

# NXP Semiconductor Problem: Model Order Reduction

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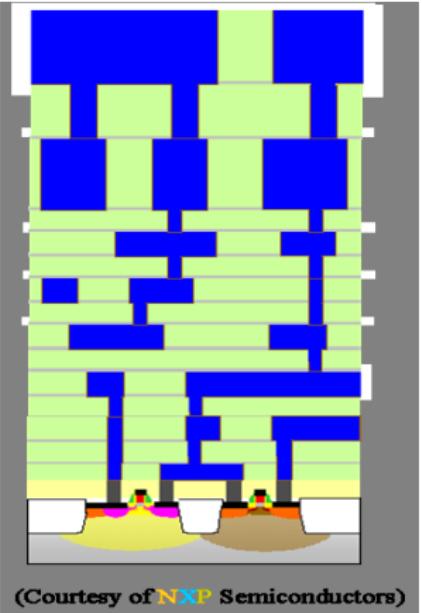
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# The Mathematics of Semiconductors Simulation of printed circuit boards (PCB's) and interconnects

NXP  
Semiconductor  
Problem:  
Model Order  
Reduction



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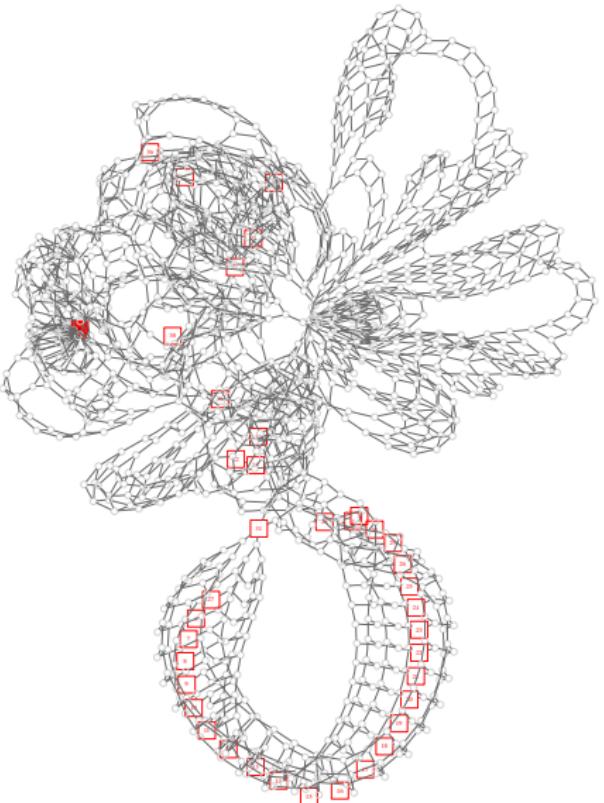
Model: Maxwell's equations

Problem 1: The substrate of the silicon layer can be modeled as a purely resistive network. Since the network is produced automatically by a commercial software package, it is cumbersome and huge. Therefore, one needs mathematical tools to reduce the network in order to test it.

Problem 2: The interconnect network above the silicon layer is not a pure resistive network. In this case, one must address issues of stability, passivity, and realizability.

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# Example: Graph Network



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# Mathematical Model Order Reduction

## Model reduction in the “resistive case”

From the system

$$\begin{pmatrix} R & -P \\ P^t & 0 \end{pmatrix} \begin{pmatrix} I \\ V \end{pmatrix} = \begin{pmatrix} 0 \\ J \end{pmatrix}$$

one can deduce that

$$(P^t R^{-1} P) \cdot V = J.$$

One can then obtain the values of the resistances between pairs of external nodes by solving this system for different vectors  $J$ . To “reduce” the model, we must find a way to remove some variables (nodes) from  $V$  and modify  $P$  and  $R$  in such a way that the resistances between pairs of external nodes remain the same.

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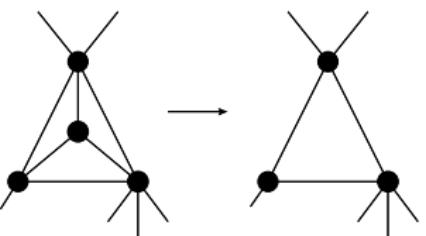
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## Graph theory and model reduction

Here is the famous “star to  $\Delta$ ” reduction.



This transformation is valid because the resistances on the edges of the  $\Delta$  can be modified in such a way that every resistance between two external nodes is unchanged.

Question: Can one find other such reductions (3-node cutsets, regions that are not traversed by any shortest path between external nodes, etc.)?

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## Motivation for Model Reduction

- ▶ Large models are hard to simulate
- ▶ Objective: To reduce the order of the model
- ▶ Any final graph MUST have all external nodes
- ▶ There exist large areas with no external nodes

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## Strategy

- ▶ Locate the “barren” (sparse) regions by finding shortest paths (SPs) between all external nodes
- ▶ There exists a transformation rule which Ortho will explain

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## Sketch of the Algorithm

► Step 1

Determine the SPs between all the external nodes,  
 $E_1, \dots, E_n$

► Step 2

List all edges  $(e_1, \dots, e_i)$  which have exactly one end point belonging to any SP

► Step 3

Remove all edges stated in Step 2

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## Sketch of the Algorithm (cont.)

- ▶ Step 4

Identify all components  $C_k$ ,  $k = 1, \dots, t$  in this new graph  $G'$

- ▶ Step 5

Apply the replacement rule if: # of edges in  $K_i \leq$   
# edges in  $C_k$

- ▶ Step 6

Replace the edges from Step 2 for the components that did not get replaced with a complete graph

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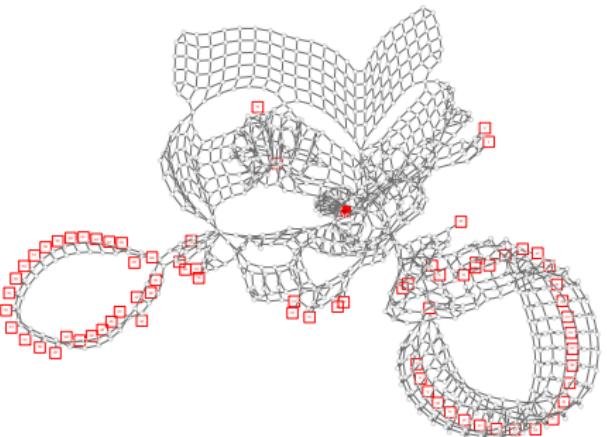


Figure 3: Graph of component 2 of network A.

## The Power of Network Reduction

Table: Capability of Network Reduction

Network Element	Before	After
ext	274	274
int	5384	205
resistors	8997	1505

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# Physical Model Order Reduction

# Stability of the Supernode Algorithm

Using BEM to discretize Maxwell's equations leads to:

$$\begin{aligned} (\mathbf{R} + s\mathbf{L})\mathbf{I} - \mathbf{PV} &= \mathbf{0} \\ \mathbf{P}^T + s\mathbf{CV} &= \mathbf{J} \quad (s = j\omega) \end{aligned} \tag{1}$$

Rewriting:

$$\left[ \begin{pmatrix} \mathbf{R} & -\mathbf{P} \\ \mathbf{P}^T & \mathbf{0} \end{pmatrix} + j\omega \begin{pmatrix} \mathbf{L} & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{pmatrix} \right] \begin{pmatrix} \mathbf{I} \\ \mathbf{V} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{J} \end{pmatrix} \tag{2}$$

**Goal:** To determine if reduced matrix (after non-super nodes are eliminated) is stable, given  $\mathbf{R}$ ,  $\mathbf{L}$  and  $\mathbf{C}$  are symmetric positive definite (SPD).

Reduced system is stable if eigenvalues of the matrix in (2) all lie in right half plane for all  $s$ .

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Case 1:  $s = 0$

System(2) becomes:

$$\begin{bmatrix} \mathbf{R} & -\mathbf{P} \\ \mathbf{P}^T & \mathbf{0} \end{bmatrix} \begin{pmatrix} \mathbf{I} \\ \mathbf{V} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{J} \end{pmatrix} \quad (3)$$

which can be reduced by elimination to:

$$\mathbf{P}^T \mathbf{R}^{-1} \mathbf{P} \mathbf{V} = \mathbf{J}. \quad (4)$$

$\mathbf{P}^T \mathbf{R}^{-1} \mathbf{P}$  = admittance matrix and is SPD.

Note: Reduced matrix is a principal submatrix of admittance matrix so is also SPD. Hence eigenvalues are in the right half plane.

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Case:  $s = j\omega$ ;  $\omega \neq 0$

Matrix in (2) can be written:

$$\mathbf{A} + j\mathbf{B}, \quad (5)$$

where

$$\mathbf{A} = \begin{pmatrix} \mathbf{R} & -\mathbf{P} \\ \mathbf{P}^T & \mathbf{0} \end{pmatrix} \quad (6)$$

$$\mathbf{B} = \omega \begin{pmatrix} \mathbf{L} & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{pmatrix} \quad (7)$$

Goal: To prove all eigenvalues of (5) lie in right half plane.

Proof: By contradiction. Suppose there is an eigenvalue,  $\epsilon$ , in the left half plane.

$$\det(\mathbf{A} + j\mathbf{B} - \epsilon\mathbf{I}) = \det \left( \begin{bmatrix} \mathbf{R} + j\omega\mathbf{L} & -\mathbf{P} \\ \mathbf{P}^T & j\omega\mathbf{C} \end{bmatrix} - \epsilon\mathbf{I} \right) = 0 \quad (8)$$

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Eigenvalues depend continuously on the parameter.

So, there exists  $\omega$  for which  $\epsilon$  lies on the imaginary axis.

By Schur complement approach:

$$\det(\mathbf{A} + j\mathbf{B} + \epsilon_0 j\mathbf{I}) = \det(j\omega C - \epsilon_0 j\mathbf{I}) \cdot$$

$$\det \left[ (\mathbf{R} + j\omega \mathbf{L} + \epsilon_0 j\mathbf{I}) + \mathbf{P} (j\omega \mathbf{C} + \epsilon_0 j\mathbf{I})^{-1} \mathbf{P}^T \right]$$

Consider the matrix

$$\left[ (\mathbf{R} + j\omega \mathbf{L} + \epsilon_0 j\mathbf{I}) + \mathbf{P} (j\omega \mathbf{C} + \epsilon_0 j\mathbf{I})^{-1} \mathbf{P}^T \right]$$

$$= \mathbf{R} + j \left[ \omega \mathbf{L} + \epsilon_0 \mathbf{I} - \mathbf{P} (\omega \mathbf{C} + \epsilon_0 \mathbf{I})^{-1} \mathbf{P}^T \right]. \quad (9)$$

Since  $\mathbf{R}$  is SPD and  $\left[ \omega \mathbf{L} + \epsilon_0 \mathbf{I} - \mathbf{P} (\omega \mathbf{C} + \epsilon_0 \mathbf{I})^{-1} \mathbf{P}^T \right]$  is symmetric, it follows that  $\det(\mathbf{A} + j\mathbf{B} - \epsilon\mathbf{I})$  is nonzero.

# Passivity of System

Definition of Passivity: For a given component there can be no electronic "power gain". The component consumes energy but does not produce it; it "cannot generate energy".

Mathematically:  $H^*(s) + H(s)$  is positive definite.

Important Concept: ALL passive systems are stable.  
However, NOT all stable systems are passive.

Using rank-one correction and Gaussian elimination, we were able to solve for the transfer function (i.e, the current and voltage) and showed the reduced system is passive.

Conclusions:

1. Current from the transform on the voltage is eliminated.
2. This holds for a general system not just for the reduced system only.

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## Realizability

It is highly desirable that the reduced model be *realizable*,  
i.e. consistent with Ohm's and Kirchhoff's laws.

$$\left\{ \begin{pmatrix} R & -P \\ P^t & 0 \end{pmatrix} + j\omega \begin{pmatrix} L & 0 \\ 0 & C \end{pmatrix} \right\} \begin{pmatrix} I \\ V \end{pmatrix} = \begin{pmatrix} 0 \\ J \end{pmatrix}$$

In other words, if some nodes (variables in  $V$ ) are removed from  $V$ , the structure of the system should not change.

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## The background (for real matrices)

In general, to eliminate variables from a system whose matrix is

$$\begin{pmatrix} A & B \\ B^t & C \end{pmatrix},$$

one can use the Schur complement representations

$$\begin{pmatrix} A - BC^{-1}B^t & B \\ 0 & C \end{pmatrix}, \begin{pmatrix} A & B \\ 0 & C - BA^{-1}B^t \end{pmatrix}.$$

Ideally, if the original matrix is positive definite, the Schur complements should also be positive definite. This is not a straightforward matter. Even if the Schur complements are positive definite, the new system might not be realizable.

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## Eliminating one node at a time

This amounts to performing a rank-one correction, corresponding to the case where  $C$  is a  $1 \times 1$  matrix on the previous slide. First we rewrite the Ohm's and Kirchhoff's laws as follows.

$$\left\{ \begin{pmatrix} R & -P & -p \\ P^t & 0 & 0 \\ p^t & 0 & 0 \end{pmatrix} + j\omega \begin{pmatrix} L & 0 & 0 \\ 0 & C & c \\ 0 & c^t & d \end{pmatrix} \right\} \begin{pmatrix} I \\ V \\ v \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix}$$

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## Application of the Schur complement process

The Schur complement process then yields a system consisting of the matrix equation

$$\left\{ \begin{pmatrix} R & -P \\ P^t & 0 \end{pmatrix} + \begin{pmatrix} p \\ 0 \end{pmatrix} v + j\omega \begin{pmatrix} L & 0 \\ 0 & C \end{pmatrix} + j\omega \begin{pmatrix} 0 \\ c \end{pmatrix} v \right\} \begin{pmatrix} I \\ V \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}$$

and the constraint

$$p^t I + j\omega (c^t V + d v) = r_3.$$

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## The final (realizable) system

Some careful algebraic manipulations yield the following reduced system, which is realizable.

$$\left\{ \begin{pmatrix} R & -P - \frac{pc^t}{d} \\ P^t + \frac{cp^t}{d} & 0 \end{pmatrix} + j\omega \begin{pmatrix} L + \frac{pp^t}{\omega^2 d} & 0 \\ 0 & C - \frac{cc^t}{d} \end{pmatrix} \right\} \begin{pmatrix} I \\ V \end{pmatrix} = \begin{pmatrix} \tilde{r}_1 \\ \tilde{r}_2 \end{pmatrix}$$

This process can be applied inductively to remove all the nodes that are not supernodes. Rank-one corrections can also be used to eliminate the currents flowing in and out of a node, and the resulting systems are realizable.

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## Eliminating Current Unknown

Original system:

$$\left[ \begin{pmatrix} \mathbf{R} & -\mathbf{P} \\ \mathbf{P}^T & \mathbf{0} \end{pmatrix} + j\omega \begin{pmatrix} \mathbf{L} & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{pmatrix} \right] \begin{pmatrix} \mathbf{I} \\ \mathbf{V} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{J} \end{pmatrix}$$

Reduced system:

$$\left[ \begin{pmatrix} \tilde{\mathbf{R}} & \tilde{\mathbf{r}} & -\tilde{\mathbf{P}} \\ \tilde{\mathbf{r}}^T & \rho & -\tilde{\mathbf{p}}^T \\ \tilde{\mathbf{P}}^T & \tilde{\mathbf{p}} & \mathbf{0} \end{pmatrix} + j\omega \begin{pmatrix} \tilde{\mathbf{L}} & \tilde{\mathbf{I}} & \mathbf{0} \\ \tilde{\mathbf{I}}^T & \gamma & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C} \end{pmatrix} \right] \begin{pmatrix} \tilde{\mathbf{I}} \\ i \\ \mathbf{V} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{J} \end{pmatrix}$$

Equation for  $i$ :

$$i = \frac{1}{\rho + j\omega\gamma} \left( -(\tilde{\mathbf{r}}^T + j\omega\tilde{\mathbf{I}}^T) \tilde{\mathbf{I}} + \tilde{\mathbf{p}}^T \mathbf{V} \right) = \mathbf{0}$$

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Equation for  $\tilde{\mathbf{I}}$ :

$$\left( \tilde{\mathbf{R}} + j\omega \tilde{\mathbf{L}} - \frac{\tilde{\mathbf{r}} + j\omega \tilde{\mathbf{I}}}{\rho + j\omega\gamma} (\tilde{\mathbf{r}}^T + j\omega \mathbf{I}^T) \right) \tilde{\mathbf{I}}$$

$$+ \left( \frac{\tilde{\mathbf{r}} + j\omega \tilde{\mathbf{I}}}{\rho + j\omega\gamma} \tilde{\mathbf{P}}^T - \tilde{\mathbf{P}} \right) \mathbf{V} = \mathbf{0}$$

Equation for  $V$  is:

$$\left( \tilde{\mathbf{P}}^T - \frac{\tilde{\mathbf{r}}^T + j\omega \tilde{\mathbf{I}}^T}{\rho + j\omega\gamma} \tilde{\mathbf{P}} \right) \tilde{\mathbf{I}} + \left( \frac{\mathbf{R}\mathbf{R}^T}{\rho + j\omega\gamma} + j\omega \mathbf{C} \right) \mathbf{V} = \mathbf{J}.$$

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By Gaussian elimination, we find the resulting system:

$$\begin{bmatrix} \begin{pmatrix} \tilde{\mathbf{R}} + K(\rho\mathbf{M}_1 + \gamma\omega^2\mathbf{M}_2) & -\tilde{\mathbf{P}} + K(\rho\mathbf{N}_1 + \gamma\omega^2\mathbf{N}_2) \\ \tilde{\mathbf{P}}^T - K(\rho\mathbf{N}_1^T + \gamma\omega^2\mathbf{N}_2^T) & \rho K \tilde{\mathbf{P}} \tilde{\mathbf{P}}^T \end{pmatrix} \\ + j\omega \begin{pmatrix} \tilde{\mathbf{L}} + K(\rho\mathbf{M}_2 - \gamma\mathbf{M}_1) & K(\rho\mathbf{N}_2 - \gamma\mathbf{N}_1) \\ K(\rho\mathbf{N}_2^T - \gamma\mathbf{N}_1^T) & \mathbf{C} - K\gamma\tilde{\mathbf{p}}\tilde{\mathbf{p}}^T \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{I}} \\ \mathbf{V} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{J} \end{pmatrix}. \end{bmatrix}$$

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Notation:

$$K = \frac{1}{\rho^2 + (w\gamma)^2},$$

$$\mathbf{M}_1 = \tilde{\mathbf{r}}\tilde{\mathbf{r}}^T - \omega^2 \tilde{\mathbf{l}}\tilde{\mathbf{l}}^T,$$

$$\mathbf{M}_2 = \tilde{\mathbf{l}}\tilde{\mathbf{l}}^T + \tilde{\mathbf{r}}\tilde{\mathbf{r}}^T,$$

$$\mathbf{N}_1 = \tilde{\mathbf{r}}\tilde{\mathbf{p}}^T,$$

$$\mathbf{N}_2 = \tilde{\mathbf{l}}\tilde{\mathbf{p}}^T.$$

In order to have almost the original form we assume that

$$\rho\mathbf{N}_2 - \gamma\mathbf{N}_1 = 0??$$

## Conclusions

- ▶ For the substrate, where we need to drastically reduce a large resistive network, an algorithm has been formulated.

New features:

- ▶ elimination of large sets of internal nodes that are far from external nodes
- ▶ the use of cutsets

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- ▶ For the interconnect structures, a thorough investigation has been performed to see whether stability and passivity can be preserved whenever non-supernodes (and corresponding current branches) are eliminated.
  - ▶ This is an entirely new way of looking at the problem.
  - ▶ The resulting reduced systems are all stable and passive.
  - ▶ Future work needs to concentrate on investigating the nonzero offdiagonal blocks in the  $\begin{pmatrix} L & 0 \\ 0 & C \end{pmatrix}$  matrix.
  - ▶ Suspicion we have: By correct choice of nodes/branches to be deleted, these blocks may end up being 0 automatically.

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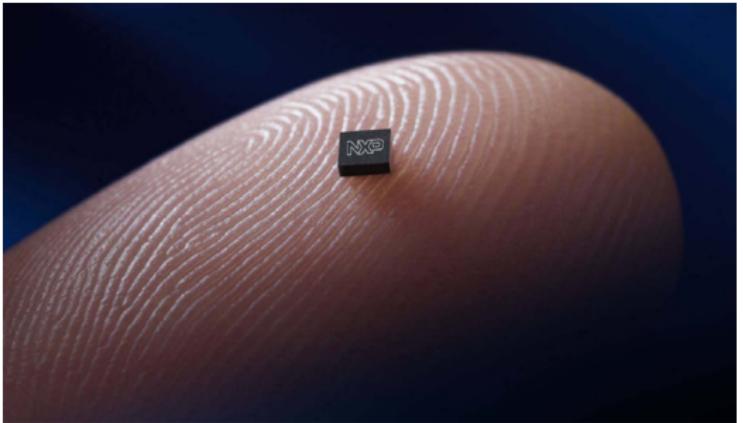
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NXP  
Semiconductor  
Problem:  
Model Order  
Reduction



The industry representative is very happy!

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