STANDARD & POOR'S

Modeling Correlation in Credit Risk Management

William Morokoff
Managing Director
Global Head of Quantitative Analytics
Standard & Poor's

The Fields Institute Seminar Series on Quantitative Finance

September 24, 2008

The McGraw-Hill Companies

Disclaimer

- Any model or analysis presented here may be exclusively part of a research effort intended to better understand the risks and modeling challenges associated with credit measures and the quantitative analysis thereof.
- No comment or representation is intended or should be inferred regarding Standard & Poor's ratings criteria or models that have been used in the ratings process for any security.
- The terms 'Credit Quality Level' or 'Ratings' refer to a general process of assigning financial securities to distinct, discreet categories of varying credit quality that may correspond to a bank's internal ratings process, agency ratings, etc. Unless explicitly stated, they do not refer to actual S&P ratings or credit opinions on any company or financial security.

Agenda

- Credit Portfolio Risk Assessment
- Overview of Correlation
- Impact of Correlation on Default Rates
- Modeling Correlation
- Estimating Correlations
- Other Forms of Dependence Modeling
- Correlation and Diversity

This presentation is a collaboration with Saiyid Islam of S&P's Quantitative Analytics and Research Group. Thanks also to Jun Wang and Joe McQuown.

STANDARD & POOR'S

Credit Portfolio Risk Assessment

Credit Portfolio Risk Assessment

Goal :

- **To estimate the distribution of potential losses in a credit portfolio over a future period of time**
  - Economic capital calculations
  - Capital allocation
  - Active Credit Portfolio Management
  - Determining appropriate subordination levels for securitizations such as ABS and CDOs; surveillance of Structured Finance debt

Means :

- **Build models that incorporate empirically observed phenomena…**
  - Dynamic credit quality of obligors that evolves through time
  - Risk concentration and diversification effects in credit portfolios (dependency)
  - Valuation depends on credit quality, market price of credit risk, liquidity, etc.
  - Contagion: distress in key names triggers distress in related names
  - Systemic recovery: realized recovery inversely related to default rates
- **…and are consistent with (i.e. calibrated to) historical data**

STANDARD & POOR'S

Credit Portfolio Risk Assessment

Typical approach to portfolio analysis is to model the main drivers of portfolio credit risk as separate components

- Default likelihood : modeling obligor specific PD term structures
- Correlation : for modeling co-movements in credit quality levels, including joint defaults, that produce fat tails in the loss distribution (i.e. extreme losses are many multiples of typical losses)
- Recovery :
  - Assume deterministic recovery profiles
  - Model stochastically
    - Common approach is to model loss given default independently of the default process by sampling from a beta distribution
    - Empirical evidence however suggests dependence between default and recovery processes

Although model components may fit historical data quite well, it remains a challenge to validate their performance in terms of portfolio loss prediction over longish horizons (5 years or more) – modeling framework typically embeds strong assumptions.

STANDARD & POOR'S

Overview of Correlations

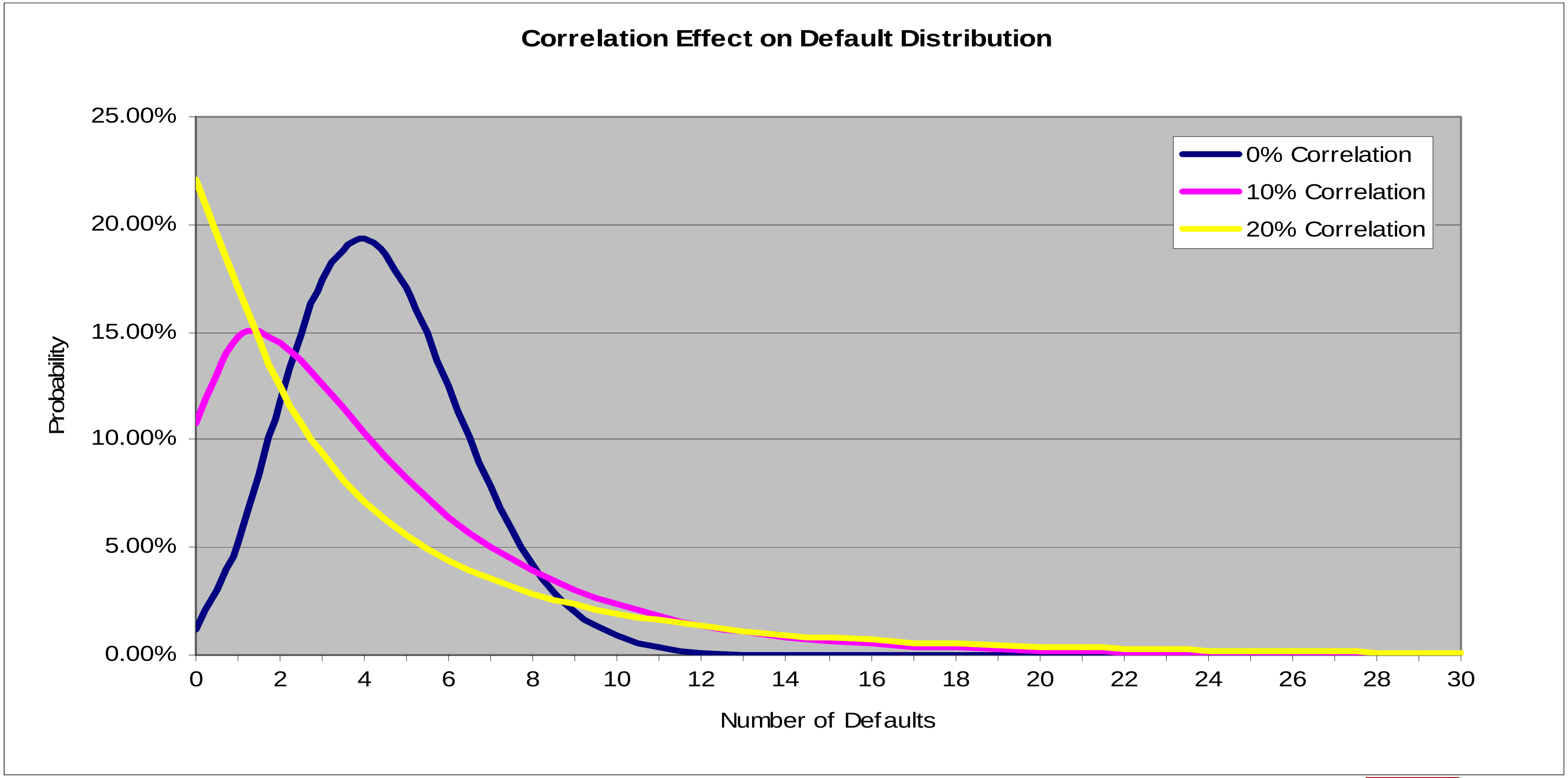
Overview of Correlation

Importance of correlation for a credit portfolio:

- Correlation determines the shape of the credit portfolio’s loss distribution
- Increases (decreases) in correlation increase (decrease) the likelihood of extreme events
  - Extreme events can be both positive (i.e. zero or few defaults) or negative (large no. of defaults)
  - Correlation does not affect the mean or Expected Loss of the credit portfolio.
- Correlation (along with Credit Transition Probabilities, and Recovery/Valuation) determines the likelihood of losses in a credit portfolio exceeding a given threshold
  - What is the likelihood that capital held by a bank would not be sufficient to withstand future losses on it’s credit portfolio
  - How much ‘cushion’ is required for achieving a specific target rating in a structured transaction
- Note: Losses in credit portfolios can be caused purely by credit quality deterioration and/or liquidity reasons even in the absence of defaults. A more appropriate approach is therefore to model correlated credit transitions explicitly, with default being just one specific transition from the current credit state to the default state.

Example: Effect of correlation on two hypothetical mezzanine attachment points for a synthetic CDO securitization

Assumptions:
- Single factor Gaussian copula model
- Maturity of 5 years
- Static underlying pool of 200 “BBB” corporate CDS (default probability → 2.32%)
- Base case recovery of 50%



Example: Impact of Correlation and Recovery on Subordination

Correlation	Recovery	Default Threshold for Mezzanine (lower quality)	Lower Mezz Subordination	Default Threshold for Mezzanine (higher quality)	Higher Mezz Subordination
0%	25%	9 of 200	3.38%	11 of 200	4.13%
0%	50%	9 of 200	2.25%	11 of 200	2.75%
0%	75%	9 of 200	1.13%	11 of 200	1.38%
10%	25%	16 of 200	6.00%	23 of 200	8.63%
10%	50%	16 of 200	4.00%	23 of 200	5.75%
10%	75%	16 of 200	2.00%	23 of 200	2.88%
20%	25%	23 of 200	8.63%	33 of 200	12.38%
20%	50%	23 of 200	5.75%	33 of 200	8.25%
20%	75%	23 of 200	2.88%	33 of 200	4.13%

Note: Shaded rows correspond to the base case recovery of 50%

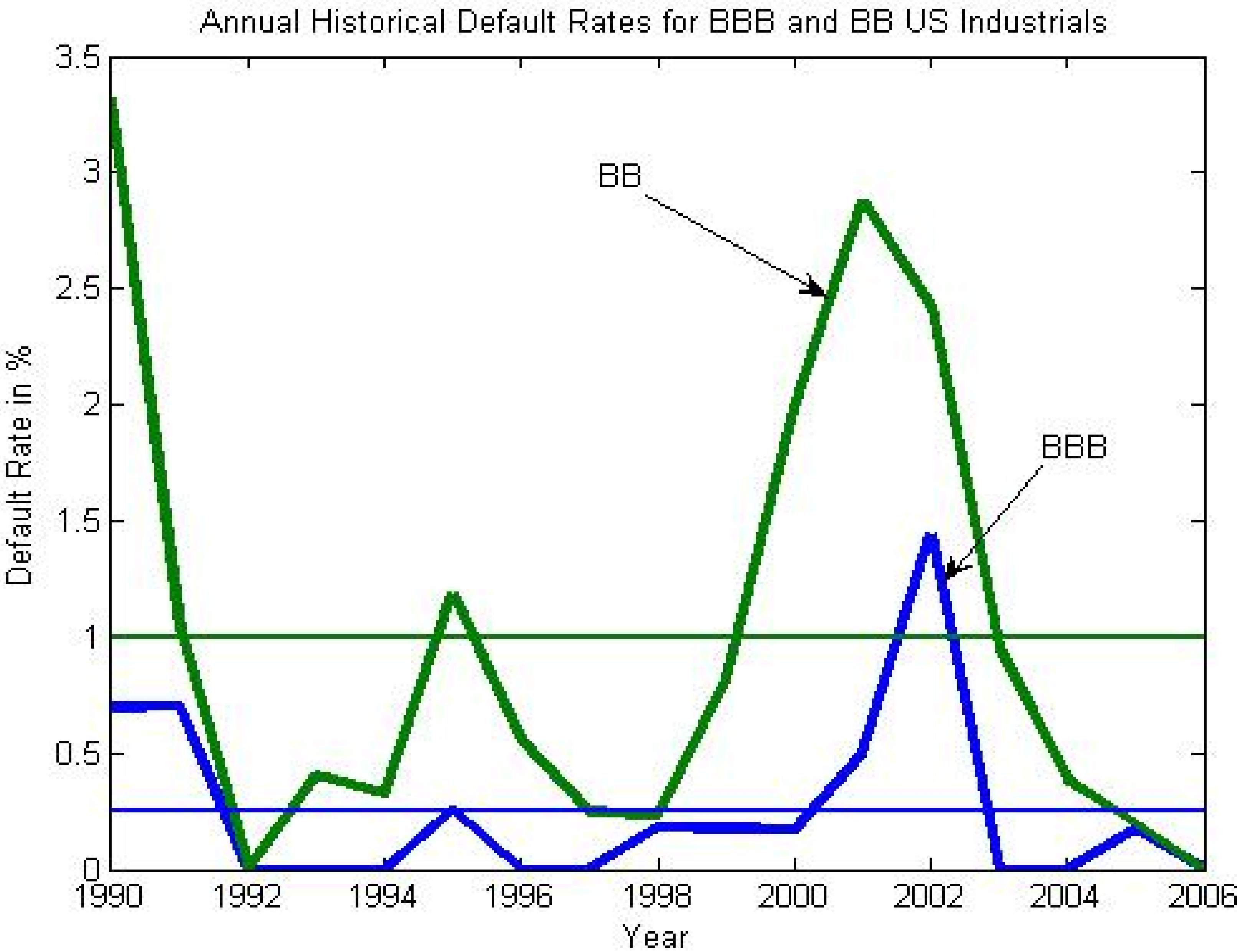
STANDARD &POOR'S

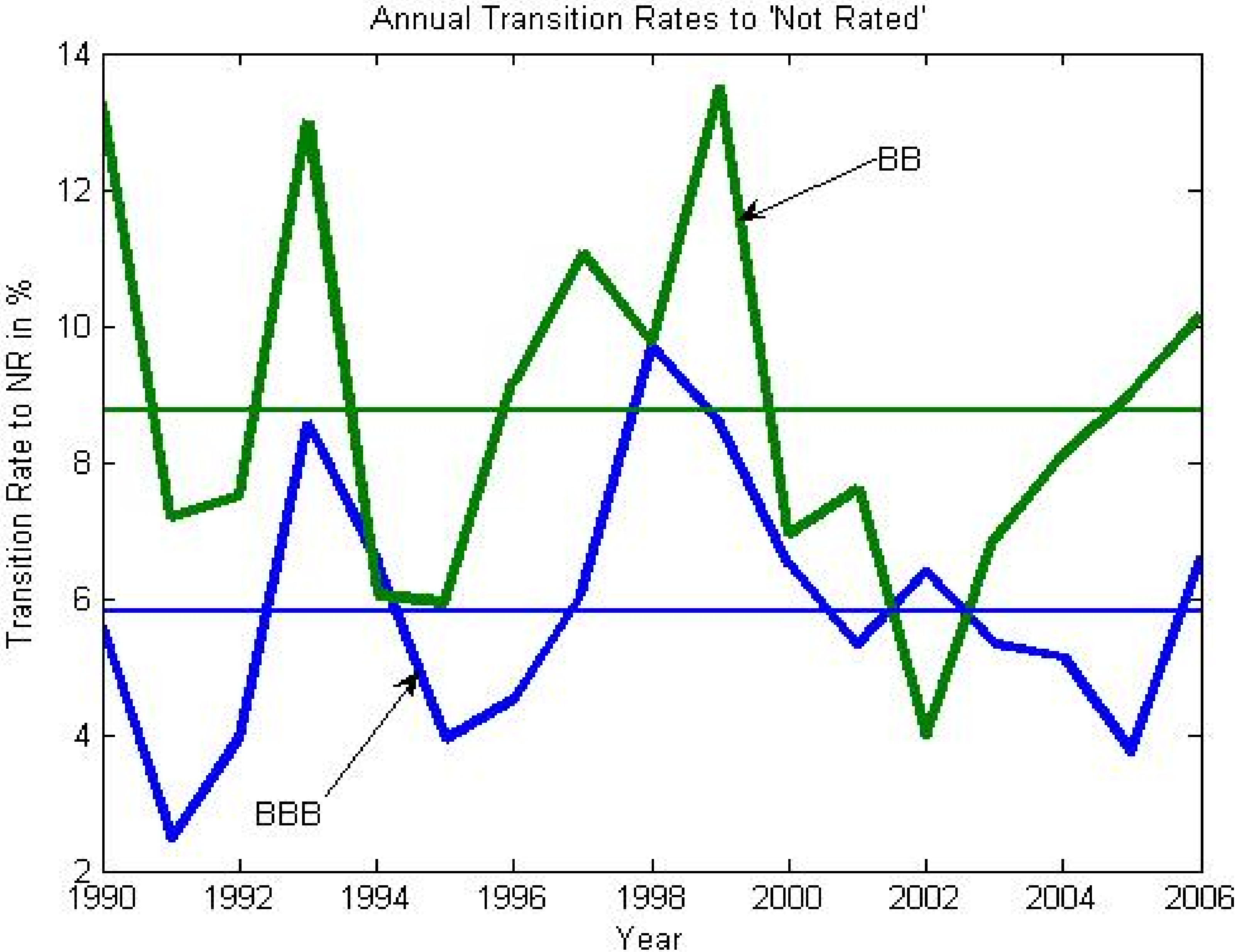
Impact of Correlation on Default Rates

What Do Historical Default Rates Tell Us About Correlations?

- For a given rating category, historical default rates count the number of defaults in a given year.
- Annual default rates vary year-by-year, following a credit cycle.
- Does this imply that default probabilities for different rating categories change through time purely due to cyclical effects or are there correlation effects (due to common systematic risk factors affecting sectors, regions or the whole economy)?
- Difficult to say whether higher/lower realized defaults are a credit cycle (PDs changing for a given rating category) or correlation effect, or a mixture of these effects.

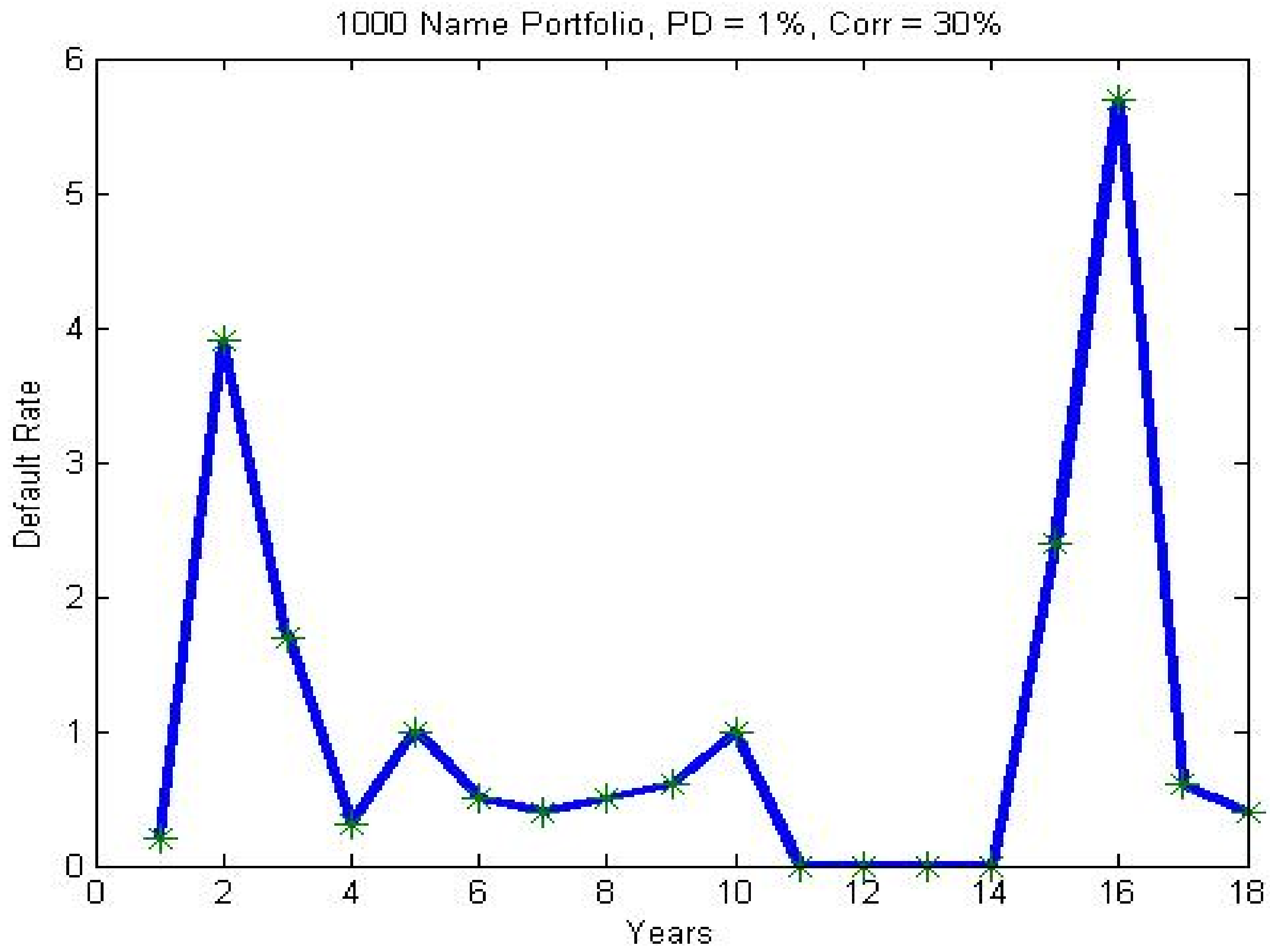
STANDARD & POOR'S


Annual Historical Default Rates for BBB and BB US Industrials
Default Rate in %
3.5
3
2.5
2
1.5
1
0.5
0
1990
1992
1994
1996
1998
2000
2002
2004
2006
Year
BB
BBB



Annual Transition Rates to 'Not Rated'
Transition Rate to NR in %
Year
BB
BBB
14
12
10
8
6
4
2
1990
1992
1994
1996
1998
2000
2002
2004
2006

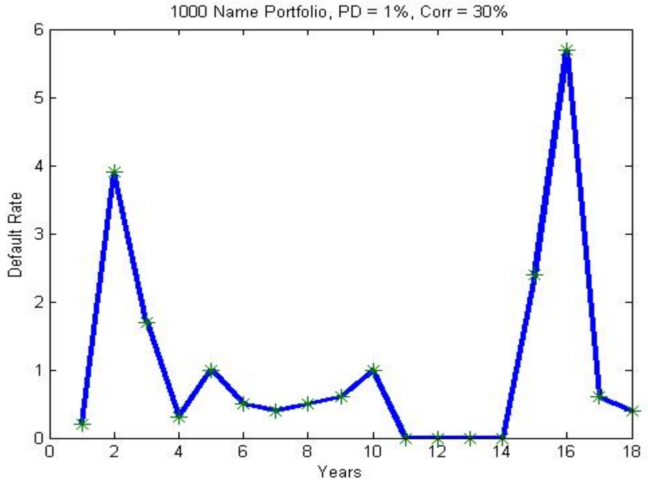
Illustration: Correlation Effect on Default Rates (model results)



- Even a pure correlation effect appears to be a cyclical effect in this example.
- With real data, and given the paucity of defaults, untying the various effects is a challenge

Modeling Correlation

Modeling Correlation



- **Implicit methods**
  - Model a portfolio level default rate directly (Top Down Method)
  - Use historical simulation methods
- **Factor models**
  - Use principal component type analyses that retain the dependence information in time series of explanatory variables or construct economic factors (indices) and determine factor loadings through econometric tools such as regression analysis
    - Factors can be statistical (as in PCA) or economic (indices)
  - Implies pair-wise correlations for portfolio constituents (Bottom Up Method)
- **Explicit methods**
  - Model pair-wise correlations of portfolio constituents directly (Bottom Up Method)

STANDARD & POOR'S

Modeling Correlation: Top Down vs. Bottom Up

- **Top Down**
  - Portfolio default intensity specified without reference to portfolio constituents
    - Defaults can be observed but identity of defaulter may not (in other words, model filtration is coarser)
  - Correlation modeled implicitly: $\lambda = X(m - N(t))$
    - Risk factor X generates variation in portfolio default intensity
  - Typically used for modeling fairly homogeneous portfolios or credit derivatives with prices across maturities and subordination levels
- **Bottom Up**
  - Portfolio default intensity aggregate of constituent default intensities
    - Both defaults as well as identity of defaulter identifiable
  - Correlation modeled explicitly: $\lambda^k = \alpha^k X + Z^k$
    - Movements in systematic factor X generate correlated changes in individual asset default intensities
  - Appropriate for modeling all types of portfolios (heterogeneous & homogeneous) but require correlation estimation at the individual asset level

STANDARD & POOR'S

Modeling Correlation: Asset Return and Copulas

- Bottom Up Approach
- Merton theory (Structural Model): Firms have asset value (value of equity plus debt) that evolves through time. Asset returns are correlated (similar to correlated equity returns). Credit quality of a firm related to how much the asset value exceeds the ‘Default Point’ (determined by book value of debt). Default occurs when the asset value falls below the default point.
- Multi-variate distribution of joint asset returns therefore determines the credit portfolio loss distribution. Credit transition process and PD term structures combined with the marginal distribution of asset return determines individual credit evolution.
- Copula of distribution (generally parameterized by a correlation matrix or factor model) determines joint behavior.

STANDARD &POOR'S

Modeling Correlations: Infinitely Granular Single Factor

- Divide portfolio into K classes, assume the large homogeneous portfolio approximation in each class:

$$N \to \infty, \quad N_k \to \infty, \quad w \to 0, \quad N_k / N = a_k$$

- Portfolio Loss is only due to defaults. Valuation is par if no default, and LGD if an obligor defaults.
- Default dependence (correlation) is based on a single factor Gaussian copula model. Normalized firm asset value return is given by

$$\varepsilon_i^k = \sqrt{\rho_k}\, z + \sqrt{1-\rho_k}\, \phi_i$$

Correlation within a class is ρ_k while correlation between classes is $\sqrt{\rho_k \rho_j}$

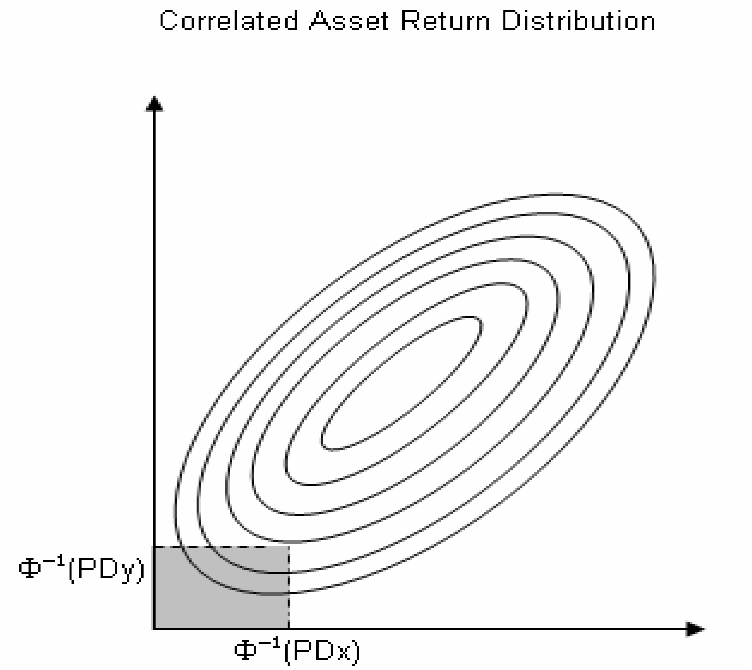
- Analytic formula for loss distribution
- Similar to Basel II treatment of correlation

STANDARD & POOR'S

Estimating Correlations

Estimating Correlations

- **From Defaults**
  - Impose a structural model, typically Gaussian Copula
  - Back out asset correlations from observations of joint defaults



  - Challenge: Defaults are rare events. Estimation errors arise due to a paucity of data.

Correlation Estimation Errors with Joint Default Analysis

- **Experiment based on Simulated Data:**
  - Rating scale used only as indicative of relative credit quality
  - Generate defaults with a multi-period, single factor Gaussian copula model assuming a certain correlation.
    - Horizon of 10 yrs, 1,000 names per rating category, quarterly time steps for the example below
  - Back out correlations from the simulated data and compare to initial correlation assumptions

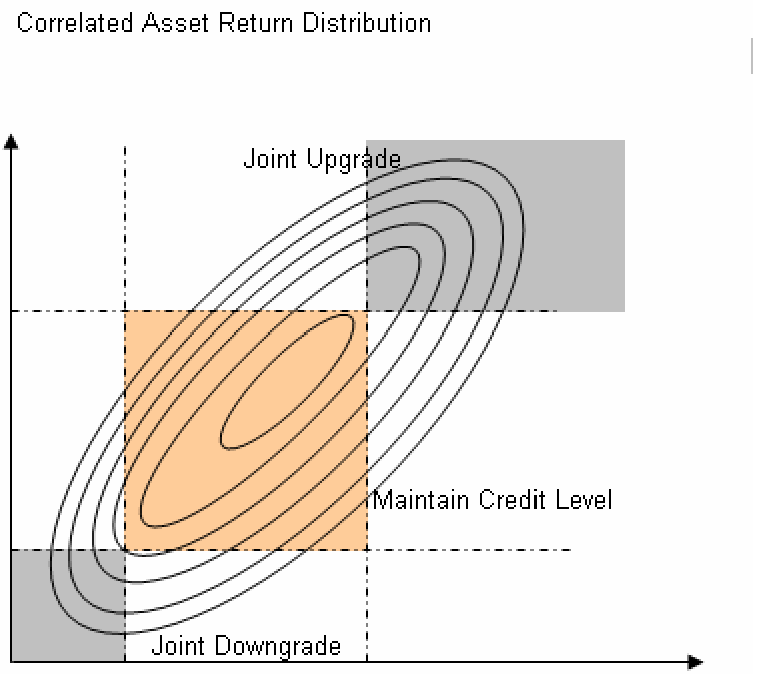
		Asset Credit Quality								
		BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
	0.1	0.0803	0.0957	0.0897	0.095	0.0929	0.1063	0.0804	0.1042	0.0954
	0.2	0.1296	0.1543	0.152	0.1621	0.1634	0.1692	0.1658	0.1679	0.171
	0.3	0.1886	0.2353	0.2332	0.2462	0.2441	0.2741	0.2278	0.2874	0.2579
	0.4	0.2154	0.2762	0.3052	0.3166	0.3035	0.3422	0.2953	0.369	0.3328
‘True’ Correlation	0.5	0.2517	0.3033	0.3537	0.3406	0.3817	0.4178	0.4179	0.4123	0.416

- **Correlation estimates show a downward bias**
  - Bias is higher as credit quality improves
  - Instances of joint defaults become rarer leading to increase in estimation errors

STANDARD & POOR’S

Estimating Correlations: From Rating Transitions

- **Rank correlation based estimates appear to be very poor measures of ‘true’ correlation**
- **Utilize a Gaussian framework and estimate correlations as the average of co-movements to a credit downgrade state, upgrade state, or credit maintenance (no movement)**



STANDARD & POOR’S

Estimating Correlations: From Rating Transitions (Simulated)

	Asset Credit Quality									
		BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
	0.1	0.0908	0.0804	0.0887	0.0977	0.0815	0.0977	0.0913	0.0895	0.0836
	0.2	0.1952	0.1902	0.1869	0.1890	0.1844	0.1875	0.1768	0.1676	0.1785
	0.3	0.2595	0.2748	0.2499	0.2543	0.2487	0.2681	0.2845	0.2723	0.2420
	0.4	0.3474	0.3891	0.3685	0.3676	0.3360	0.3957	0.3592	0.3622	0.3626
‘True’ Correlation	0.5	0.4832	0.4746	0.4346	0.4073	0.4870	0.4543	0.4270	0.4724	0.4390

- **Correlation estimates based on joint movements in credit quality are better measures of ‘true’ correlation**
- **A bias in the estimates still persists due to the non-linear relationship**

$$\hat{\rho} = f(\hat{P}_{i,i\to j,j}), \text{ where } \hat{P}_{i,i\to j,j} = P_{i,i\to j,j} + \varepsilon^{simulation}$$

$$E(\hat{\rho}) = E[f(\hat{P}_{i,i\to j,j})] \neq E[f(P_{i,i\to j,j})]$$

- **Even in a controlled experiment, correlation estimation is problematic. With real data, there are further issues:**
  - Various rating categories may not have sufficient number of observations for a good estimation
  - Real ratings are very frequently withdrawn or transition to an “NR” state.
  - Close to half of all entities rated by S&P over the past decade had at least one transition to the “NR” state.

STANDARD & POOR’S

Estimating Correlations: From Equity Markets (Merton)

- Given market price of a firm's equity, estimate an implied market value (and volatility) of a firm's assets using a BSM option pricing framework

- Assume a lognormal distribution for the firm's asset value process

$$dA = \mu A dt + \sigma_A A dz \qquad \text{(i)}$$

- Value of equity,

$$E = A \bullet N(d_1) - F \bullet e^{-rt} N(d_2) \qquad \text{(ii)}$$

$$\text{where } d_1 = \frac{\ln\left(\frac{A}{F}\right) + \left(r + \frac{\sigma_A^2}{2}\right)t}{\sigma_A \sqrt{t}}, \; d_2 = d_1 - \sigma_A \sqrt{t}$$

- Applying Ito's lemma to (ii) and equating variance terms

$$\sigma_E = \frac{A}{E} \Delta \sigma_A \qquad \text{(iii)}$$

- Solve for asset value A and asset volatility

STANDARD &POOR'S

Estimating Correlations: From Equity Markets (Merton)

- Generate a time series of asset returns from a time series of equity prices
- Estimate correlations directly between pairs of firms or construct a factor model from the asset return time series

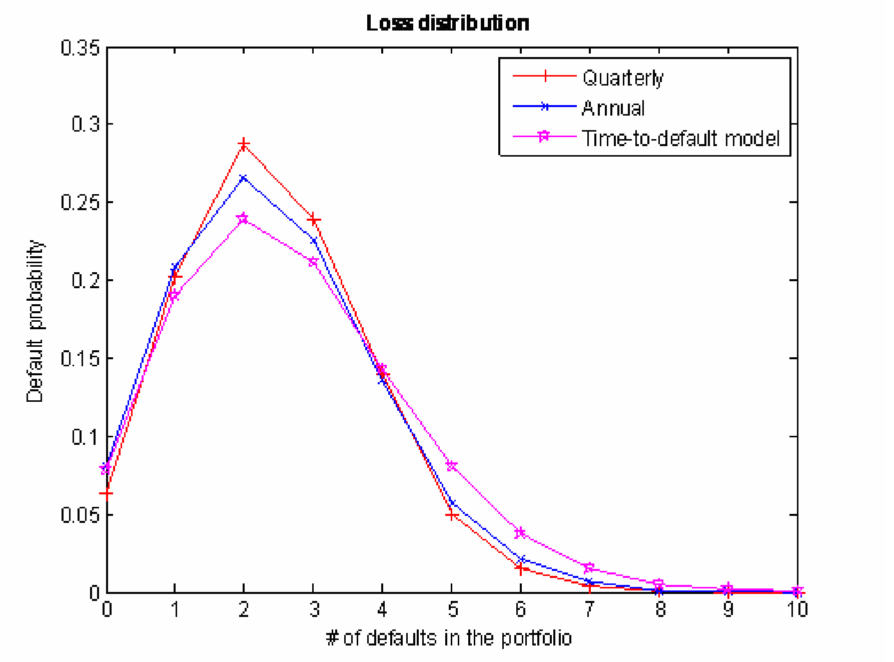
Challenges:

- Firms issue various types of debts and of varying maturities (short term vs long term, bullet vs amortizing, revolving lines of credit, convertible debt, preferred equity, etc). Defining default barrier is tricky, debt structure is dynamic.
- Estimates for correlation usually based on daily or weekly equity/asset returns. Is this appropriate for long horizons?
- How stable are correlations over time?
- Imposes substantial faith in modeling framework for longer horizons.

STANDARD &POOR'S

Single Step vs Multi-Step

- Correlations are typically estimated from multi-period data i.e. over short horizons, but applied over longer, single time step in the 'standard' GCDT model
- How do loss distributions generated from single and multi-step models compare over longer horizons?



- Experiment :
  - 10 names, randomly assigned NIG rating levels (1 BB+, 2 BB, 1 BB-, 2 B+, 2 B, 2 CCC)
  - Pair-wise correlation of 15%
  - Horizon of 5 years
  - Generate default distributions using multi-steps (quarterly & annual), and single step (standard default time model)

STANDARD & POOR'S

Estimating Correlations: From Credit Derivatives Markets

- Various correlation products trade actively in the market
  - Index tranches on the CDX North American indices and iTraxx Europe indices are the most liquid
- Can credit derivative prices be used to back out accurate correlation estimates?
  - **Note: These would be risk-neutral correlations more appropriate for pricing purposes**
- Experiment: Generate tranche spreads using a double t-distribution model and certain correlation (and spread) assumptions for asset return factor model (correlation specified by ‘a’)

$$X_i = a_i . S + \sqrt{1 - a_i^2} Z_i, \text{ where } S \text{ and } Z_i \sim \mathrm{St}(0,1,\nu)$$

- Use the fair spreads estimated above to back out an implied correlation using the standard Gaussian default time model.
- Assumptions:

ν = 4, 7 or 12

LGD = 50%

Constant spread, s = 50 Bps on each underlying name

Default intensity, $\lambda = s$ / LGD

$P_i(t) = 1 - e^{-\lambda t}$

Tranching similar to CDX.NA.IG (125 names)

Maturity of 5 years

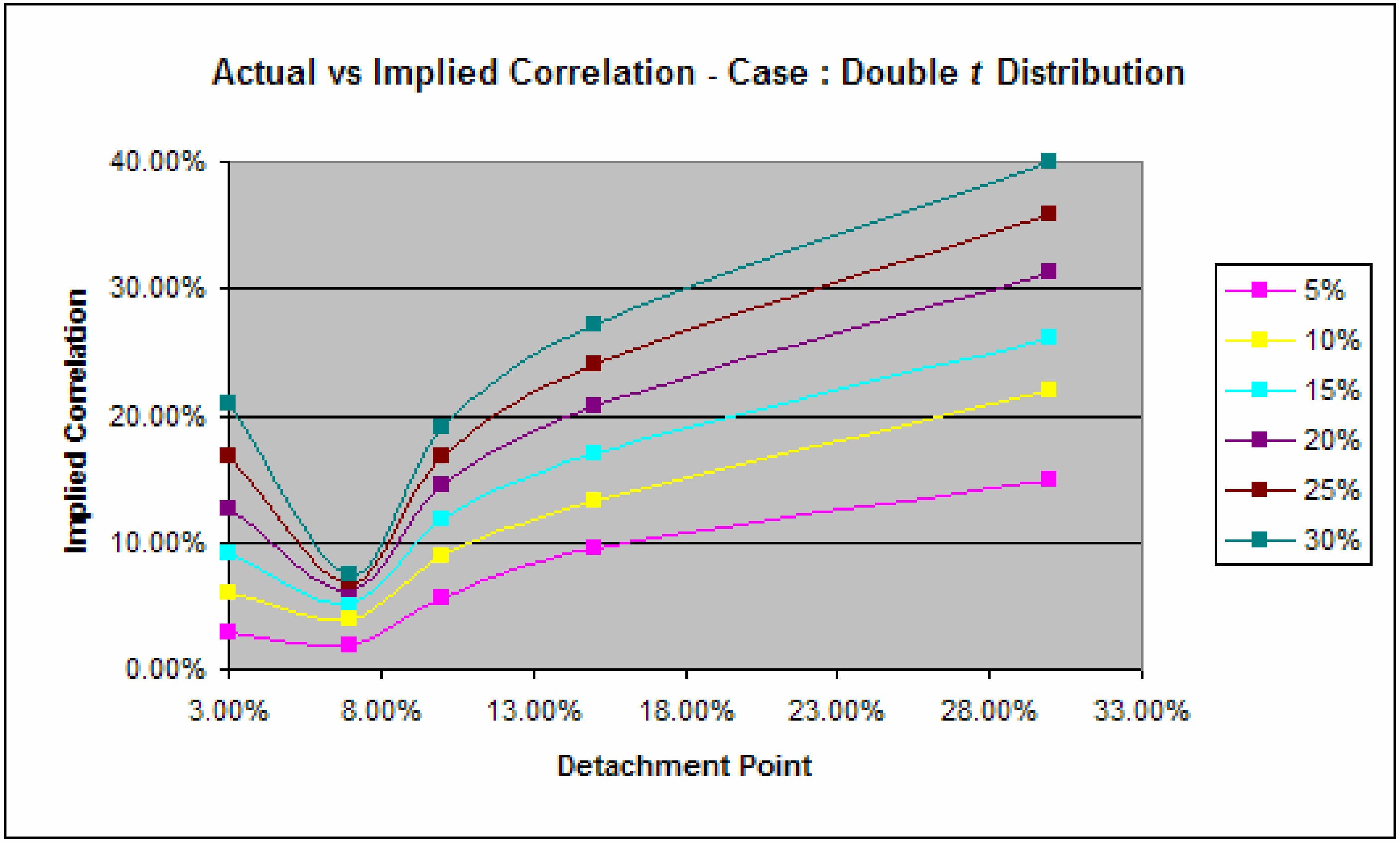
STANDARD & POOR’S

Estimating Correlations: From Credit Derivatives Markets

	Fair values of tranches using a double t-distribution (v = 4)				
Correlation	0%-3% Tranche (Upfront)	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0	53.24%	77.67	0	0	0
0.05	49.12%	126.92	8.85	3.1	0.89
0.10	45.41%	158.52	24.01	9.77	3.11
0.15	41.45%	176.6	40.76	18.95	6.7
0.20	37.66%	187.72	53.46	26.62	9.83
0.25	34.66%	199.81	68.44	36.39	14.97
0.30	31.21%	201.93	78.34	45.14	20.13
Implied correlations computed from the fair values above					
Correlation	0%-3% Tranche	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0	0	0	0	0	0
0.05	0.034	0.024	0.06	0.099	0.171
0.10	0.067	0.041	0.094	0.145	0.235
0.15	0.108	0.053	0.127	0.186	0.292
0.20	0.152	0.062	0.149	0.217	0.325
0.25	0.188	0.07	0.178	0.252	0.383
0.30	0.234	0.072	0.199	0.288	0.422

STANDARD & POOR'S

Fair spreads from a t-distribution model produce a correlation smile



- Implied correlations are very different from ‘true’ underlying correlations and vary across the CDO capital structure
- Mezzanine tranche (3-7) implied correlations cluster together regardless of actual underlying correlations
  - **Equity tranches are long correlation, senior tranches are short correlation. Thus mezzanine tranches, due to their location in the capital structure, typically display correlation insensitivity.**

STANDARD
&POOR’S

Other Forms of Dependence Modeling

STANDARD & POOR'S

Implicit Modeling of Correlation : Historical Simulations

- **Transitions between various credit quality levels (e.g. rating transitions) can be modeled through transition intensities**
- **Explicit modeling approach**
  - Estimate average (annualized) transition probabilities from time series of credit quality changes.
  - Transitions over periods other than a year obtained by taking appropriate powers
  - Simulate correlated asset returns and determine credit quality changes from the appropriate transition matrix
- **Historical (implicit) modeling approach**
  - Estimate a series of credit quality transitions, say annually, over a period of time. Transition probabilities will typically change from one year and retain the impact of cyclical or correlation effects
  - Sample a random point in time and select the corresponding (series of) transition matrices from the sampled point to the relevant horizon.
    - Example: If the horizon constitutes n time periods, we select n transition matrices starting with the sampled point in time. The product of the n transition matrices gives the effective transitions over horizon
    - Repeat the process by sampling new points in time
    - Simulation results capture the historical performance of the credit portfolio

STANDARD & POOR'S

Implicit Modeling of Correlation : Transitions with Covariates

- **Historical simulation example → time was the only covariate describing credit quality transitions**
- **Transitions with Covariates → relate changes in transition intensities to other explanatory variables: GDP growth rates, interest rates, shape of the yield curve, equity returns (S&P 500), equity return volatility (VIX), etc.**
    - Covariates can be global or sector and region specific
- **Conditional on macroeconomic covariates, performance of credit portfolios over multiple future periods can be ascertained.**
- **Example: Duffie et al. (2007) "Multi-period corporate default prediction with stochastic covariates" examines transitions to the default state and finds the following variables to have explanatory power:**
    - Firm's distance to default, firm's trailing one year stock return, the 3-month treasury bill rate, and the trailing one-Year return on the S&P 500
- **Note: For economic capital calculations, it is not only default prediction but also credit quality changes that are important.**
    - Case in point: Vast majority of the structured securities that have recently been downgraded have lost value but have not defaulted yet.

STANDARD & POOR'S

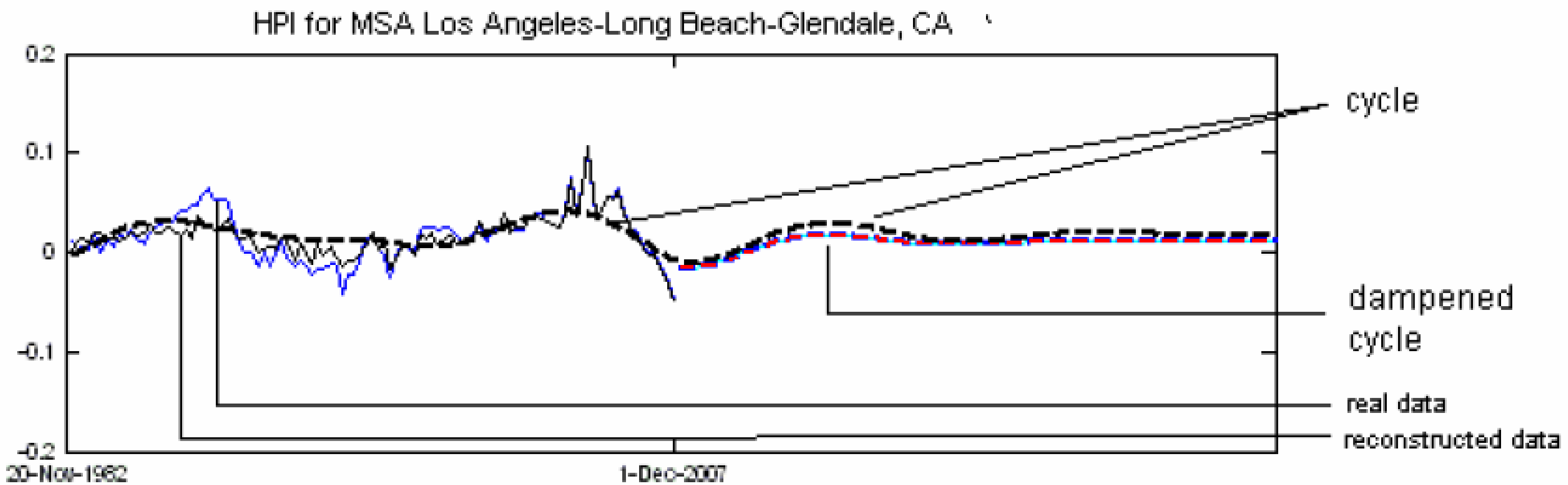
Factor Modeling of Correlation : RMBS analysis

- **HPA (Home Price Appreciation) is a primary risk driver of RMBS securitizations**
- **Modeling correlated changes in house price movements for different regions is key**
- **Start with a matrix, D, of quarterly growth rates for housing price indices (HPI) for different regions**
- **Other explanatory variables such as quarterly growth rate for LIBOR, CPI index etc. can also be included**

$$
\mathrm{D}_{1\ldots T} = \begin{pmatrix}
\Delta LIBOR_1 & \Delta HPI_{1,1} & \Delta HPI_{2,1} & \ldots & \Delta HPI_{381,1} & \Delta HInc_{1,1} & \ldots & \Delta HInc_{50,1} & \Delta CPI_1 \\
\Delta LIBOR_2 & \Delta HPI_{1,2} & \Delta HPI_{2,2} & \ldots & \Delta HPI_{381,2} & \Delta HInc_{1,2} & \ldots & \Delta HInc_{50,2} & \Delta CPI_2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\Delta LIBOR_T & \Delta HPI_{1,T} & \Delta HPI_{2,T} & \ldots & \Delta HPI_{381,T} & \Delta HInc_{1,T} & \ldots & \Delta HInc_{50,T} & \Delta CPI_T
\end{pmatrix}
$$

Factor Modeling of Correlation : RMBS analysis

- Use principal component analysis (PCA) to project the data on to *N* number of linearly independent vectors or factors
- Each of the *N* dimensions being linearly independent can be modeled in a 'univariate' manner.
- Typically the data tend to exhibit cyclical trends. One can take higher ordered differences or fit cycles (through cosine and sine functions) to the data using optimization schemes.



Estimated cycles for Los Angeles MSA

- Deviations from the trend can be fitted to an ARMA/GARCH process for simulating HPI scenarios in the future

STANDARD & POOR'S

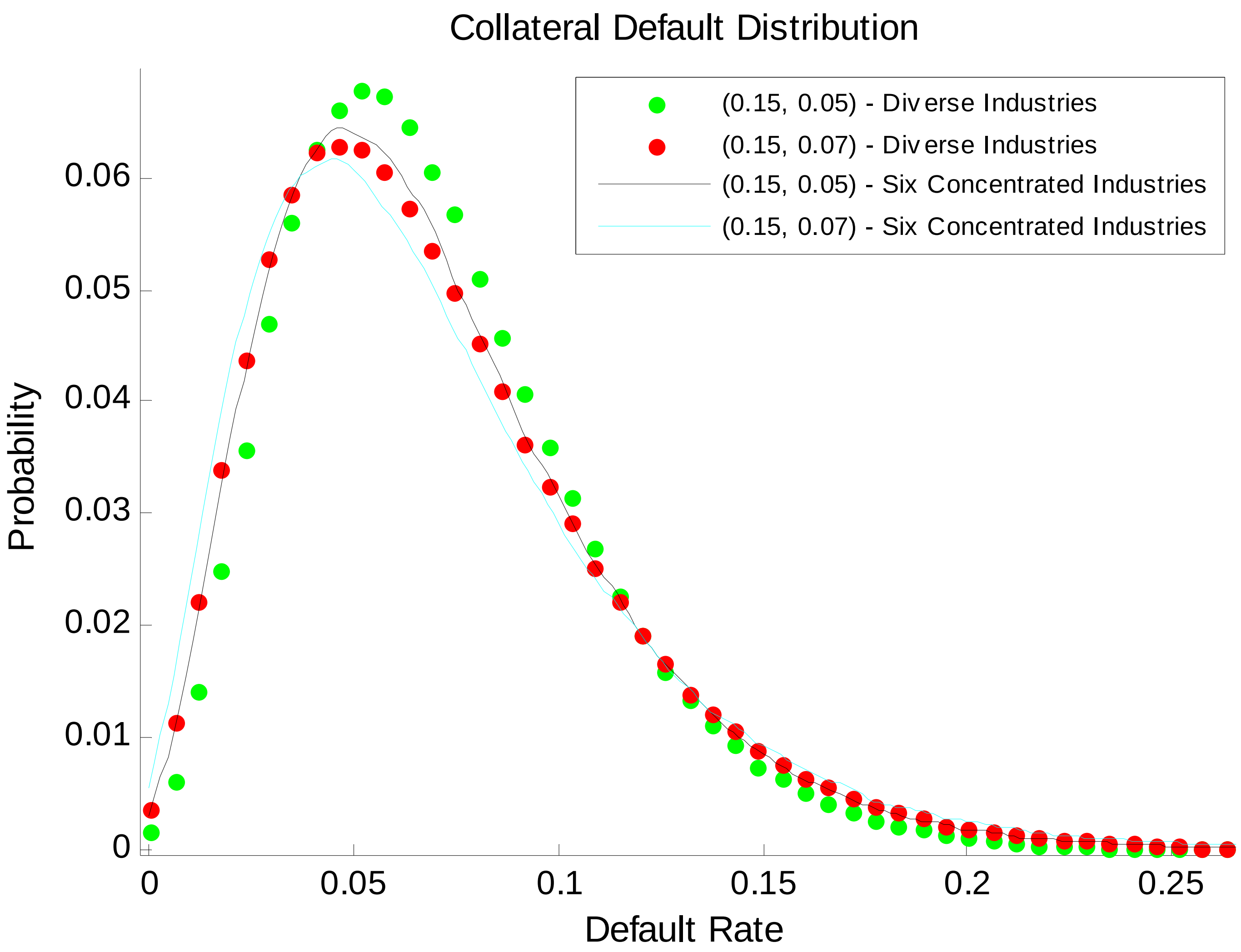
Correlation and Diversity

Within Industry Concentrations vs Across Industry Correlations

- **Within Industry correlations are found to be higher than across industry correlations**
  - Observance of joint defaults / rating downgrades in specific sectors
    - Telecoms in the early 2000s
    - Financials in the current crisis
- **Increased correlation risk in a portfolio can arise due to**
  - Concentrations in a few industries
  - Increased correlation across industries
    - Tighter integration of different sectors in the economy
    - Contagion
- **Experiment:**
  - 175 name portfolio distributed across 25 industries (diverse case) and 6 industries (concentrated case)
  - 5 year maturity; PD = 7% (similar to historical performance of BB+ bonds)
  - Within Industry correlation = 15%; Across industry correlation = 5%
  - Gaussian Copula Model

STANDARD & POOR'S

Within Industry Concentrations vs Across Industry Correlation



STANDARD & POOR'S

www.standardandpoors.com