), Detemple & Serrat (03)
- Asset Allocation with Margin/VaR Constraints
  - Cuoco (97), Teplá (00), Cuoco & Liu (00,05), Davis, Kubler & Willen (06)
- General Equilibrium and Financial Constraints
  - Black (72), Constantinides, Donaldson and Mehra (02)

Individual Portfolios — Empirical Evidence

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- Kelly (95): $\overline{N} = 1$, Rich individuals $\overline{N} = 10$
- Ivkovic, Sialm & Weisbeinner (04): $\overline{N} = 3.9, \overline{N} = 2.4(W \leq \$25,000), \overline{N} = 7(W \geq \$25,000), \overline{N} = 11.7(W \geq \$100,000)$
- Polkovnichenko (05): $\overline{N} \leq 4(W \leq \$1M), \overline{N} = 14(W \geq \$1M)$
- Goetzmann & Kumar (05): $\overline{N} = 4, \overline{N} \geq 10(W \geq \$100,000)$
- Calvet, Campbell & Sodini (06)

Partial Equilibrium — Preview of Results

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- When the constraints bind the investor faces a tradeoff between diversification and higher expected return
- The optimal asset allocation strategy is characterized by thresholds in the ratio of financial wealth to income — as the ratio decreases the investor holds progressively fewer assets and less diversified positions.
- For very low levels of the financial wealth to income ratio, the investor holds only one asset: the asset that provides the highest leveraged expected return, adjusted for labor income correlation, regardless of volatility

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(joint with Hervé Roche and Chunyu Yang)

- Mutual Fund Separation Theorem:
  - Differences in risk aversion across investors imply differences in allocation between risk-free and risky assets
  - Within the risky part of a portfolio, all investors should hold exactly the same mix of risky assets (fully diversified portfolio)
- Intuition:
  - Increasing allocation to risky assets scales up both risk and return — investor wants to maximize the return/risk ratio within the risky part of the portfolio
  - Fully diversified portfolio has the highest return/risk ratio

Intuition for Key Result

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- Consider an investor for whom the financial constraint is binding:
  - Holding a fully diversified portfolio gives too low a return; investor would like to increase leverage but can not because of the financial constraint
  - Therefore the investor shifts away from a fully diversified portfolio, increasing holdings of stocks with highest expected returns
  - This shift makes the investor better off but exposes him/her to idiosyncratic risk

Two period model, mean-variance utility

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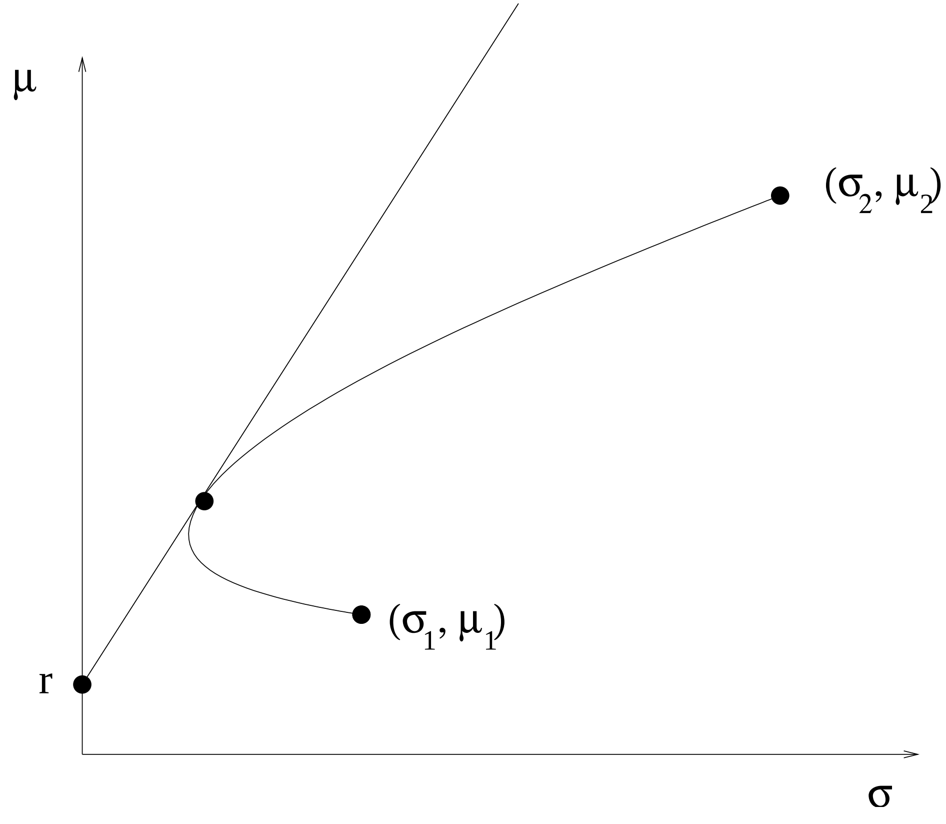
- Two independent risky assets, $\mu_1 \neq \mu_2$
- Mean-variance utility from terminal wealth
- Initial wealth W in period 1, income Y received in period 2
- No borrowing/shorting allowed



- High ratio of W/Y, both assets held
- Lower ratio of W/Y, shadow riskfree rate increases
- Low ratio of W/Y, only one asset held

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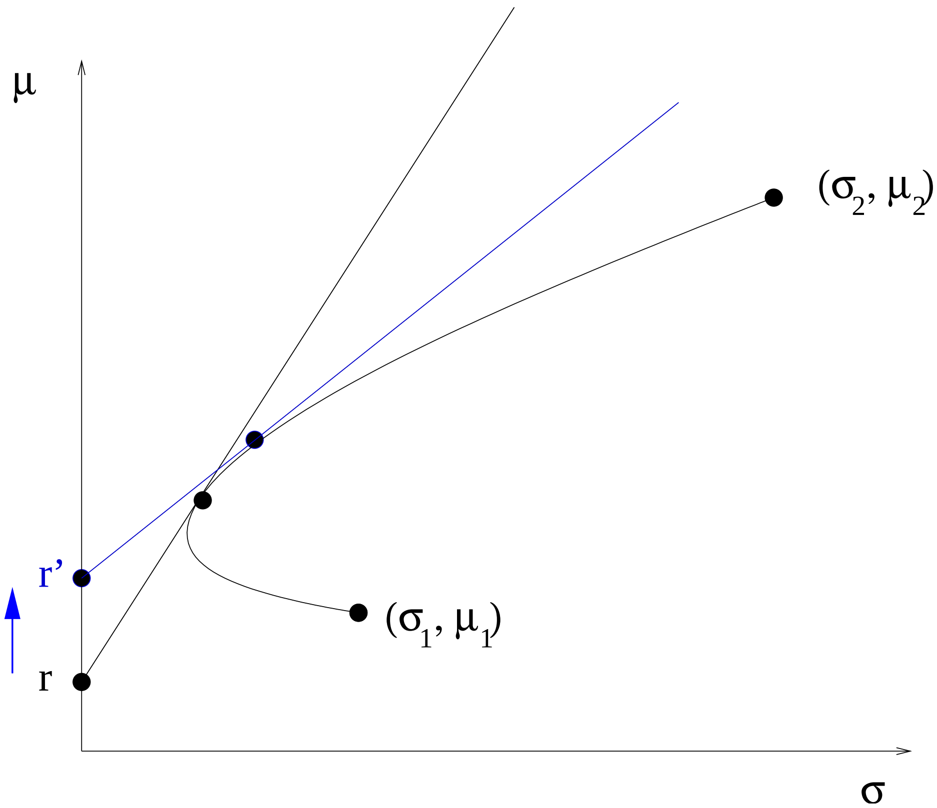

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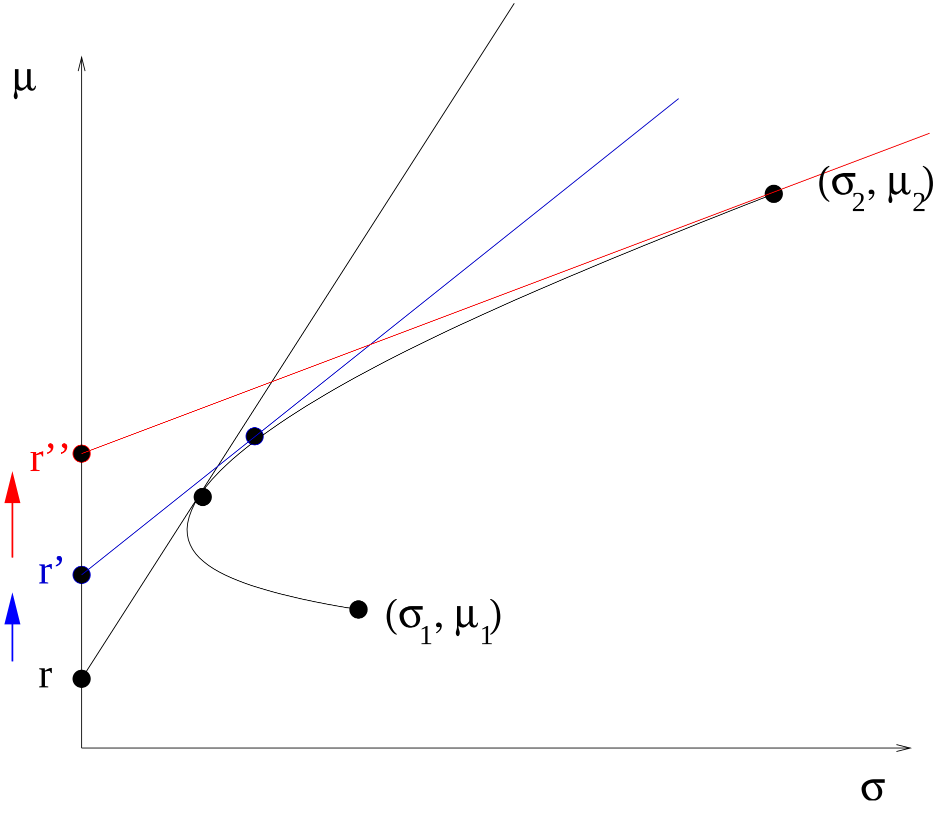

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- Standard Merton optimum consumption-portfolio problem
- Infinite investor horizon
- Three financial assets
  - riskless bond B

$$dB_t = rB_t dt$$

  - two (independent) risky securities S_i

$$dS_{it} = S_{it}\left(\mu_i dt + \sigma_i dw_{it}\right)$$

- Margin requirements:

$$Q = \{(z_1, z_2) \in \mathbb{R}^2, \lambda^+(z_1^+ + z_2^+) + \lambda^-(z_1^- + z_2^-) \leq W\},$$

with z_i dollar amount invested in asset i (regulation T: 50% margin for long positions, 150% for short positions)

Economic Setting, cont.

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- Labor income spanned by the risky assets

$$dY_t = Y_t \left(m dt + \Sigma_1 dw_{1t} + \Sigma_2 dw_{2t} \right)$$

- CRRA investor with CRRA coefficient $\gamma > 0$ and time discount factor $\theta > 0$
- Parameter restrictions
  - margin constraints not binding when initial wealth is high relative to income
  - demand for the risky assets increases as income level increases

$$\sum_{i=1}^{2} \frac{\Sigma_i}{\sigma_i} \lambda_i < \sum_{i=1}^{2} \frac{\mu_i - r}{\gamma \sigma_i^2} \lambda_i < 1$$

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Maximize discounted utility from consumption:

$$F(W_t, Y_t) = \max_{(c,\ (x,z) \in Q)} E_t \left[\int_t^{\infty} \frac{c_s^{1-\gamma}}{1-\gamma} e^{-\theta(s-t)} ds \right]$$

where Q is the set of admissible portfolios

Wealth evolution

$$dW_s = \left(rW_s - c_s + Y_s + z_s^{\mathsf{T}} (\mu - r\bar{1}) \right) ds + \sigma z_s^{\mathsf{T}} dw_s$$

Income

$$dY_s = Y_s \left(m ds + \Sigma^{\mathsf{T}} dw_s \right)$$

Initial conditions

$$W_t > 0, Y_t > 0$$

No Financial Constraints

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Intuition: add discounted income to financial wealth and choose allocation according to the sum of financial wealth plus discounted income, adjusting for correlations between income and the returns of the risky assets

$$
\begin{aligned}
z_{it}^f &= \frac{\mu_i - r}{\gamma \sigma_i^2} W_t + B \left(\frac{\mu_i - r}{\gamma \sigma_i^2} - \frac{\Sigma_i}{\sigma_i} \right) Y_t \\
c_t^f &= \frac{W_t + B Y_t}{A},
\end{aligned}
$$

where

$$
\begin{aligned}
A^{-1} &= \frac{\theta}{\gamma} + \frac{\gamma - 1}{\gamma} \left(r + \frac{(\mu - r)^2}{2\gamma\sigma^2} \right) > 0 \\
B &= \frac{1}{r - m + \sum_{i=1}^{2} \frac{\Sigma_i (\mu_i - r)}{\sigma_i}} > 0
\end{aligned}
$$

Financial Constraints

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- The value function, F, is strictly increasing and concave in W and homogeneous of degree $1-\gamma$ in (W, Y), so

$$F(W, Y) = Y^{1-\gamma} f(v),$$

with $v = \frac{W}{Y}$

- Three regions characterized by cutoffs of the *effective* relative risk aversion coefficient

$$y = -\frac{W F_{11}}{F_1},$$

or equivalently by ratios of financial wealth to income

Region 1: Non-Binding Region

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- High levels of relative risk aversion
- High levels of financial wealth to income ratios
- Both risky and riskless assets held in nonzero amounts

$$\gamma < \left(1 - \sum_{i=1}^{2} \frac{\Sigma_i}{\sigma_i} \lambda_i\right)^{-1} \gamma \sum_{i=1}^{2} \left(\frac{\mu_i - r}{\gamma \sigma_i^2} - \frac{\Sigma_i}{\sigma_i}\right) \lambda_i < y$$

Region 2: Binding Region, Two Assets Held

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- Intermediate levels of relative risk aversion
- Intermediate levels of financial wealth to income ratios
- Only risky assets held in nonzero amounts

$$\max\ \{y_i^*, y_j^*\} < y < \left(1 - \sum_{i=1}^{2} \frac{\Sigma_i}{\sigma_i}\lambda_i\right)^{-1} \gamma \sum_{i=1}^{2} \left(\frac{\mu_i - r}{\gamma\sigma_i^2} - \frac{\Sigma_i}{\sigma_i}\right)\lambda_i,$$

with

$$y_k^* = \frac{\lambda_k(\lambda_h(\mu_k - r - \gamma\sigma_k\Sigma_k) - \lambda_k(\mu_h - r - \gamma\sigma_h\Sigma_h))}{\lambda_h\sigma_k^2 + \lambda_k(\lambda_k\sigma_h\Sigma_h - \lambda_h\sigma_k\Sigma_k)}, \quad k \neq h$$

Region 3: Binding Region, One Asset Held

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- Low levels of relative risk aversion
- Low levels of financial wealth to income ratios
- Only one risky asset held

$$0 \leq y \leq \max\{y_i^*, y_j^*\}.$$

In the case of more than 2 risky assets, the results generalize and one obtains $n+1$ regions, as assets are progressively dropped from the optimal portfolio.

Properties of Optimal Strategy

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- Given (W, Y), risky asset positions always smaller than in the case without constraints
- Asset ultimately selected: the one with highest leveraged expected excess return adjusted for labor correlation:

$$\frac{\mu_i - r - \gamma \sigma_i \Sigma_i}{\lambda_i}$$

- Given the same initial wealth and income, consumption is always smaller in the case with financial constraints

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- The five risky assets correspond to the stock indices of five industries: consumer, manufacturing, high tech, health, other.
- The expected returns, variances, and correlations between these five stock indices are estimated using the data from the website of Ken French.
- The drift and volatility of the income stream is calibrated as in Viceira(2001).

Model Parameters

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Periods					45 years (age 20 to age 65)
Risk aversion					3
Long margin					0.5
Short margin					0.5
Discount rate					0.98
Interest rate					1.4%
Income growth					3%
Asset drift	8.7%	9.7%	10.3%	10.8%	10.0%
Volatility	28.9%	25.8%	33.3%	26.6%	29.7%
Sharpe ratio	25.3%	37.6%	31.0%	40.6%	33.7%
Correlations	1.000	0.898	0.832	0.732	0.932
		1.000	0.848	0.698	0.930
			1.000	0.772	0.856
				1.000	0.727
					1.000

Discretization

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- Stock prices $\left\{X_t^i\right\}_{i=1}^5$ and income Y_t follow geometric Brownian motion with correlation matrix ρ
- Standard discretization scheme $Z_{t+\Delta t} - Z_t = \sqrt{\Delta t} AN$
  - $A_{6\times 6}$ is the Cholesky factorization of ρ
  - $N_{6\times 1} \sim MVN\left(0_{6\times 1}, I_{6\times 6}\right)$
- The drawback of sampling from MVN is that extreme returns that can cause negative wealth may be generated
- He(1990): discretize a m-dimensional MVN with a m-dimensional $(m+1)$-nomial discrete distribution
  - Deterministic Income (5 factors): sampling from a 5-D discrete distribution with 6 states
  - Stochastic Income (6 factors): sampling from a 6-D discrete distribution with 7 states

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- Step 1: Simulate n_p paths of stock excess returns and income growth rate
- Step 2: Set the terminal condition at time $T : V_T\left(W_T\right) = \phi u\left(W_T + 1\right)$
- Step 3: Find the optimal portfolio and consumption $t = T - 1, T - 2, \cdots, 0$
  - Step 3.1: Construct a grid for the state variable wealth minus consumption n_g grid points
  - Step 3.2: Find the optimal portfolio and consumption at each grid point by solving the KKT conditions

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$$
\begin{aligned}
J_t\left(I_t\right) = \quad & \max_{z_t^+, z_t^-} \beta E_t\left[g_t^{1-\gamma} V_{t+1}\left(W_{t+1}\right)\right] \\
s.t. \quad & W_{t+1} = g_t^{-1} I_t\left[\sum_{i=1}^{n_a}\left(z_t^{i+} - z_t^{i-}\right) R_t^{e,i} + R^f\right] \\
& \lambda^+ \sum_{i=1}^{n_a} z_t^{i+} + \lambda^- \sum_{i=1}^{n_a} z_t^{i-} \leq 1 \\
& z_t^{i+}, z_t^{i-} \geq 0, i = 1, \cdots, n_a
\end{aligned}
$$

Lagrangian

$$
\begin{aligned}
\mathcal{L}\left(z_t^+, z_t^-, \mu_t^+, \mu_t^-, l_t\right) = \quad & \beta E_t\left[g_t^{1-\gamma} V_{t+1}\left(W_{t+1}\right)\right] \\
& + \sum_{i=1}^{n_a} \mu_t^{i+} z_t^{i+} + \sum_{i=1}^{n_a} \mu_t^{i-} z_t^{i-} \\
& + l_t\left(1 - \lambda^+ \sum_{i=1}^{n_a} z_t^{i+} - \lambda^- \sum_{i=1}^{n_a} z_t^{i-}\right)
\end{aligned}
$$

The Karush-Kuhn-Tucker Conditions

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$0 = \beta I_t E_t \left\{ g_t^{-\gamma} \frac{\partial V_{t+1}(W_{t+1})}{\partial W_{t+1}} R_t^{e,i} \right\} + \mu_t^{i+} - l_t \lambda^+$	FOCs
$0 = -\beta I_t E_t \left\{ g_t^{-\gamma} \frac{\partial V_{t+1}(W_{t+1})}{\partial W_{t+1}} R_t^{e,i} \right\} + \mu_t^{i-} - l_t \lambda^-$	FOCs
$0 = \mu_t^{i+} z_t^{i+}$	Complementarity
$0 = \mu_t^{i-} z_t^{i-}$	Complementarity
$0 = l_t \left(1 - \lambda^+ \sum_{i=1}^{n_a} z_t^{i+} - \lambda^- \sum_{i=1}^{n_a} z_t^{i-}\right)$	Complementarity
$1 \geq \lambda^+ \sum_{i=1}^{n_a} z_t^{i+} + \lambda^- \sum_{i=1}^{n_a} z_t^{i-}$	Feasibility
$0 \leq z_t^{i+}, z_t^{i-}, \mu_t^{i+}, \mu_t^{i-}, l_t$	Feasibility

Cases

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Case	Income	Correlation	Constraint
1	Deterministic		Margin
2	Stochastic	Uncorrelated	Margin
3	Stochastic	Correlated/4	Margin
4	Stochastic	Uncorrelated	No-short-sale-no-borrowing
5	Stochastic	Uncorrelated	Non-negative wealth

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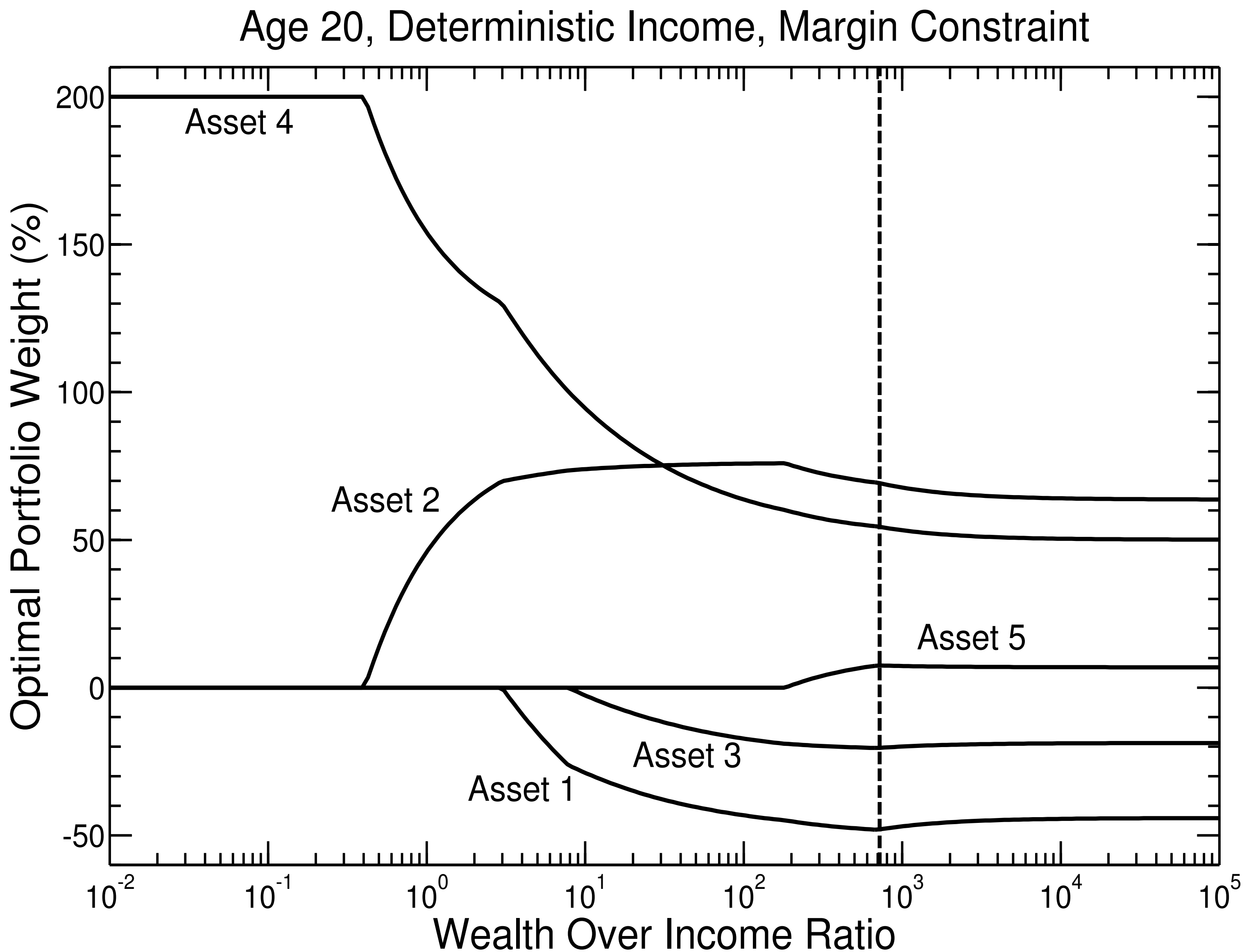

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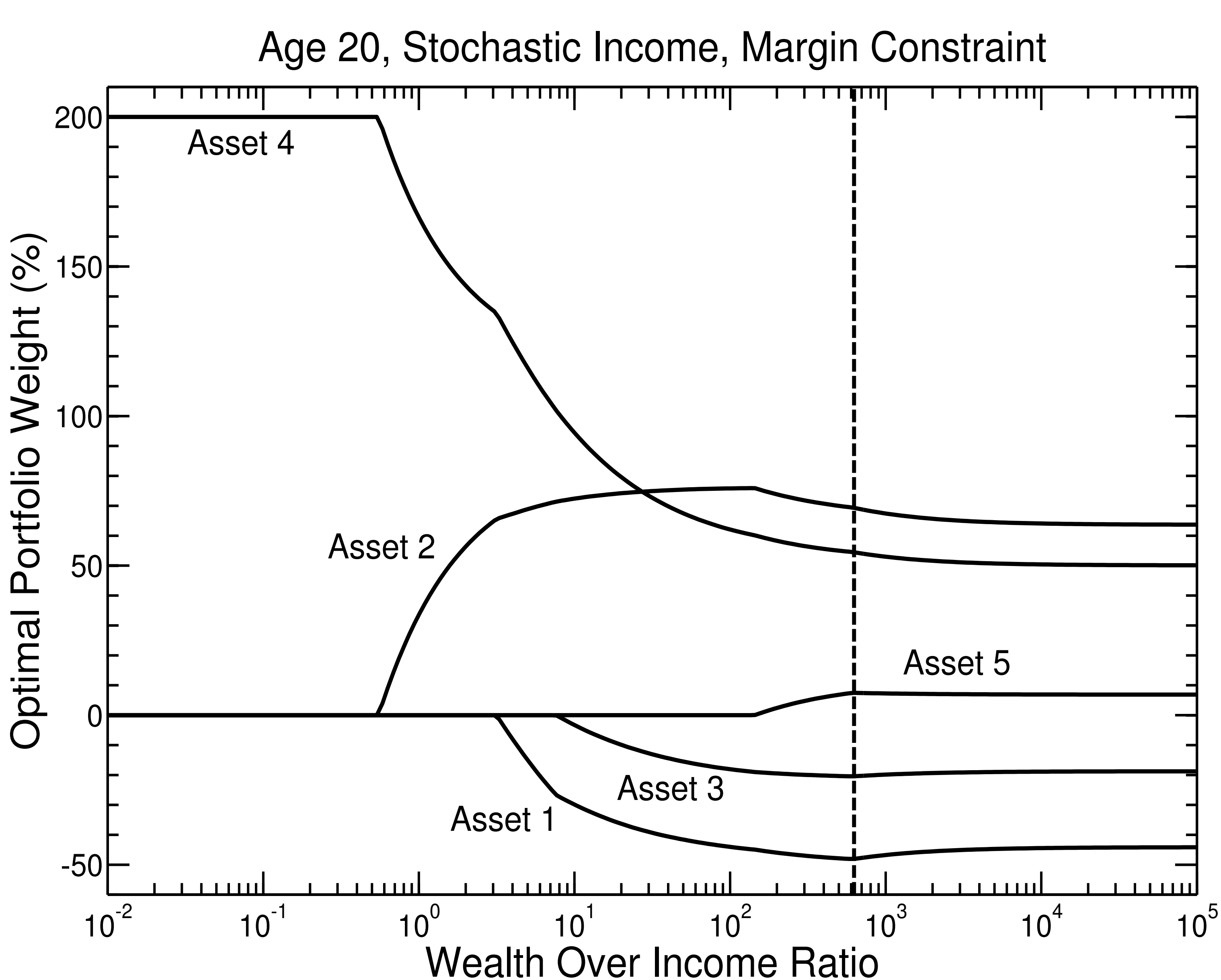

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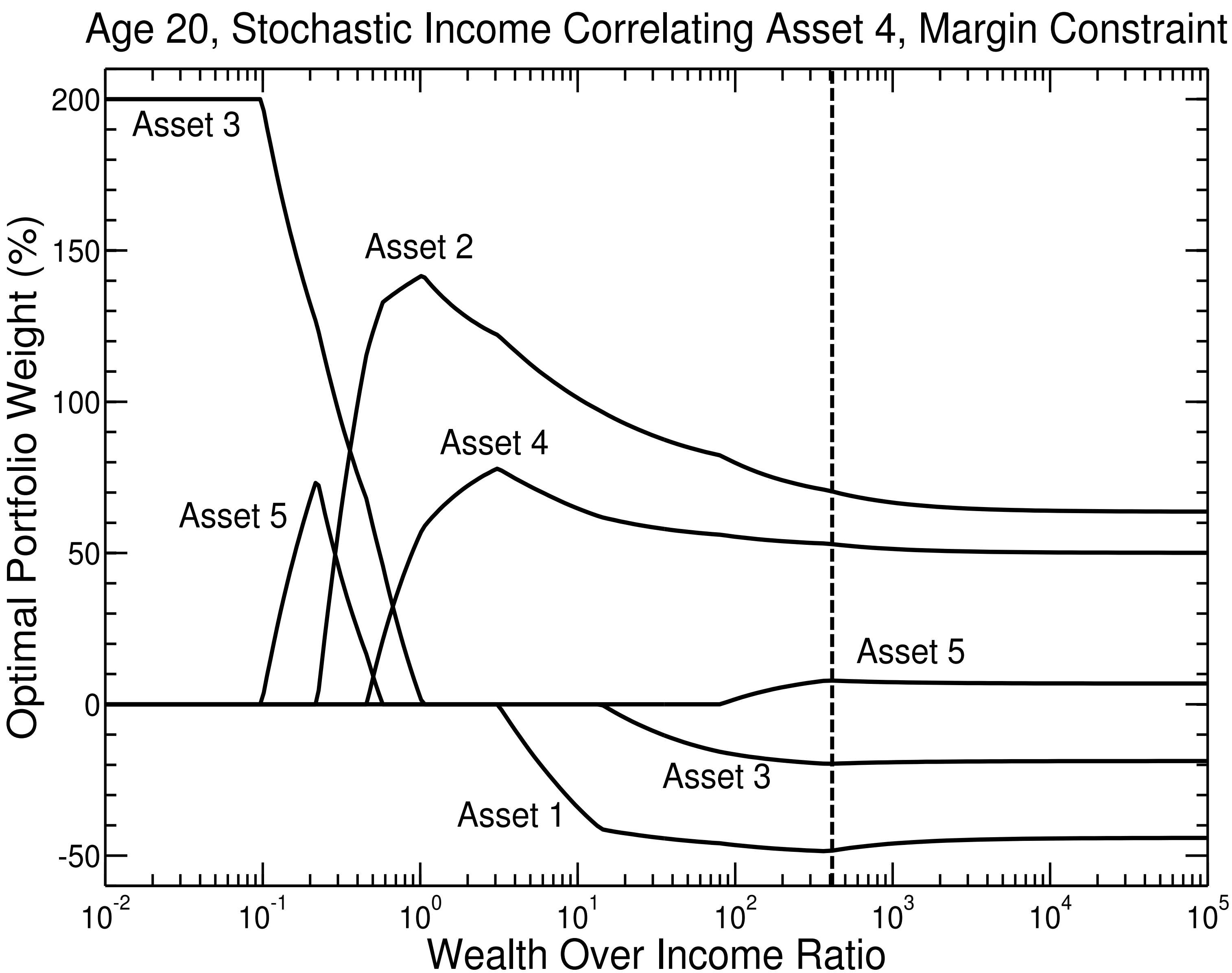

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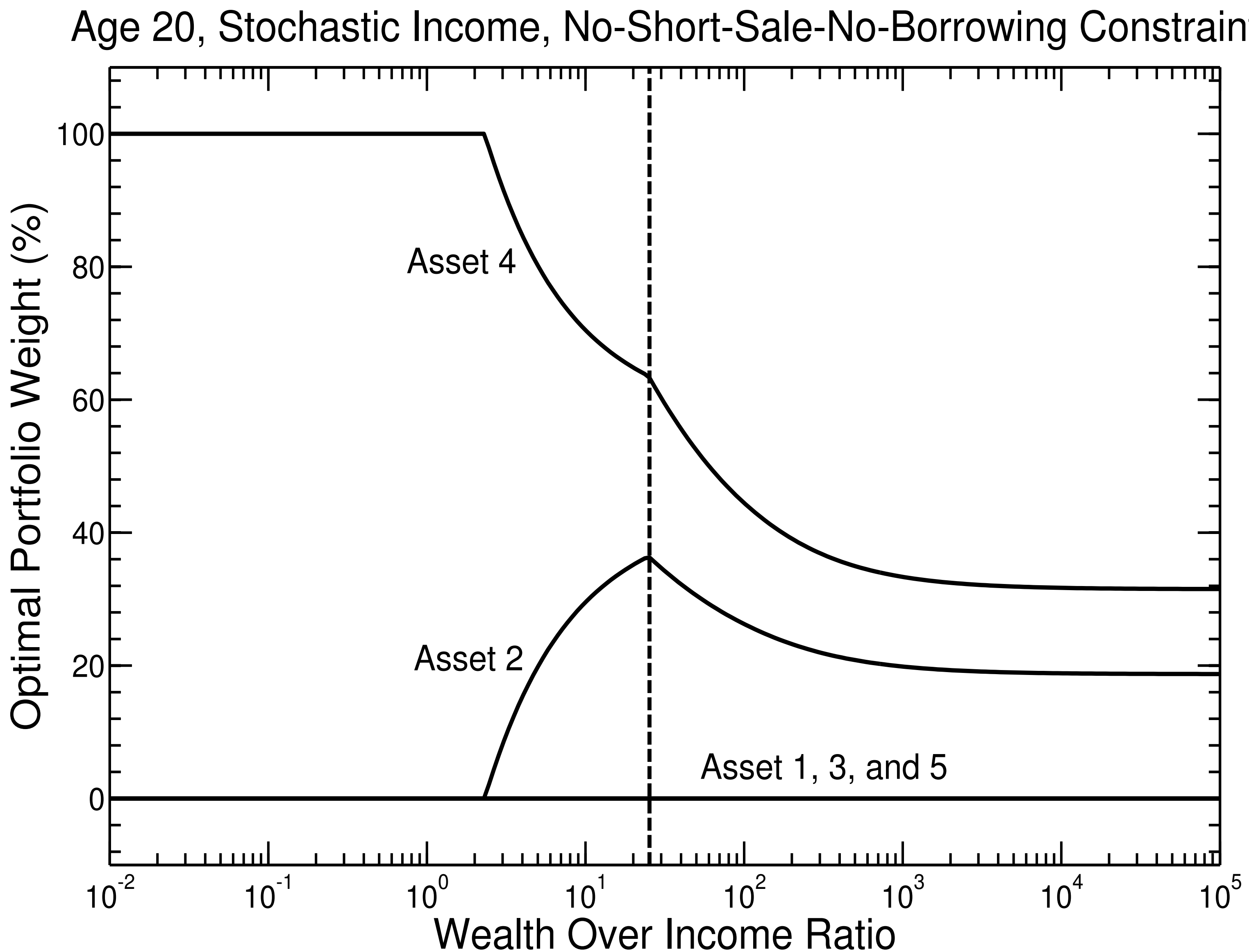

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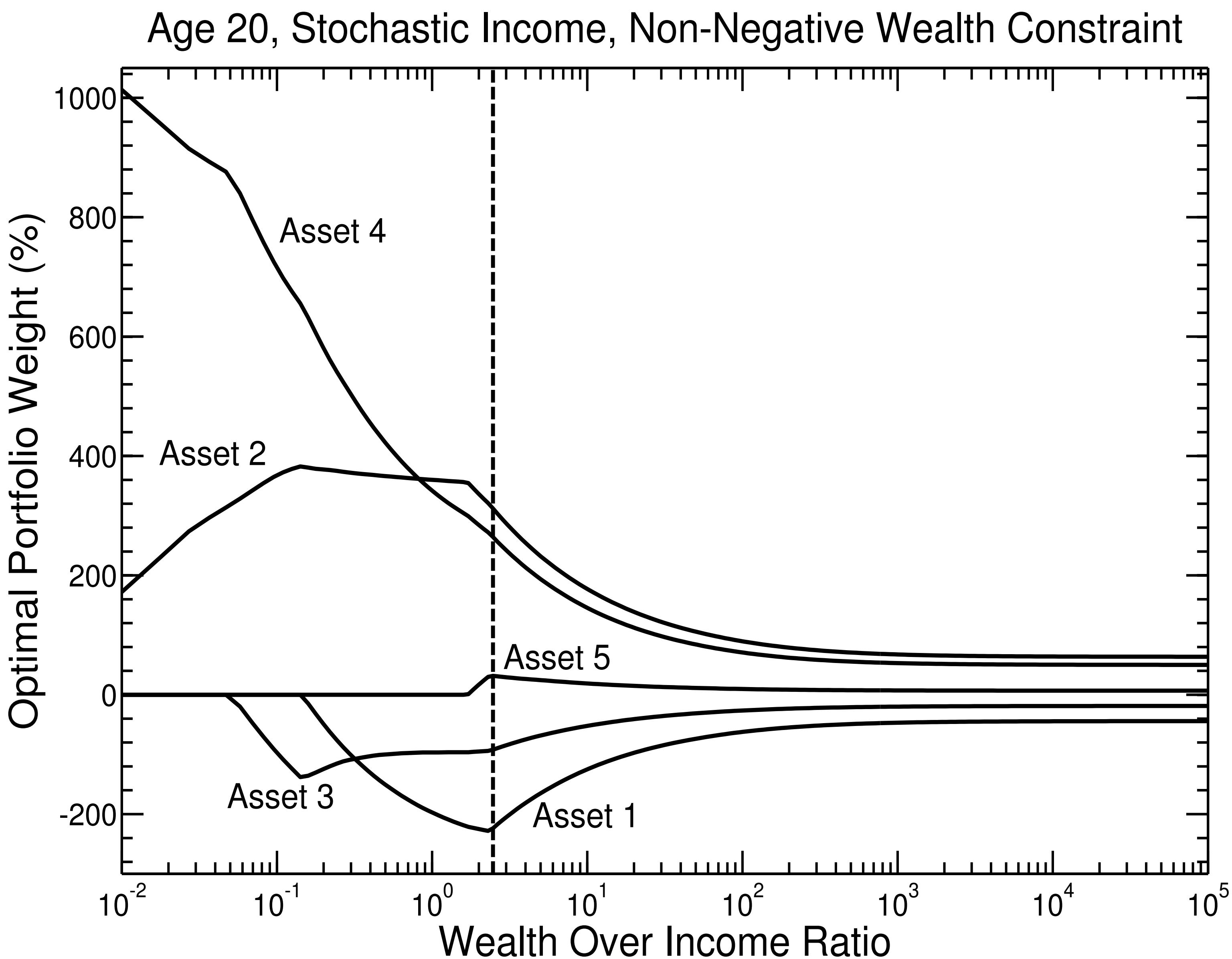

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- The model predicts that investors that find the constraint more binding will hold less diversified portfolios; e.g., young investors with stable income
- The model predicts that constrained investors will concentrate their holdings on stocks with, when leveraged, offer the highest expected return; e.g., real estate investments, high beta stocks
- The model predicts that investors with relatively low wealth and high (and long) income streams, will hold few stocks and under-diversified financial portfolios
- The model identifies the current financial wealth to income ratio as a determinant of the degree of portfolio under-diversification.
- While other models can also justify holding undiversified portfolios — e.g. facing capital gains taxes — they do not generate the same empirical predictions.

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(in progress — joint with Michael Gallmeyer, Hanjiang Zhang)

- Extension of Constantinides, Donaldson, Mehra (02) to two risky dividend streams and one riskless dividend stream
- Want to explore the impact of the financial constraints on the young to the relationship between a stock's expected return and its covariance with the market.
- Three generations:
  - The young: low financial wealth, uncertain future income, inability to borrow against future income
  - The middle aged: no uncertainty regarding income
  - The old: no income, consume their accumulated wealth

Overlapping Generations Model, cont.

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- Intuition: the young concentrate their portfolio. They prefer the asset with the higher expected return, but their demand for it lowers the return (although not below the returns of the other assets)
- Complications: correlations between labor income, dividend streams creates a strong hedging motive
- Calibration: match model parameters to empirically observed ones; e.g., correlation between dividends and income, correlation between asset returns for different asset classes, share of financial wealth for each generation.

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- When the young can borrow freely, the relationship between the expected excess return of the financial assets and their beta with the market portfolio is very close to linear (the market portfolio in this case includes a portion of the labor income)
- When the young are constrained from borrowing, the relationship between the expected excess return of the financial assets and their beta with the market portfolio is non-linear
- When the young are constrained, they change their portfolio composition away from a diversified portfolio.
- Still to do: determine how to adjust for risk, measure risk-adjusted expected returns, strength of motive to hedge labor income, impact on volatility of consumption

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- Financial constraints offer a rational explanation for holding an under-diversified portfolio
- When a significant fraction of the market is constrained
  - Assets with higher expected returns have lower return/risk ratio
  - Unconstrained investors overweight assets with the lowest expected returns (because they have the highest return/risk ratio)
- Overcoming financial constraints by other means: relationship between choice of profession and age

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KKT Conditions

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- The KKT conditions of this problem are both necessary and sufficient for optimality because
  - The objective function is concave in $\left(z_t^+, z_t^-\right)$
  - All constraints are linear in $\left(z_t^+, z_t^-\right)$
- The complementary conditions have the following structure
  - If the margin constraint is not binding ($l_t = 0$), only need to solve the FOCs without splitting z_t into z_t^+ and z_t^-
  - If the margin constraint is binding ($l_t > 0$), for $i = 1, \cdots, n_a$

μ_t^{i+}	μ_t^{i-}	z_t^{i+}	z_t^{i-}	z_t^i
> 0	> 0	$= 0$	$= 0$	$= 0$
> 0	$= 0$	$= 0$	> 0	< 0
$= 0$	> 0	> 0	$= 0$	> 0
$= 0$	$= 0$	> 0	> 0	not optimal

  - Overall there are mostly $3^{n_a} + 1$ specifications to check.

Conditional Expectations

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- Approximate conditional expectations as a linear combination of basis functions $\{b_j(z_t)\}_{j=1}^{n_b}$ and estimate the basis weights through cross-test-solution regression for $i = 1, \cdots, n_a$
- Generate n_s test solutions quasi-randomly within the test region Θ, where $\Theta \subseteq \bar{\Theta}$ (the whole feasible region)
- The accuracy of the approximation is determined by
  - number of regressors (number of basis functions): n_b
  - number of data points (number of test solutions): n_s
  - size of test region Θ

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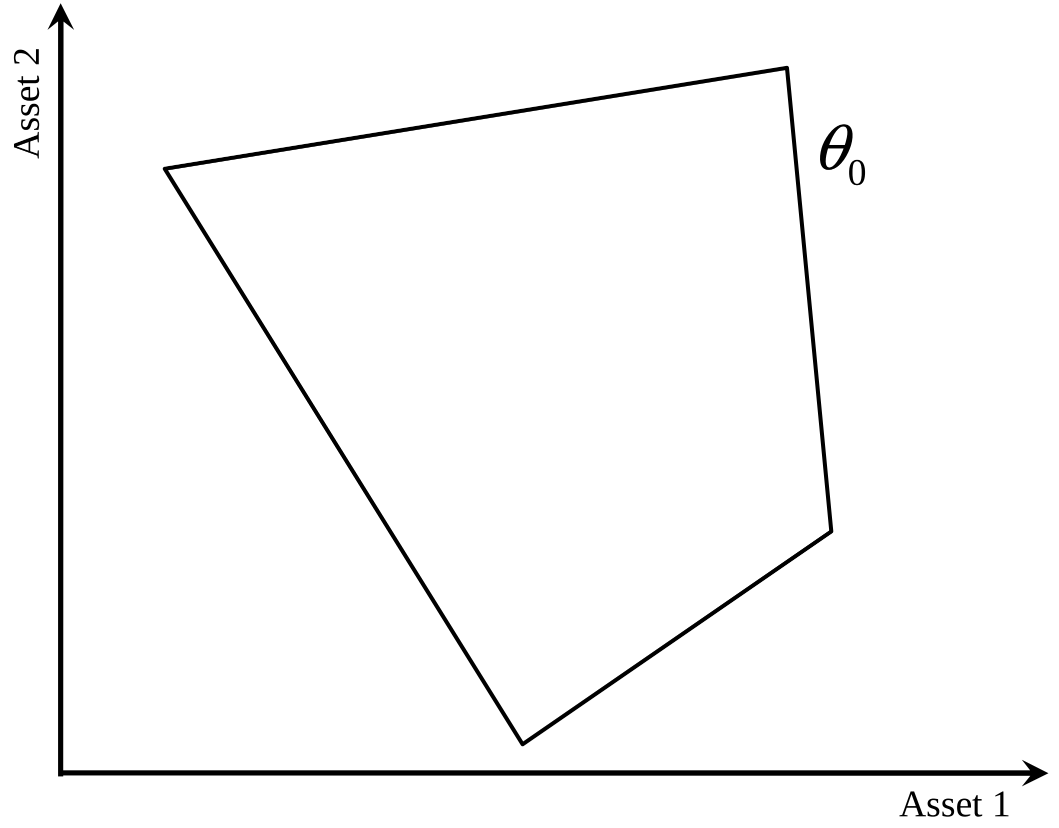

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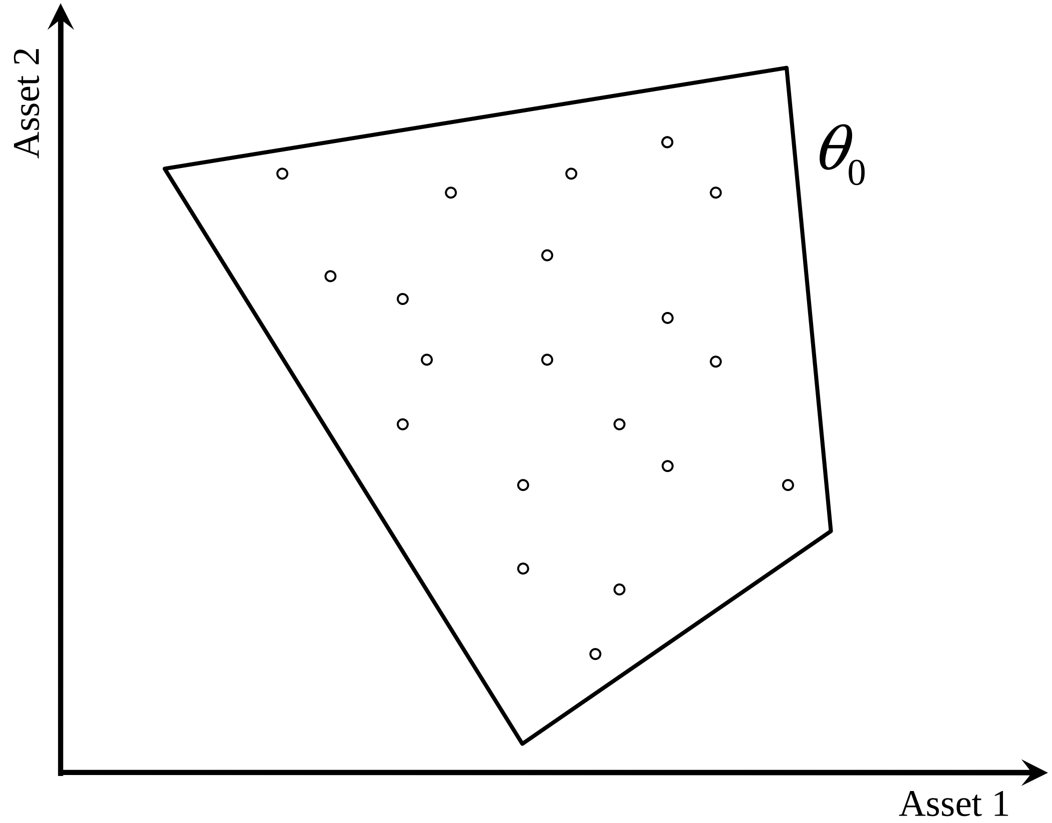

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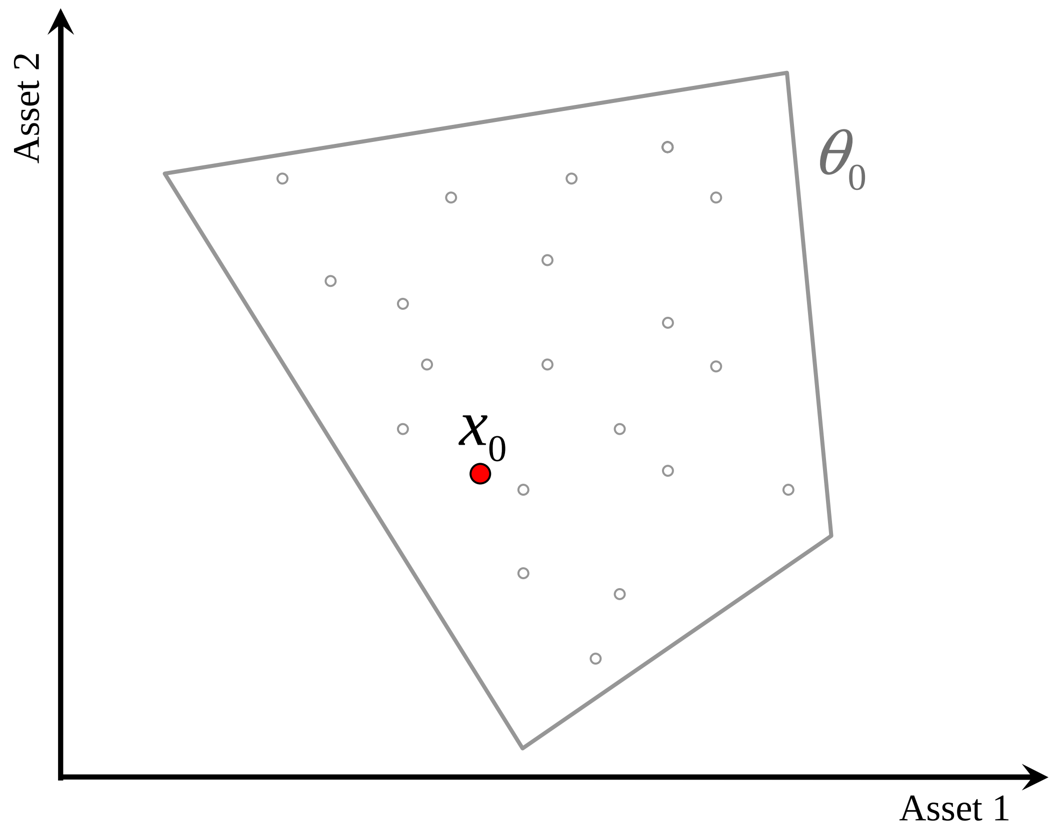

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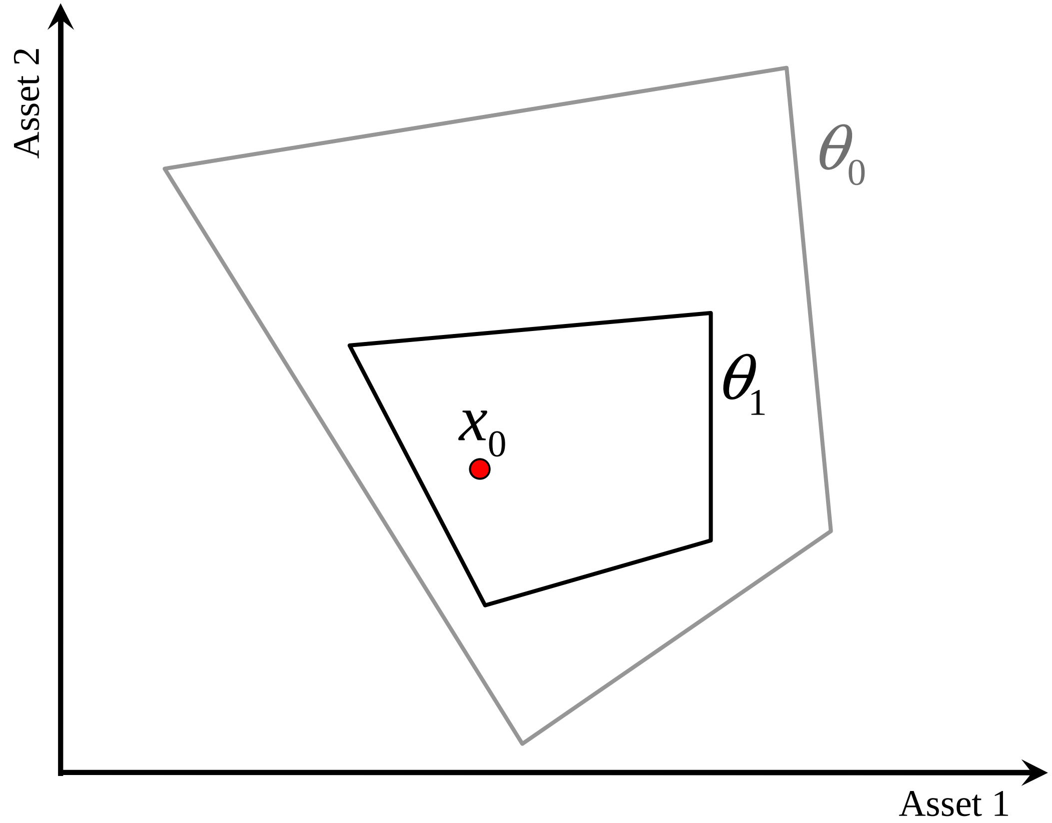

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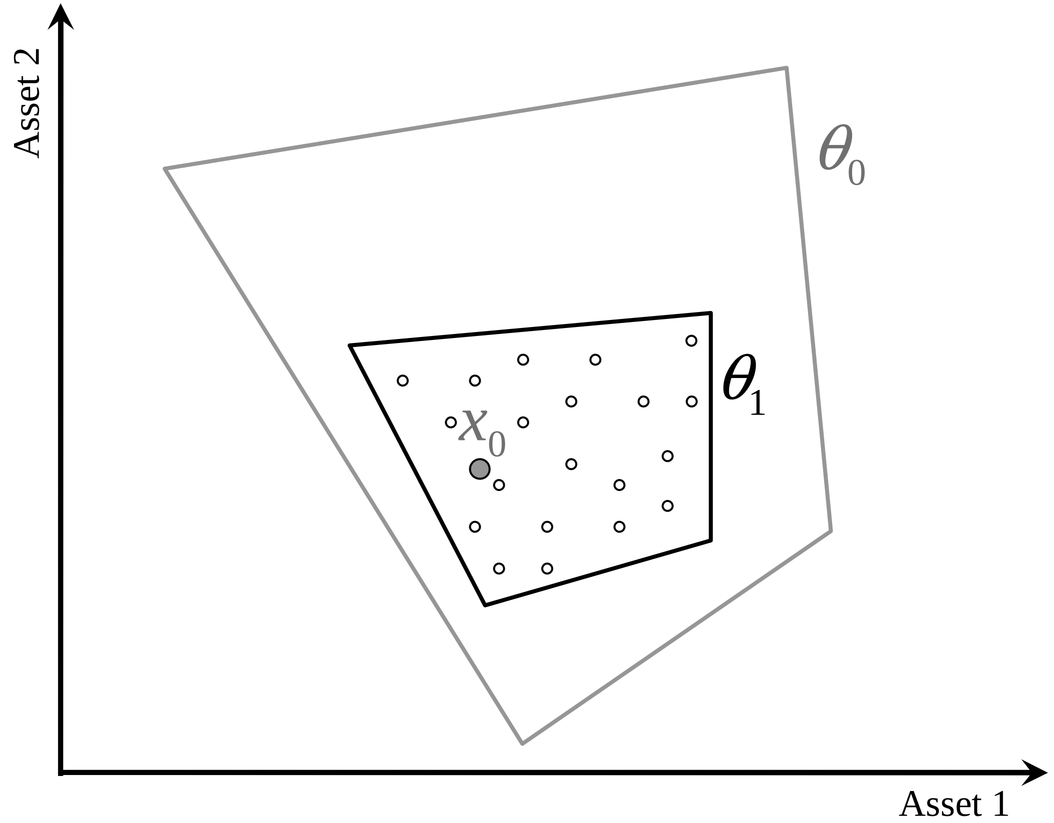

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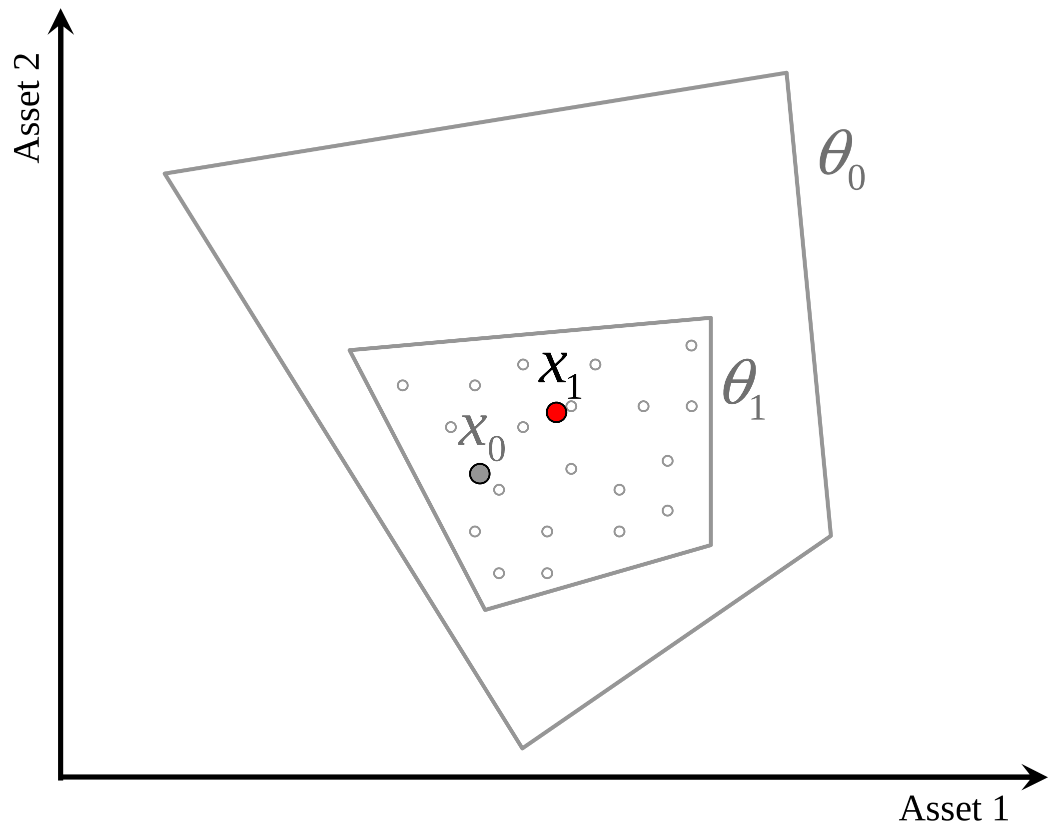

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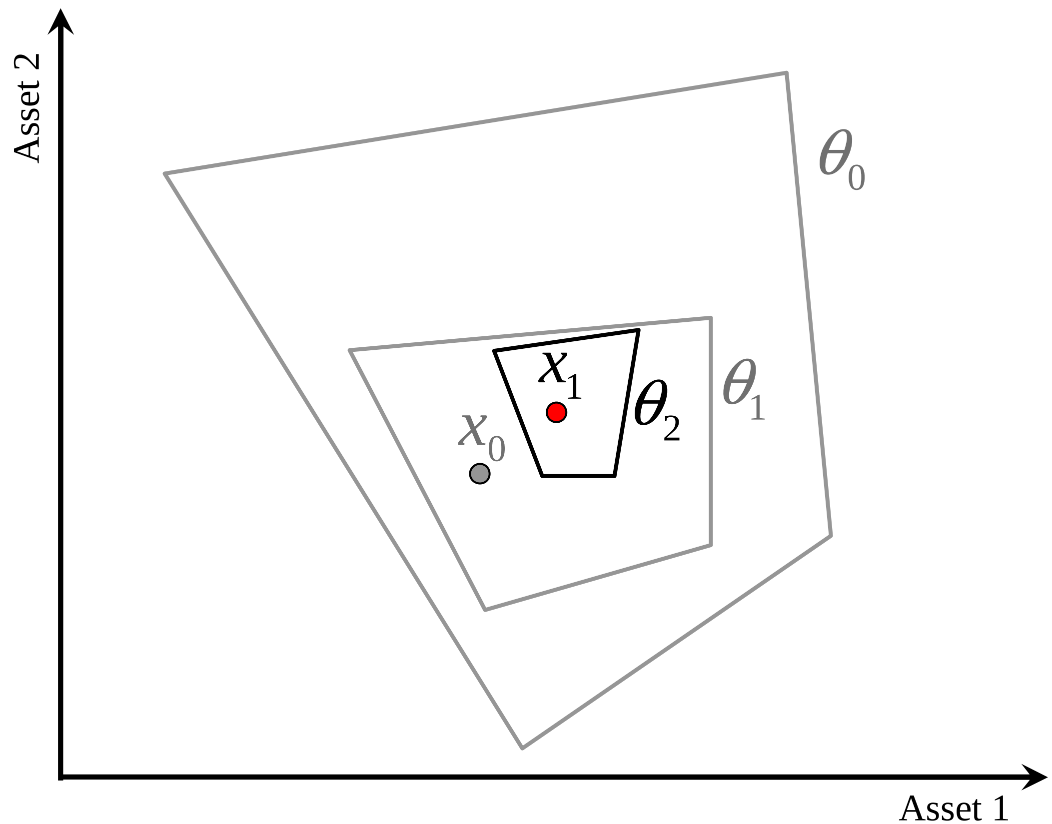

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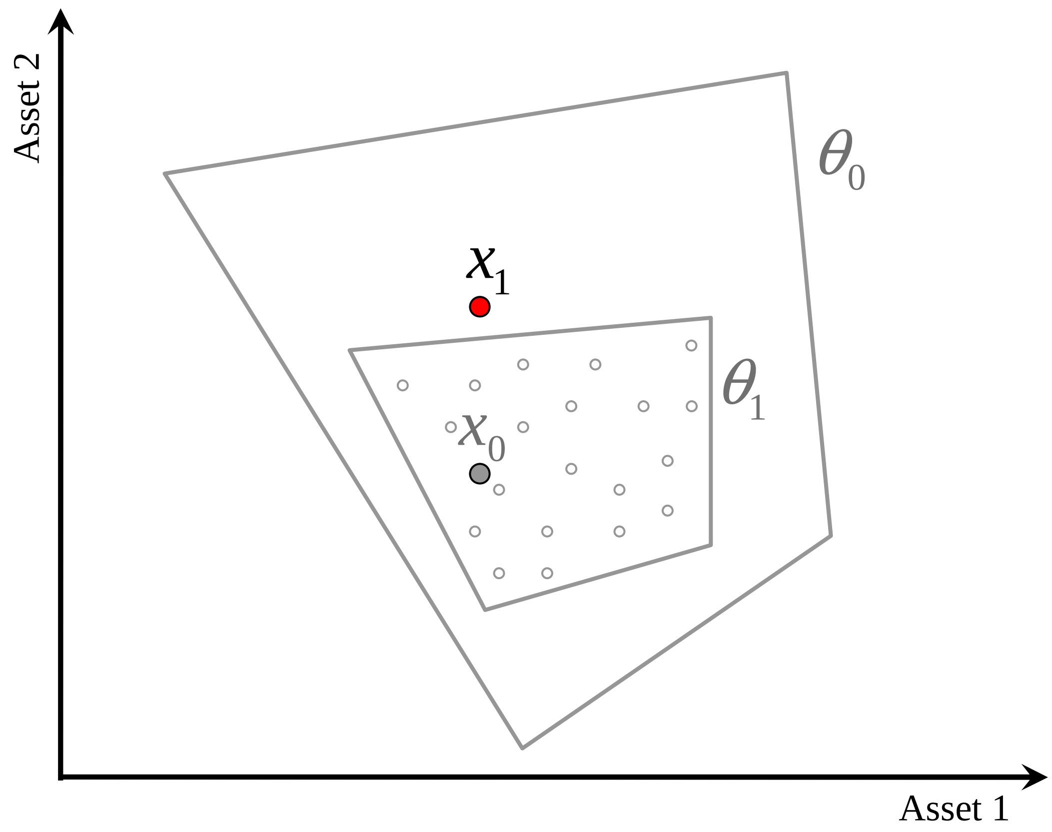

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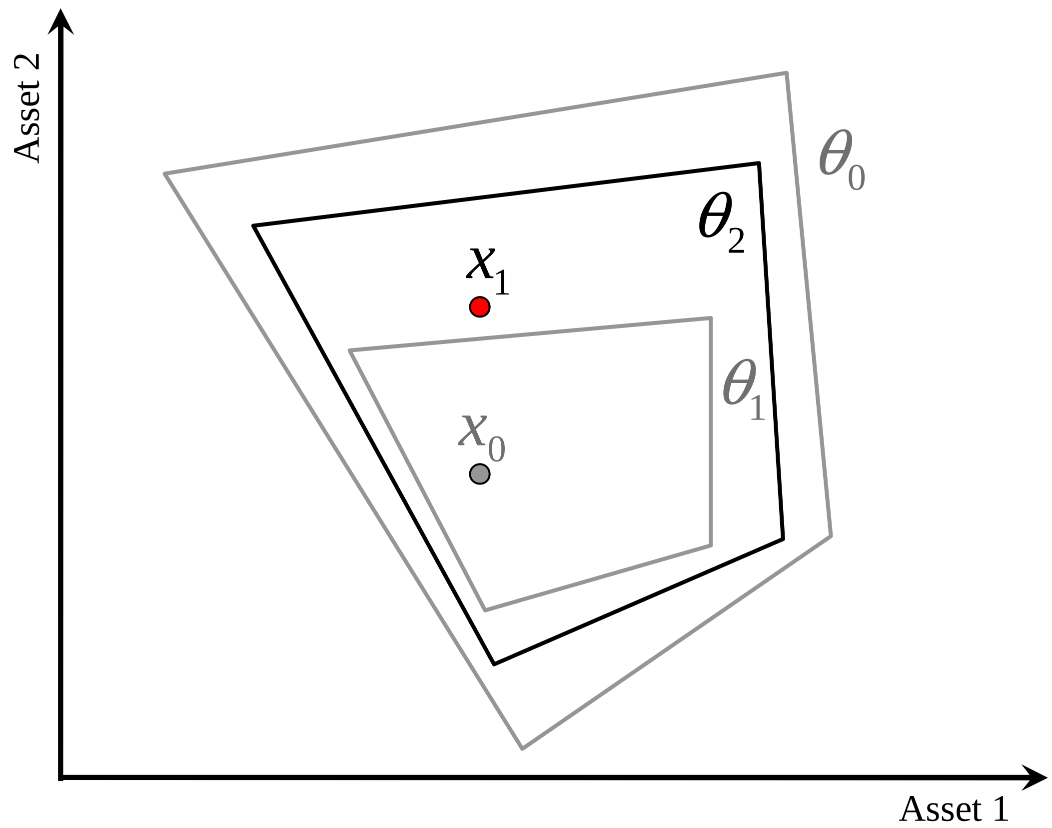

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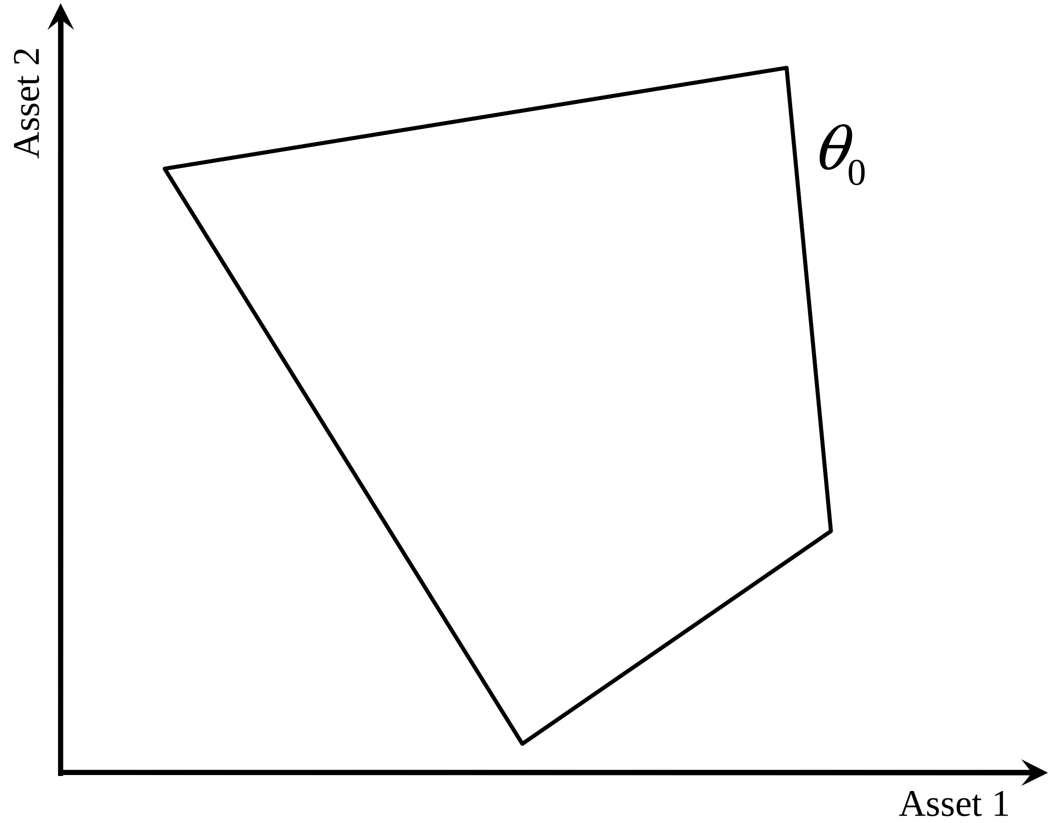

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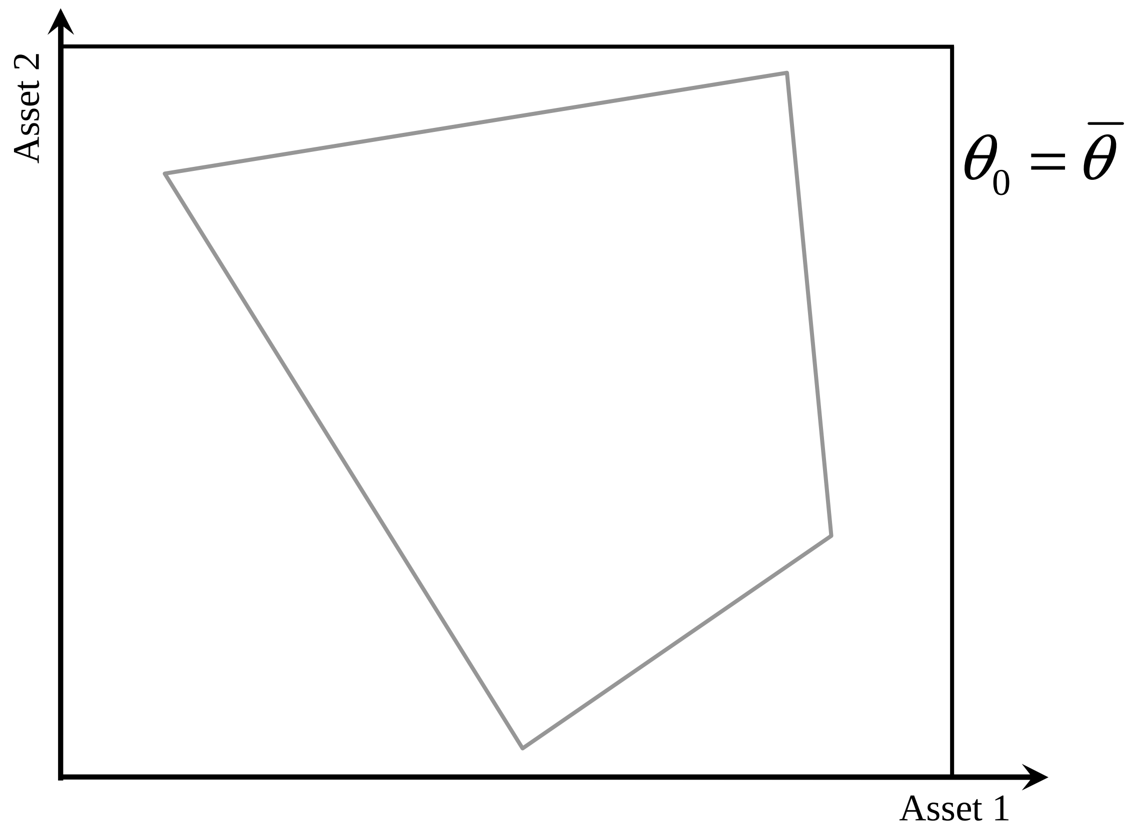

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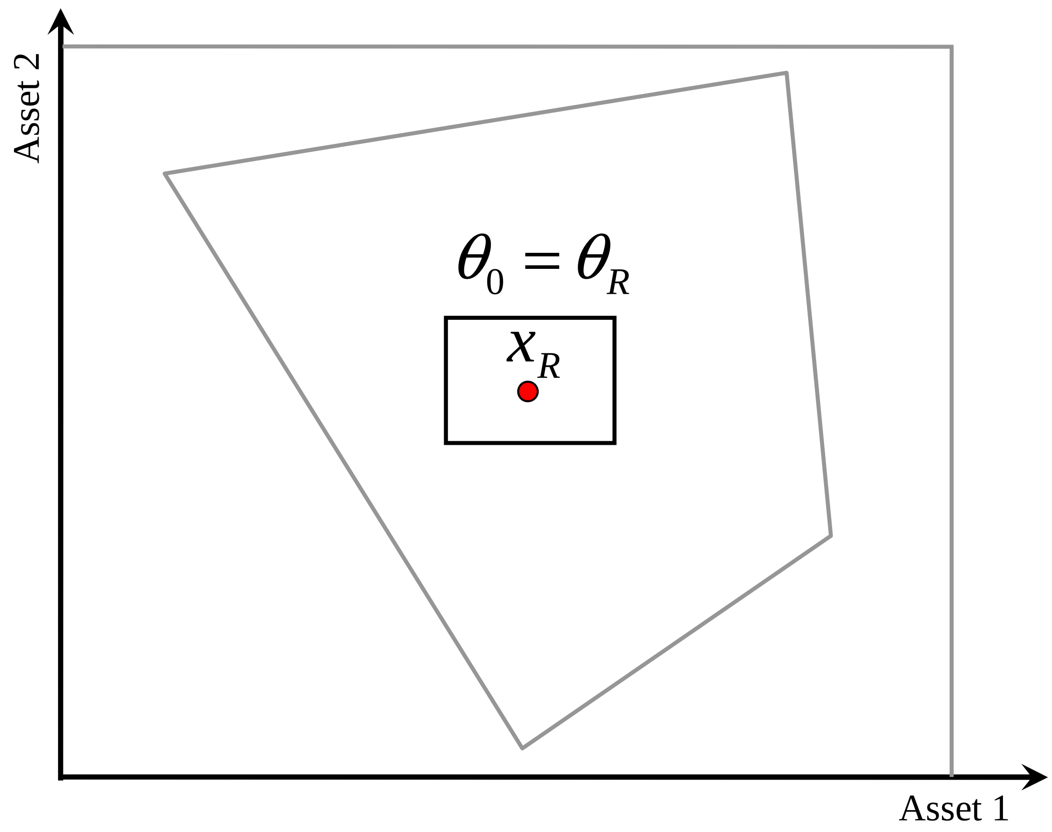

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- Step 1: Initialization ($i = 0$): set the initial portfolio $z^{(0)} = z^R$ (using the reference portfolio) and the initial test interval width $\delta^{(0)}$
- Step 2: Construct test region $\Theta^{(i)}$ by

$$\Theta^{(i)} = \bar{\Theta} \cap \left\{ z : z_j^{(i)} - \delta^{(i)} \leq z_j \leq z_j^{(i)} + \delta^{(i)}, j = 1, \cdots, n_a \right\}$$

- Step 3: Construct n_s test solutions quasi-randomly within $\Theta^{(i)}$
- Step 4: Estimate conditional expectations through cross-test-solution regression with n_b regressors

Algorithm, cont.

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- Step 5: Find portfolio $z^{(i+1)}$:
  - Step 5.1: Solve the KKT conditions under the specification of complementary conditions corresponding to $z^{(i)}$. If a solution is found, go to Step 6.
  - Step 5.2: Solve the KKT conditions under the other 3^{n_a} specifications. Whenever a solution is found, ignore the remaining specifications and go to Step 6.
- Step 6: Test convergence: If $\left\| z^{(i+1)} - z^{(i)} \right\| / \left\| z^{(i)} \right\| < \varepsilon$, stop and report $z^{(i+1)}$.
- Step 7: Update test interval width

$$\delta^{(i+1)} = \begin{cases} a_s \delta^{(i)} & \text{If } z^{(i+1)} \in \Theta^{(i)} \\ a_e \delta^{(i)} & \text{If } z^{(i+1)} \notin \Theta^{(i)} \end{cases}$$

- Step 8: Update iteration number $i = i + 1$ and go to Step 2.