



Mathematical problem from the Electronics Industry

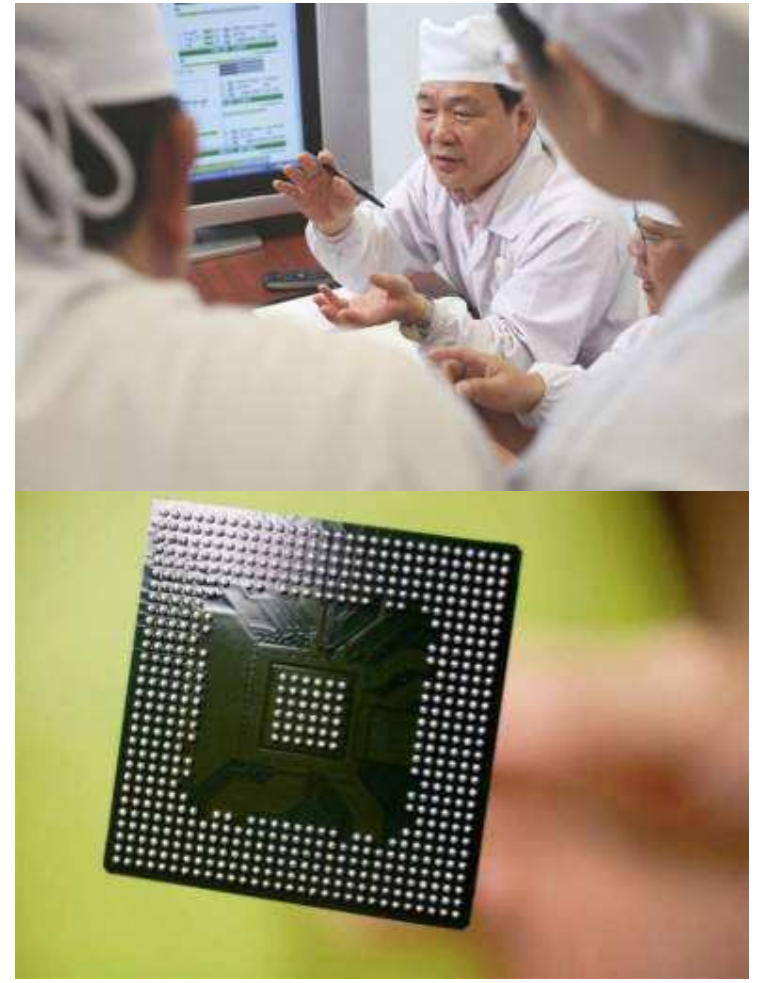
Wil Schilders
NXP Semiconductors

August 11-15, 2008
Fields-MITACS Industrial Problem-Solving Workshop
Toronto, Canada



Outline

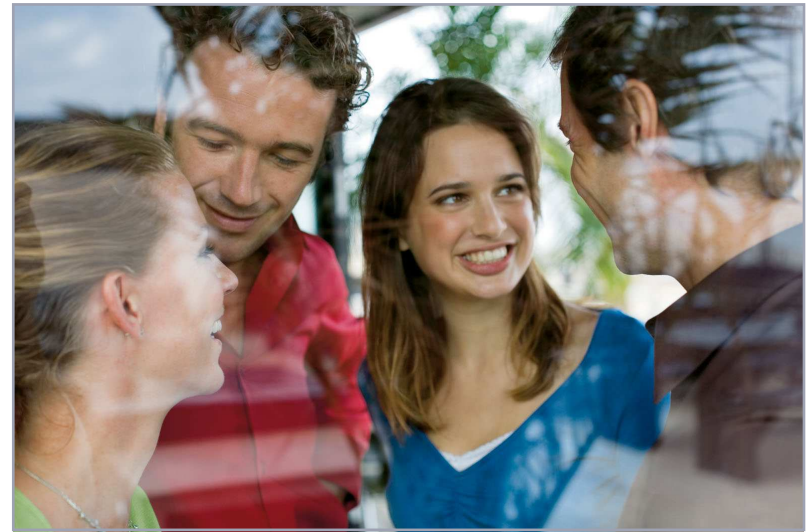
- ▶ NXP Semiconductors and mathematics
- ▶ Model Order Reduction
- ▶ The Problem



Introducing NXP Semiconductors

Leader in Vibrant Media Technologies

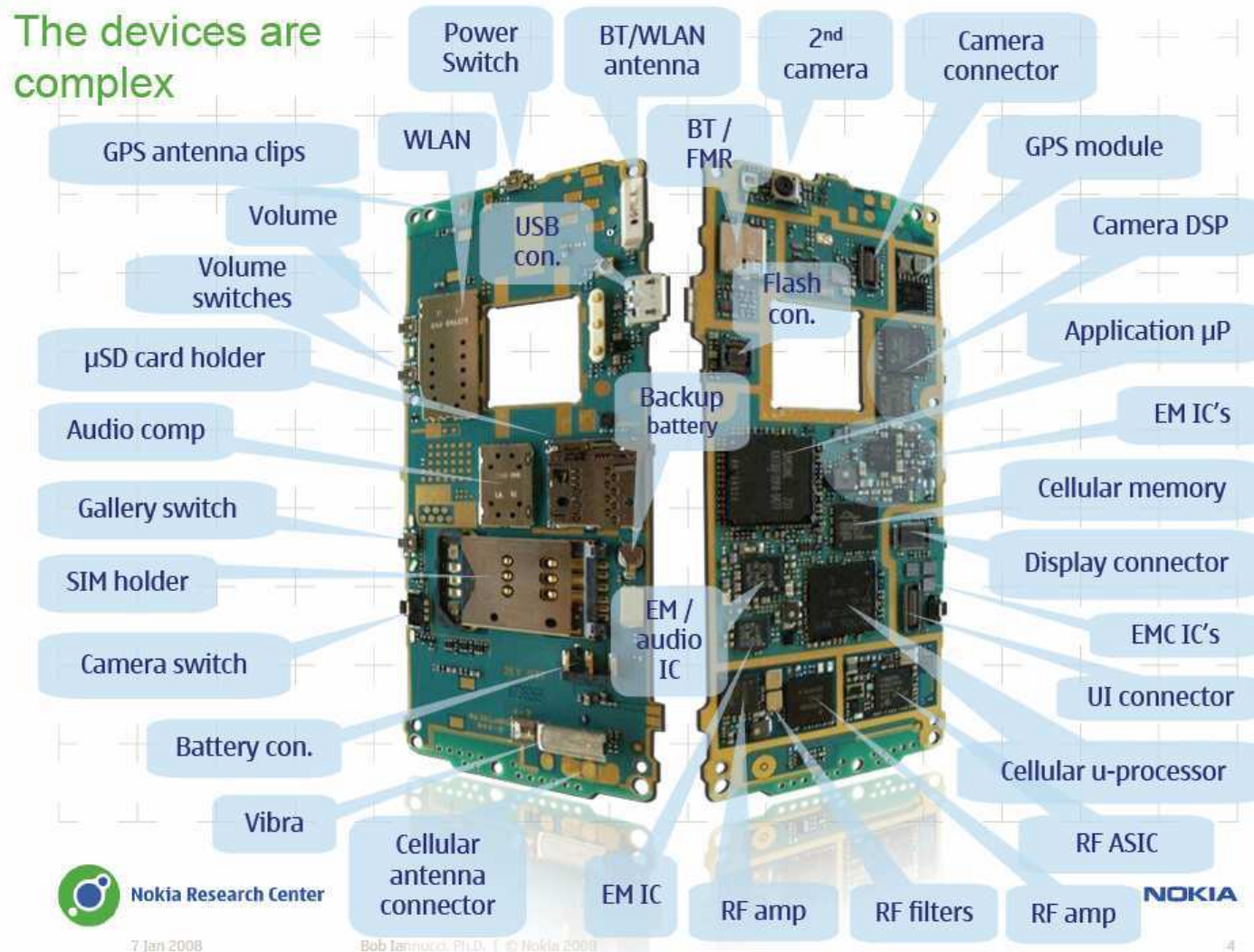
- ▶ Established in 2006
(formerly a division of Royal Philips)
- ▶ 53 years semiconductor experience
- ▶ Semiconductors and software that enable better sensory experiences
- ▶ Key focus areas:
 - Mobile & Personal
 - Home
 - Automotive
 - Identification
 - Multi market Semiconductors
- ▶ 37,000 employees / 7,500 engineers
- ▶ Investing € 950 million in R&D annually
- ▶ 25,000+ patents
- ▶ Private Equity Backed



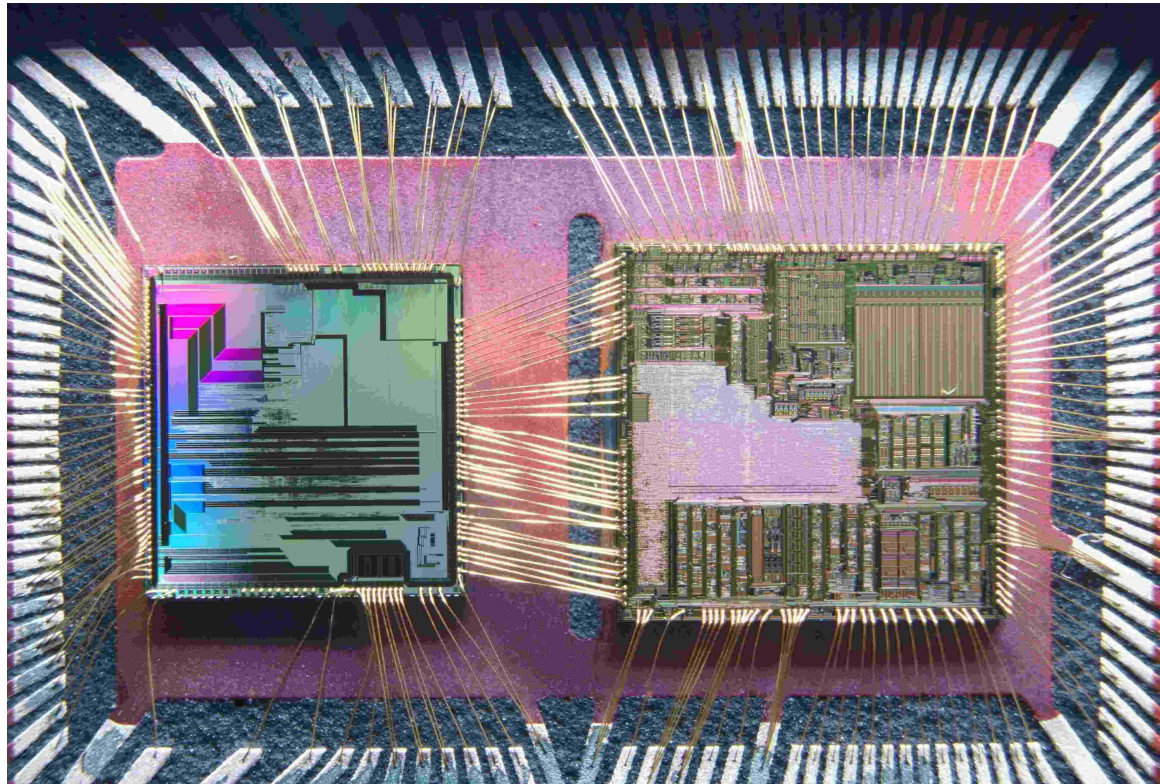
Next eXPerience



ICs in Mobile Phone: Nokia's view...



Our offering



Mixed signal chip

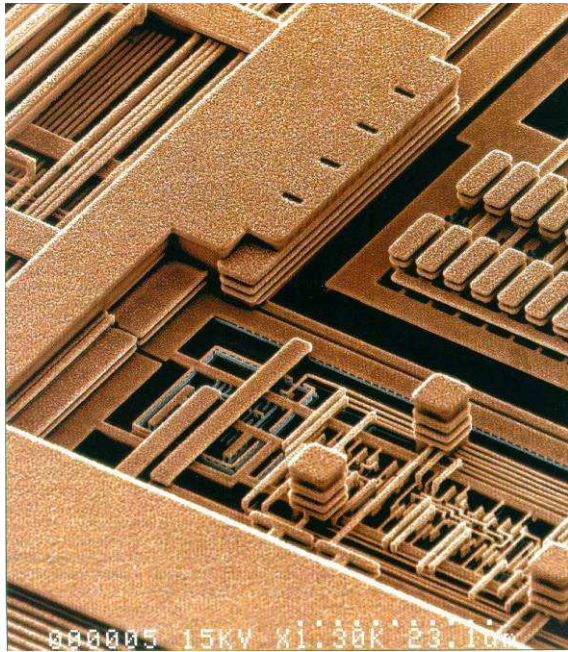
- video signal processing
- 50,000 components in analog part
- 50,000 components in digital part

Digital chip

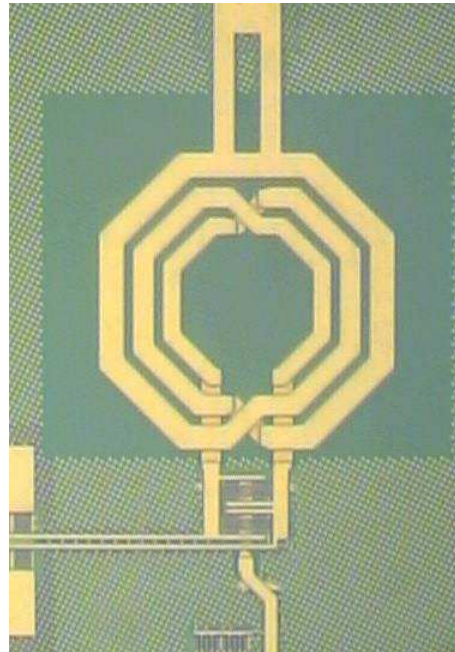
- TV control; -teletext decoder
- stereo decoder
- 50,000,000 transistors



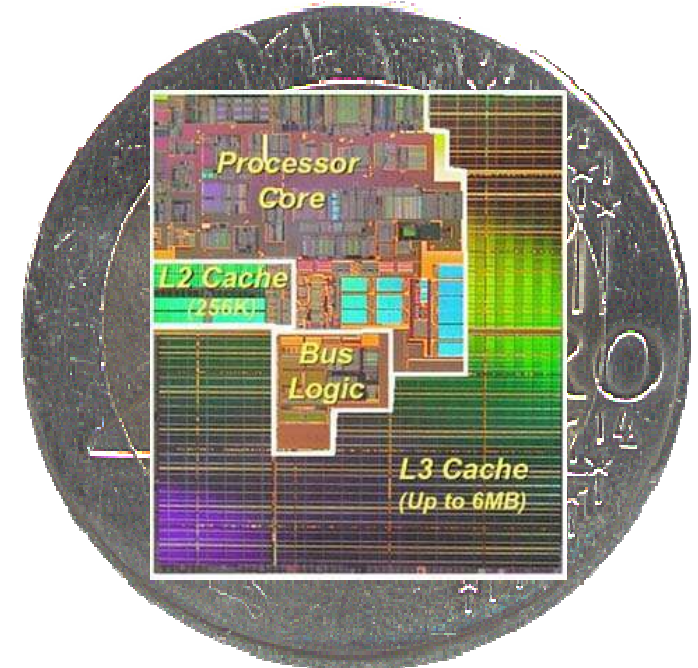
Computer chips: dealing with complexity



Close-up of wiring
~100 nm pattern
details



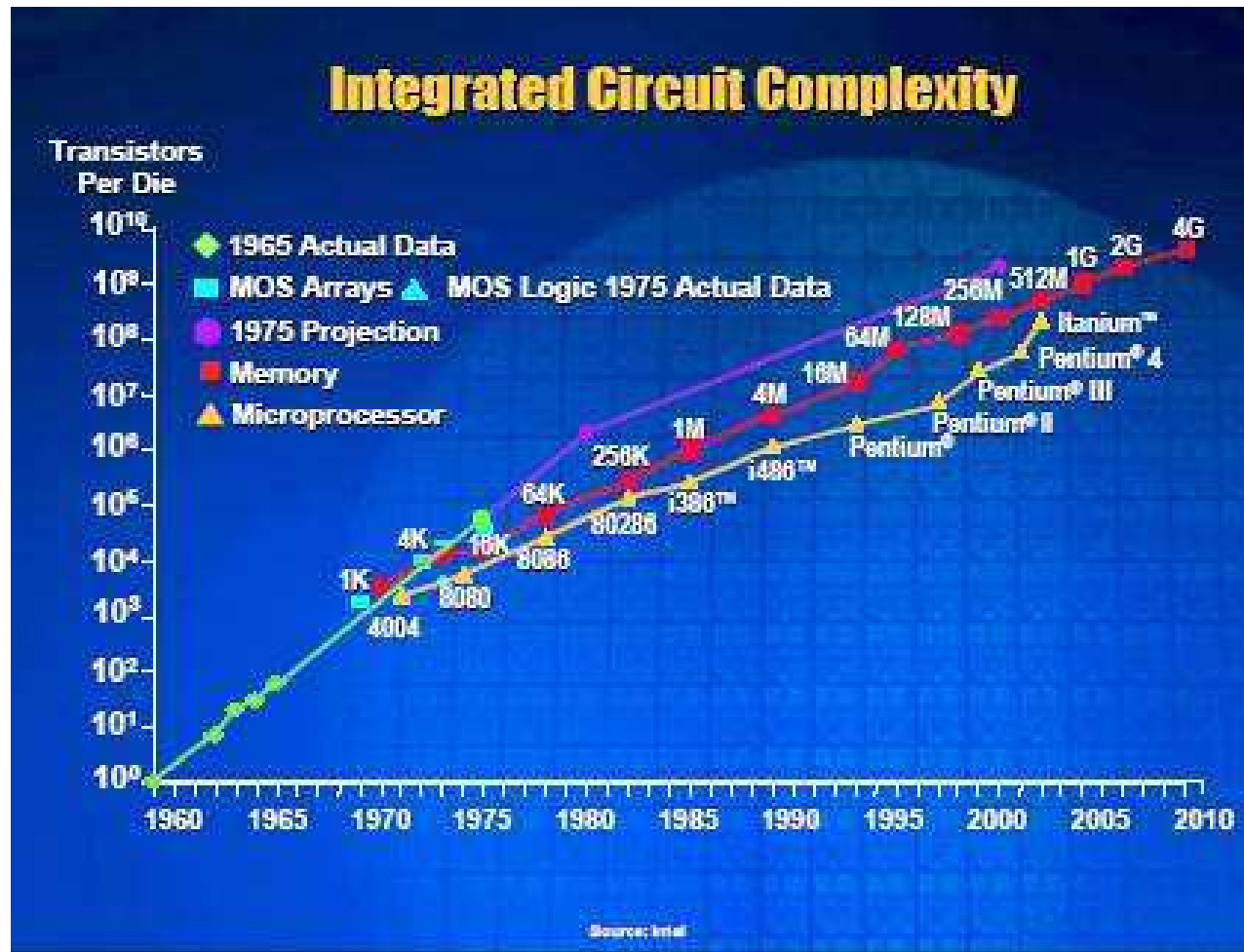
**Infineon 50 GHz
VCO coil**
~500 μm of
interconnect



**Intel Itanium
(McKinley, 2002)**
~10 km interconnect

IC Modeling deals with $> 10^{11}$ scale range

Enabled by Moore's law...



From the ITRS 2007

DIFFICULT CHALLENGES

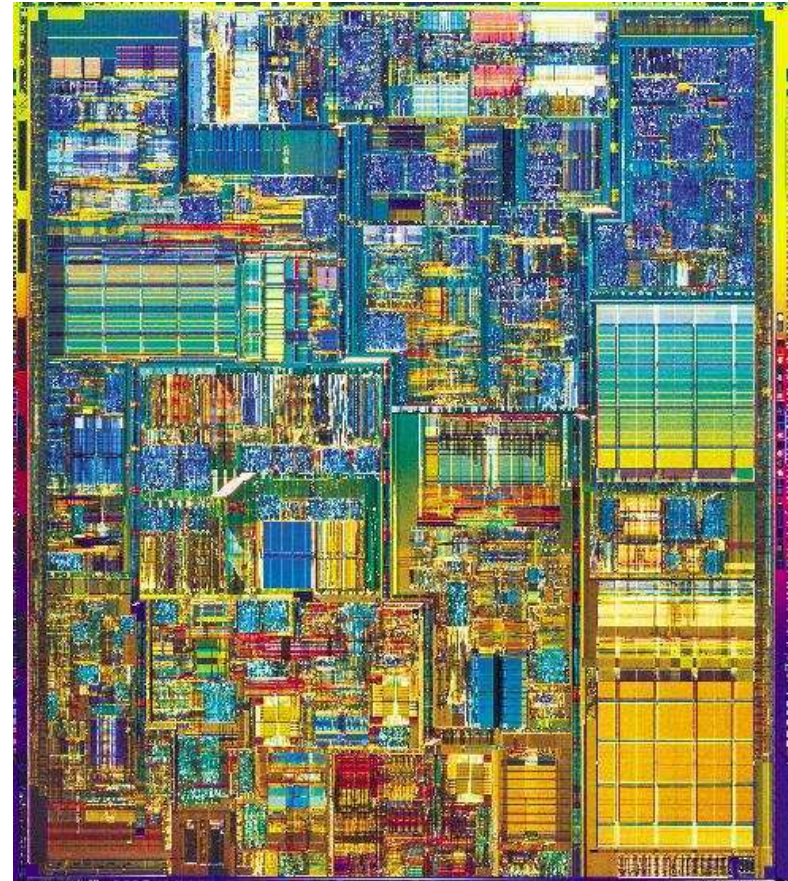
*Table ITWG4 RF and Analog Mixed-Signal (RF and AMS) Technologies for Wireless Communications
Difficult Challenges*

| <i>Difficult Challenges</i> | <i>Summary of Issues</i> |
|-----------------------------|--|
| Radio Integration | Performance and cost trade-offs for SoC versus SiP solutions Signal isolation and integrity are challenges to technologists, designers, and EDA tool providers for both analog and digital domains CAD solutions for integrated radio SiP designs (chip, passive, MEMS, package, tool compatibility, and model accuracies) |
| Device Technology | Optimizing analog/RF CMOS devices with scaled technologies. Fundamental changes in CMOS device structure may lead to the need for separate process/chip to support conventional precision analog/RF devices Increasing Ft of silicon bipolar devices by more aggressive vertical profiles Managing higher current and power densities that result from aggressive vertical profiles in silicon bipolar devices Performance and cost trade-offs for integrating passive devices Predictability of battery technology (end-of-life) and its impact on PA roadmap Compound semiconductor substrate quality, reliability, thermal management, particularly for GaN Low-cost processing equipment for compound semiconductors |
| Design | Design approach for wider range of supply voltages Digitizing analog functions in the software define radio (SDR) Non-linear and 3D Electromagnetic models for accurate design and simulation Computationally efficient physical models for compound semiconductors Thermal modeling and simulations that are integrated with RF and digital design tools. |

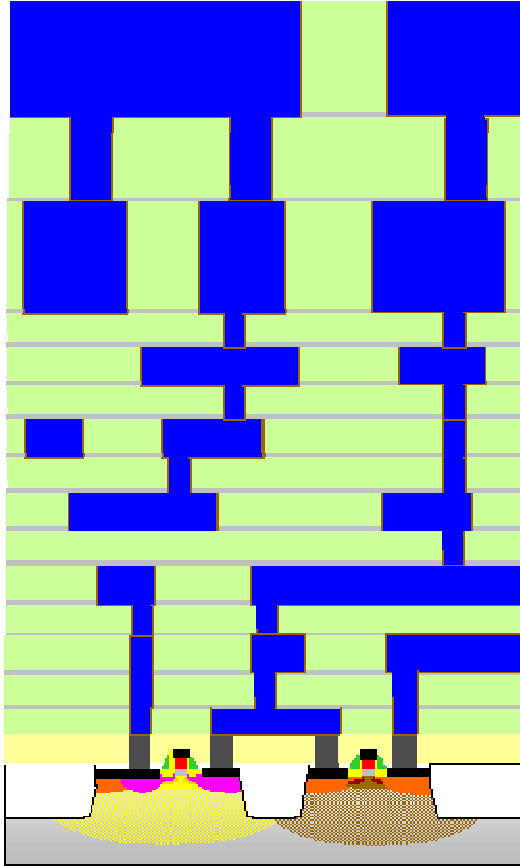


Challenges translated into mathematics (1)

- ▶ Electronic circuit simulation
 - Large (millions) discrete networks containing resistors, capacitors, inductors, diodes, transistors,.....
 - “Discretisation” simple using Kirchhoff’s laws
 - Extremely nonlinear models for diodes and transistors
 - Sparse linear systems with hierarchical structure
 - Various formulations leading to definite or indefinite systems
 - The substrate is modeled by using extremely large extracted R, RC and RLC networks; need to be reduced to acceptable size
 - Pole-zero analysis



Challenges translated into mathematics (2)

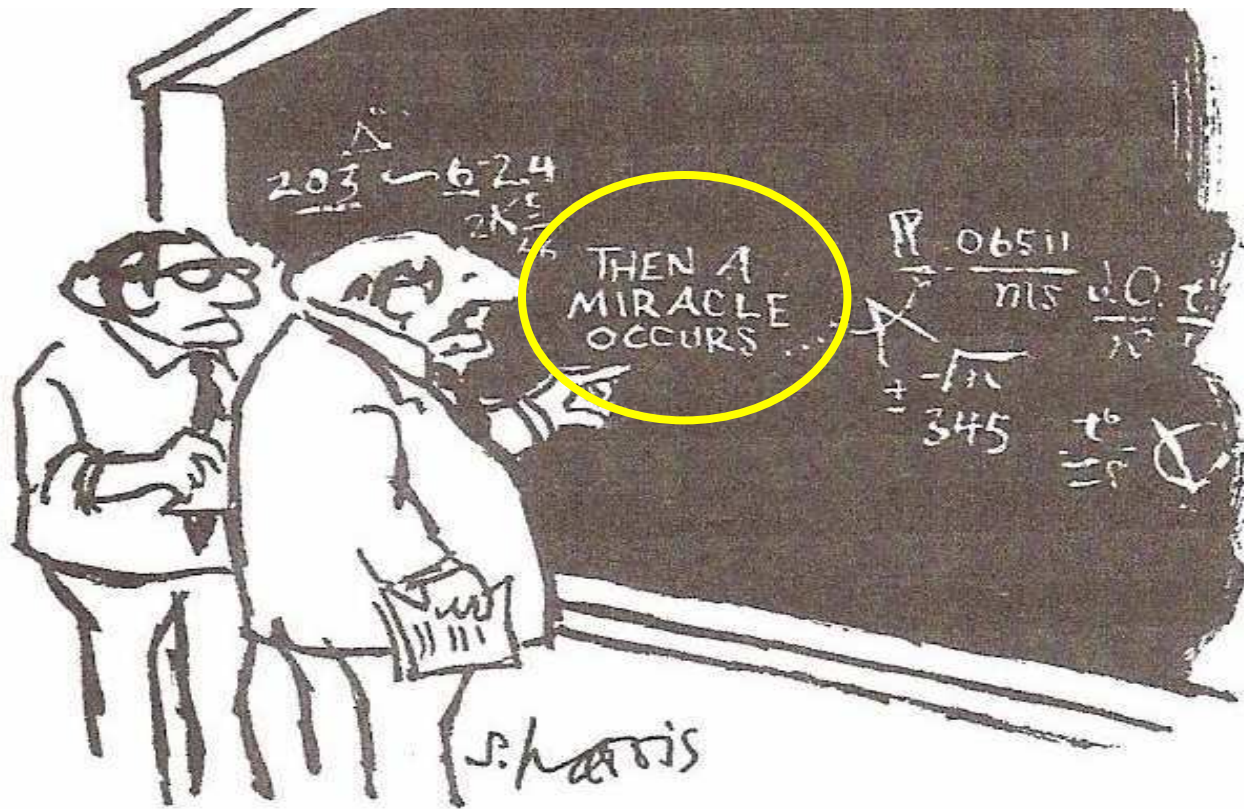


(Courtesy of NXP Semiconductors)

► Electromagnetics

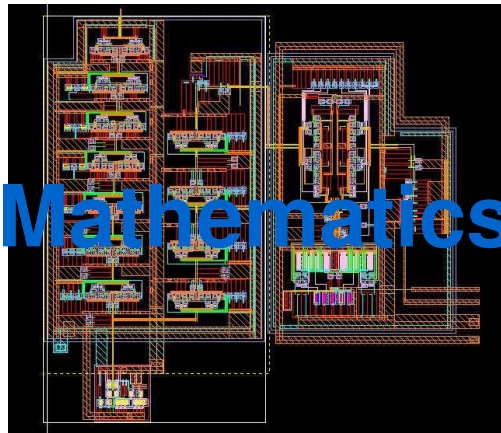
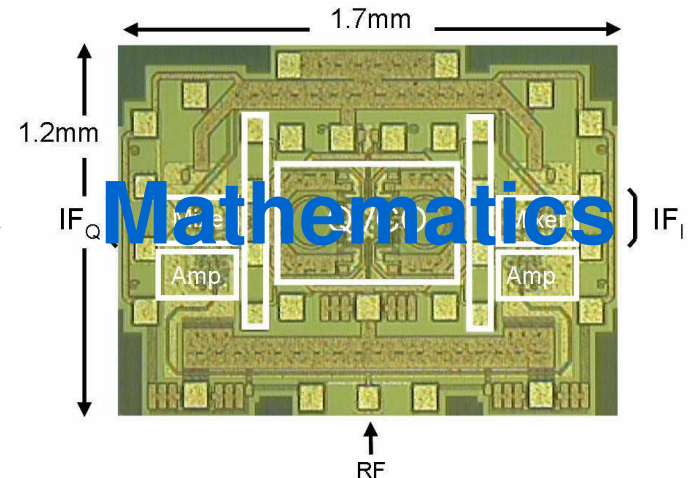
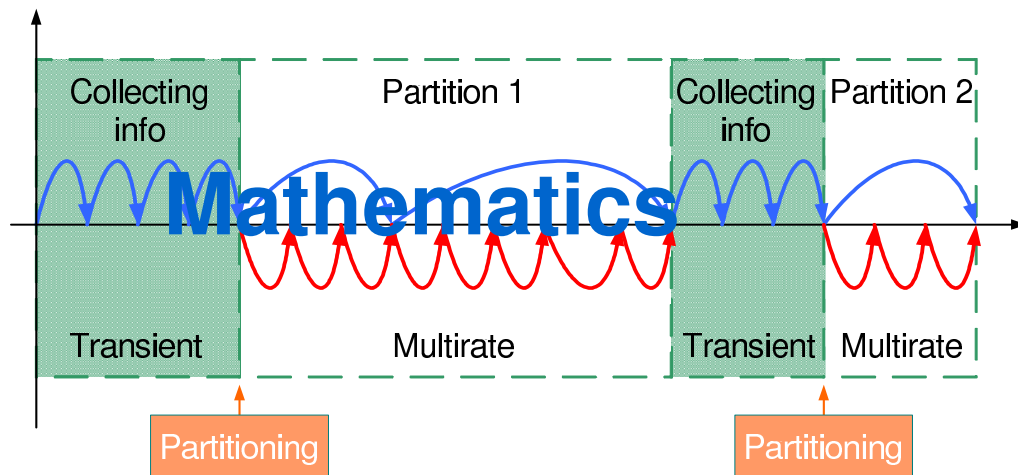
- At high frequencies (RF), electromagnetic effects may start to influence behaviour
- This leads to delay of signals, malfunctioning devices, undesired effects in substrate
- Maxwell equations need to be solved, either full wave (using FDTD, FIT, BEM, FVM, ...) or approximately (PEEC)
- Resulting large systems must be reduced to acceptable size (to be coupled with circuit)
- Complex linear systems of large size
- Iterative methods cannot cope with high frequencies
- Dependence of eigenvalues on frequency

The work of mathematicians may not always be very transparant.....



"I think you should be more explicit here in step two."

But it is present everywhere in our business



**Invisible contribution,
visible success**

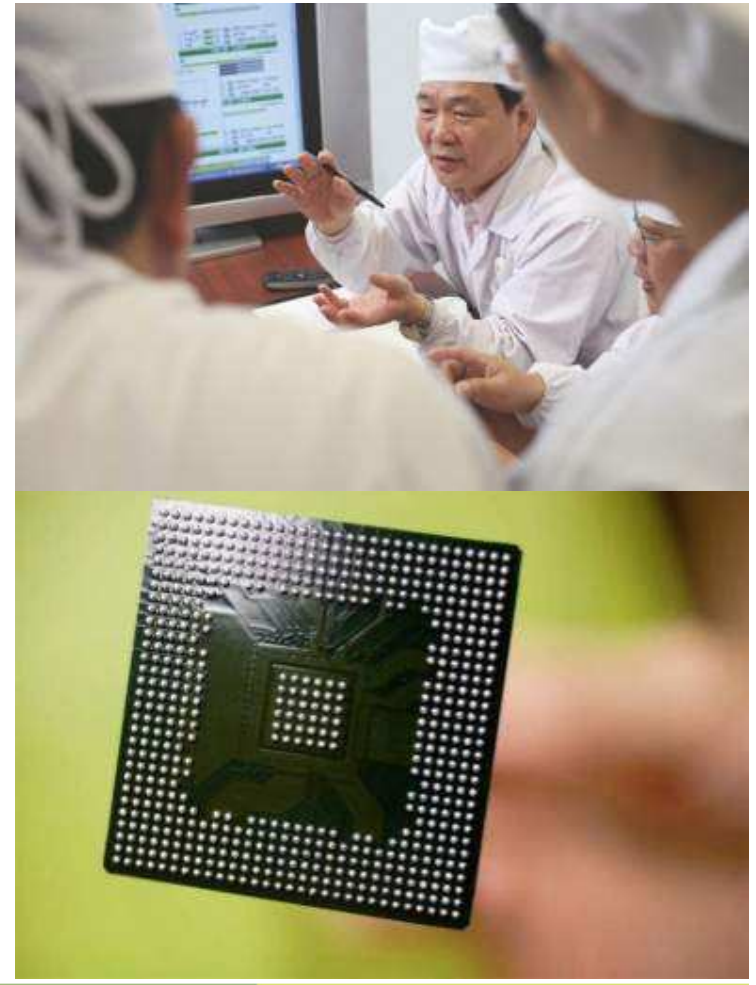




**~~Mathematics in Industry –
Cost Factor or Key for Profits ?~~ !**

Outline

- ▶ NXP Semiconductors and mathematics
- ▶ Model Order Reduction
- ▶ The Problem



- ▶ In many areas of application one has reached a point where **more realistic** simulations are needed and feasible, but only at the price of **far more complex** computational models.
- ▶ For instance, the **dynamic coupling** between **formerly decoupled** and methodologically different computational models.

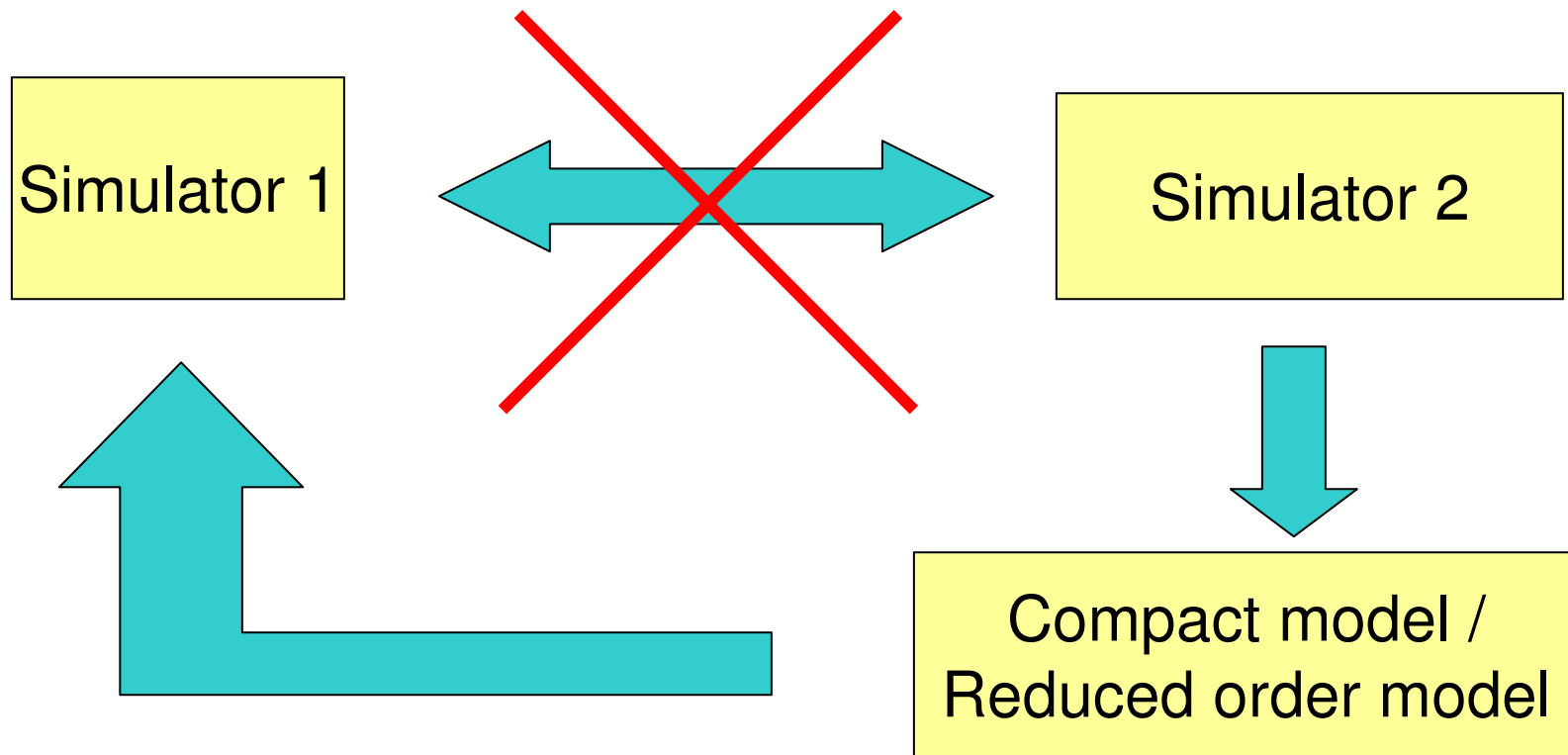
Multi-physics simulations

Straightforward coupling of simulation tools:

- ▶ leads to accurate results....
- ▶ ...and answers to design questions....
- ▶ ...but is often too time-consuming....
- ▶ ...and hence not feasible in practice (only for gauging)



Coupled simulations

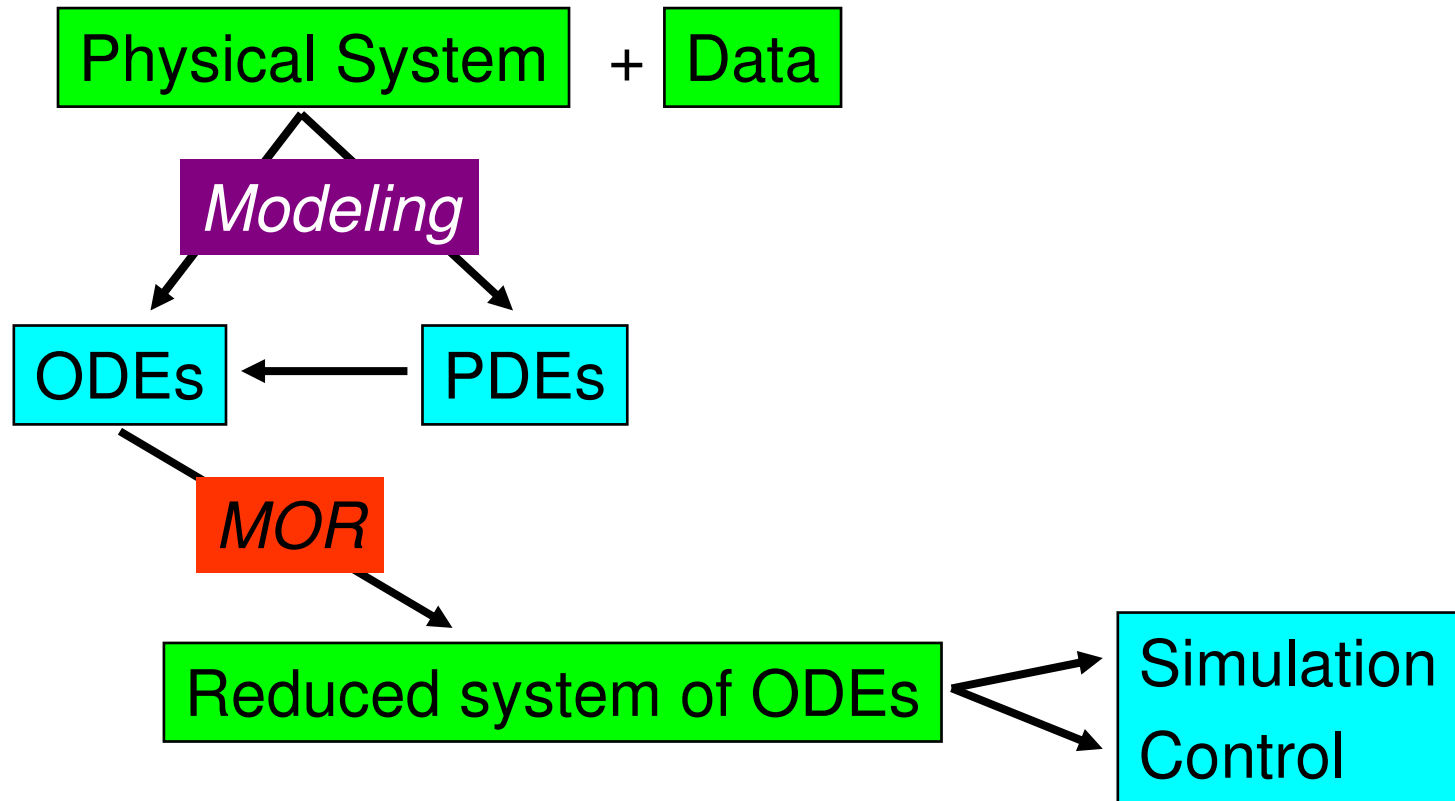


Use of low order models

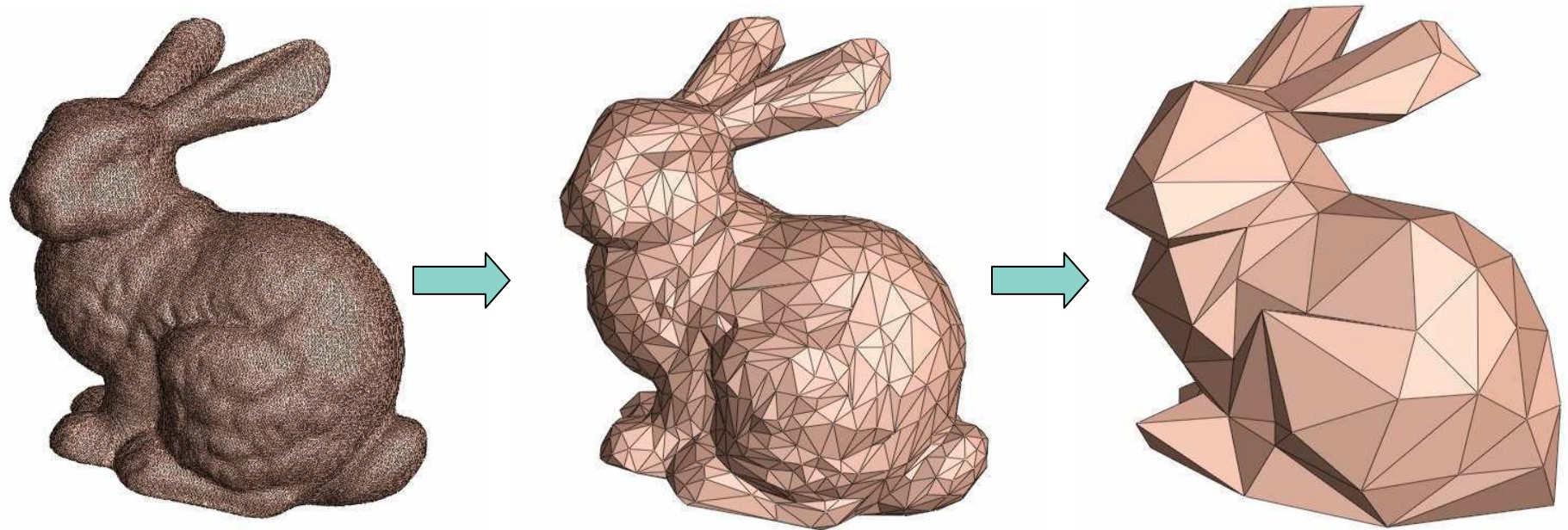


Model Order Reduction (MOR)

Overall picture

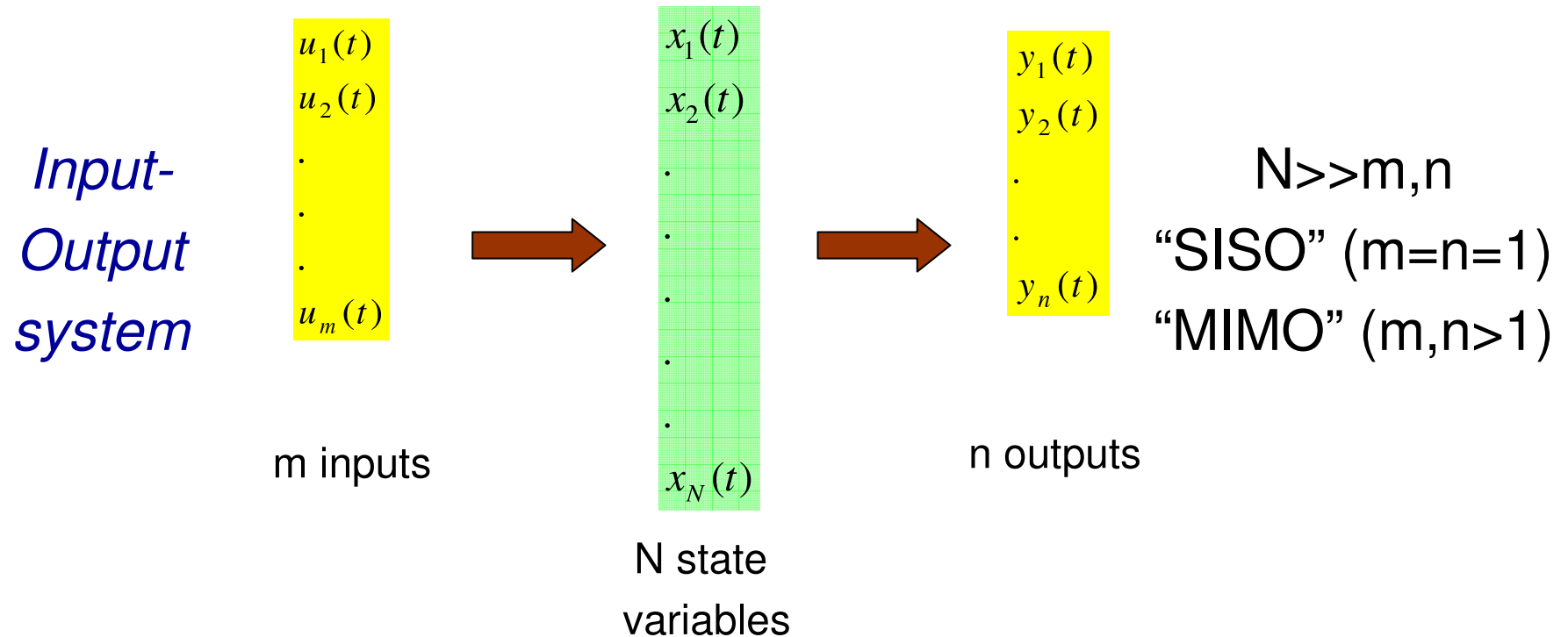


Model Order Reduction



Capture main features

Dynamical Systems



$$\Sigma : \quad \frac{dx(t)}{dt} = f(x(t), u(t))$$

$$y(t) = h(x(t), u(t))$$

State space description

Reduction

Given: a dynamical system $\Sigma = (f, h)$ with

$$\begin{aligned} u &\in R^m, \\ y &\in R^n, \\ x &\in R^N, \end{aligned}$$

Problem: approximate this with

$$\hat{\Sigma} = (\hat{f}, \hat{h}), u(t) \in R^m, y(t) \in R^n, x(t) \in R^k, k \ll N$$

such that

- approximation error small
- preservation of essential properties (stability, passivity, structure)
- procedure computationally efficient



Approximation by projection

Unifying feature of approximation methods: **projection**

Let $V, W \in R^{N \times k}$ such that

$$W^* V = I_k \Rightarrow \Pi \equiv VW^* \text{ is a projection}$$

Define $\hat{x} = W^* x$. **Then**

$$\hat{\Sigma} : \begin{cases} \frac{d\hat{x}(t)}{dt} = W^* f(V\hat{x}(t), u(t)) \\ y(t) = h(V\hat{x}(t), u(t)) \end{cases}$$

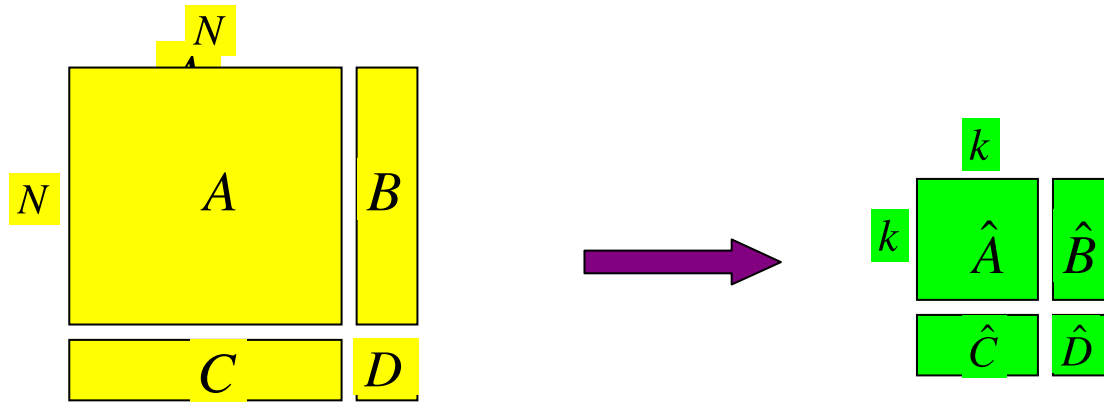
Thus, approximation error small if $x - \Pi x$ small.



Linear dynamical systems

Special case:
$$\left. \begin{aligned} f(x,u) &= Ax + Bu \\ h(x,u) &= Cx + Du \end{aligned} \right\} \Rightarrow \Sigma = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Then:
$$\hat{\Sigma} = \begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} = \begin{pmatrix} W^*AV & W^*B \\ CV & D \end{pmatrix}$$

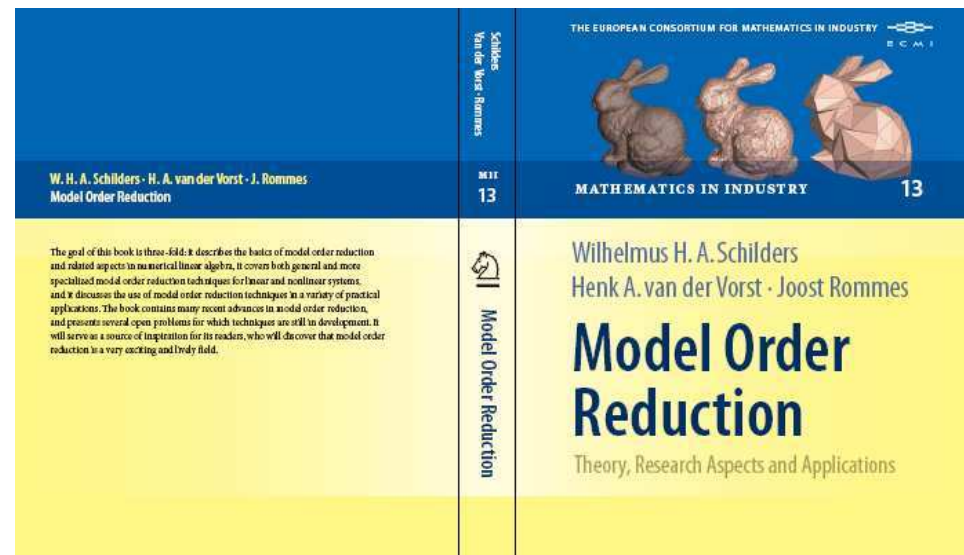


Motivating examples

| | |
|---|---|
| Electronics | <ul style="list-style-type: none">▶ <i>VLSI circuits</i>▶ <i>Thermal issues</i>▶ <i>Power delivery networks</i> |
| Data assimilation | <ul style="list-style-type: none">▶ <i>North sea forecast</i>▶ <i>Air quality forecast</i> |
| Molecular systems | <ul style="list-style-type: none">▶ <i>MD simulations</i>▶ <i>Heat capacity</i> |
| CVD reactor | <ul style="list-style-type: none">▶ <i>Bifurcations</i> |
| Mechanical systems | <ul style="list-style-type: none">▶ <i>Windscreen vibrations</i>▶ <i>Buildings (earthquakes)</i> |
| Optimal cooling | <ul style="list-style-type: none">▶ <i>Steel profile</i> |
| Micro Electro-Mechanical Systems (MEMS) | <ul style="list-style-type: none">▶ <i>Elk sensor</i>▶ <i>Heat generation</i> |

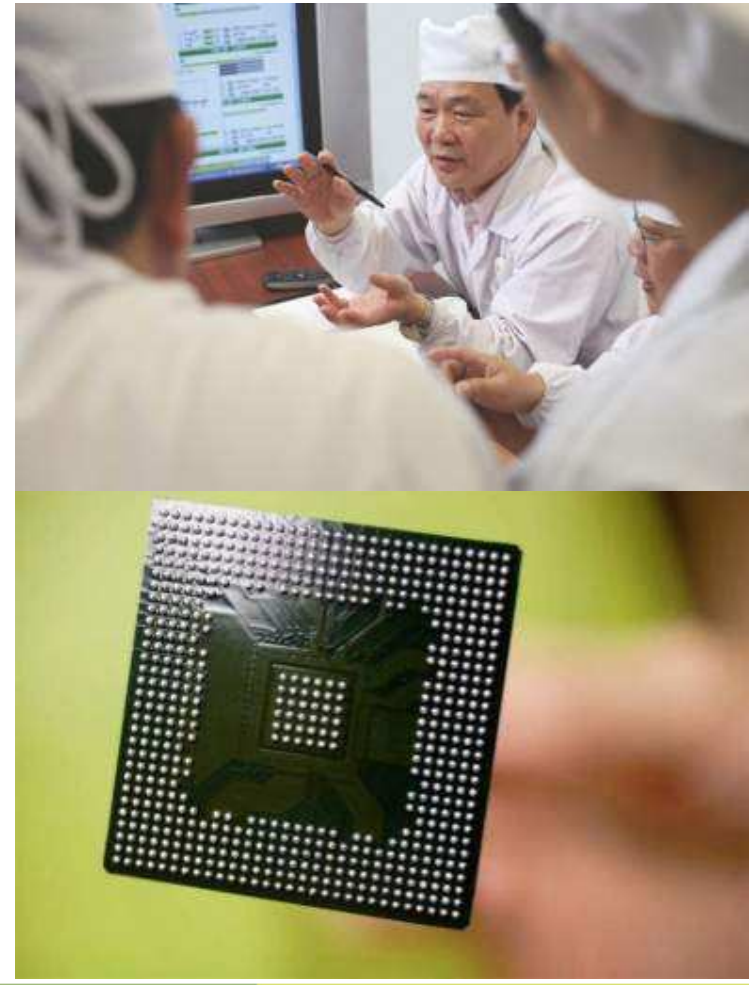
Types of MOR techniques

- ▶ **Krylov-based**
 - Pade-via-Lanczos (PVL), PRIMA
- ▶ **SVD-based**
 - Proper orthogonal decomposition (POD)
- ▶ **Balanced truncation (TBR)**
 - Systems and control theory
 - Observability, controllability
 - PMTBR
- ▶ **Combinations of the above**
 - Lyapunov equations
 - Schur decompositions
- ▶ **Physically motivated algorithms**
 - SUPERNODE ALGORITHM



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Brief version of problem

For the Model Order Reduction algorithm that has been designed specifically for electromagnetic systems, we are not able to show passivity and/or stability

Passivity

- ▶ In [circuit design](#), informally, passive components refer to ones that are not capable of [power gain](#)
- ▶ Under this definition, passive components include capacitors, inductors, resistors, transformers, voltage sources, and current sources.
- ▶ Given an n -port R with a state representation S , and initial state x , define available energy E_A as:

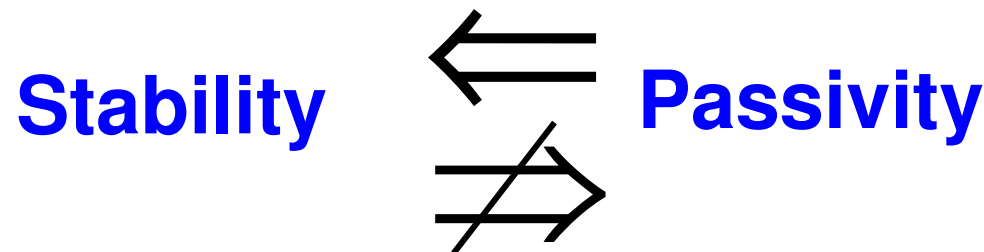
$$E_A \equiv \sup_{x \rightarrow T > 0} \left[\int_0^T - \langle v(t), i(t) \rangle dt \right]$$

- ▶ A system is considered passive if E_A is finite for all initial states x . Otherwise, the system is considered active.
- ▶ The concept of [positive real](#) plays an important role in showing passivity



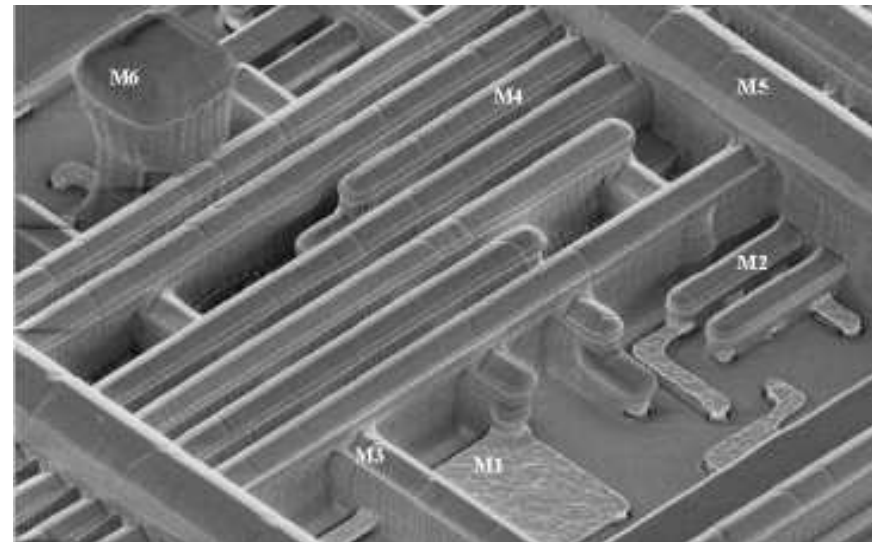
Stability

- ▶ Passivity, in most cases, can be used to demonstrate that passive circuits will be stable under specific criteria.
- ▶ In addition, passive circuits will not necessarily be stable under all stability criteria. For instance, a resonant series [LC circuit](#) will have unbounded voltage output for a bounded voltage input, but will be stable in the sense of [Lyapunov](#), and given bounded energy input will have bounded energy output.
- ▶ Passivity is frequently used in control systems to design stable control systems or to show stability in control systems. Passivity is also used in some areas of circuit design, especially filter design.

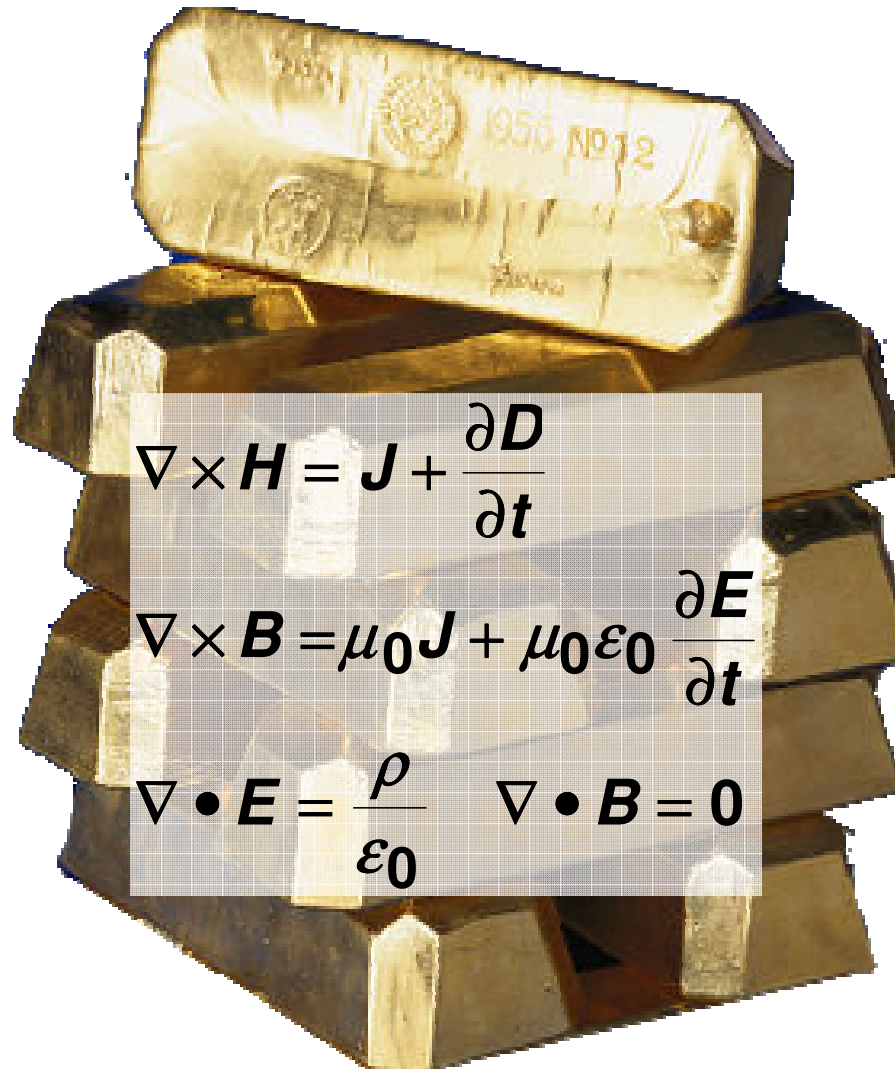


Simulation of PCB's and interconnects

- ▶ Electromagnetic behaviour of printed circuit boards and interconnects influences circuit operation
- ▶ NXP's in-house code: FASTERIX, based on BEM
- ▶ Reduced order model of EM behaviour suffices MOR
- ▶ Model order reduction using Krylov or Laguerre methods successful
 - preservation of stability and passivity
 - Realization in terms of RLC circuit cumbersome
- ▶ Supernode algorithm implemented
 - Advantage: physical basis, RLC repr
 - Disadvantage: stability, passivity not guaranteed



Fundamental Equations: Maxwell



After BEM discretization

$$(\mathbf{R} + s\mathbf{L})\mathbf{I} - \mathbf{P}\mathbf{V} = \mathbf{0}$$

$$\mathbf{P}^T\mathbf{I} + s\mathbf{C}\mathbf{V} = \mathbf{J}$$

- ▶ P denotes connection between branches and nodes (entries: -1,0,1)
- ▶ Properties of R,C,L:
 - Symmetric
 - Positive definite
 - Dense

- ▶ Resistance matrix R
- ▶ Capacitance matrix C
- ▶ Inductance matrix L
- ▶ Incidence matrix P
- ▶ V is the vector of nodal voltages
- ▶ I is the vector of branch currents
- ▶ J denotes (given) terminal currents

$$s = -j\omega$$



Super node algorithm (1)

- ▶ Based on maximum frequency (characteristic distance), a subset of the nodes is selected as the so-called super nodes
- ▶ The remaining nodes are eliminated from the system, so that a reduced system in the super nodes remains
- ▶ Complication: only super nodes, no super branches
- ▶ Denote set of super nodes by N, remaining set by N'

$$\left(\underbrace{\begin{pmatrix} \mathbf{R} & -\mathbf{P}_{N'} \\ \mathbf{P}_{N'}^T & 0 \end{pmatrix}}_{\mathbf{G}} + s \underbrace{\begin{pmatrix} \mathbf{L} & 0 \\ 0 & \mathbf{C}_{N'N'} \end{pmatrix}}_{\mathbf{C}} \right) x = \begin{pmatrix} \mathbf{P}_N \\ -s\mathbf{C}_{NN'} \end{pmatrix} v^{(p)}, \quad x = (I, V_{N'})^T$$

$$i^{(p)}(s) = (\mathbf{P}_N^T s\mathbf{C}_{NN'}^T) x + s\mathbf{C}_{NN'} v^{(p)},$$

Super node algorithm (2)

$$\left(\underbrace{\begin{pmatrix} \mathbf{R} & -\mathbf{P}_N \\ \mathbf{P}_N^T & 0 \end{pmatrix}}_{\mathbf{G}} + s \underbrace{\begin{pmatrix} \mathbf{L} & 0 \\ 0 & \mathbf{C}_{NN} \end{pmatrix}}_{\mathbf{C}} \right) \mathbf{x} = \begin{pmatrix} \mathbf{P}_N \\ -s\mathbf{C}_{NN} \end{pmatrix} \mathbf{v}^{(p)}, \quad \mathbf{x} = (\mathbf{I}, \mathbf{V}_N)^T$$

$$\mathbf{i}^{(p)}(s) = (\mathbf{P}_N^T \ s\mathbf{C}_{NN}^T) \mathbf{x} + s\mathbf{C}_{NN} \mathbf{v}^{(p)},$$

- ▶ The relation between terminal currents and voltages defines the admittance matrix:

$$\mathbf{Y}\mathbf{v} = \mathbf{i},$$

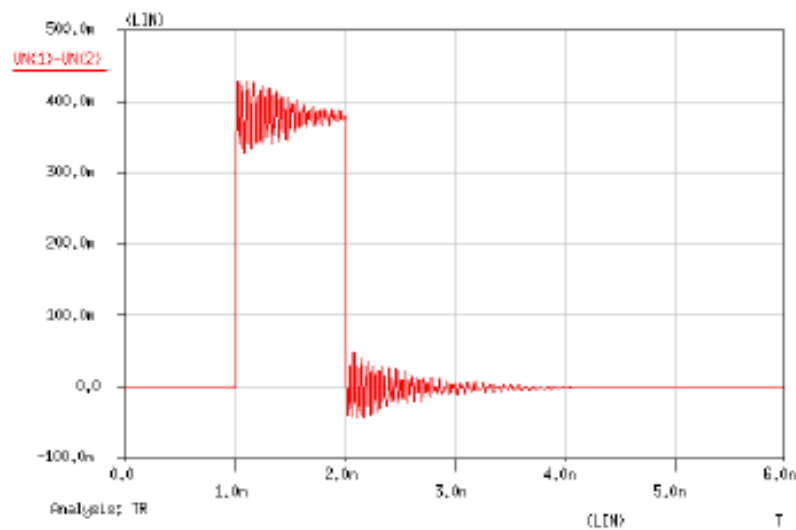
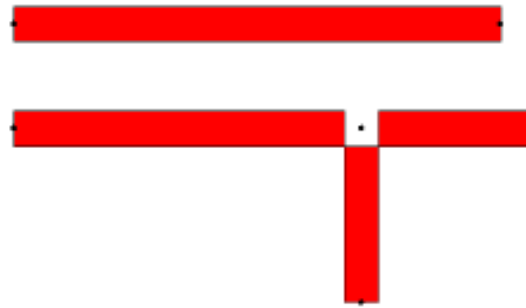
- ▶ This matrix describes the behaviour of the reduced circuit
 - Frequency-dependent

Super node algorithm (3)

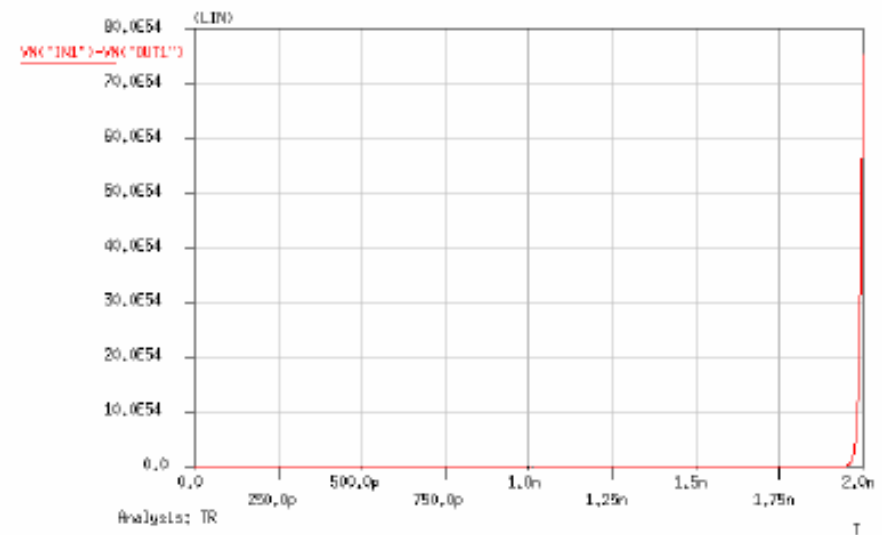
- ▶ Approximations of the admittance matrix can be obtained in various ways
 - Assuming high frequencies only
 - For the entire frequency range
- ▶ Important constraint: define an approximate Y that originates from an RLC circuit
- ▶ Algorithms suggested produce RLC circuits, but cannot guarantee stability and passivity



Example



original
(unreduced)



super node
(reduced)



Main question

Can the super node algorithm be turned into a provably stable and passive method?

Alternative question

Find a way to produce a realizable representation of mathematically reduced systems

In the small working group.....

- ▶ More detail will be given on passivity, stability, realizability, and the super node algorithm.....
- ▶but not too much detail in order to avoid bias in certain directions!
- ▶ Need good definitions of stability and passivity, both for SISO and MIMO case
- ▶ Documents on MOR, EM simulation, super node algorithm are available
- ▶ The problem is very challenging!



Thank you for your attention

