Understanding Index Option Returns

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Expected option returns

- What is the expected return from buying a one-month maturity European put option on the S&P500 index (normalize to $S = 100$ and $K = 96$)?
- Assume the Black-Scholes model holds
- Is the expected option return:
  - Positive?
  - Zero?
  - Negative?

“Puzzling” index put option returns

1. *“… empirical evidence on option returns suggest that stock index options markets are operating inefficiently.”*

2. *“The most likely explanation is mispricing.... A simulated trading strategy yields risk-adjusted expected excess returns during the post-crash period…even when we account for transaction costs and hedge the downside.”*

3. *“We find significantly positive abnormal returns when selling options across the range of exercise prices, with the lowest exercise prices being most profitable.”*

“Puzzling” index put option returns

4. *“…volatility risk and possibly jump risk are priced in the cross-section of index options, but that these systematic risks are insufficient for explaining option returns. … short-term, deep OTM money put options appear overpriced relative to longer-term OTM puts and calls, often generating negative abnormal returns in excess of half a percent per day.”*
5. *“…we find that a number of strategies that involve shorting options have offered extremely high returns. These returns are hard to justify as compensation for risk, even after taking into account the nonlinear nature of option risks and their exposure to infrequent-jump risks.”*

The evidence

- Fact 1: Historical average put returns are very negative
  - -60% per month for 6% OTM options and -30% per month for ATM options

- Fact 2: Puts have larger historical Sharpe ratios than the underlying index
  - Roughly 3-4 times as large

- Fact 3: Puts have large historical CAPM alphas
  - -50% per month for 6% OTM options

An alternative view

"If you write a CAB, soon enough you'll be driving a cab"

(*Experienced option trader on the CME*)

- CAB: "cabinet trades," option trades occurring at the minimum offer price (bid price is zero). Deep OTM transactions.

- Numerous examples of blow-ups with deep OTM options: Victor Niederhoffer
  - Mini-crash of 1997: wrote deep OTM S&P puts for roughly \$1, covered at roughly \$30.
  - 2001 and 9-11: "*I was exposed. It was nip and tuck. Two-planes crashing into the World Trade Center, that was a totally unexpected event.*"
  - Summer 2007: "*The market was not as liquid as I anticipated. The movements in volatility were greater than I had anticipated. We were prepared for many different contingencies, but this kind of one we were not prepared for.*

What are the issues?

- Evidence indicates that some *statistical* sense, average returns, CAPM alphas, and Sharpe ratios are excessive given "risks"

- Concerns
  1. Interpreting these statistical metrics. How to quantify the 'risks' in options?
     - Shouldn't average returns be quite negative for puts (insurance)? Why use t-tests with a null value of zero?
     - Shouldn't CAPM alphas be different from zero for options?
     - Why is the Sharpe ratio relevant? Option returns are highly non-normal.

  2. "Noisy" data and statistical uncertainty
     - Finite sample issues: how can we estimate average option returns if we have a hard time estimating the equity premium?

- More general issue: how to evaluate returns generated by non-normal, non-linear securities like options?

Our approach

- What do standard option pricing models such as Black-Scholes-Merton and extensions with jumps or stochastic volatility imply for expected and realized option returns?

- Advantages

  1. Study how different factors affect expected and realized option returns.
  2. Anchor hypothesis tests at reasonable values (i.e. account for the fact that puts should have negative expected returns)
  3. Accounts for non-linear/non-normal option returns
  4. Easy to address finite sample issues
  5. Formal framework for evaluating explanations such as risk aversion or estimation risk

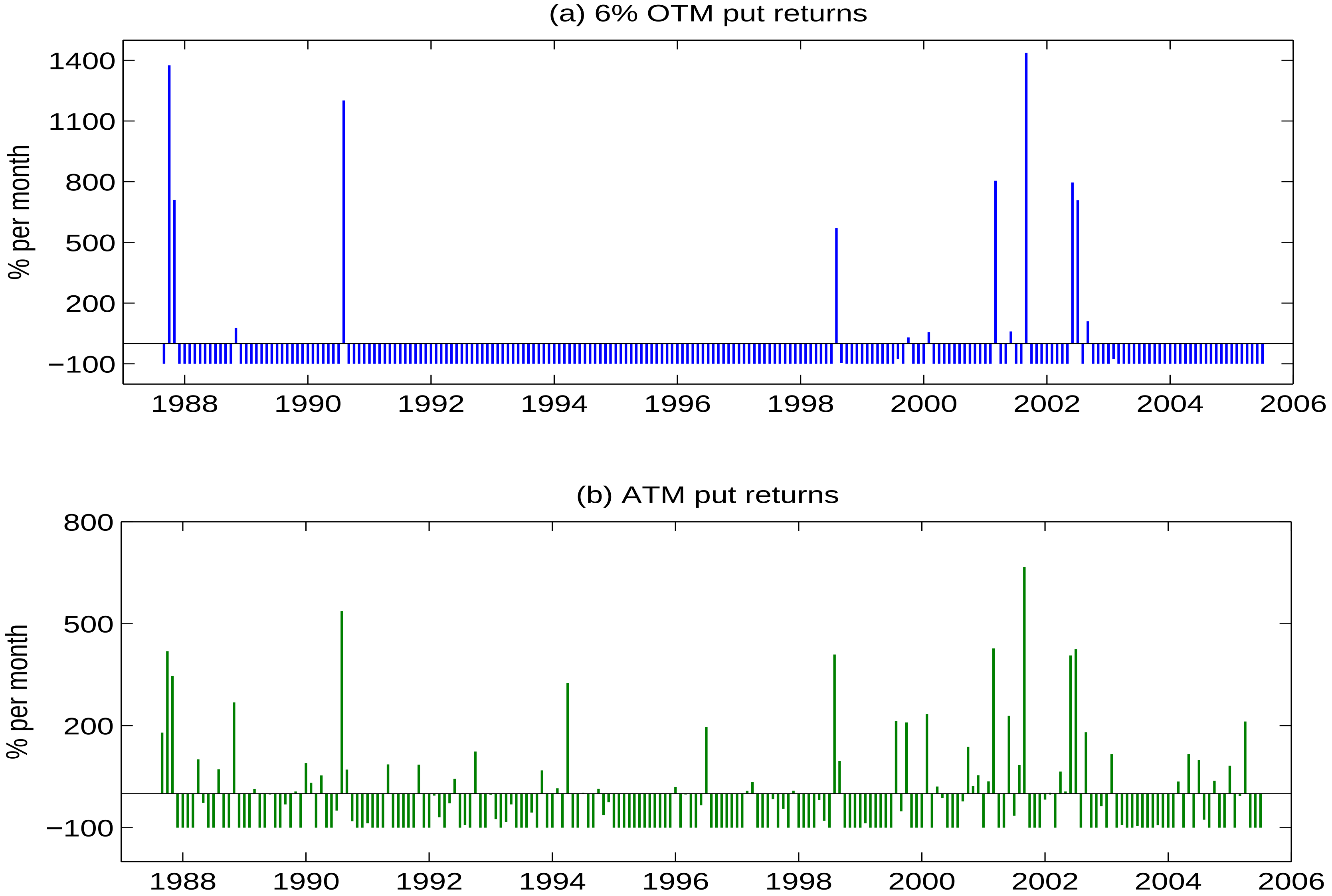
Historical option returns

- We focus on hold to expiration monthly returns: for puts

$$r_{t,T} = \frac{(S_T - K)^+}{P(t,T,K)}$$

- 215 months of S&P 500 futures options from 08/1987 to 06/2005
  - Serial contracts starting in 08/1987
  - Longer sample than existing studies: Jackwerth (2000): 86 months, Bollen and Whaley (2003): 60 and 144 months, Bondarenko (2003): 161 months
  - S&P 500 futures options and follow Broadie, Chernov, and Johannes (2007) in preparing the dataset

Put returns (Figure 1)



Evidence 1: Average returns

Moneyness	0.94	0.96	0.98	1.00	1.02
08/1987 to 06/2005	−56.8	−52.3	−44.7	−29.9	−19.0
Standard error	14.2	12.3	10.6	8.8	7.1
t-stat	−3.9	−4.2	−4.2	−3.3	−2.6
p-value, %	0.0	0.0	0.0	0.0	0.4
Skew	5.5	4.5	3.6	2.5	1.8
Kurt	34.2	25.1	16.7	10.5	7.1
Subsamples					
01/1988 to 06/2005	−65.2	−60.6	−51.5	−34.1	−21.6
01/1995 to 09/2000	−85.5	−71.6	−63.5	−50.5	−37.5
10/2000 to 02/2003	+67.2	+54.3	+44.5	+48.2	+40.4
08/1987 to 01/2000	−83.9	−63.2	−55.7	−39.5	−25.5

Table 2: Average put option returns. The first panel contains the full sample, with standard errors, t-statistics, and skewness and kurtosis statistics. The second panel analyzes subsamples. All relevant statistics are in percentages per month.

Evidence 2: Risk adjustments

Moneyness	0.94	0.96	0.98	1.00	1.02
CAPM α, %	-48.3	-44.1	-36.8	-22.5	-12.5
Std.err., %	11.6	9.3	7.1	4.8	2.9
t-stat	-4.1	-4.7	-5.1	-4.6	-4.2
p-value, %	0.0	0.0	0.0	0.0	0.0
Sharpe ratio	-0.27	-0.29	-0.29	-0.23	-0.18

Table 3: Risk-corrected measures of average put option returns. The first panel provides CAPM α's with standard errors and the second panel provides put option Sharpe ratios. All relevant statistics except for the Sharpe ratios are in percentages per month. Sharpe ratios are monthly. The p-values are computed under the (incorrect) assumption that t-statistics are t-distributed.

Our strategy

- Compute exact buy and hold expected returns

$$E_t^{\mathsf{P}}(r_{t,T}) = \frac{E_t^{\mathsf{P}}(S_T - K)^+}{P(t,T,K)} - 1 = \frac{E_t^{\mathsf{P}}(S_T - K)^+}{E_t^{\mathsf{Q}}(e^{-rT}(S_T - K)^+)} - 1$$

- Compute finite sample distributions
  - Simulate returns and return statistics (avg returns, CAPM alphas, SRs)
  - How close are they to the expected returns?
  - Parametric bootstrap

- What can we learn from Black-Scholes-Merton (and then progressively more complicated models)?

- Assumption: parameters match historical experience over our sample (equity premium, volatility, etc.)

Sensitivity of expected option returns

- Black-Scholes model
  - μ is the equity premium and σ is volatility

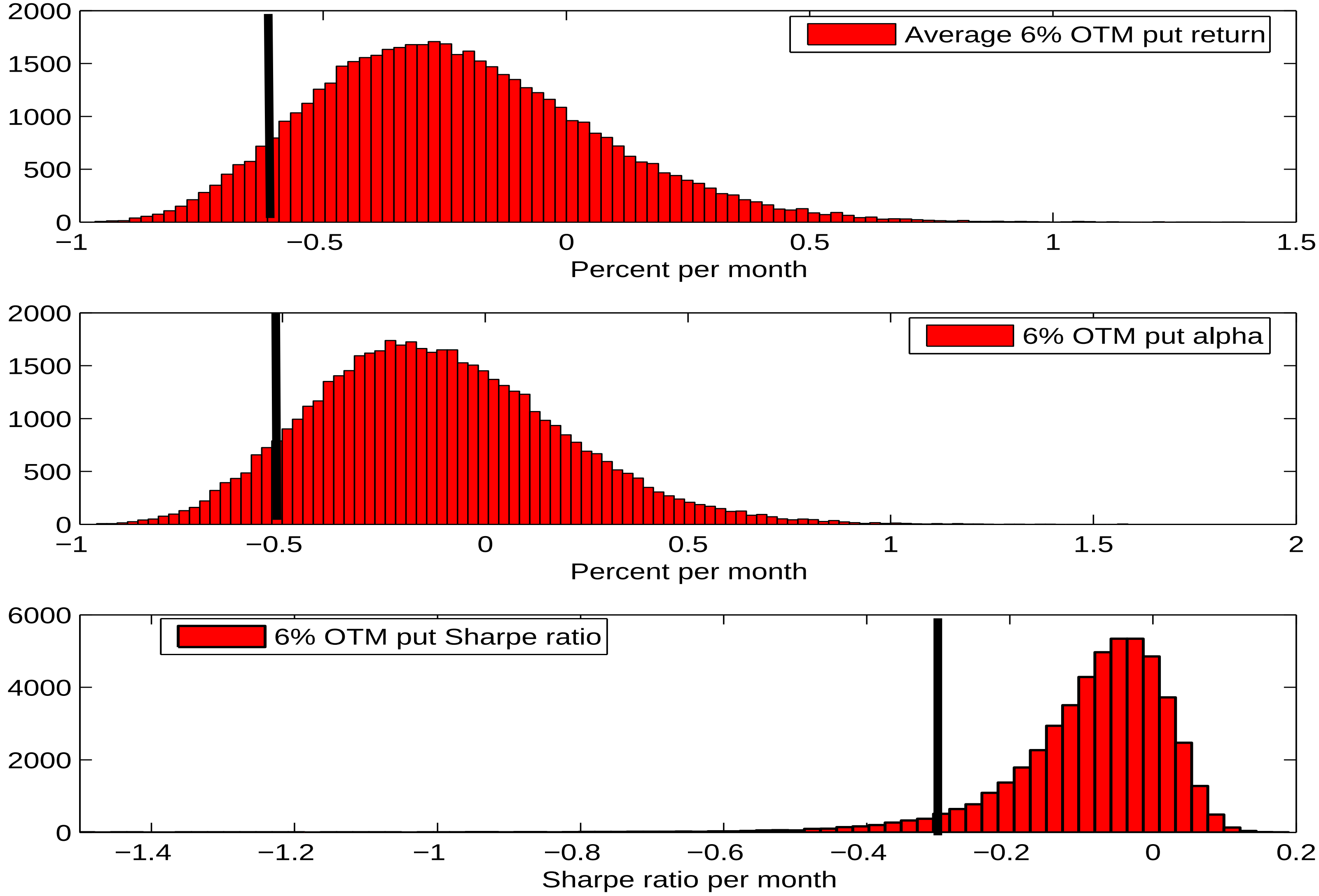
		Moneyness	
σ	μ	0.94	1.00
10%	4%	-28	-13
	6%	-39	-20
	8%	-48	-26
15%	4%	-15	-9
	6%	-23	-13
	8%	-29	-18
20%	4%	-10	-7
	6%	-15	-10
	8%	-20	-13

Black-Scholes-Merton returns in finite samples

- The parameter values are set to match historical returns on S&P 500 over our sample period

$$\mu = 5.4\%, \ r = 4.5\%, \ \sigma = 15\%$$

BSM: Finite sample distribution of average OTM put returns



Average returns and finite sample significance

	Moneyness		0.94	0.96	0.98	1.00
Average returns	Data, %		−56.8	−52.3	−44.7	−29.9
	BS	$E^{\mathbb{P}}$,%	−20.6	−17.6	−14.6	−12.0
		p-value,%	8.1	1.7	0.4	2.2
CAPM αs	Data, %		−48.3	−44.1	−36.8	−22.5
	BS	$E^{\mathbb{P}}$,%	−17.9	−15.3	−12.7	−10.4
		p-value,%	12.6	2.7	0.3	1.2
Sharpe ratios	Data		−0.27	−0.29	−0.29	−0.23
	BS	$E^{\mathbb{P}}$	−0.05	−0.07	−0.08	−0.09
		p-value,%	4.9	1.9	1.2	4.0

Borderline insignificant

Table 5: This table reports population expected option returns, CAPM α's, and Sharpe ratios and finite sample distribution p-values for the Black-Scholes (BS) and stochastic volatility (SV) models. We assume that all risk premia (except for the equity premium) are equal to zero.

Stochastic volatility

- The Heston SV model

$$dV_t = \kappa_v(\theta_v - V_t)dt + \sigma_v\sqrt{V_t}dW_t^{\mathbb{P}}$$

  - No volatility risk premia
  - Parameters estimated to match historical index returns over our sample

Stochastic volatility

OTM options:
Insignificant.
One in 4 paths
generates more
negative average
returns than data

	Moneyness		0.94	0.96	0.98	1.00
Average returns	Data, %		−56.8	−52.3	−44.7	−29.9
	BS	$E^{\mathbb{P}}$,%	−20.6	−17.6	−14.6	−12.0
		p-value,%	8.1	1.7	0.4	2.2
	SV	$E^{\mathbb{P}}$,%	−25.8	−21.5	−17.5	−13.7
		p-value,%	24.1	9.3	3.0	7.3
s	Data, %		−48.3	−44.1	−36.8	−22.5
	BS	$E^{\mathbb{P}}$,%	−17.9	−15.3	−12.7	−10.4
		p-value,%	12.6	2.7	0.3	1.2
	SV	$E^{\mathbb{P}}$,%	−23.6	−19.5	−15.8	−12.4
		p-value,%	39.1	14.1	3.4	8.7
Sharpe ratios	Data		−0.27	−0.29	−0.29	−0.23
	BS	$E^{\mathbb{P}}$	−0.05	−0.07	−0.08	−0.09
		p-value,%	4.9	1.9	1.2	4.0
	SV	$E^{\mathbb{P}}$	−0.04	−0.07	−0.09	−0.10
		p-value,%	21.5	12.0	7.7	14.3

Table 5: This table reports population expected option returns, CAPM α's, and Sharpe ratios and finite sample distribution p-values for the Black-Scholes (BS) and stochastic volatility (SV) models. We assume that all risk premia (except for the equity premium) are equal to zero.

Lessons

- Based on the SV model with no risk premia, we conclude that there is nothing anomalous about put returns
  - This is particularly true of deep OTM puts

- The effect of statistical uncertainty
  - It is very hard to measure average returns of highly-leveraged securities
  - Raw put returns (CAPM alphas and Sharpe ratios) are too noisy to use for tests: extreme statistical uncertainty

- What about portfolio strategies?

Alternative test portfolios

- Some interesting option portfolios
  - Delta-hedged puts: buy put and delta shares of index
  - ATM straddles (ATMS): buy ATM call and ATM put
  - Crash-neutral straddles (CNS): buy ATM straddle, sell deep OTM put
  - Put spread (PSP): buy ATM put, sell deep OTM put

- Why hold to maturity returns? Why not higher frequency strategies?
  1. Transactions costs
  2. Statistical properties
  3. Data requirements/liquidity

Alternate test portfolios

Strategy		Delta-hedged puts				ATMS	CNS	PSP
Moneyness		0.94	0.96	0.98	1.00			
Data, %		−1.3	−1.2	−1.0	−0.6	−15.7	−9.9	−21.2
BS	$E^{\mathbb{P}}$, %	0.0	0.1	0.0	0.0	1.1	2.2	−11.1
	p-val, %	0.0	0.0	0.0	0.0	0.0	0.8	12.5
SV	$E^{\mathbb{P}}$, %	−0.3	−0.1	−0.0	0.1	1.4	2.2	−13.1
	p-val, %	0.3	0.0	0.0	0.0	0.0	0.9	17.1

Table 6: Returns on option portfolios. This table reports sample average returns for various put-based portfolios. Population expected returns and finite sample p-values are computed from the Black-Scholes (BS) and stochastic volatility (SV) models. We assume that volatility risk premia are equal to zero. ATMS, CNS and PSP refer to the statistics associated with at-the-money straddles, crash-neutral straddles and put spreads, respectively.

Explanations

- Effect is largely due to difference between ATM implied volatility and subsequent realized volatility: “volatility gaps”
  - Historical volatility: 15%
  - Implied volatility: 17%
  - 2% gap largely generates returns

- What is the source of the gap?
  1. Genuine, persistent mispricing
  2. Jump risk premia
  3. Estimation risk/Peso problems (increase or decrease parameters by 1 standard deviation)

Stochastic Volatility and Jumps (SVJ)

- Add jump in prices to the Heston SV model (the Bates and Scott SVJ model)

- This is a rich model
  - Realistic description of historical index dynamics
  - Provides a lab for analyzing explanations for observed option returns:
    1. Jump risk premia
    2. Estimation risk

Calibration

- Main issue is how to calibrate parameters under both measures

- Historical measure
  - Straightforward to replicate historical experience. Estimate the model using historical returns over our sample.
  - Models fit equity premium, interest rate, dividend yield, and total volatility.
  - Jump and stochastic volatility parameters: simulate posterior distribution in SVJ model using Markov Chain Monte Carlo (MCMC) methods
    - These parameters provide a model-based statistical summary of the behavior of stochastic volatility and jumps in prices

- Risk-neutral measure: two distinct explanations
  - Jump risk premia
  - Estimation risk

Calibration

- P-parameters: Use S&P 500 index over our sample
  - Match the equity premium and volatility

- Q-parameters:
  1. Jump risk premia (γ=10)
     - Bates (1988); Naik and Lee (1990)
  2. Estimation risk: adjust parameters by one standard deviation

- In both cases, <u>no</u> options are used in the calibration

Option portfolio returns and significance

Strategy		Delta-hedged puts				ATMS	CNS
Moneyness		0.94	0.96	0.98	1.00		
Data, %		−1.3	−1.2	−1.0	−0.6	−15.7	−9.9
Jump risk premia	$E^{\mathbb{P}}$, %	−1.3	−1.0	−0.7	−0.4	−11.1	−4.6
	p-val, %	57.0	35.2	10.9	7.7	8.3	6.9
Estimation risk	$E^{\mathbb{P}}$, %	−0.9	−0.7	−0.6	−0.4	−11.0	−7.7
	p-val, %	30.8	18.2	7.6	10.6	12.4	17.5

- Nothing significant
- Can imagine both contribute (some jump risk premia and some estimation risk)

Conclusions

1. Deep OTM puts are not inconsistent with the SV model (without any risk premia beyond the equity premium)
   - Extreme statistical uncertainty: difficult to draw any conclusions from raw put returns

2. Returns on ATM straddles, delta-hedged and other portfolio strategies can be explained by
   - Jump risk premia
   - Estimation risk